7. Areas

Exercise 7A

1. Question

Find the area of the triangle whose base measures 24 cm and the corresponding height measures 14.5 cm.

Answer

Given,

Base of triangle, b = 24 cm

Height of triangle = 14.5 cm

We have to find out the area of the given triangle

We know that,

Area of triangle = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times 24 \times 14.5$

= 174 cm^2

Hence, the area of the given triangle is 174 cm^2

2. Question

The base of a triangular field is three times its altitude. If the cost of sowing the field at Rs58 per hectare is Rs783, find its base and height.

Answer

It is given that the base of the triangular field is three times greater than its altitude

Let us assume height of the triangular field be x and base be 3x

We know that,

Area of triangle = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times x \times 3x$

$= \frac{3}{2} x^2$

We know that,

1 hectare = 10,000 sq metre

Given,

Rate of sowing the field per hectare = Rs. 58

Total cost of sowing the triangular field = Rs. 783

Therefore,

Total cost = Area of the triangular field × Rs. 58

$$\frac{3}{2}x^2 \times \frac{58}{10000} = 783$$

$$x^2 = \frac{783}{58} \times \frac{2}{3} \times 10000$$

$$x^2 = 90000\,m^2$$

x = 300 m

Hence,

Height of the triangular field = x = 300 m

Base of triangular field = 3x = 3 × 300 = 900 m

3. Question

Find the area of triangle whose sides are 42cm,34 cm and 20cm in length. Hence find the height corresponding to the longest side.

Answer

Given,

a = 42 cm

b = 34 cm

c = 20 cm

Therefore,

$$S = \frac{42+34+20}{2}$$

$$= \frac{96}{2}$$

= 48

We know that,

Area = $\sqrt{S\,(S-a)(S-b)(S-c)}$

Putting the values of a, b and c in the formula, we get

$= \sqrt{48\,(48-42)(48-34)(48-20)}$

$= \sqrt{48 \times 6 \times 14 \times 28}$

$= \sqrt{4 \times 4 \times 3 \times 3 \times 2 \times 14 \times 14 \times 2}$

= 4 × 3 × 2 × 14

= 336 cm^2

Longest side of the triangle = b = 42 cm

Let h be the corresponding height to the longest side

Therefore,

Area of triangle = $\frac{1}{2} \times b \times h$

336 = $\frac{1}{2} \times b \times h$

42 × h = 336 × 2

h = $\frac{336 \times 2}{42}$

= 16 cm

Hence, corresponding height of the triangle is 16 cm

4. Question

Calculate the area of the triangle whose sides are 18cm, 24cm and 30 cm in length. Also, find the length of the altitude corresponding to the smallest side.

Answer

Given,

a = 18 cm

b = 24 cm

c = 30 cm

Therefore,

s = $\frac{18+24+30}{2}$

= 36

We know that,

Area = $\sqrt{s\,(s-a)(s-b)\,(s-c)}$

$= \sqrt{36\,(36-18)(36-24)(36-30)}$

$= \sqrt{36 \times 18 \times 12 \times 6}$

$= \sqrt{6 \times 6 \times 6 \times 3 \times 3 \times 4 \times 6}$

= 6 × 6 × 3 × 2

= 216 cm^2

Smallest side = a = 18 cm

Let, h be the height corresponding to the smallest side of the triangle

Therefore,

Area of triangle = $\frac{1}{2} \times b \times h$

216 = $\frac{1}{2} \times b \times h$

18 × h = 216 × 2

h = $\frac{216 \times 2}{18}$

= 24 cm

5. Question

Find the area of a triangular field whose sides are 91m, 98m and 105m in length. Find the height corresponding to the longest side.

Answer

Given,

a = 91 m

b = 98 m

c = 105 m

Therefore,

s = $\frac{91+98+105}{2}$

= $\frac{294}{2}$

= 147

We know that,

Area = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{147\ (147-91)(147-98)(147-105)}$

= $\sqrt{147 \times 56 \times 49 \times 42}$

= $\sqrt{49 \times 3 \times 7 \times 2 \times 2 \times 2 \times 49 \times 7 \times 3 \times 2}$

= 49 × 3 × 2 × 2 × 7

= 4116 m^2

Longest side = c = 105 cm

Let, h be the height corresponding to the longest side of the triangle

Area of triangle = $\frac{1}{2} \times b \times h$

4116 = $\frac{1}{2} \times b \times h$

4116 × 2 = 2 × 4116

h = $\frac{2 \times 4116}{105}$

= 78.4 m

6. Question

The sides of triangle are in the ratio 5 : 12 : 13 and its perimeter is 150m. Find the area of triangle.

Answer

Let the sides of the given triangle be 5x, 12x and 13x

Given,

Perimeter of the triangle = 150m

Perimeter of the triangle = (5x + 12x + 13x)

150 = 30x

Therefore,

x = $\frac{150}{30} = 5\ m$

Thus,

Sides of the triangle are:

5x = 5 × 5 = 25 m

12x = 12 × 5 = 60 m

13x = 13 × 5 = 65 m

Let,

a = 25 m, b = 60 m and c = 65 m

Therefore,

s = $\frac{1}{2}(a+b+c)$

= $\frac{1}{2}(25+60+65)$

= $\frac{1}{2}(150)$

= 75 m

We know that,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{75(75-25)(75-60)(75-65)}$

= $\sqrt{75 \times 50 \times 15 \times 10}$

= $\sqrt{25 \times 3 \times 25 \times 2 \times 5 \times 3 \times 5 \times 2}$

= $\sqrt{25 \times 25 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2}$

= 25 × 5 × 3 × 2

= 750 sq m

Hence, area of triangle is 750 sq m.

7. Question

The perimeter of a triangular field is 540m and its sides are in the ratio 25 : 17 : 12. Find the area of the triangle. Also, find the cost of ploughing the field at Rs. 18.80 per 10m^2.

Answer

Let the sides of the given triangle be 25x, 17x and 12x

Given,

Perimeter of the triangle = 540 m

540 = 25x + 17x + 12x

540 = 54x

x = $\frac{540}{54}$

x = 10 m

Thus, sides of the triangle are:

$25x = 25 \times 10 = 250$ m

$17x = 17 \times 10 = 170$ m

$12x = 12 \times 10 = 120$ m

Let,

a = 250 m, b = 170 m and c = 120 m

Therefore,

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(250 + 170 + 120)$

$= \frac{1}{2}(540)$

= 270 m

Therefore,

Area of triangle= $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-250)(270-170)(270-120)}$

$= \sqrt{3 \times 3 \times 3 \times 10 \times 10 \times 2 \times 10 \times 10 \times 10 \times 5 \times 3}$

$= 3 \times 3 \times 10 \times 10 \times 10$

= 9000 m^2

Cost of ploughing the field at the rate of Rs. 18.80 per 10 m^2 = $\frac{18.80}{10} \times 9000$

= Rs. 16920

Therefore, cost of ploughing the field is Rs. 16920

8. Question

Two sides of a triangular field are 85m and 154m in length and its perimeter is 324m. Find:

(i) The area of the field and

(ii) The length of the perpendicular from the opposite vertex of the side measuring 154m.

Answer

Given,

First side of the triangular field = 85 m

Second side of the triangular field = 154 m

Let the third side be x

Perimeter of the triangular field = 324 m

85 m + 154 m + x = 324 m

x = 324 – 239

x = 85 m

Let the three sides of the triangle be:

a = 85 m, b = 154n m and c = 85 m

Now,

s = $\frac{1}{2}(a+b+c)$

= $\frac{(85+154+85)}{2}$

= $\frac{324}{2}$

= 162

We know that,

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{162 \times 77 \times 8 \times 77}$

= $\sqrt{2 \times 9 \times 9 \times 11 \times 2 \times 2 \times 2 \times 7 \times 11}$

= $\sqrt{11 \times 11 \times 9 \times 9 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2}$

= 11 × 9 × 7 × 2 × 2

= 2771 m^2

We also know that,

Area of triangle = $\frac{1}{2} \times base \times height$

2772 = $\frac{1}{2} \times 154 \times h$

2772 = 77h

h = $\frac{2772}{77}$

h = 36 m

Therefore,

The length of the perpendicular from the opposite vertex on the side measuring 154 m is 36 m.

9. Question

Find the area of an isosceles triangle each of whose equal sides measures 13cm and whose base measures 20cm.

Answer

Let,

a = 13 cm

b = 13 cm

And,

C = 20 cm

Now,

s = $\frac{1}{2}$ $(a + b + c)$

= $\frac{(13+13+20)}{2}$

= $\frac{46}{2}$ = 23 cm

We know that,

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{23\,(23-13)(23-13)(23-20)}$

= $\sqrt{23 \times 10 \times 10 \times 3}$

= $10\sqrt{69}$

= 10 × 8.306

= 83.06 cm^2

Therefore,

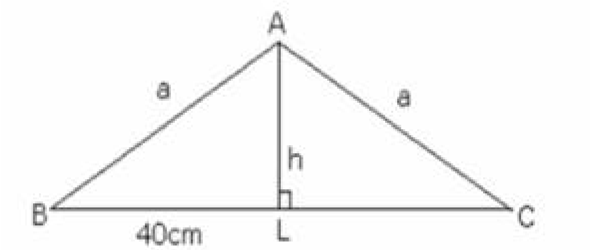
Area of isosceles triangle = 83.06 cm^2

10. Question

The base of an isosceles triangle measures 80cm and its area is 360cm^2 . Find the perimeter of the triangle.

Answer

Let us assume ΔABC be an isosceles triangle and let AL perpendicular BC



It is given that,

BC = 80 cm

Area of triangle ABC = 360 cm^2

We know that,

Area of triangle = $\frac{1}{2} \times base \times height$

$\frac{1}{2} \times BC \times AL$ = 360 cm^2

$\frac{1}{2} \times 80 \times h$ = 360 cm^2

$40 \times h$ = 360 cm^2

$h = \frac{360}{40}$

= 9 cm

Now,

BL = $\frac{1}{2}\ (BC)$

= $(\frac{1}{2} \times 80)$

= 40 cm

a = $\sqrt{BL^2 + AL^2}$

= $\sqrt{(40)^2 + (9)^2}$

= $\sqrt{1600 + 81}$

= $\sqrt{1681}$

= 41 cm

Therefore,

Perimeter of the triangle = (41 + 41 + 80) = 162 cm

11. Question

The perimeter of an isosceles triangle is 42cm and its base is $1\frac{1}{2}$ times, each of the equal sides.

Find:

(i) The length of each side of the triangle

(ii) The area of the triangle

(iii) The height of the triangle.

Answer

We know that,

In any isosceles triangle, the lateral sides are of equal length

Let,

The lateral side of the triangle be x

Given,

Base of the triangle = $\frac{3}{2} \times x$

(i) We have to find out length of each side of the triangle:

Perimeter of the triangle = 42 cm (Given)

x + x + $\frac{3}{2}x$ = 42 cm

2x + 2x + 3x = 84 cm

7x = 84 cm

x = $\frac{84}{7}\ cm$

x = 12 cm

Therefore,

Length of lateral side of the triangle = x = 12 cm

Base = $\frac{3}{2} \times x = \frac{3}{2} \times 12$

= 18 cm

Hence,

Length of each side of the triangle is 12 cm, 12 cm and 18 cm

(ii) Now, we have to find out area of the triangle:

Let,

a = 12 cm

b = 12 cm

And,

c= 18 cm

Now,

s = $\frac{1}{2}(a+b+c)$

= $\frac{1}{2}(12+12+18)$

= $\frac{1}{2}(42)$

= 21 cm

We know that,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{21(21-12)(21-12)(21-18)}$

= $\sqrt{21\times 9\times 9\times 3}$

= $\sqrt{3\times 7\times 9\times 9\times 3}$

= $27\sqrt{7}$

= 71.42 cm^2

Therefore, area of the given triangle is 71.42 cm^2

(iii) We have to calculate height of the triangle:

We know that,

Area of triangle = $\frac{1}{2}\times base\times height$

71.42 cm^2 = $\frac{1}{2}\times 18\times h$

71.42 cm^2 = 9 × h

h = $\frac{71.42}{9}$ = 7.94 cm

Therefore, height of the triangle is 7.94 cm

12. Question

If the area of the equilateral triangle is $36\sqrt{3}$ cm^2, find its perimeter.

Answer

Given,

Area of the equilateral triangle = $36\sqrt{3}$ cm^2

Let us assume a be the length of the side of an equilateral triangle

We know that,

Area of an equilateral triangle = $\frac{\sqrt{3}\times a^2}{4}$ sq units

$36\sqrt{3} = \frac{\sqrt{3}\times a^2}{4}$

$a^2 = \frac{36\times\sqrt{3}\times4}{\sqrt{3}}$

$a^2 = 36 \times 4$

$a^2 = 144$

a = 12 cm

We know that,

Perimeter of an equilateral triangle = 3 × a

= 3 × 12

= 36 cm

Hence, perimeter of the given equilateral triangle is 36 cm.

13. Question

If the area of the equilateral triangle is $81\sqrt{3}$ cm^2, find its height.

Answer

Let us assume a be the side of the equilateral triangle

We know that,

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ sq units

It is given that,

Area of the equilateral triangle = $81\sqrt{3}$ cm^2

$81\sqrt{3}$ cm^2 = $\frac{\sqrt{3}}{4}a^2$

$a^2 = \frac{81\sqrt{3}\times4}{\sqrt{3}} = 324$

$a = \sqrt{324} = 18$ cm

Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ a

Since, the value of a is 18 cm

Therefore,

Height = $\frac{\sqrt{3}}{2} \times 18$

= $9\sqrt{3}$ cm

14. Question

The base of a right – angles triangle measures 48cm and its hypotenuse measures 50cm. Find the area of the triangle.

Answer

Given that,

Base = BC = 48 cm

Hypotenuse = AC = 50 cm

Let us assume AB = x cm

By using Pythagoras theorem, we get

$AC^2 = AB^2 + BC^2$

Putting the value of BC, AC and AB we get:

$50^2 = x^2 + 48^2$

$x^2 = 50^2 - 48^2$

$x^2 = 2500 - 2304$

$x^2 = 196$

$x = \sqrt{196}$

x = 14 cm

We know that,

Area of right angle triangle = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times 48 \times 14$

= 24×14

= $336\ cm^2$

15. Question

Each side of an equilateral triangle measures 8cm. Find:

(i) The area of the triangle, correct to 2 places of decimal

(ii) The height of the triangle, correct to 2 places of decimal.

Take $\sqrt{3}$ =1.732

Answer

(i) We know that,

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ sq units

It is given that, each side of equilateral triangle is of 8 cm

Therefore,

Area = $\frac{\sqrt{3}}{4} \times 8^2$

= $\frac{\sqrt{3}}{4} \times 64$

= $\sqrt{3} \times 16$

= 1.732 × 16

= 27.712

= 27.71 cm^2 (Up to 2 decimal places)

(ii) We also know that,

Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ a

= $\frac{\sqrt{3}}{2} \times 8$

= $\sqrt{3} \times 4$

= 1.732 × 4

= 6.928

= 6.93 cm (Up to 2 decimal places)

16. Question

The height of an equilateral triangle measures 9 cm. Find its area, correct to 2 decimal places. Take $\sqrt{3}$ =1.732.

Answer

Let us assume a be the side of the equilateral triangle

We know that,

Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ a units

Height 0f the equilateral triangle = 9 cm (Given)

$\frac{\sqrt{3}}{2}a = 9$

$a = \frac{9\times2}{\sqrt{3}}$

= $\frac{9\times2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$ (Rationalizing the denominator)

= $\frac{9\times2\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

= $6\sqrt{3}$

Base of the triangle = $6\sqrt{3}$

We know that,

Area of triangle = $\frac{1}{2}\times base \times height$

= $\frac{1}{2}$ × $6\sqrt{3}$ × 9

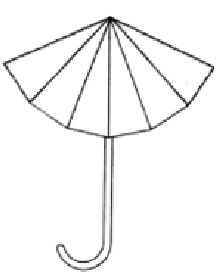
= $27\sqrt{3}$

= 27 × 1.732

= 46.76 cm^2 (Up to 2 decimal places)

17. Question

An umbrella is made by stitching 12 triangular pieces of cloth, each measuring (50 cm x 20 cm x 50 cm). Find the area of the cloth used in it.



Answer

Let the sides of the triangle be,

a = 50 cm

b = 20 cm

And

C = 50 cm

Now, let us find the value of s:

s = $\frac{1}{2}(a + b + c)$

= $\frac{1}{2}(50 + 20 + 50)$

= 60 cm

We know that,

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Area of one triangular piece of cloth = $\sqrt{60(60-50)(60-20)(60-50)}$

= $\sqrt{60 \times 10 \times 40 \times 10}$

= $\sqrt{6 \times 10 \times 10 \times 4 \times 10 \times 10}$

= $\sqrt{10 \times 10 \times 10 \times 10 \times 2 \times 2 \times 2 \times 3}$

= 10 × 10 × 2$\sqrt{6}$

= 200$\sqrt{6}$

= 200 × 2.45

= 490 cm^2

Therefore,

Area of one piece of cloth = 490 cm^2

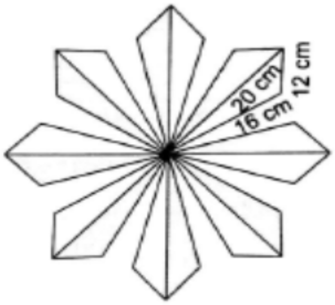
Hence,

Area of 12 pieces of cloth = 12 × 490

= 5880 cm^2

18. Question

A floral design on a floor is made up of 16 tiles, each triangular in shape having sides 16 cm, 12 cm, and 20cm. Find the cost of polishing the tiles at Re 1 per sq cm.



Answer

Let the sides of the triangle be:

a = 16 cm

b = 12 cm

And,

c = 20 cm

Now we have to find out the value of s:

s = $\frac{1}{2}(a + b + c)$

= $\frac{1}{2}(16 + 12 + 20)$

= $\frac{48}{2}$ = 24 cm

We know that,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Therefore,

Area of triangular tile = $\sqrt{24(24-16)(24-12)(24-20)}$

= $\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}$

= 2 × 2 × 2 × 2 × 2 × 3

= 96 cm^2

Therefore,

Area of one tile = 96 cm^2

Hence,

Area of 16 such tiles = 96 × 16 = 1536 cm^2

Now,

Cost of polishing the tiles per square cm = Rs. 1

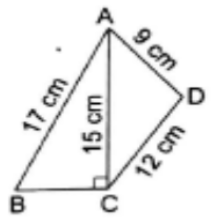
Therefore,

Total cost of polishing the tiles = 1 × 1536

= Rs. 1536

19. Question

Find the perimeter and area of the quadrilateral ABCD in which AB = 17cm, AD = 9cm, CD = 12cm, $\angle ACB$ =90^0 and AC = 15cm.



Answer

By using Pythagoras theorem in right triangle ABC, we get

BC= $\sqrt{AB^2 - AC^2}$

= $\sqrt{17^2 - 15^2}$

= $\sqrt{289 - 225}$

= $\sqrt{64}$

= 8 cm

Let us first find out the perimeter of the given quadrilateral

Perimeter of quadrilateral ABCD = 17 + 9 + 12 + 8 = 46 cm

We know that,

Area of triangle ABC = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times BC \times AC$

= $\frac{1}{2} \times 8 \times 15$

= 60 cm^2

Now,

For area of triangle ACD, we have

a = 15 cm

b = 12 cm

And,

c = 9 cm

Therefore,

s = $\frac{a+b+c}{2}$

= $\frac{15+12+9}{2}$

= 18 cm

Now,

Area of triangle ACD = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{18\,(18-15)(18-12)(18-9)}$

= $\sqrt{18 \times 3 \times 6 \times 9}$

= $\sqrt{18 \times 18 \times 3 \times 3}$

= 18 × 3

= 54 cm^2

Therefore,

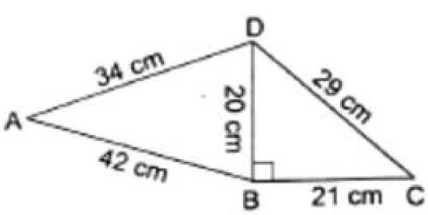
Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD

= 60 + 54

= 114 cm^2

20. Question

Find the perimeter and area of the quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and $\angle CBD$ =90^0.



Answer

Firstly, let us calculate the perimeter of the given quadrilateral

Perimeter of quadrilateral ABCD = 34 + 29 + 21 + 42 = 126 cm

We know that,

Area of triangle = $\frac{1}{2} \times base \times height$

Area of triangle BCD = $\frac{1}{2} \times 20 \times 21$

= 210 cm^2

Now, we have to calculate the area of triangle ABD,

For this, we have

a = 42 cm

b = 20 cm

c = 34 cm

Therefore,

s = $\frac{42+20+34}{2}$

= $\frac{96}{2}$

= 48 cm

We know that,

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

Therefore,

Area of triangle ABD = $\sqrt{48\,(48-42)(48-20)(48-34)}$

= $\sqrt{48 \times 6 \times 28 \times 14}$

= $\sqrt{16 \times 3 \times 3 \times 2 \times 2 \times 14 \times 14}$

= 4 × 3 × 2 × 14

= 336 cm^2

Hence,

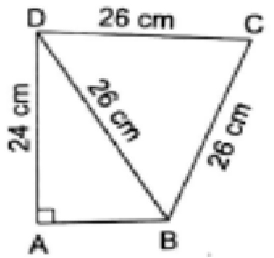
Area of quadrilateral ABCD = Area of triangle ABD + Area of triangle BCD

= 336 + 210

= 546 cm^2

21. Question

Find the area of the quadrilateral ABCD in which AD = 42 cm , $\angle BAD$ = 90^0 and $\triangle BCD$ is an equilateral triangle having each side equal to 26 cm. Also, find the perimeter of the quadrilateral. [Given $\sqrt{3}$ = 1.73]



Answer

Let us consider a right triangle ABD,

By using Pythagoras theorem in this, we get

AB = $\sqrt{AB^2 - AD^2}$

= $\sqrt{26^2 - 24^2}$

= $\sqrt{676 - 576}$

= 10 cm

We know that,

Area of triangle = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times 10 \times 24$

= 120 cm^2

We also know that,

Area of an equilateral triangle BCD = $\frac{\sqrt{3}}{4} a^2$ sq units

= $\frac{1.73}{4} \times (26)^2$

= 292.37 cm^2

Therefore,

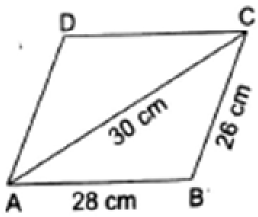
Area of quadrilateral ABCD = Area of triangle ABD + Area of triangle BCD

= 120 + 292.37

= 412.37 cm^2

22. Question

Find the area of a parallelogram ABCD in which AB = 28cm, BC = 26cm and diagonal AC = 30 cm.



Answer

Let the sides of the triangle ABC be:

a = 26 cm

b = 30 cm

And

c = 28 cm

Let us find out the value of s

We know that,

s = $\frac{1}{2}(a+b+c)$

= $\frac{1}{2}(26+30+28)$

= $\frac{84}{2}$

= 42 cm

We know that,

Area of triangle ABC = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{42\,(42-26)(42-30)(42-28)}$

= $\sqrt{42 \times 16 \times 12 \times 14}$

= $\sqrt{14 \times 3 \times 16 \times 4 \times 3 \times 14}$

= $\sqrt{14 \times 14 \times 3 \times 3 \times 16 \times 4}$

= 14 × 3 × 4 × 2

= 336 cm^2

We know that,

In a parallelogram, the diagonal divides the parallelogram in two equal area

Therefore,

Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD

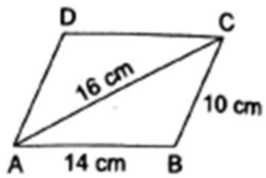
= Area of triangle ABC × 2

= 336 × 2

= 672 cm^2

23. Question

Find the area of the parallelogram ABCD in which AB = 14 cm, BC = 10 cm and AC = 16cm. [Given $\sqrt{3}$ = 1.73]



Answer

According to the question,

In order to find the area of quadrilateral ABCD,

At first,

Let us consider triangle ABC,

Say,

a = 10 cm, b = 16 cm and c = 14 cm

Now,

Semi perimeter of ΔABC, s = $\frac{a+b+c}{2}$

= $\frac{10+16+14}{2}$

= $\frac{40}{2}$

= 20 cm

Now,

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{20(20-10)(20-16)(20-14)}$

= $\sqrt{20 \times 10 \times 6 \times 4}$

= $40\sqrt{3}$ cm^2

We know that, the diagonal of a parallelogram divides it into two triangles of equal areas.

Hence,

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

= Area of ΔABC × 2

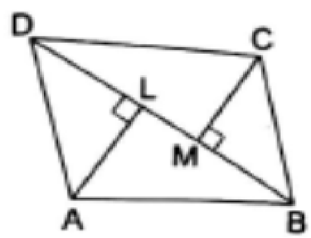
= $40\sqrt{3} \times 2$

= $80\sqrt{3}$ cm^2

= 138.4 cm^2

24. Question

In the figure ABCD is a quadrilateral in which diagonal BD = 64 cm, AL $\perp$ BD and CM $\perp$ BD such that AL = 16.8 cm and CM = 13.2 cm. Calculate the area of the quadrilateral ABCD.



Answer

According to the question,

In order to find the area of quadrilateral ABCD,

At first,

We will find the area of triangle ABD and triangle BCD respectively.

And, then we'll add them.

Hence,

Area of ΔABD = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times BD \times AL$

= $\frac{1}{2} \times 64 \times 16.8$

= 537.6 cm^2

Area of ΔBCD = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times BD \times CM$

= $\frac{1}{2} \times 64 \times 13.2$

= 422.4 cm^2

Now,

Area of quadrilateral ABCD = Area of ΔABD + Area of ΔBCD

= 537 + 422.4

= 960 cm^2

CCE Questions

1. Question

In a ΔABC it is given that base = 12cm and height = 5cm. Its area is

A. 60cm^2

B. 30 cm^2

C. $15\sqrt{3}$ cm^2

D. 45cm^2

Answer

We have,

Base of triangle = 12 cm

Height of triangle = 5 cm

We know that,

Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

= $\frac{1}{2} \times 12 \times 5$

= 6×5

= 30 cm^2

Hence, option (b) is correct

2. Question

The length of three sides of a triangle are 20 cm, 16 cm and 12 cm. The area of the triangle is –

A. 96cm^2

B. 120cm^2

C. 144cm^2

D. 160cm^2

Answer

Let the threes ides of the triangle be,

a = 20 cm, b = 16 cm and c = 12 cm

Now, s = $\frac{a+b+c}{2}$

= $\frac{20+16+12}{2}$

= $\frac{48}{2}$

= 24 cm

Now, by using Heron's formula we have:

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{24\,(24-20)(24-16)(24-12)}$

= $\sqrt{24 \times 4 \times 8 \times 12}$

= $\sqrt{6 \times 4 \times 4 \times 4 \times 4 \times 6}$

= 6 × 4 × 4

= 96 cm^2

Hence, option (a) is correct

3. Question

Each side of an equilateral triangle measures 8 cm. The area of the triangle is

A. $8\sqrt{3}$ cm^2

B. $16\sqrt{3}$ cm^2

C. $32\sqrt{3}$ cm^2

D. 48cm^2

Answer

It is given in the question that,

Side of equilateral triangle = 8 cm

We know that,

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

= $\frac{\sqrt{3}}{4} \times (8)^2$

$= \frac{\sqrt{3}}{4} \times 64$

$= 16\sqrt{3}$ cm^2

Hence, option (b) is correct

4. Question

The base of an isosceles triangle is 8cm long and each of its equal sides measures 6cm. The area of the triangle is –

A. $16\sqrt{5}$ cm^2

B. $8\sqrt{5}$ cm^2

C. $16\sqrt{3}$ cm^2

D. $8\sqrt{3}$ cm^2

Answer

We know that,

Area of an isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$

It is given that,

a = 6 cm and b = 8 cm

∴ we have:

$\frac{8}{4} \times \sqrt{4(6)^2 - 8^2}$

$= \frac{8}{4} \times \sqrt{144 - 64}$

$= \frac{8}{4} \times \sqrt{80}$

$= \frac{8}{4} \times 4\sqrt{5}$

$= 8\sqrt{5}$ cm^2

Hence, option (b) is correct

5. Question

The base of an isosceles triangle is 6cm and each of its equal sides is 5cm. The height of the triangle is –

A. 8 cm

B. $\sqrt{30}$ cm

C. 4 cm

D. $\sqrt{11}$ cm

Answer

It is given in the question that,

Base of the isosceles triangle = b = 6 cm

Two equal sides = a = 5 cm

We know that,

Height of an isosceles triangle = $\frac{1}{2} \times \sqrt{4a^2 - b^2}$

= $\frac{1}{2} \times \sqrt{4(5)^2 - 6^2}$

= $\frac{1}{2} \times \sqrt{100 - 36}$

= $\frac{1}{2} \times \sqrt{64}$

= $\frac{1}{2} \times 8$

= 4 cm

Hence, option (c) is correct

6. Question

Each of the two equal sides of an isosceles right triangle is 10cm long. Its area is –

A. $5\sqrt{10}$ cm^2

B. 50cm^2

C. $10\sqrt{3}$ cm^2

D. 75cm^2

Answer

From the given question, we have

Base of triangle = 10 cm

Height of triangle = 10 cm

∴ Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 10 \times 10$

= 5 × 10

= 50 cm^2

Hence, option (b) is correct

7. Question

Each side of an equilateral triangle is 10 cm long. The height of the triangle is –

A. $10\sqrt{3}$ cm

B. $5\sqrt{3}$ cm

C. $10\sqrt{2}$ cm

D. 5cm

Answer

We have,

Each side of the equilateral triangle = 10 cm

We know that,

In an equilateral triangle altitude divides its base into 2 equal parts

$\therefore \frac{1}{2} \times 10 = 5 \text{ cm}$

Let the height be h

Now, by using Pythagoras theorem

$10^2 = 5^2 + h^2$

$100 = 25 + h^2$

$h^2 = 100 - 25$

$h^2 = 75$

$h = \sqrt{75}$

$h = 5\sqrt{3}$ cm

Hence, height of the triangle is $5\sqrt{3}$ cm

Thus, option (b) is correct

8. Question

The height of an equilateral triangle is 6cm. Its area is –

A. $12\sqrt{3}$ cm^2

B. $6\sqrt{3}$ cm^2

C. $12\sqrt{2}$ cm^2

D. 18cm^2

Answer

It is given in the question that,

Height of an equilateral triangle = 6 cm

Let the side of triangle be a

Then, the altitude of the equilateral triangle is given as:

$\therefore \text{Altitude} = \frac{\sqrt{3}}{2}a$

Put altitude = 6 cm we get,

$6 = \frac{\sqrt{3}}{2} \times a$

a = $\frac{12}{\sqrt{3}}$

a = $4\sqrt{3}$ cm

$\therefore$ Area of triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

= $\frac{\sqrt{3}}{4} \times \left(4\sqrt{3}\right)^2$

= $\frac{\sqrt{3}}{4} \times 16 \times 3$

= $12\sqrt{3}$ cm^2

Hence, option (a) is correct

9. Question

The length of the three sides of the triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is –

A. 480m^2

B. 320m^2

C. $384m^2$

D. $360m^2$

Answer

It is given in the question that,

Sides of the triangle = 40 m, 24 m and 32 m

∴ Semi-perimeter, s = $\frac{40+24+32}{2}$

= $\frac{96}{2}$

= 48 cm

Now, by using Heron's formula we get:

Area of triangle = $\sqrt{48\,(48-40)(48-24)(48-32)}$

= $\sqrt{48 \times 8 \times 24 \times 16}$

= $\sqrt{147456}$

= 384 m^2

Hence, option (c) is correct

10. Question

The sides of the triangle are in the ratio 5: 12:13 and its perimeter is 150 cm. The area of the triangle is –

A. $375cm^2$

B. $750cm^2$

C. $250cm^2$

D. $500cm^2$

Answer

It is given in the question that,

The sides of given triangle are in the ratio 5: 12: 13

Let the sides be 5x, 12x and 13x

According to the question,

5x + 12x + 13 x = 150

30x = 150

$x = \frac{150}{30}$

x = 5

So, 5x = 25

12x = 60

13x = 65

Semi-perimeter = $\frac{25+60+65}{2}$

= $\frac{150}{2}$

= 75 cm

Now, by using Heron's formula we get:

Area of triangle = $\sqrt{75\,(75-25)(75-60)(75-65)}$

= $\sqrt{75 \times 50 \times 15 \times 10}$

= $\sqrt{562500}$

= 750 cm^2

Hence, option (b) is correct

11. Question

The lengths of the three sides of the triangle are 30 cm, 24 cm and 18 cm respectively. The length of the altitude of the triangle corresponding to the smallest side is-

A. 24 cm

B. 18 cm

C. 30 cm

D. 12 cm

Answer

It is given in the question that,

Sides of the triangle = 30 cm, 24 cm and 18 cm

Let h be the altitude of the triangle

∴ Semi-perimeter = $\frac{30+24+18}{2}$

= $\frac{72}{2}$

= 36 cm

Now, Area of triangle = $\sqrt{36\,(36-30)(36-24)(36-18)}$

= $\sqrt{36 \times 6 \times 12 \times 18}$

= $\sqrt{46656}$

= 216 cm^2

Also, Area = $\frac{1}{2} \times \text{Base} \times \text{Height}$

216 = $\frac{1}{2} \times 18 \times h$

216 = 9 × h

h = $\frac{216}{9}$

= 24 cm

Hence, option (a) is correct

12. Question

The base of an isosceles triangle is 16 cm and its area is 48cm^2. The perimeter of the triangle is –

A. 41 cm

B. 36 cm

C. 48 cm

D. 324 cm

Answer

It is given in the question that,

Base of the triangle = 16 cm

Area of the triangle = 48 cm^2

Let the height of the triangle be h

We know that,

Area of the triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

48 = $\frac{1}{2} \times 16 \times h$

48 = 8 × h

h = $\frac{48}{8}$

h = 6 cm

Now, half of the base = $\frac{16}{2}$ = 8 cm

∴ By using Pythagoras theorem, we have

$Side^2 = 8^2 + 6^2$

= 64 + 36

= 100

= 10 cm

Now, perimeter of the triangle = Sum of all sides

= 10 + 10 + 16

= 36 cm

Hence, option (b) is correct

13. Question

The area of an equilateral triangle is $36\sqrt{3}$ cm^2. Its perimeter is –

A. 36cm

B. $12\sqrt{3}$ cm

C. 24cm

D. 30 cm

Answer

It is given in the question that,

Area of an equilateral triangle = $36\sqrt{3}$ cm^2

We know that,

Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

$36\sqrt{3} = \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$(\text{Side})^2 = 144$

Side = 12 cm

∴ Perimeter of equilateral triangle = 3 × Side

= 3 × 12

= 36 cm

Hence, option (a) is correct

14. Question

Each of the equal sides of an isosceles triangle is 13cm and base is 24cm. The area of the triangle is –

A. 156 cm^2

B. 78 cm^2

C. 60 cm^2

D. 120 cm^2

Answer

It is given in the question that,

Equal sides of isosceles triangle = 13cm

Base = 24 cm and $\frac{1}{2}(\text{Base})$ = 12 cm

Let the height of the triangle be h

$\therefore (13)^2 = (12)^2 + h^2$

$169 = 144 + h^2$

$h^2 = 169 - 144$

$h^2 = 25$

h = 5

Thus, area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 24 \times 5$

$= 12 \times 5$

$= 60 \text{ cm}^2$

Hence, option (c) is correct

15. Question

The base of a right triangle is 48 cm and its hypotenuse is 50 cm long. The area of the triangle is –

A. $168cm^2$

B. $252cm^2$

C. $336cm^2$

D. $504cm^2$

Answer

Base of right angled triangle = 48 cm

Hypotenuse of triangle = 50 cm

Now, by using pythagoras theorem we get:

$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Height}^2$

$(50)^2 = (48)^2 - h^2$

$2500 = 2304 - h^2$

$h^2 = 2500 - 2304$

$h^2 = 196$

h = 14 cm

Now, Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 14 \times 48$

= 7 × 48

= 336 cm^2

Hence, option (c) Is correct

16. Question

The area of an equilateral triangle is $81\sqrt{3}$ cm^2. Its height is-

A. $9\sqrt{3}$ cm

B. $6\sqrt{3}$

C. $18\sqrt{3}$ cm

D. 9 cm

Answer

It is given in the question that,

Area of an equilateral triangle = $81\sqrt{3}$ cm^2

Let a be the side of the triangle and h be the height

We know that,

Area of an equilateral triangle = $\frac{\sqrt{3}}{4} a^2$

$81\sqrt{3} = \frac{\sqrt{3}}{4} \times a^2$

$a^2 = 81 \times 4$

a = 18

Also, Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$81\sqrt{3} = \frac{1}{2} \times 18 \times h$

$h = \frac{81\sqrt{3}}{9}$

$h = 9\sqrt{3}$ cm

Hence, option (a) is correct

17. Question

The difference between the semi- perimeter and the sides of a Δ ABC are 8cm, 7cm and 5cm respectively. The area of the triangle is –

A. $20\sqrt{7}$ cm^2

B. $10\sqrt{14}$ cm^2

C. $20\sqrt{14}$ cm^2

D. 140 cm^2

Answer

Let the semi-perimeter be s

Let the sides of the triangle be a, b and c

It is given in the question that,

s – a = 8 ...(i)

s – b = 7 ...(ii)

s – c = 5 ...(iii)

Now, by adding (i), (ii) and (iii) we get:

(s – a) + (s – b) + (s – c) = 8 + 7 + 5

3s – a – b – c = 20

3s – (a + b + c) = 20

We know that,

$s = \frac{a+b+c}{2}$

∴ 3s – 2s = 20

s = 20 cm

Now, area of the triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{20(8)(7)(5)}$

= $20\sqrt{14}$ cm^2

Hence, option (c) is correct

18. Question

For an isosceles right angles triangle having each of equal sides 'a' , we have

I. Area = $\frac{1}{2}a^2$

II. Perimeter = $(2 + \sqrt{2})a$

III. Hypotenuse = 2a

Which of the following is true?

A. I only

B. II only

C. I and II

D. I and III

Answer

We know that,

Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times a \times a$

$= \frac{1}{2} \times a^2$

Now, Hypotenuse = $\sqrt{a^2 + a^2}$

$= \sqrt{2a^2}$

$= \sqrt{2}a$

Perimeter = a + a + $\sqrt{2}a$

= 2a + $\sqrt{2}a$

= a (2 + $\sqrt{2}$)

∴ I and II are true

Hence, option (c) is correct

19. Question

For an isosceles triangle having base b and each of the equal sides as a, we have:

I. Area = $\frac{b\sqrt{4a^2 - b^2}}{4}$

II. Perimeter = (2a +b)

III. Height = $\frac{1}{2}\sqrt{4a^2 - b^2}$

Which of the following is true?

A. I only

B. I and II only

C. II and III only

D. I, II and III

Answer

According to question, we have:

Base of triangle = b

Equal sides of triangle = a

∴ Area = $\frac{b\sqrt{4a^2 - b^2}}{4}$

Perimeter = (2a + b)

And, Height = $\frac{1}{2}\sqrt{4a^2 - b^2}$

∴ I, II and III are true

Hence, option (d) is correct

20. Question

The question consists of two statements namely, Assertion (a) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
Area of an equilateral triangle having each side equal to 4cm is $4\sqrt{3}$ sq cm.	Area of an equilateral triangle having each side a is $\frac{\sqrt{3}}{4}a^2$ sq units.

A. Both Assertion (A) and Reason (B) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (B) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (a) is false and Reason (R) is true.

Answer

In the given question, we have:

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

= $\frac{\sqrt{3}}{4} \times (4)^2$

= $\frac{\sqrt{3}}{4} \times 16$

= $4\sqrt{3}$ cm^2

Also, Area of an equilateral triangle having each side a = $\frac{\sqrt{3}}{4}a^2$ sq units

∴ Both Assertion and Reason are true

Hence, option (a) is correct

21. Question

The question consists of two statements namely, Assertion (a) and Reason (R). Please select the correct answer.

Assertion(A)	Reason (R)
The area of an isosceles triangle having base = 8cm and each of the equal sides = 5cm is $12cm^2$.	The area of an isosceles triangle having each of the equal sides as a and base =b is $\frac{1}{4}b\sqrt{4a^2-b^2}$.

A. Both Assertion (A) and Reason (B) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (B) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (a) is false and Reason (R) is true.

Answer

In the given question, we have

Area of isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$

Here, we have:

a = 5 cm and b = 8 cm

$\therefore \frac{8}{4} \times \sqrt{4(5)^2 - 8^2}$

= 2 × $\sqrt{100-64}$

= 2 × $\sqrt{36}$

= 2 × 6

= 12 cm^2

Also, Area of an isosceles triangle having each of the equal sides as a and base b = $\frac{1}{4}b\sqrt{4a^2 - b^2}$

∴ Both Assertion and Reason are true

Hence, option (a) is correct

22. Question

The question consists of two statements namely, Assertion (a) and Reason (R). Please select the correct answer.

Assertion(A)	**Reason (R)**
The area of an equilateral triangle having side 4cm is $3cm^2$	The area of an equilateral triangle having each side a is ($\frac{\sqrt{3}}{4}a^2$) sq units.

A. Both Assertion (A) and Reason (B) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (B) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (a) is false and Reason (R) is true.

Answer

In this question, we have

Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

= $\frac{\sqrt{3}}{4} \times (4)^2$

= $\frac{\sqrt{3}}{4} \times 16$

= $4\sqrt{3}$ cm^2

Also, Area of an equilateral triangle having each side a = $\frac{\sqrt{3}}{4}a^2 \text{sq units}$

Thus, assertion is false whereas reason is true

Hence, option (d) is correct

23. Question

The question consists of two statements namely, Assertion (a) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
The sides of the triangle ABC are in the ratio 2 : 3 : 4 and its perimeter is 36 cm. Then ar(Δ ABC) = $12\sqrt{15}$ cm^2.	If 2s = (a + b +c) where a, b, c are the sides of the triangle, then its area is = $\sqrt{(s-a)(s-b)(s-c)}$.

A. Both Assertion (A) and Reason (B) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (B) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (a) is false and Reason (R) is true.

Answer

In the given question,

Let us assume the sides of the triangle be 2x, 3x and 4x

We know that,

Perimeter of triangle = Sum of all sides

36 = 2x + 3x + 4x

36 = 9x

x = $\frac{36}{9}$

x = 4

∴ Sides of the triangle are:

2x = 2 × 4 = 8 cm

3x = 3 × 4 = 12 cm

4x = 4 × 4 = 16 cm

Let, a = 8 cm, b = 12 cm and c = 16 cm

So, s = $\frac{a+b+c}{2}$

$= \frac{8+12+16}{2}$

$= \frac{36}{2}$

= 18 cm

Now, by using Heron's formula we have:

Area of triangle = $\sqrt{s\ (s-a)(s-b)(s-c)}$

= $\sqrt{18\ (18-8)(18-12)(18-16)}$

= $\sqrt{18 \times 10 \times 6 \times 2}$

= $\sqrt{6 \times 3 \times 5 \times 2 \times 6 \times 2}$

= 6 × 2$\sqrt{15}$

= 12$\sqrt{15}$ cm^2

Also, if 2s = (a + b + c)

Where a, b and c are the sides of the triangle then:

Area = $\sqrt{(s-a)(s-b(\ (s-c)}$ which is false as it should be:

Area = $\sqrt{s\ (s-a)(s-b(\ (s-c)}$

∴ Assertion is true whereas reason is false

Hence, option (c) is correct

24. Question

The question consists of two statements namely, Assertion (a) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
The area of an isosceles triangle having base = 24 cm and each of the equal sides equal to 13cm is 60cm^2.	If 2s = (a + b+ c) where a, b, c are the sides of a triangle, then area = $\sqrt{s(s-a)(s-b)(s-c)}$.

A. Both Assertion (A) and Reason (B) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (B) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (a) is false and Reason (R) is true.

Answer

From the given question, we have

a = 24 cm, b = 13 cm and c = 13 cm

$\therefore$ s = $\frac{a+b+c}{2}$

= $\frac{24+13+13}{2}$

= $\frac{50}{2}$

= 25 cm

Now, by using heron's formula we have:

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{25\,(25-24)(25-13)(25-13)}$

= $\sqrt{25 \times 1 \times 12 \times 12}$

= 5 × 12

= 60 cm^2

Also, if 2s = (a + b + c) where a, b and c are the sides of the triangle then:

Area = $\sqrt{s\,(s-a)(s-b(\,(s-c)}$

$\therefore$ Assertion and reason both are correct

Hence, option (a) is correct

25. Question

If the base of an isosceles triangle is 6cm and its perimeter is 16cm, then its area is 12cm^2.

Answer

It is given in the question that,

Base of the triangle, b = 6 cm

Equal sides of the isosceles triangle = a cm

Perimeter = 16 cm

We know that,

Perimeter = Sum of all sides

16 = a + a + 6

16 = 2a + 6

2a = 10

a = $\frac{10}{2}$

a = 5 cm

∴ Area of an isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$

= $\frac{6}{4}\sqrt{4(5)^2 - 6^2}$

= 1.5 × $\sqrt{100 - 36}$

= 1.5 × $\sqrt{64}$

= 1.5 × 8

= 12 cm^2

Hence, the given statement is true

26. Question

If each side of an equilateral triangle is 8cm long, then its area is $20\sqrt{3}$ cm^2.

Answer

It is given in the question that,

Each side of an equi8lateral triangle = 8 cm

Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2$

= $\frac{\sqrt{3}}{4} \times (8)^2$

= $\frac{\sqrt{3}}{4} \times 64$

= $16\sqrt{3}$ cm^2

Hence, the given statement is false

27. Question

If the sides of a triangular field measures 52m, 37 m and 20 m, then the cost of leveling at Rs 5 per m^2 is Rs 1530.

Answer

Let the sides of the triangular field be:

a = 52 m, b = 37 m and c = 20 m

$\therefore$ s = $\frac{a+b+c}{2}$

= $\frac{51+37+20}{2}$

= $\frac{108}{2}$

= 54 m

Now, by using Heron's formula we get:

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{54\,(54-51)(54-37)(54-20)}$

= $\sqrt{54 \times 3 \times 17 \times 34}$

= $\sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2}$

= 17 × 2 × 3 × 3

= 306 m^2

It is given that,

Cost of leveling 1 m^2 area = Rs 5

$\therefore$ Cost of leveling 306 m^2 area = 5 × 306

= Rs 1530

Hence, the given statement is true

28. Question

Match the following columns.

Column I	Column II
(a) The lengths of three sides of a triangle are 26 cm, 28 cm and 30cm. The height corresponding to base 28cm is……cm	(p) 6
(b) The area of an equilateral triangle is $4\sqrt{3}$ cm^2. The perimeter of the triangle is…..cm	(q) 4
(c) If the height of an equilateral triangle is $3\sqrt{3}$ cm, then each side of the triangle measures …. Cm	(r) 24
(d) Let the base of an isosceles triangle be 6 cm and each of the equal side be 5cm. Then, its height is ….cm	(s) 12

The correct answer is :

(a)-…….. (b)-……..

(c)-……… (d)-………

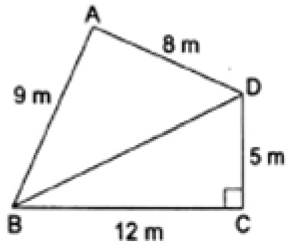
Answer

The correct match for the table is as follows:

Column I	Column II
(a) The lengths of three sides of a triangle are 26 cm, 28 cm and 30cm. The height corresponding to base 28cm is……cm	(r) 24
(b) The area of an equilateral triangle is $4\sqrt{3}$ cm^2. The perimeter of the triangle is…..cm	(s) 12
(c) If the height of an equilateral triangle is $3\sqrt{3}$ cm, then each side of the triangle measures …. Cm	(p) 6
(d) Let the base of an isosceles triangle be 6 cm and each of the equal side be 5cm. Then, its height is ….cm	(q) 4

29. Question

A park in the shape of a quadrilateral ABCD has AB = 9m, BC = 12m, CD = 5 cm, AD = 8 m and c= 90°. Find the area of the park. [Given: $\sqrt{35}$ = 5.9]



Answer

rom the given figure, it is clear that:

BCD is a right triangle

∴ BD = $\sqrt{BC^2 + CD^2}$

= $\sqrt{12^2 + 5^2}$

= $\sqrt{144 + 25}$

= $\sqrt{169}$

= 13 m

Now, area ofΔBCD = $\frac{1}{2} \times \text{Base} \times \text{Height}$

= $\frac{1}{2} \times BC \times CD$

= $\frac{1}{2} \times 12 \times 5$

= 6 × 5 = 30 m^2

Let the sides of the triangle be: a = 9 m, b = 8 m and c = 13 m

∴ s = $\frac{a+b+c}{2}$

= $\frac{9+8+13}{2}$

= $\frac{30}{2}$ = 15 m

Thus, by using Heron's formula we get:

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{15(15-9)(15-8)(15-13)}$

= $\sqrt{15 \times 6 \times 7 \times 2}$

= $\sqrt{5 \times 3 \times 3 \times 2 \times 7 \times 2}$

$= 3 \times 2\sqrt{35}$

$= 6\sqrt{35}$

$= 6 \times 5.9 = 35.4$ m^2

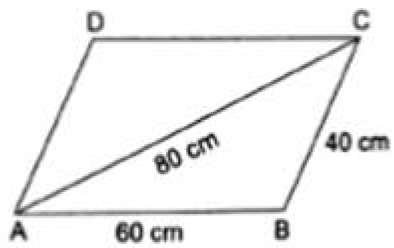
∴ Area of quadrilateral ABCD = Area of ΔBCD + Area of ΔABD

= 30 + 35.4

= 65.4 m^2

30. Question

Find the area of a parallelogram ABCD in which AB = 60cm, BC = 40cm and AC = 80 cm. [Given: $\sqrt{5}$ = 3.87]



Answer

Let the sides of the triangle ABC be:

a = 40 cm, b = 80 cm and c = 60 cm

∴ s = $\frac{a+b+c}{2}$

$= \frac{40+80+60}{2}$

$= \frac{180}{2}$

= 90 cm

Now, by using Heron's formula we get:

Area of $\Delta ABC = \sqrt{s\ (s-a)(s-b)(s-c)}$

$= \sqrt{90\ (90-40)(90-80)(90-60)}$

$= \sqrt{90 \times 50 \times 10 \times 30}$

$= \sqrt{30 \times 3 \times 10 \times 5 \times 10 \times 30}$

$= 30 \times 10\sqrt{15}$

= 300 × 3.87

= 1161 cm^2

As we know that, the diagonal of a parallelogram divides it into two triangles of equal areas

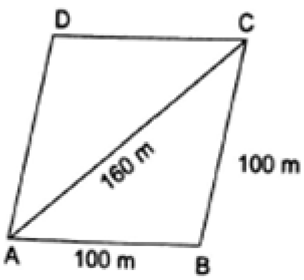
∴ Area of parallelogram (ABCD) = 2 × Area of (ΔABC)

= 2 × 1161

= 2322 cm^2

31. Question

A piece of land is in the shape of a rhombus ABCD in which each side measures 100m and diagonal AC is 160m long. Find the area of the rhombus.



Answer

Let the sides of triangle be 100m, 160m, and 100m

Semi perimeter, $s = \frac{100+160+100}{2} = \frac{360}{2} = 180 \text{ m}$

Now, using Heron's formula,

$$Area\ of\ \Delta\text{ABC} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{180(180-100)(180-160)(180-100)}$$

$$= \sqrt{180(80)(20)(80)}$$

$$= \sqrt{4800 \times 4800}$$

= 4800 m^2

Now, we know that, diagonal divides a parallelogram into two triangles of equal areas.

Area of parallelogram ABCD = 2(area of ΔABC)

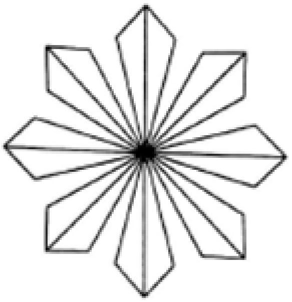
= 2 × 4800

= 9600 m^2

32. Question

A floral design on a floor is made up of 16 triangular tiles, each having sides 9cm, 28 cm and 35 cm. Find the cost of polishing the tiles at the rate of Rs. 2.50 per cm^2

[Take $\sqrt{6}$ =2.454]



Answer

Let the sides of triangle be 9 cm, 28 cm, and 35 cm

Semi perimeter, $s = \frac{9+28+35}{2} = \frac{72}{2} = 36 \text{ cm}$

Now, using Heron's formula,

$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (Area of 1 tile)

$= \sqrt{36(36-9)(36-28)(36-35)}$

$= \sqrt{36(8)(27)(1)}$

$= \sqrt{4 \times 9 \times 3 \times 9 \times 2 \times 4}$

$= 9 \times 4\sqrt{3}$ = 88.2cm^2

∴ Area of 16 tiles = 16 × area of one tile

= 16 × 88.2 cm^2

= 1411.2 cm^2

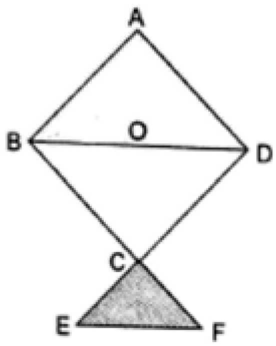
Now, cost of polishing 1cm^2 area = Rs 2.5

∴ Cost of polishing 1411.2 cm^2 = 2.5 × 1411.2 = Rs 3528

33. Question

A kite in the shape of a square with each diagonal 32 cm and having a tail in the shape of an isosceles triangle of base 8cm and each side 6cm, is made of three different shades as shown in the figure. How much paper of each shade has been used in it?

[Given : $\sqrt{5}$ = 2.24]



Answer

We know that every square is a rhombus.

And, area of rhombus $= \frac{1}{2}$(product of diagonals)

Each of the equal diagonals = 32 cm

$\therefore$ Area of square ABCD $= \frac{1}{2}(\text{diagonal})^2$

$= \frac{1}{2} \times 32 \times 32$ = 512 cm^2

Note: Diagonal of a parallelogram divides it into two triangles of equal areas and square is a parallelogram.

$\therefore$ Area of ΔABD = Area of ΔBDC = 1/2 area of ABCD

$= \frac{1}{2} \times 512 = 256\ cm^2$

Area of isosceles triangle CEF $= \frac{b}{4}\sqrt{4a^2 - b^2}$

Whereas, a = 6 cm and b = 8 cm

$= \frac{8}{4}\sqrt{4(6)^2 - 8^2}$

$= \frac{8}{4}\sqrt{144 - 64}$

= $2\sqrt{80}$

= $8\sqrt{5}$

= 17.92 cm^2

Formative Assessment (Unit Test)

1. Question

Each side of an equilateral triangle is 8 cm. Its altitude is

A. $2\sqrt{2}$ cm

B. $2\sqrt{3}$ cm

C. $4\sqrt{3}$ cm

D. $2\sqrt{6}$ cm

Answer

It is given that,

Each side of an equilateral triangle, a = 8 cm

We know that,

Area of equilateral triangle = $\frac{\sqrt{3}}{4}\ a^2$

= $\frac{\sqrt{3}}{4} \times (8)^2$

= $\frac{\sqrt{3}}{4} \times 64$

= $\sqrt{3} \times 16$

= $16\sqrt{3}$ cm^2

Also,

Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$

$16\sqrt{3} = \frac{1}{2} \times a \times \text{Altitude}$

$16\sqrt{3} = \frac{1}{2} \times 8 \times \text{Altitude}$

∴ Altitude = $\frac{16\sqrt{3}}{4}$

= $4\sqrt{3}$ cm

Hence, altitude of the triangle is $4\sqrt{3}$ cm

Thus, option (c) is correct

2. Question

The perimeter of an isosceles right- angled triangle having a as each of the equal sides is

A. $(1+\sqrt{2})a$

B. $(2+\sqrt{2})a$

C. 3a

D. $(3+\sqrt{2})a$

Answer

It is given in the question that, equal sides of isosceles triangle is a

It is also given that, the given triangle is isosceles right-angled triangle

∴ AC = $\sqrt{AB^2 + BC^2}$

AC = $\sqrt{a^2 + a^2}$

AC = $\sqrt{2a^2}$

AC = a$\sqrt{2}$

We know that,

Perimeter of triangle = Sum of all sides

∴ Perimeter = (AB + BC + AC)

= (a + a + a$\sqrt{2}$)

= 2a + a$\sqrt{2}$

= a (2 + $\sqrt{2}$)

Hence, option (b) is correct

3. Question

For an isosceles triangle having base = 12 cm and each of the equal sides equal to 10 cm, the height is

A. 12 cm

B. 16 cm

C. 6 cm

D. 8 cm

Answer

Let us assume ABC be an isosceles triangle having,

Base, AC = 12 cm

AB = AC = 10 cm

BD = $\frac{1}{2} \times BC$

$= \frac{1}{2} \times 12$

= 6 cm

We know that,

In right angled triangle, ABC

AD = $\sqrt{AB^2 - BD^2}$

$= \sqrt{(10)^2 - (6)^2}$

$= \sqrt{100 - 36}$

$= \sqrt{64}$

= 8 cm

Thus, height of the triangle is 8 cm

Hence, option (d) is correct

4. Question

Find the area of an equilateral triangle having each side 6cm.

Answer

Let us assume each side of the equilateral triangle be a

It is given that,

Side of equilateral triangle = 6 cm

We know that,

Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times (6)^2$

$= \frac{\sqrt{3}}{4} \times 36$

$= \sqrt{3} \times 9$

$= 9\sqrt{3}$ cm^2

5. Question

Using Heron's formula find the area of Δ ABC in which BC = 13 cm, AC = 14 cm and AB = 15cm.

Answer

It is given in the question that,

Sides or triangle ABC are:

BC = 13 cm

AC = 14 cm

AB = 15 cm

We know that,

Perimeter of triangle = Sum of all sides

= AB + BC + AC

= 15 + 13 + 14

= 42 cm

∴ s = $\frac{1}{2} \times$ Perimeter of triangle ABC

= $\frac{1}{2} \times 42$

= 21 cm

Hence,

Area of ΔABC = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{21\,(21-13)(21-14)(21-15)}$

= $\sqrt{21 \times 8 \times 7 \times 6}$

= 84 cm^2

6. Question

The sides of a triangle are in the ratio 13: 14: 15 and its perimeter is 84 cm. Find the area of the triangle.

Answer

It is given in the question that,

Perimeter of triangle = 84 cm

Also, sides or triangle are: in ratio 13: 14: 15

Let, a = 13x

b = 14x

c = 15x

We know that,

Perimeter of triangle = Sum of all sides

$84 = a + b + c$

$84 = 13x + 14x + 15x$

$84 = 42x$

$x = \frac{84}{42}$

x = 2 cm

Thus, $a = 13 \times 2 = 26$ cm

$b = 14 \times 2 = 28$ cm

$c = 15 \times 2 = 30$ cm

$\therefore s = \frac{1}{2} \times \text{Perimeter of triangle ABC}$

$= \frac{1}{2} \times 84$

= 42 cm

Hence,

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

$= \sqrt{42\,(42-26)(42-28)(42-30)}$

$= \sqrt{42 \times 16 \times 14 \times 12}$

= 336 cm^2

7. Question

Find the area of ABC in which BC = 8cm, AC = 15cm and AB = 17 cm. Find the length of altitude drawn on AB.

Answer

It is given in the question that,

Sides or triangle ABC is:

BC = a = 8 cm

AC = b = 15 cm

AB = c = 17 cm

We know that,

Perimeter of triangle = Sum of all sides

= a + b + c

= 8 + 15 + 17

= 40 cm

∴ s = $\frac{1}{2} \times$ Perimeter of triangle ABC

= $\frac{1}{2} \times 40$

= 20 cm

Hence,

Area of ΔABC = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{20\,(20-8)(20-15)(20-17)}$

= $\sqrt{20 \times 12 \times 5 \times 3}$

= 60 cm^2

Also, Area of triangle ABC = $\frac{1}{2} \times$ Base $\times$ Height

60 = $\frac{1}{2} \times AB \times$ Height

120 = 17 × Height

Height = $\frac{120}{17}$

= 7.06 cm

Hence, area of triangle is 60 cm^2 and length of altitude is 7.06 cm

8. Question

An isosceles triangle has perimeter 30cm and each of its equal sides is 12cm. Find the area of the triangle.

Answer

It is given in the question that,

Equal sides of isosceles triangle = a = b = 12 cm

Also, perimeter = 30 cm

We know that perimeter of triangle = Sum of all sides

(a + b + c) = 30 cm

12 + 12 + c = 30

24 + c = 30

c = 30 – 24

= 6 cm

Hence, s = $\frac{1}{2} \times Perimeter$

s = $\frac{1}{2} \times 30$

s = 15 cm

∴ Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{15\,(15-12)(15-12)(15-6)}$

= $\sqrt{15 \times 3 \times 3 \times 9}$

= $9\sqrt{15}$ cm^2

9. Question

The perimeter of an isosceles triangle is 32cm. The ratio of one of the equal side to its base is 3: 2. Find the area of the triangle.

Answer

It is given in the question that,

Perimeter of an isosceles triangle = 32 cm

Let us assume the sides of the triangle be a, b, c and a = b

We know that,

Perimeter = a + b + c

32 = a + b + c

32 = a + a + c

32 = 2a + c (i)

According to the condition given in the question, we have:

a: c = 3: 2

So, a = 3x and c = 2x

Now putting values of a and c in (i), we get

2 × 3x + 2x = 32

6x + 2x = 32

8x = 32

x = $\frac{32}{8}$

x = 4

Thus, a = 3 × 4 = 12 cm

b = 12 cm

c = 2 × 4 = 8 cm

Now, s = $\frac{1}{2} \times \text{Perimeter}$

= $\frac{1}{2} \times 32$

= 16 cm

Area of triangle = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{16\,(16-12)(16-12)(16-8)}$

= $\sqrt{16 \times 4 \times 4 \times 8}$

= 4 × 4 × 2$\sqrt{5}$

= 32$\sqrt{2}$ cm^2

10. Question

Given a ABC in which

I. A, B and C are in the ratio 3: 2 :1.

II. AB , AC and BC are in the ratio 3 : 3 : 2 3 and AB = 3 3 cm.

Is ABC a right triangle?

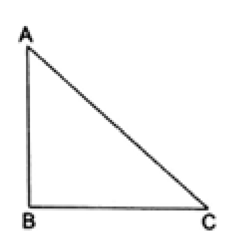
The question give above has two Statements I and II. Answer the questions by using instructions given below:

(a) If the question can be answered by one of the given statements only and not by the other.

(b) If the question can be answered by using either statement alone.

(c) If the question can be answered by using both the statements but cannot be answered by using either statement.

(d) If the question cannot be answered even by using both the statements together.



Answer

I. It is given in the question that,

$\angle A$, $\angle B$ and $\angle C$ are in the ratio 3: 2: 1

Let $\angle A = 3x$

$\angle B = 2x$

$\angle C = x$

We know that, sum of angles of a triangle = 180^o

$\angle A + \angle B + \angle C = 180^o$

$3x + 2x + x = 180^o$

$6x = 180^o$

$x = \frac{180^o}{6}$

$x = 30^o$

Hence, $\angle A = 3 \times 30^o = 90^o$

∴ Δ ABC is a right-angled triangle

II. It is also given that:

AB, AC and BC are in the ratio $3: \sqrt{3}: 2\sqrt{3}$

Now, $AB = 3x$, $AC = \sqrt{3}x$ and $BC = 2\sqrt{3}x$

As it is given that,

$AB = 3\sqrt{3}$

∴ $x = \sqrt{3}$

$AC = 3$

$BC = 6$

Now, by using Pythagoras theorem in ΔABC we get:

$AC = \sqrt{AB^2 + BC^2}$

$3 = \sqrt{(3\sqrt{3})^2 + (6)^2}$

$3 = \sqrt{27 + 36}$

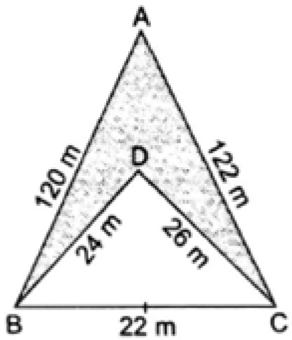
$3 \neq \sqrt{63}$

∴ The question can be answered by using either statement alone

Hence, option (b) is correct

11. Question

In the given figure ABC and DBC have the same base BC such that AB = 120m, AC = 122m, BC = 22m, BD = 24m and CD = 26m. Find the area of the shaded region. (Take $\sqrt{105}$ = 10.25)



Answer

It is given in the question that,

AB = 120 m

AC = 122 m

BC = 22 m

BD = 24 m

And, CD = 26 m

We know that,

Perimeter of triangle = Sum of all sides

∴ Perimeter of ΔABC = AB + BC + AC

= 120 + 22 + 122

= 264 m

s = $\frac{1}{2} \times \text{Perimeter } (\Delta\text{ABC})$

= $\frac{1}{2} \times 264$

= 132 m

Now, Area (ΔABC) = $\sqrt{s\,(s-a)(s-b)(s-c)}$

= $\sqrt{132\,(132-22)(132-122)(132-120)}$

= $\sqrt{132 \times 110 \times 10 \times 12}$

= 11 × 12 × 10

$= 1320\ m^2$

Now, in ΔBCD

BC = a, BD = b and CD = c

∴ Perimeter of ΔBCD = 22 + 24 + 26

= 72 m

s = $\frac{1}{2} \times$ Perimeter of ΔBCD

$= \frac{1}{2} \times 72$

= 36 m

Hence, area $(\Delta BCD) = \sqrt{s\,(s-a)(s-b)(s-c)}$

$= \sqrt{36\,(36-22)(36-24)(36-26)}$

$= \sqrt{36 \times 14 \times 12 \times 10}$

$= 6 \times 2\sqrt{420}$

$= 6 \times 2 \times 2\sqrt{105}$

$= 24\sqrt{105}$

$= 24 \times 10.25$

$= 246\ m^2$

∴ Area of shaded region = Area (ΔABC) – Area (ΔBCD)

= 1320 – 246

$= 1074\ m^2$

12. Question

A point O is taken inside an equilateral ΔABC. If OL $\perp$ BC, OM $\perp$ AC and ON $\perp$ AB such that OL = 14 cm, OM = 10 cm and ON = 6 cm, find the area of $\triangle$ ABC.

Answer

Let each side of ΔABC be a cm

So, area (ΔABC) = Area (ΔAOB) + Area (ΔAOC) + Area (ΔBOC)

$= \frac{1}{2} \times a \times ON + \frac{1}{2} \times a \times OM + \frac{1}{2} \times a \times OL$

On taking "a" as common, we get,

$= \frac{1}{2} a\ (ON + OM + OL)$

$= \frac{1}{2} \times a\ (6 + 10 + 14)$

$= \frac{1}{2} \times a \times 30$

= 15a cm^2 (i)

As, triangle ABC is an equilateral triangle and we know that:

Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$ cm^2 (ii)

Now, from (i) and (ii) we get:

15a = $\frac{\sqrt{3}}{4} a^2$

15 × 4 = $\sqrt{3}a$

60 = $\sqrt{3}a$

a = $\frac{60}{\sqrt{3}}$

a = 20√3 cm

Now, putting the value of a in (i), we get

Area (ΔABC) = 15 × 20√3

= 300√3 cm^2

7. Summative Assessment I

Sample Paper 1

1. Question

Which of the following is a rational number?

A. 2/(√3)

B. √2/3

C. 3√5

D. -3/5

Answer

A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q.

Since for option D numerator, p = -3 and denominator q = 5 both are integers.

-3/5 is a rational number.

2. Question

The value of k for which the polynomial $x^3 - 4x^2 + 2x + k$ has 3 as its zero, is

A. 3

B. -3

C. 6

D. -6

Answer

If 3 is the solution for the equation. It must satisfy the expression.

So, putting x = 3 it must be zero.

33 - 4 × 32 + 2 × 3 + k = 0

27 – 4 × 9 + 6 + k = 0

k - 3 = 0

k = 3

3. Question

Which of the following is a zero of the polynomial $x^3 + 2x^2 - 5x - 6$?

A. -2

B. 2

C. -4

D. 3

Answer

We need to do hit and trial to find root of a cubic equation.

If it is a root of equation, it must satisfy the equation.

So, let’s start with option A.

$(-2)^3 + 2(-2)^2 - 5(-2)-6 = -8 + 8 + 10 - 6 = 4$

Let’s try option B

$(2)^3 + 2(2)^2 - 5(2)-6 = 8 + 8 - 10 - 6 = 0$

Let’s try option C

$(-3)^3 + 2(-3)^2 - 5(-3)-6 = -27 + 18 + 15 - 6 = 0$

For option D

$(3)^3 + 2(3)^2 - 5(3)-6 = 27 + 18 - 15 - 6 = 24$

Hence Option B and C are correct

Verifying –

Factors of the given equation is $(x-2)(x + 3)(x + 1) = x^3 + 2x^2 - 5x - 6$.

4. Question

The factorization of $-x^2 + 7x - 12$ yields

A. (x - 3)(x - 4)

B. (3 + x)(4 - x)

C. (x - 4)(3 - x)

D. (4 - x)(3 - x)

Answer

$-x^2 + 7x - 12$ can be factorized as-

$-x^2 + 4x + 3x - 12$

$-x(x - 4) + 3(x - 4)$

$(x - 4)(3 - x)$

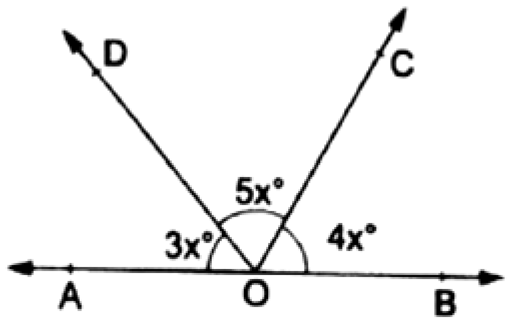
Also recheck by-

Sum of roots = 7 {-coefficient of x/ coefficient of x^2}

Product of roots = 12 {constant/ coefficient of x^2}

5. Question

In the given figure, ∠BOC = ?



A. 45°

B. 60°

C. 75°

D. 56°

Answer

Sum of angles in a straight line is 180°

So, ∠AOD + ∠DOC + ∠BOC = 180°

3x + 5x + 4x = 180

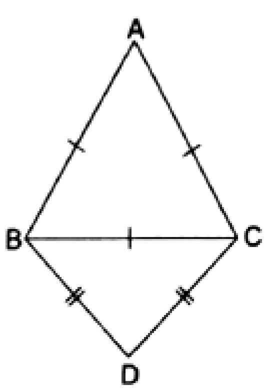
12x = 180

x = 15

∠BOC = 4x = 4×15 = 60°.

6. Question

In the given figure, ΔABC is an equilateral triangle and ΔBDC is an isosceles right triangle, right-angled at D. Then ∠ACD = ?



A. 60°

B. 90°

C. 120°

D. 105°

Answer

Since we know all the angles in an equilateral triangle is of 60°.

So, ∠ABC = ∠ACB = ∠CAB = 60° ...(i)

Also for an isosceles triangle, the angles opposite to equal sides are equal.

So, ∠DBC = ∠DCB = x (let’s say)

Also sum of all angles in a triangle = 180°.

So, in ΔBDC,

∠DBC + ∠DCB + ∠BDC = 180°

x + x + 90 = 180 {since ∠BDC = 90°}

2x = 90

x = 45°

so ∠DCB = 45 ...(ii)

And ∠ACD = ∠ACB + ∠DCB = 60° + 45° = 105° {from (i) and (ii)}

7. Question

Each of the equal sides of an isosceles triangle is 13 cm and its base is 24 cm. The area of the triangle is

A. 30 cm^2

B. 45 cm^2

C. 60 cm^2

D. 78 cm^2

Answer

Applying heron’s formula-

We know,

$s = \frac{a + b + c}{2}$ here a, b and c are sides of a triangle

$$\text{So, } s = \frac{13 + 13 + 24}{2} = 25$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

So, Area = $= \sqrt{25(25-13)(25-13)(25-24)}$

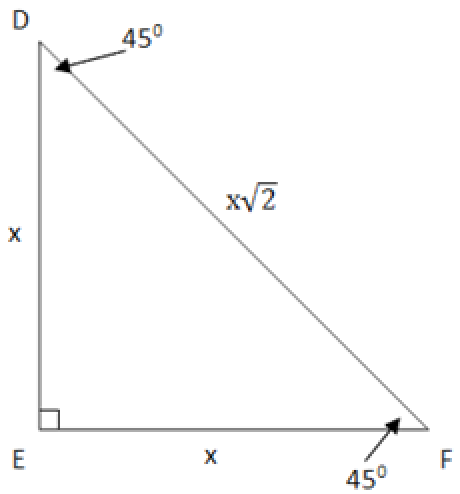
Hence Area = $\sqrt{25(12)(12)(1)}$

= √3600

= 60 square units

8. Question

In an isosceles right triangle, the length of the hypotenuse is 4√2 cm. The length of each of the equal sides is



A. 4√3 cm

B. 6 cm

C. 5 cm

D. 4 cm

Answer

For a right-angled triangle,

Applying Pythagoras theorem,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

Since triangle is isosceles.

So, base = perpendicular = x (let's say)

Hence $(\text{hypotenuse})^2 = (x)^2 + (x)^2$

$(4\sqrt{2})^2 = 2x^2$

$32 = 2x^2$

$x^2 = 16$

so, x = 4 cm.

9. Question

If, $x = 7 + 4\sqrt{3}$ find the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$

Answer

Let $\sqrt{x} + \frac{1}{\sqrt{x}}$ to be y.

So $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

Squaring both sides,

$$y^2 = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

$$= \left(\sqrt{x}\right)^2 + \left(\frac{1}{\sqrt{x}}\right)^2 + 2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right) = x + \frac{1}{x} + 2$$

Also, $x = 7 + 4\sqrt{3}$

So $y^2 = 7 + 4\sqrt{3} + \frac{1}{7 + 4\sqrt{3}} + 2$

$$= 9 + 4\sqrt{3} + \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} \text{ (on rationalizing)}$$

$$= 9 + 4\sqrt{3} + \frac{7 - 4\sqrt{3}}{(7)^2 - (4\sqrt{3})^2}$$

$$= 9 + 4\sqrt{3} + \frac{7 - 4\sqrt{3}}{49 - 48}$$

$$= 9 + 4\sqrt{3} + 7 - 4\sqrt{3}$$

= 16

So, $y = \sqrt{16} = 4$

Hence $y = \sqrt{x} + \frac{1}{\sqrt{x}} = 4$

10. Question

Factorize: $(7a^3 + 56b^3)$

Answer

$(7a^3 + 56b^3)$

$= 7(a^3 + 8b^3)$

$= 7(a^3 + (2b)^3)$

$= 7(a + (2b))(a^2 + (2b)^2 - a(2b))$

[since $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$= 7(a + 2b)(a^2 + 4b^2 - 2ab)$

11. Question

Find the value of a for which (x - 1) is a factor of the polynomial $(a^2x^3 - 4ax + 4a - 1)$.

Answer

If (x - 1) is a factor of the polynomial $(a^2x^3 - 4ax + 4a - 1)$.

then it must satisfy it.

So, putting x = 1 the polynomial must be zero.

Putting x = 1 and equating to zero.

$= (a^2(1)^3 - 4a(1) + 4a - 1)$

$= a^2 - 4a + 4a - 1 = 0$

$= a^2 = 1$

So, a = �1.

12. Question

In the given figure, if AC = BD show that AB = CD. State the Euclid’s axiom used for it.

Answer

Given- AC = BD

Subtracting BC on both sides-

(AC - BC) = (BD - BC)

AB = CD

13. Question

In a ΔABC if 2∠A = 3∠B = 6∠C, calculate the measure of ∠B.

Answer

In a triangle sum of all angles = 180°

So, ∠A + ∠B + ∠C = 180°

It is given that-

∠A = 3/2 ∠B

∠C = � ∠B

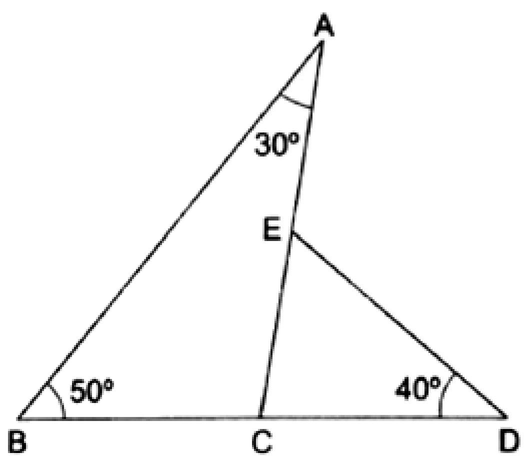
So, ∠A + ∠B + ∠C = (3/2) ∠B + ∠B + (1/2) ∠B = 180°

3∠B = 180°

∠B = 60°

14. Question

In the given figure ∠BAC = 30°, ∠ABC = 50° and ∠CDE = 40° Find ∠AED?



Answer

In ΔABC sum of all angles = 180°.

So, ∠BAC + ∠ABC + ∠ACB = 180°

30 + 50 + ∠ACB = 180

∠ACB = 100°

Since BCD represents a straight line ∠ACB + ∠ECD = 180°

So, ∠ECD = 80°

In ΔECD sum of all angles = 180°

So, ∠ECD + ∠EDC + ∠CED = 180°

60 + 40 + ∠CED = 180

∠CED = 80°

Since AEC represents a straight line, ∠CED + ∠AED = 180°

So, ∠AED = 120°

15. Question

If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ find the value of $(x^2 + y^2)$

Or

Simplify: $\frac{7\ +3\sqrt{5}}{3\ +\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Answer

$x = \frac{\sqrt{5}\ +\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}\ +\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}\ +\sqrt{3}}{\sqrt{5}\ +\sqrt{3}}$ (on rationalizing we get)

$$= \frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\sqrt{5}^2-\sqrt{3}^2} \quad \left\{\text{since } (a+b)(a-b) = a^2 - b^2\right\}$$

$$= \frac{\sqrt{5}^2+\sqrt{3}^2+2\times\sqrt{5}\times\sqrt{3}}{5-3}$$

$$= \frac{5+3+2(\sqrt{5})(\sqrt{3})}{2}$$

= 4 + (√5)(√3)

= 4 + √15

Similarly $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}\ +\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}\ +\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (rationalising)

$$= \frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\sqrt{5}^2-\sqrt{3}^2} \quad \{\text{since } (a+b)(a-b)=a^2-b^2\}$$

$$= \frac{\sqrt{5}^2+\sqrt{3}^2-2\times\sqrt{5}\times\sqrt{3}}{5-3} = \frac{5+3-2\left(\sqrt{5}\right)\left(\sqrt{3}\right)}{2}$$

= (5 + 3-2(√5)(√3))/2

= 4- (√5) (√3)

= 4- √15

So, $x^2+y^2=\left(4+\sqrt{15}\right)^2+\left(4-\sqrt{15}\right)^2$

$$=\left(4^2+\sqrt{15}^2+2\times4\times\sqrt{15}\right)+\left(4^2+\sqrt{15}^2-2\times4\times\sqrt{15}\right)$$

$$=\left(16+15+8\sqrt{15}\right)+\left(16+15-8\sqrt{15}\right)$$

= 32 + 30

= 62

(II) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Taking LCM as (3 + √5)(3-√5)

$$=\frac{\left(7+3\sqrt{5}\right)\left(3-\sqrt{5}\right)-\left(7-3\sqrt{5}\right)\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}$$

$$=\frac{\left(21-7\sqrt{5}+9\sqrt{5}-3\sqrt{5}\times\sqrt{5}\right)-\left(21-9\sqrt{5}+7\sqrt{5}-3\sqrt{5}\times\sqrt{5}\right)}{3^2-\sqrt{5}^2}$$

(since (a + b)(a - b) = a^2 - b^2)

$$=\frac{4\sqrt{5}}{(9-5)}$$

$$=\frac{4\sqrt{5}}{4}=\sqrt{5}$$

16. Question

If 2 and -1/3 are the zeros of the polynomial $3x^3 - 2x^2 - 7x - 2$ find the third zero of the polynomial.

Answer

We know for a cubic polynomial, sum of roots $= -\dfrac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$

Let the third root be x.

$$\text{So, } x + 2 + \left(-\frac{1}{3}\right) = -\left(-\frac{2}{3}\right)$$

$$x + \frac{5}{3} = \frac{2}{3}$$

$$x = \frac{2}{3} - \frac{5}{3}$$

$$x = -1$$

17. Question

Find the remainder when the polynomial $f(x) = 4x^2 - 12x^2 + 14x - 3$ is divided by (2x - 1).

Answer

If we divide $f(x) = 4x^2 - 12x^2 + 14x - 3$ by (2x - 1) remainder can be find at value of –

(2x-1) = 0

Or x = 1/2

So, we will put x = 1/2 in $f(x) = 4x^2 - 12x^2 + 14x - 3$

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + \frac{14}{2} - 3$$

$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 7 - 3$$

$$= \frac{1}{2} - 3 + 7 - 3$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

18. Question

Factorize: $(p-q)^3 + (q-r)^3 + (r-p)^3$

Answer

We know that –

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.

here if a + b + c = 0

$a^3 + b^3 + c^3 = 3abc$.

So, $(p-q)^3 + (q-r)^3 + (r-p)^3 = 3(p-q)(q-r)(r-p)$ {since (p-q) + (q-r) + (r-p) = 0}

19. Question

In the given figure, in ΔABC it is given that ∠B = 40°and ∠C = 50°, DE || BC, and EF || AB Find: (i) ∠ADE + ∠MEN (ii) ∠BDE and (iii) ∠BFE

Answer

Since DE || BC and AB acts as transversal.

So, ∠ADE = ∠ABC {corresponding angles}

since ∠ABC = 40°

So, ∠ADE = 40°

Since EF || AB and DN acts as transversal.

So, ∠ADE = ∠MEN {corresponding angles}

∠MEN = 40°

Hence, ∠ADE + ∠MEN = 80°

(ii) 140°

Since AB represents a straight line. Sum of angles in line AB = 180°

So, ∠BDE + ∠ADE = 180°

since, ∠ADE = 40°

So, ∠BDE = 140°

(iii) 140°

Since DE || BC and FM acts as transversal.

So, ∠EFC = ∠ MEN = 40°

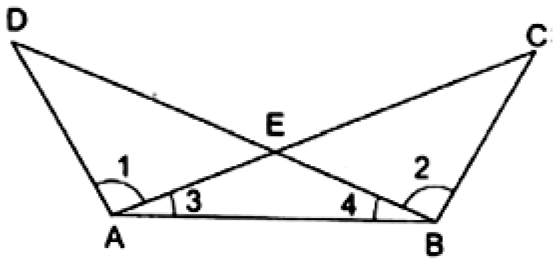
And BC represents a straight line. Sum of angles in line BC = 180°

= ∠EFC + ∠BFE = 180°

= ∠BFE = 140°

20. Question

In the given figure, ΔABC and ΔABD are such that AD = BC, ∠1 = ∠2 and ∠3 = ∠4. Prove that BD = AC.



Answer

Taking ΔABC and ΔABD in consideration-

AD = BC

Since, it is given that

∠1 = ∠2 and ∠3 = ∠4

Adding them –

∠1 + ∠3 = ∠2 + ∠4.

= ∠DAB = ∠ABC

And AB is the common side on both triangle.

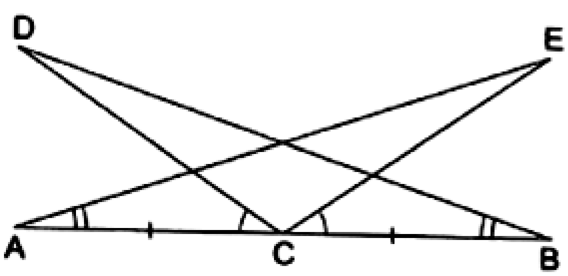
So, by side angle side(SAS) criteria-

Triangle ΔABC and ΔABD are congruent.

So, BD = AC (by congruency criteria).

21. Question

In the given figure, C is the mid-point of AB. If ∠DCA = ∠ECB and ∠DBC = ∠EAC prove that DC = EC.



Answer

Since C is the mid-point of AB.

So, AC = BC.

Taking ΔACE and ΔBCD in consideration-

∠DBC = ∠EAC

AC = BC

Also ∠DCA = ∠ECB

Adding ∠DCE on both sides-

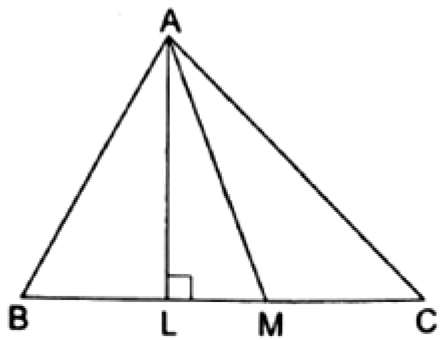
∠DCB = ∠ECA

So, by Angle side Angle(ASA) criteria ΔACE and ΔBCD are congruent.

And hence DC = EC (by congruency criteria).

22. Question

In ΔABC if AL ⊥ BC and AM is the bisector of ∠A. Show that $\angle LAM = \frac{\angle B}{2} - \frac{\angle C}{2}$



Answer

Sum of all angles in a triangle = 180°

∠A + ∠B + ∠C = 180°

∠A = 2 ∠CAM = 2 ∠MAB {since AM is bisector of ∠A}

= 2∠CAM + ∠B + ∠C = 180°

= 2∠CAM = 180 - (∠B + ∠C)

$$= \angle CAM = 90 - \frac{\angle B + \angle C}{2}$$

∠AML = ∠CAM + ∠C {Exterior Angle theorem}

$$= 90 - \frac{\angle B + \angle C}{2} + \angle C$$

$$= 90 + \frac{\angle C}{2} - \frac{\angle B}{2}$$

In Triangle ΔALM, Sum of all angles must be 180°

So, ∠LAM + ∠AML + 90 = 180

∠LAM + ∠AML = 90

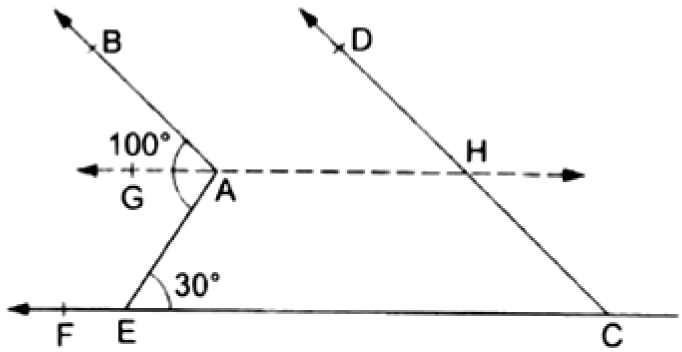
∠LAM = 90 -∠AML

$$= 90 - \left(90 + \frac{\angle C}{2} - \frac{\angle B}{2}\right)$$

$$= \frac{\angle B}{2} - \frac{\angle C}{2}$$

23. Question

In the given figure, AB || CD, ∠BAE = 100° and ∠AEC = 30°. Find ∠DCE.



Answer

Since AH || EC

So, ∠GAE = ∠AEC = 30° {alternate angle}

Also ∠BAG = 100°-∠GAE

∠BAG = 70°

Here also, AB || DC and GH acts as transversal.

So, ∠BAG = ∠DHA = 70° {corresponding angles}

Similarly,

AH || EC and DC acts as transversal.

So, ∠DCE = ∠DHA = 70° {corresponding angles}

24. Question

Factorize: $a^3 - b^3 + 1 + 3ab$.

Answer

$a^3 - b^3 + 1 + 3ab$

$= a^3 + (-b)^3 + 13 - 3\{1 \times a \times (-b)\}$

$= \{a + (-b) + 1\} \{a^2 + (-b)^2 + 12 - a(-b) - (-b)1 - 1a\}$

using identity $\{a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)\} + (a-b + 1)(a^2 + b^2 + 1 + ab + b - a)$

25. Question

If $x = \frac{1}{2-\sqrt{3}}$ show that the value of $x^3 - 2x^2 - 7x + 5$ is 3.

Or

Simplify:

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots.. + \frac{1}{\sqrt{8}+\sqrt{9}}$$

Answer

$$x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{\left(2+\sqrt{3}\right)}{2+\sqrt{3}} \ \{\text{rationalizing}\}$$

$$x = \frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$

$$x = \frac{2+\sqrt{3}}{4-3}$$

$$x = 2+\sqrt{3}$$

Now, $x^2 = (2 + \sqrt{3})^2 = 4 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}$

Also, $x^3 = x \times x^2 = (2 + \sqrt{3})(7 + 4\sqrt{3})$

$= 2(7) + 7(\sqrt{3}) + 2(4\sqrt{3}) + (\sqrt{3})(4\sqrt{3})$

$= 14 + 15\sqrt{3} + 12$

$= 26 + 15\sqrt{3}$

Put all the values in the expression: $x^3 - 2x^2 - 7x + 5$

$= (26 + 15\sqrt{3})-2(7 + 4\sqrt{3})-7(2 + \sqrt{3}) + 5$

$= 3$

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots.. + \frac{1}{\sqrt{8}+\sqrt{9}}$$

rationalize-

$$\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} + \ldots.. + \frac{1}{\sqrt{8}+\sqrt{9}} \times \frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}-\sqrt{9}}$$

$$= \frac{1-\sqrt{2}}{1^2-\sqrt{2}^2} + \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} + \ldots$$

$$= \frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \ldots$$

= √2-1 + √3-√2 + …√8-√7 + √9-√8

= √9-1

= 3-1

= 2

26. Question

If $x = \frac{\sqrt{a + 2b} + \sqrt{a-2b}}{\sqrt{a + 2b} - \sqrt{a-2b}}$ then show that $bx^2 - ax + b = 0$.

Answer

$$x = \frac{\sqrt{a + 2b} + \sqrt{a-2b}}{\sqrt{a + 2b} - \sqrt{a-2b}}$$

$$x = \frac{\sqrt{a + 2b} + \sqrt{a-2b}}{\sqrt{a + 2b} - \sqrt{a-2b}} \times \frac{\sqrt{a + 2b} + \sqrt{a-2b}}{\sqrt{a + 2b} + \sqrt{a-2b}} \{\text{rationalizing}\}$$

$$x = \frac{\left(\sqrt{a + 2b} + \sqrt{a-2b}\right)^2}{\sqrt{a + 2b}^2 - \sqrt{a-2b}^2}$$

$$x = \frac{\sqrt{a + 2b}^2 + \sqrt{a-2b}^2 + 2\left(\sqrt{a + 2b}\right)\left(\sqrt{a-2b}\right)}{(a + 2b)-(a-2b)}$$

$$\frac{a + 2b + a-2b + 2\sqrt{(a + 2b)(a-2b)}}{4b}$$

$$\frac{2a + 2\sqrt{(a + 2b)(a-2b)}}{4b}$$

$$x = \frac{a + \sqrt{a^2-(2b)^2}}{2b}\{\text{since } (a+b)(a-b) = a^2-b^2\}$$

$$\text{So, } 2bx - a = \sqrt{a^2 - (2b)^2}$$

$$= (2bx - a)^2 = \sqrt{a^2 - (2b)^2}^2 \quad \{\text{squaring both sides}\}$$

$= 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

$= 4b^2x^2 - 4abx + 4b^2 = 0$ {rearranging terms and cancelling a^2}

Dividing the expression by $4b - bx^2 - ax + b = 0$

27. Question

If $(x^3 + mx^2 - x + 6)$ has $(x - 2)$ as a factor and leaves a remainder r, when divided by $(x - 3)$, find the values of m and r.

Answer

If $(x - 2)$ is a factor of the polynomial $(x^3 + mx^2 - x + 6)$ then it must satisfy it.

So, putting $x = 2$ the polynomial must be zero.

Putting $x = 2$ and equating to zero.

$= (23 + m2^2 - 2 + 6)$

$= 4m + 12 = 0$

$= m = -3$

If we divide $f(x) = (x^3 + mx^2 - x + 6)$ by $(x - 3)$ remainder can be find at value of –

$(x - 3) = 0$

Or $x = 3$

So we will put $x = 3$ in $f(x) = (x^3 + mx^2 - x + 6)$

$f(3) = (3^3 + m3^2 - 3 + 6)$

$= 30 + 9m$

So remainder $= 30 + 9m$

$= 30 + 9(-3) = 30 - 27 = 3$

So, $r = 3$.

28. Question

If r and s be the remainders when the polynomials $(x^3 + 2x^2 - 5ax - 7)$ and $(x^3 + ax^2 - 12x + 6)$ are divided by $(x + 1)$ and $(x - 2)$ respectively and $2r + s = 6$ find the value of a.

Answer

If we divide $f(x) = (x^3 + 2x^2 - 5ax - 7)$ by $(x + 1)$ remainder can be find at value of –

$(x + 1) = 0$

Or $x = -1$

So, we will put $x = -1$ in $f(x) = (x^3 + 2x^2 - 5ax - 7)$

$f(-1) = ((-1)^3 + 2(-1)^2 - 5a(-1)-7)$

$= -6 + 5a$

So, remainder $= r = -6 + 5a$

Also if we divide $f(x) = (x^3 + ax^2 - 12x + 6)$ by $(x - 2)$ remainder can be find at value of –

$(x - 2) = 0$

Or $x = 2$

So we will put $x = 2$ in $f(x) = (x^3 + ax^2 - 12x + 6)$

$f(2) = (2^3 + a2^2 - 12(2) + 6)$

$= 4a - 10$

So, remainder $= s = 4a - 10$

Also it is given that $2r + s = 6$

So putting r and s from above expressions-

$2(-6 + 5a) + (4a - 10) = 6$

$= 14a = 28$

$= a = 2$

29. Question

Prove that: $(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = 2(a^3 + b^3 + c^3 - 3abc)$

Answer

We know that –

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.

So applying the theorem here,

$(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = ((a + b) + (b + c) + (c + a))((a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c) - (b + c)(c + a)-(c + a)(a + b))$

$= (2(a + b + c))((a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c)-(b + c)(c + a)-(c + a)(a + b))$

{since $((a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c) - (b + c)(c + a) - (c + a)(a + b))$

$= (a^2 + b^2 + c^2 - ab - bc - ca)$}

$= 2(a^3 + b^3 + c^3 - 3(a)(b)(c))$

{using this theorem again: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$}

30. Question

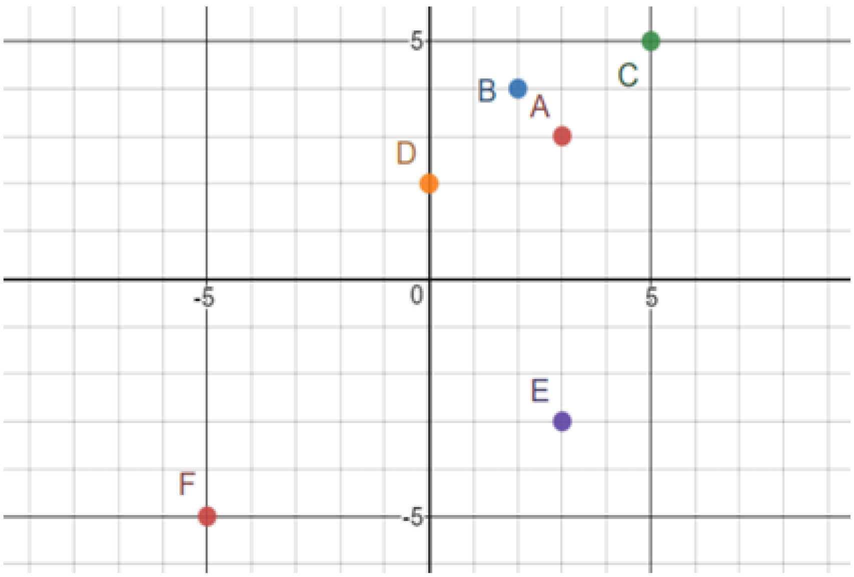
On a graph paper plot the following points:

A(3, 3), B(2, 4), C(5, 5), D(0, 2), E(3, -3) and F(-5, -5).

Which of these points are the mirror images in (i) x-axis (ii) y-axis?

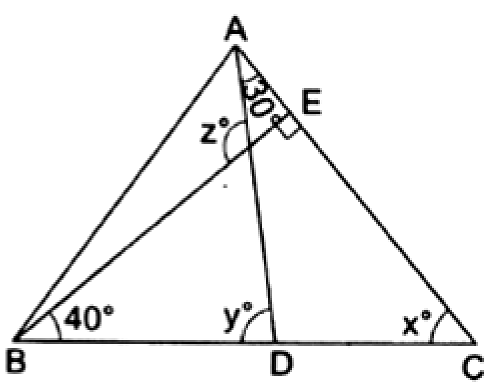
Answer

It is clear from the graph A and E are mirror image wrt. x-axis and there is no mirror image points wrt. y-axis.



31. Question

In the given figure, in a ΔABC, BE ⊥ AC, ∠EBC = 40° and ∠DAC = 30. ∠DAC = 30°. Find the values of x, y and z.



Answer

We know that,

Sum of all angles in a triangle = 180°

So, in ΔBEC

= 40 + x + 90 = 180

So, x = 50°

Now, in ΔADC-

= 50 + 30 + ∠ADC = 180

= ∠ADC = 100°

Since BC represents a straight line, sum of angles = 180°.

So, ∠ADC + y = 180

hence y = 80° since ∠ADC = 100°

By exterior angle sum theorem of the smaller triangle formed-

z = ∠DAE + ∠BEA = 90° + 30° = 120°

32. Question

In the given figure, ABC is a triangle in which AB = AC. D is a point in the interior of ΔABC such that ∠DBC = ∠DCB. Prove that AD bisects ∠BAC.

Answer

In ΔBDC ∠DBC = ∠DCB so

BD = DC …(i)

{sides opposite to equal angles in a triangle are equal}

Now let's consider that ΔABD and ΔADC –

AB = AC {given}

AD is a common side.

And BD = DC {from equation (i)}

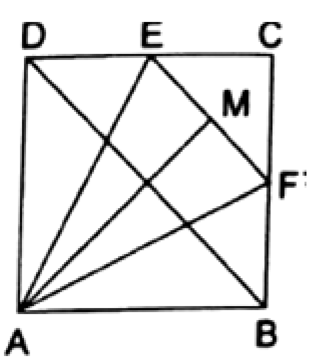
Hence ΔABD and ΔADC are congruent.

So ∠BAD = ∠DAC (congruency criteria)

Hence AD bisects ∠BAC.

33. Question

In the given figure, ABCD is a square and EF is parallel to diagonal DB and EM = FM. Prove that: (i) BF = DE (ii) AM bisects ∠BAD.



Answer

Since diagonal of square bisects the angles.

So, ∠CBD = ∠CDB = 45° [Also all angles of square are right angles i.e. half of all is 45°] (1)

Also similarly ∠ABD = ∠ADB = 45°

Since lines EF || BD

By corresponding angles-

∠CEF = ∠CDB = 45°

Also ∠CFE = ∠CBD = 45°

So, CE = CF {since sides opposite to equal angles are equal} …(i)

And CD = BC {sides of a square are equal} …(ii)

Subtracting I from II

CD-CE = BC-CF

So, BF = DE

Also let's consider ΔADX and ΔABX {where X is intersection point of AM and BD}

∠ABD = ∠ADB = 45°

AX is a common side.

AD = AB {sides of a square are equal}

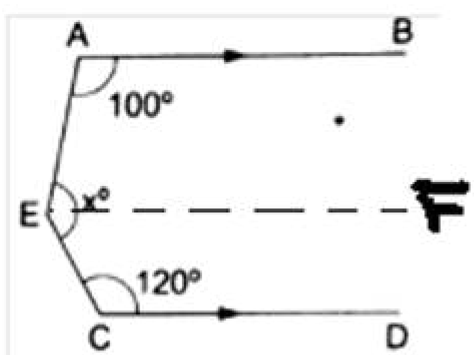
The triangles are congruent by SAS (side angle side) criteria.

So, ∠DAM = ∠MAB (congruency criteria)

Hence AM bisects ∠BAD.

34. Question

In the given figure, AB || CD If ∠BAE = 100° and ∠ECD = 120° then x = ?



Answer

Draw one line EF || CD and AB.

Since EF || CD and CE is transversal.

∠FEC + ∠ECD = 180°

∠FEC = 60° {since ∠ECD = 120°}

Also, EF || AB and AE is transversal.

∠FEA + ∠BAE = 180°

∠FEA = 80° {since ∠BAE = 100°}

And x = ∠FEC + ∠FEA

= 60° + 80°

= 140°

Sample Paper 2

1. Question

An irrational number between 2 and 2.5 is

A. √3

B. 2.3

C. √5

D. $2.\overline{34}$

Answer

Irrational numbers are numbers which cannot be expressed as simple fraction or simple ratios of two integers. That leaves us with just two options A and C. So, only √5 comes in between 2 and 2.5.

2. Question

Which of the following is a polynomial in one variable?

A. $x^2 + x^{-2}$

B. √3x + 9

C. $x^2 + 2x - \sqrt{x} + 3$

D. $\sqrt{3} + 2x - x^2$

Answer

A polynomial in one variable is an algebraic expression that consists of terms in the form of ax^n, where n is either zero or positive only. Given the options all expressions except D has the value of n as negative.

3. Question

Solve the equation and choose the correct answer $\frac{1}{\sqrt{18}-\sqrt{32}}=?$

A. √2 B. 1/√2

C. -√2 D. -1/√2

Answer

Given, $\frac{1}{\sqrt{18}-\sqrt{32}}$

Rationalising the above term,

$$\therefore \frac{1}{\sqrt{18}-\sqrt{32}} \times \frac{\sqrt{18}+\sqrt{32}}{\sqrt{18}+\sqrt{32}} = \frac{\sqrt{18}+\sqrt{32}}{\left(\sqrt{18}-\sqrt{32}\right)\left(\sqrt{18}+\sqrt{32}\right)}$$

Using the formula (a + b) (a – b) = $a^2 - b^2$ for the denominator,

$$\Rightarrow \frac{3\sqrt{2}+4\sqrt{2}}{18-32} = \frac{\sqrt{2}(3+4)}{-14}$$

$$\Rightarrow \frac{7\sqrt{2}}{-14} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$$

4. Question

If p(x) = $(x^4 - x^2 + x)$, then $p\left(\frac{1}{2}\right)=?$

A. 1/16

B. 3/16

C. 5/16

D. 7/16

Answer

Given, p(x) = $(x^4 - x^2 + x)$

Substituting the value of 1/2 in place of will give,

$$\Rightarrow p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^4-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)$$

$$\Rightarrow p\left(\frac{1}{2}\right)=\frac{1}{16}-\frac{1}{4}+\frac{1}{2}$$

$$\Rightarrow p\left(\frac{1}{2}\right)=\frac{1-4+8}{16}$$

$$\therefore p\left(\frac{1}{2}\right)=\frac{5}{16}$$

5. Question

If p(x) = $x^3 + x^2 + ax + 115$ is exactly divisible by (x + 5) then a = ?

A. 8

B. 6

C. 5

D. 3

Answer

Given, p(x) = $x^3 + x^2 + ax + 115$

($x^3 + x^2 + ax + 115$) is exactly divisible by (x + 5)

Hence, substituting x = -5 will give us the value of a

⇒ $(-5)^3 + (-5)^2 + a(-5) + 115 = 0$

⇒ -125 + 25 − 5a + 115 = 0

⇒ 5a = 15

∴ a = 3

6. Question

The equation of y-axis is

A. y = 0

B. x = 0

C. y = x

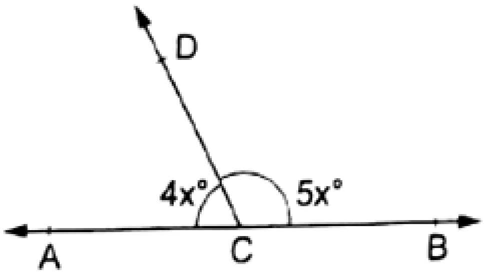
D. y = constant

Answer

We know that, the value of x is always zero on the y-axis.

7. Question

In the given figure, the value of x is



A. 10

B. 12

C. 15

D. 20

Answer

According to the figure,

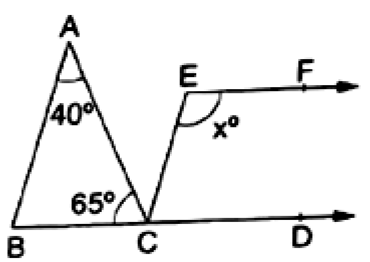
⇒ $4x + 5x = 180°$ [Angle on a straight line]

⇒ $9x = 180°$

∴ $x = 20°$

8. Question

In the given figure, CE || BA and EF || CD. If ∠BAC = 40°, ∠ACB = 65° and ∠CEF = x° then the value of x is



A. 40°

B. 65°

C. 75°

D. 105°

Answer

Given,

∠BAC = 40°

∠ACB = 65°

According to figure,

∴ ∠ACE = 40° [Alternate angles]

∴ ∠ACB + ∠ACE = x° [Alternate angles]

⇒ x° = 65° + 40°

∴ x = 105°

9. Question

Factorize: $\sqrt{2}x^2 + 3x + \sqrt{2}$

Answer

Given, $\sqrt{2}x^2 + 3x + \sqrt{2}$

By splitting the middle term,

⇒ $\sqrt{2}\,x^2 + 2x + x + \sqrt{2}$

⇒ $\sqrt{2}\,x(x + \sqrt{2}) + 1(x + \sqrt{2})$

∴ $(x + \sqrt{2})(\sqrt{2}\,x + 1)$

10. Question

Prove that √5 is an irrational number.

Answer

Let's assume that √5 is a rational number.

Hence, √5 can be written in the form a/b [where a and b (b ≠ 0) are co-prime (i.e. no common factor other than 1)]

∴ √5 = a/b

⇒ √5 b = a

Squaring both sides,

⇒ $(\sqrt{5}\,b)^2 = a^2$

⇒ $5b^2 = a^2$

⇒ $a^2/5 = b^2$

Hence, 5 divides a^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides a too

Hence, we can say a/5 = c where, c is some integer

So, a = 5c

Now we know that,

$5b^2 = a^2$

Putting a = 5c,

⇒ $5b^2 = (5c)^2$

⇒ $5b^2 = 25c^2$

⇒ $b^2 = 5c^2$

∴ $b^2/5 = c^2$

Hence, 5 divides b^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides b too

By earlier deductions, 5 divides both a and b

Hence, 5 is a factor of a and b

∴ a and b are not co-prime.

Hence, the assumption is wrong.

∴ By contradiction,

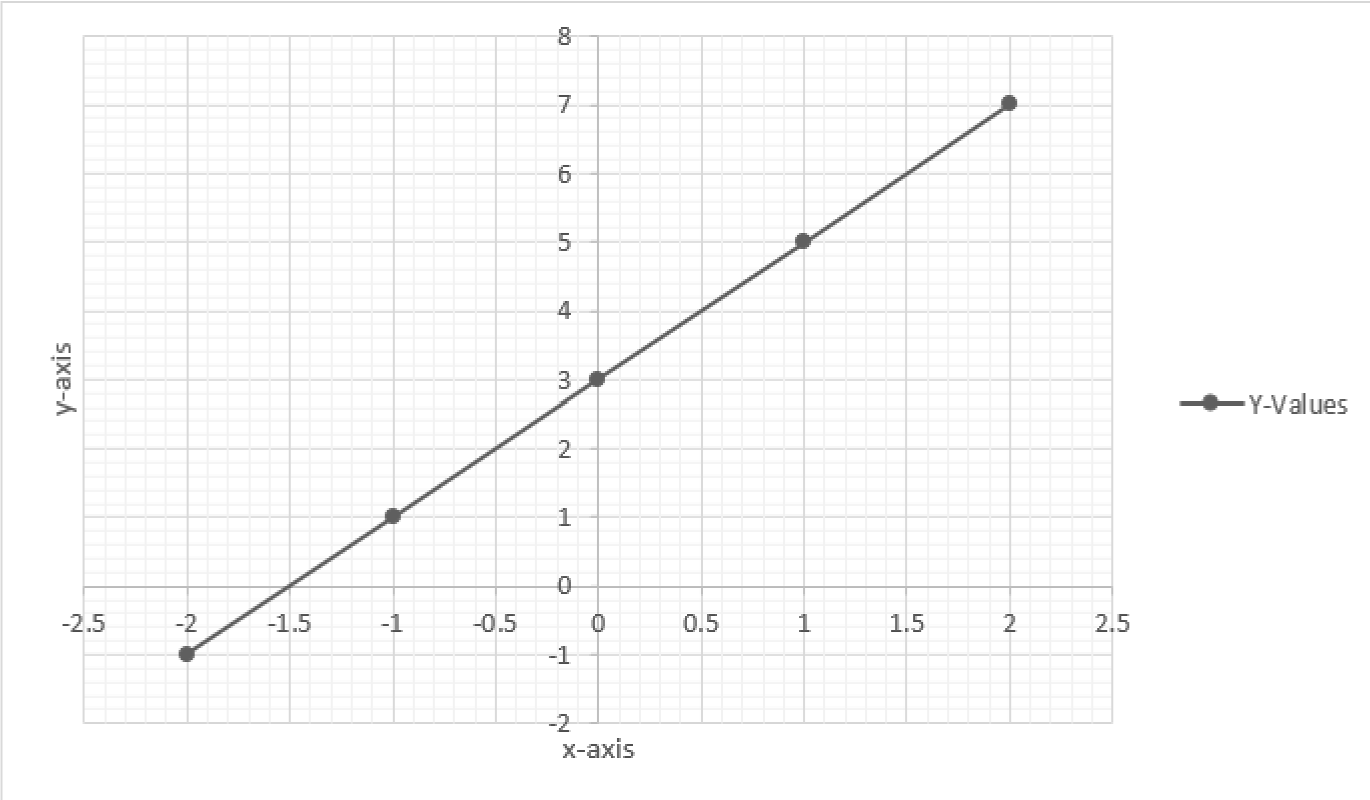
∴ √5 is irrational

11. Question

Draw the graph of the equation y = 2x + 3

x	-2	-1	0	1	2
y	-1	1	3	5	7

Answer



12. Question

If x = (3 + √8), find the value of $\left(x^2 + \frac{1}{x^2}\right)$.

Answer

Given, x = (3 + √8)

Let us calculate 1/x,

$$\Rightarrow \frac{1}{x} = \frac{1}{3+\sqrt{8}}$$

Rationalising the above term,

$$\Rightarrow \frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$$

Using the formula (a + b) (a – b) = (a^2 – b^2),

$$\Rightarrow \frac{1}{x} = \frac{3-\sqrt{8}}{9-8}$$

$$\therefore \frac{1}{x} = 3-\sqrt{8}$$

Now,

$$\left(x+\frac{1}{x}\right)=3+\sqrt{8}+3-\sqrt{8}$$

$$\therefore\left(x+\frac{1}{x}\right)=6$$

On squaring both sides, we get

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=6^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2=36$$

$$\therefore\left(x^2+\frac{1}{x^2}\right)=34$$

13. Question

Find the area of the triangle whose sides measure 52 cm, 56 cm and 60 cm respectively.

Answer

Given, three sides of a triangle 52 cm, 56 cm, 60cm

Area of a triangle is given by,

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where,

$s=\dfrac{a+b+c}{2}$ and a, b, c are the sides of the triangle

$$\Rightarrow s=\frac{52+56+60}{2}$$

$$\therefore s=\frac{168}{2}=84$$

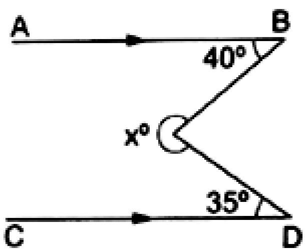
∴ Area of triangle = $\sqrt{84(84-52)(84-56)(84-60)}$

$$=\sqrt{84*32*28*24}$$

$$=\sqrt{1806336}=1344\ cm^2$$

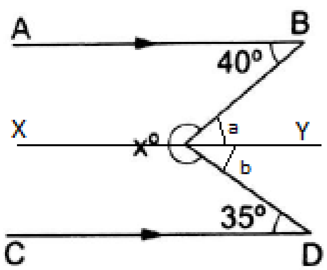
14. Question

In the given figure, AB || CD. Find the value of x.



Answer

Lets draw another line XY || AB and CD.



According to the figure,

⇒ ∠a = 40° [Alternate angles]

⇒ ∠b = 35° [Alternate angles]

∴ ∠x + ∠a + ∠b = 360° [Angle at a point = 360°]

∴ ∠x = 360° - 40° - 35° = 285°

15. Question

Find the values of a and b so that the polynomial (x^4 + ax^3 − $7x^2$ - 8x + b) is exactly divisible by (x + 2) as well as (x + 3).

Answer

Given, $x^4 + ax^3 - 7x^2 - 8x + b = 0$

∴ x = -2, -3 are a root of the above equation (∵ they are exactly divisible)

Substituting the value -2 and -3 in place of x will give,

⇒ $(-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b = 0$

⇒ 16 − 8a − 28 + 16 + b = 0

∴ 8a − b = 4 …. (i)

⇒ $(-3)^4 + a(-3)^3 - 7(-3)^2 - 8(-3) + b = 0$

⇒ 81 − 27a − 63 + 24 + b = 0

∴ 27a − b = 42 …. (ii)

Simultaneously solving eq(i) and eq(ii) we get,

∴ a = 2

∴ b = 12

16. Question

Using remainder theorem, find the remainder when $p(x) = x^3 - 3x^2 + 4x + 50$ is divided by (x + 3).

Answer

Given, $p(x) = x^3 - 3x^2 + 4x + 50$

Divisor, (x + 3)

∴ x = -3

Substituting -3 in place of x gives us,

⇒ $(-3)^3 - 3(-3)^2 + 4(-3) + 50$

= -27 − 27 − 12 + 50 = -16

17. Question

Factorize: $(2x^3 + 54)$

Answer

Given, $(2x^3 + 54)$

Taking common terms out,

⇒ $2 (x^3 + 27)$

Using the formula, $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$

⇒ $2 (x + 3) (x^2 - 3x + 3^2)$

∴ $2 (x + 3) (x^2 - 3x + 9)$

18. Question

Find the product $(a - b - c) (a^2 + b^2 + c^2 + ab + ac - bc)$

Answer

Given, $(a - b - c) (a2 + b^2 + c^2 + ab + ac - bc)$

$= a^3 + ab^2 + ac^2 + a^2b + a^2c - abc - a^2b - b^3 - bc^2 - ab^2 - abc + b^2c - a^2c - b^2c - c^3 - abc - ac^2 - bc^2$

Cancelling the terms with opposite signs,

$= a^3 - b^3 - c^3 - 3 abc$

19. Question

In a ΔABC, if ∠A - ∠B = 33° and ∠B - ∠C = 18°, find the measure of each angle of the triangle.

Answer

Let the three angles of a triangle be ∠A, ∠B, ∠C

Given, ∠A - ∠B = 33°

⇒ ∠A = ∠B + 33°

∠B - ∠C = 18°

⇒ ∠C = ∠B – 18°

Now,

∠A + ∠B + ∠C = 180° [Sum of all angles of a triangle = 180°]

⇒ ∠B + 33° + ∠B + ∠B – 18° = 180°

⇒ 3∠B = 180° - 15°

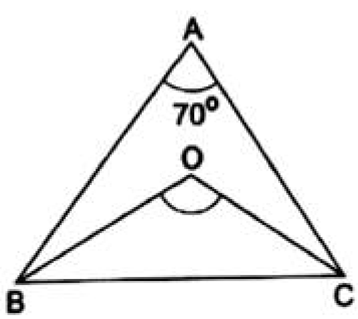
∴ ∠B = 55°

∴ ∠A = ∠B + 33° = 88°

∴ ∠C = ∠B – 18° = 37°

20. Question

In the given figure, in ΔABC, the angle bisectors of ∠B and ∠C meet at a point O. Find the measure of ∠BOC.



Answer

Given, ∠A = 70°

Let the two angles ∠B = 2x and ∠C = 2y.

Then, angle bisector of B, ∠OBC = x and angle bisector of C, ∠OCB = y

∴ ∠A + ∠B + ∠C = 180° [Sum of all angles of a triangle = 180°]

⇒ 70° + 2x + 2y = 180°

⇒ 2x + 2y = 110°

∴ x + y = 55° …. (i)

Now,

∠BOC + x + y = 180° [Sum of all angles of a triangle = 180°]

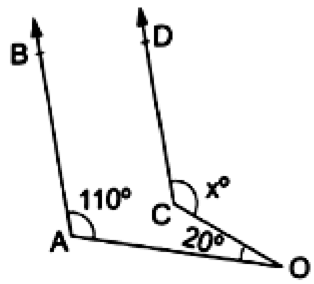
⇒ ∠BOC = 180° - (x + y)

⇒ ∠BOC = 180° - 55° [from eq. (i)]

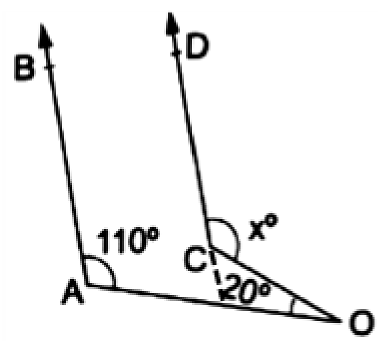
∴ ∠BOC = 125°

21. Question

In the given figure, AB || CD. If ∠BAO = 110°, ∠AOC = 20° and ∠OCD = x°, find the value of x.



Answer



Given, ∠BAO = 110°, ∠AOC = 20°

∠CEO = 110° [Corresponding angles]

∴ x° = 110° + 20° [Exterior angle = Sum of two opposite interior angles]

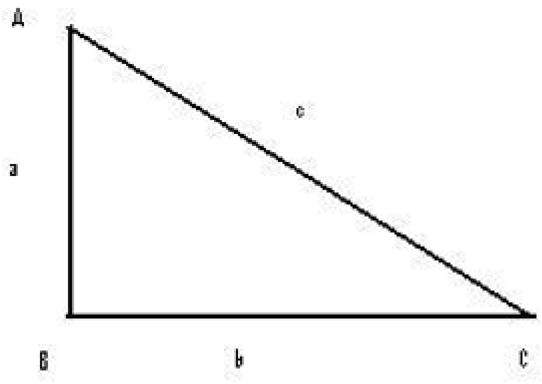
∴ x° = 130°

22. Question

In a right-angled triangle, prove that the hypotenuse is the longest side.

Answer

Given, ΔABC is a right- angled triangle at B i.e. ∠B = 90°



To prove AC is the longest side of ΔABC

Proof:

In ΔABC,

∠A + ∠B + ∠C = 180° [Sum of all angles of a triangle = 180°]

∠A + 90° + ∠C = 180° [Given ∠B = 90°]

∠A + ∠C = 180° - 90°

∴ ∠A + ∠C = 90°

Hence, ∠A < 90°

∠A < ∠B

BC < AC [Side opposite to a larger angle is longer]

Similarly,

∠C < 90°

∠C < ∠B

AB < AC [Side opposite to a larger angle is longer]

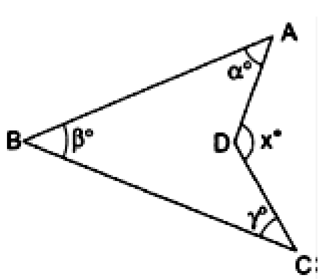
Hence,

∴ AC is the longest side of ΔABC i.e. the hypotenuse.

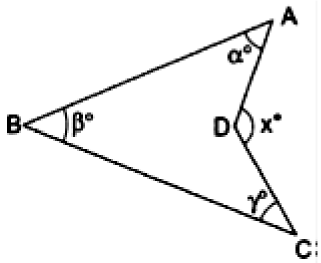
23. Question

In the given figure, prove that:

$x = \alpha + \beta + \gamma$



Answer



In ΔABC,

∠A + ∠B + ∠C = 180° [Sum of all angles of a triangle = 180°]

According to the figure,

⇒ ∠B + (α + ∠DAC) + (γ + ∠DCA) = 180°

⇒ ∠DAC + ∠DCA + α + β + γ = 180°

⇒ ∠DAC + ∠DCA = 180° - (α + β + γ) …. (i)

In ΔADC,

⇒ x + ∠DAC + ∠DCA = 180° [Sum of all angles of a triangle = 180°]

⇒ x = 180° - ∠DAC - ∠DCA

⇒ x = 180° - 180° + (α + β + γ)

∴ x = (α + β + γ)

Hence proved.

24. Question

Find six rational numbers between 3 and 4.

Answer

Since, we want six numbers, we write 1 and 2 as rational numbers with denominator 6 + 1 = 7

So, multiply in numerator and denominator by 7, we get

$3=\frac{3\times7}{1\times7}=\frac{21}{7}$ and $4=\frac{4\times7}{1\times7}=\frac{28}{7}$

We know that, 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28

$$\frac{21}{7}<\frac{22}{7}<\frac{23}{7}<\frac{24}{7}<\frac{25}{7}<\frac{26}{7}<\frac{27}{7}<\frac{28}{7}$$

Hence, six rational numbers between $3=\frac{21}{7}\text{ and }4=\frac{28}{7}$ are

$$\frac{22}{7},\frac{23}{7},\frac{24}{7},\frac{25}{7},\frac{26}{7},\frac{27}{7}$$

25. Question

If $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + \sqrt{15}b$, find the values of a and b.

OR

Factorize: $(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3$

Answer

Given, $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Rationalising the above term,

$$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} * \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Using the formula (a + b) (a - b) = $(a^2 - b^2)$

$$\Rightarrow \frac{5+3+2\sqrt{15}}{5-3} = \frac{8+2\sqrt{15}}{2}$$

∴ $4 + \sqrt{15}$

Comparing with $a + \sqrt{15}\,b$,

∴ a = 4, b = 1

OR

Solution: Given, $(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3$

Using the formula, $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$

$\Rightarrow a^3 + b^3 + c^3 = (a + b + c)^3 - 3(a + b)(b + c)(c + a)$

$\Rightarrow (5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = (5a - 7b + 9c - 5a + 7b - 9c)^3 - 3(5a - 7b + 9c - 5a)(9c - 5a + 7b - 9c)(7b - 9c + 5a - 7b)$

$\Rightarrow (5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = 0^3 - 3(-7b + 9c)(-5a + 7b)(-9c + 5a)$

∴ $(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = 3(5a - 7b)(7b - 9c)(9c - 5a)$

26. Question

Factorize:

$12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$

Answer

Given, $12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$

By splitting the middle term i.e. $8(x^2 + 7x)(2x - 1)$, we get

$= 12(x^2 + 7x)^2 - 18(x^2 + 7x)(2x - 1) + 10(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$

$= 6(x^2 + 7x)[2(x^2 + 7x) - 3(2x - 1)] + 5(2x - 1)[2(x^2 + 7x) - 3(2x - 1)]$

$= [2(x^2 + 7x) - 3(2x - 1)][6(x^2 + 7x) + 5(2x - 1)]$

$= (2x^2 + 14x - 6x + 3)(6x^2 + 42x + 10x - 5)$

$= (2x^2 + 8x + 3)(6x^2 + 52x - 5)$

27. Question

If $(x^3 + ax^2 + bx + 6)$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $(x - 3)$, find the values of a and b.

Answer

Given, $(x^3 + ax^2 + bx + 6)$ exactly divisible by $(x - 2)$

$\therefore x = 2$ is a root of the above equation.

$\Rightarrow 2^3 + a(2)^2 + b(2) + 6 = 0$

$\Rightarrow 8 + 4a + 2b + 6 = 0$

$\therefore 4a + 2b = -14$ $b = \frac{-14-4a}{2} \quad \ldots\ldots (i)$

Given, $(x^3 + ax^2 + bx + 6)$ divided by $(x - 3)$ leaves a remainder 3

$\therefore 3^3 + a(3)^2 + b(3) + 6 = 3$

$\Rightarrow 27 + 9a + 3b + 6 = 3$

$\therefore 9a + 3b = -30$ …. (ii)

Put value of b from (i) in this equation to get, $9a + 3\left(\frac{-14-4a}{2}\right) = -30$ 18a - 42 - 12

a = - 606a - 42 = -606a = -60 + 426a = -18a = -3Put the value of a in (i) to get:

$b = \frac{-14-4(-3)}{2} \quad b = \frac{-14+12}{2} \quad b = \frac{-2}{2}$

Solving simultaneously eq (i) and eq (ii), we get

a = -3, b = -1

28. Question

Without actual division, show that $(x^3 - 3x^2 - 13x + 15)$ is exactly divisible by $(x^2 + 2x - 3)$.

Answer

Let's find the roots of the equation $(x^2 + 2x - 3)$

$\Rightarrow x^2 + 3x - x - 3 = 0$

$\Rightarrow x(x + 3) - 1(x + 3) = 0$

$\therefore (x + 3)(x - 1)$

Hence, if $(x + 3)$ and $(x - 1)$ satisfies the equation $x^3 - 3x^2 - 13x + 15 = 0$, then $(x^3 - 3x^2 - 13x + 15)$ will be exactly divisible by $(x^2 + 2x - 3)$.

For x = -3,

$\Rightarrow (-3)^3 - 3(-3)^2 - 13(-3) + 15$

$\Rightarrow -27 - 27 + 39 + 15 = 0$

For x = 1,

$\Rightarrow 1^3 - 3(1)^2 - 13(1) + 15$

$\Rightarrow 1 - 3 - 13 + 15 = 0$

Hence proved.

29. Question

Factorize: $a^3 - b^3 + 1 + 3ab$

Answer

Given, $a^3 - b^3 + 1 + 3ab$

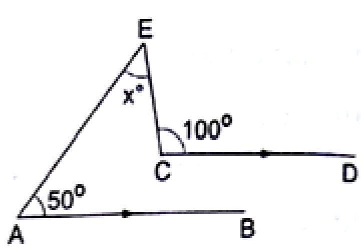
$\Rightarrow a^3 + (-b)^3 + 1^3 - 3(1 * a * (-b))$

$\Rightarrow [a + (-b) + 1] [a^2 + (-b)^2 + 1^2 - a(-b) - (-b)1 - 1a]$

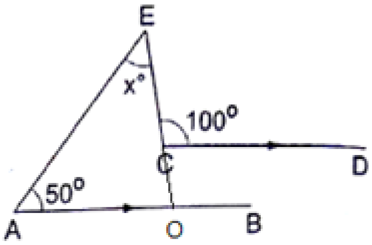
$\therefore (a - b + 1)(a^2 + b^2 + 1 + ab + b - a)$

30. Question

In the given figure, AB || CD, ∠ECD = 100°, ∠EAB = 50° and ∠AEC = x°. Find the value of x.



Answer



Given, ∠ECD = 100°, ∠EAB = 50°

∠COB = 100°

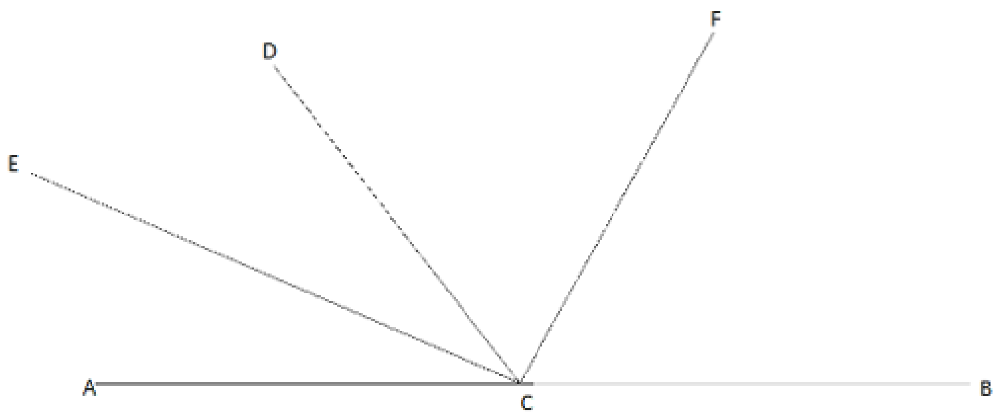
∴ x = 100° - 50° [Exterior angle = Sum of two opposite interior angles of a triangle]

∴ x = 50°

31. Question

Prove that the bisectors of the angles of a linear pair are at right angles.

Answer



Given, ∠ACD and ∠BCD are linear pairs

CE and CF bisect ∠ACD and ∠BCD respectively

To prove:

∠ECF = 90°

∴ ∠ACD + ∠BCD = 180° [Angle on a straight line]

⇒ ∠ACD/2 + ∠BCD/2 = 180°/2 = 90°

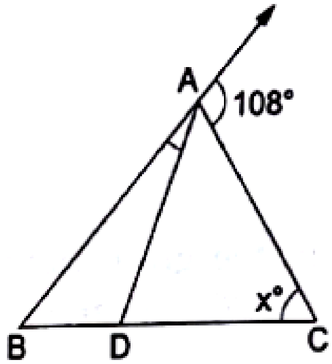
⇒ ∠ECD + ∠DCF = 90° [∵ CE and CF bisect ∠ACD and ∠BCD respectively]

∴ ∠ECD + ∠DCF = ∠ECF = 90°

Hence Proved.

32. Question

In the given figure, AD bisects ∠BAC in the ratio 1: 3 and AD = DB. Determine the value of x.



Answer

Let the ratio be y

∴ ∠DAB = y

∴ ∠DAC = 3y

∴ y + 3y + 108° = 180° [Angle on a straight line]

⇒ 4y = 72°

∴ y = 18°

∴ ∠DAC = 3y = 54°

∠ABD = 18° [∵ AD = DB, ΔABD is an isosceles triangle]

In ΔABC,

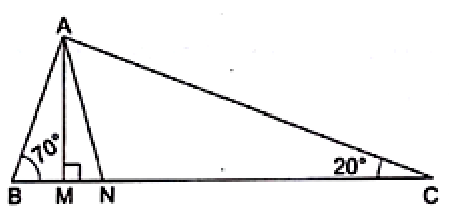
⇒ x + ∠A + ∠B = 180° [Sum of all angles of a triangle = 180°]

⇒ x = 180° - 72° - 18°

∴ x = 90°

33. Question

In the given figure, AM ⊥ BC and AN is the bisector of ∠A. If ∠ABC = 70° and ∠ACB = 20°, find ∠MAN.



Answer

In ΔABC,

∠A = 180° - 70° - 20° [Sum of all angles of a triangle = 180°]

∴ ∠A = 90°

∴ ∠BAN = 45° [∵ AN is the bisector of ∠A]

In ΔABN,

∠N = 180° - 70° - 45° [Sum of all angles of a triangle = 180°]

∴ ∠N = 65°

In ΔAMN,

∠MAN = 180° - 90° - 65° [Sum of all angles of a triangle = 180°]

∴ ∠MAN = 25°

34. Question

If the bisector of the vertical angle of a triangle bisects the base, prove that the triangle is isosceles.

Answer

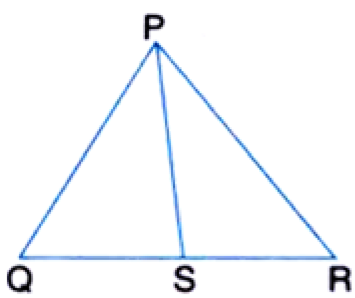
Given,

In ΔPQR,

PS bisects ∠QPR and QS = SR

To prove:

PQ = PR



In ΔPQS and ΔPRS

QS = SR [Given]

∠QPS = ∠RPS [Given]

PS = PS [Common]

ΔPQS is congruent to ΔPRS [S.A.S]

∴ PQ = PR [C.P.C.T.C]

Hence Proved.