Chapter 9

Rational numbers

Introduction to Rational Numbers

Let us all recall various types of numbers that we have learned so far, starting with natural numbers.

Natural Numbers

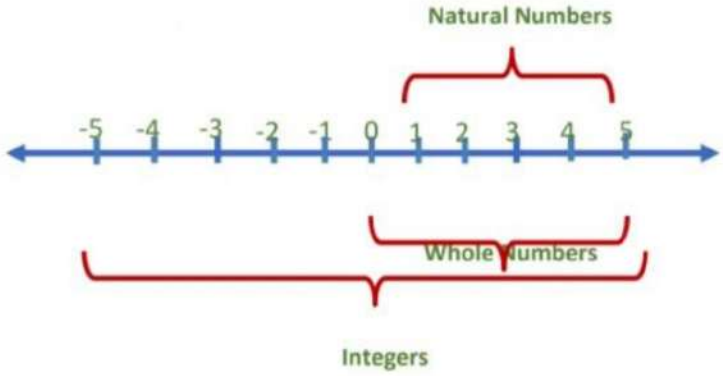
Counting numbers starting from 1 are known as natural numbers.
i.e., {1, 2, 3, 4....}

Whole numbers

The natural numbers together with 0 are called whole numbers.
i.e., {0, 1, 2, 3, 4, 5.....}

Integers

The whole numbers and negative of whole numbers together are called Integers.
i.e., {...-4, -3, -2, -1, 0, 1, 2, 3, 4...}



Need for Rational Numbers

We now know that, if we have to represent any loss of 100 it can be represented as -100, which will be done with the help of integers or profit

of 100 will be represented as 100, which will be done with the help of whole numbers.

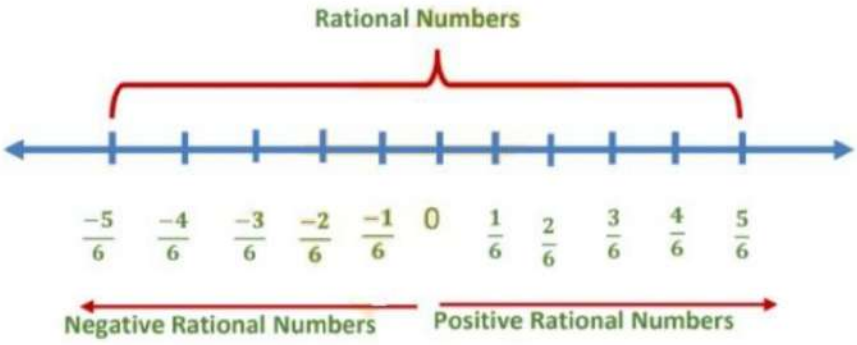
But what if we want to represent 750 m above sea level in kilometers it will be $\frac{3}{4}$ km which is a fractional value and if we want to represent 750 m below sea level in kilometres it will be $-\frac{3}{4}$ km.

So $-\frac{3}{4}$ is neither an integer nor fractional value here the rational number comes into play.

What are Rational Numbers?

A number which can be written in the form $\frac{p}{q}$, where p and q are integers, and $q \neq 0$ is called a rational number.

For example: $\frac{-2}{3}, \frac{6}{7}$



In $\frac{p}{q}$, integer p is called as the numerator and non zero integer q is called as the denominator.

Equivalent Rational Numbers

So, if we multiply or divide numerator and denominator of a rational number by same non-zero integer, then we will get another equivalent rational number.

Thus, a rational number can be written in several equivalent forms.

The general form for Equivalent rational number can be written as:

If $\frac{p}{q}$ is any rational number and $\frac{r}{s}$ be its equivalent rational number then,

$$\frac{p}{q} = \frac{r}{s}$$

such that ps = rq where p, q, r, and s are integers such that q and s are non-zero integers.
For example,

Find any 3 equivalent rational number of $\frac{1}{2}$.

Sol.
If we multiply both the numerator and denominator with the same non zero number, we will get its equivalent rational number. If we multiply given rational number by:

- 2

$$\frac{1}{2} = \frac{1X2}{2X2} = \frac{2}{4}$$

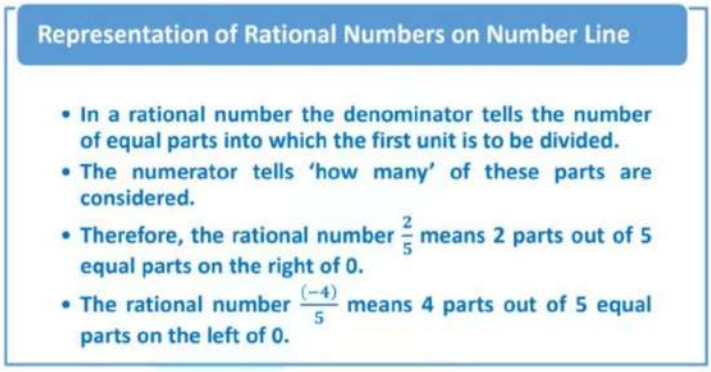
- 3

$$\frac{1}{2} = \frac{1X3}{2X3} = \frac{3}{6}$$

- 4

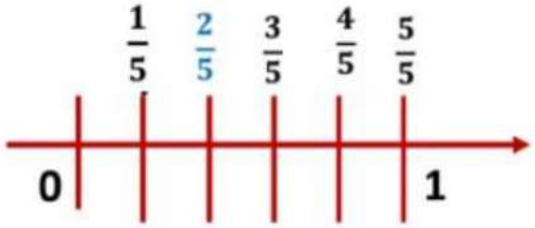
$$\frac{1}{2} = \frac{1X4}{2X4} = \frac{4}{8}$$

Rational Numbers on a Line



Represent $\frac{2}{5}$ on the number line.

i) Draw a number line and mark O on it to represent zero.
ii) As the denominator of rational number is 5.the number line is divided into 5 equal parts.
iii) The numerator is 2 and so we will consider 2 parts out of 5 equal parts.



Comparison of Rational Number

We know how to compare two fractional values and positive rational numbers are nothing but fractions

For comparing negative rational numbers, we will use the way we used to check for integers, as the number on the right is always greater than the number on the left side of the number line

To compare rational numbers, we can use both of these.

Let us understand this more with the help of an example:

Example: Which one of the following is greater?

i), $\frac{1}{2}$

ii) $-\frac{3}{10}, \frac{1}{2}$

Sol.

i) Consider, $-\frac{3}{10}$ and $-\frac{1}{2}$ to compare these we will first make the denominator of both the given rational numbers as equal.

So, to do this we will multiply both numerator and denominator of $-\frac{1}{2}$ by 5.

So,

$$\frac{1X5}{2X5} = \frac{5}{10}$$

So, $\frac{3}{10} < \frac{5}{10}$

ii) Consider, $-\frac{3}{10}$ and $-\frac{1}{2}$

to compare these we will first make the denominator of both the given rational numbers as equal.

So, to do this we will multiply both numerator and denominator of $-\frac{1}{2}$ by 5.

So,

$$-\frac{1X5}{2X5} = -\frac{5}{10}$$

As the number on the right is always greater than the number on the left side of the number line

$$-\frac{3}{10} > -\frac{5}{10}$$

Rational Number Between Two Rational Numbers

We know about different kinds of numbers. Let us understand the numbers that lie between two given numbers.

Let's take two natural numbers between 2 and 5, we can tell that 3 and 4 are natural numbers that lie between 2 and 5, and similarly if asked about integers that lie between -2 and 2 we can have -1, 0 and 1 as our answer.

But what about rational numbers,

If we are asked about rational numbers between $\frac{2}{5}$ and $\frac{4}{5}$. You might think that only $\frac{3}{5}$ will lie between them.

But $\frac{2}{5} = \frac{4}{10}$ and $\frac{4}{5} = \frac{8}{10}$

So $\frac{5}{10}$, $\frac{6}{10}$ and $\frac{7}{10}$ will also lie.

Moreover, $\frac{2}{5} = \frac{40}{100}$ and $\frac{4}{5} = \frac{80}{100}$

Now the numbers will be $\frac{41}{100}, \frac{42}{100}$, ..., $\frac{78}{100}, \frac{79}{100}$

.

So in between two rational numbers, there can be infinitely many numbers possible.

Let us see how to find such numbers: -

Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$

Method 1

We will first find the mean of the two rational numbers.

$\frac{1}{4}$ and $\frac{1}{2}$

$\left(\frac{1}{4}+\frac{1}{2}\right) \div 2$

$\left(\frac{2+4}{8}\right) \div 2$

$\frac{6}{8} \times \frac{1}{2} = \frac{3}{8}$

Method 2

We will first convert the rational numbers $\frac{1}{4}$ and $\frac{1}{2}$ to the rational number with the same denominator.

$\frac{1}{4}$ and $\frac{1}{2}$

$\frac{1\times2}{4\times2}$ and $\frac{1\times4}{2\times4}$

$\frac{2}{8}$ and $\frac{4}{8}$

$\frac{2}{8}, \frac{3}{8}, \frac{4}{8}$

Hence, the required rational number is $\frac{3}{8}$.

Operations on Rational Numbers

Let us see various operations that we can perform on rational numbers.

Starting with addition,

• Addition

Let us add two numbers with the same denominator.

$\frac{1}{2} + \frac{3}{2}$

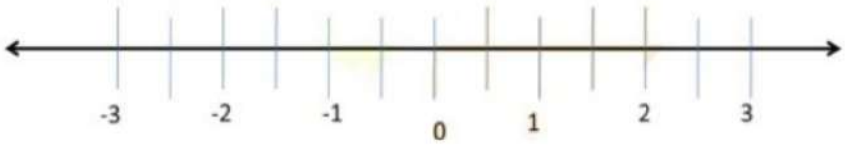
Let us have a look at the first question

We will solve it in two ways

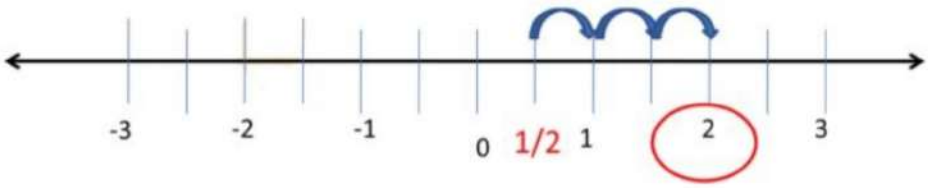
i) By number line

ii) By solving numerical values

Now let us draw a number line such that each gap represents $\frac{1}{2}$.



Now, we mark $\frac{1}{2}$ and to add $\frac{3}{2}$ as each step is of $\frac{1}{2}$ we will move 3 steps towards the right as we have to add both the numbers.



$\frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

• Subtraction

Let us subtract two numbers with the same denominator.

$\frac{5}{2} - \frac{3}{2}$

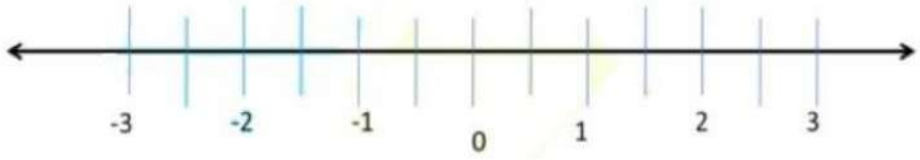
Let us have a look at the first question

We will solve it in two ways

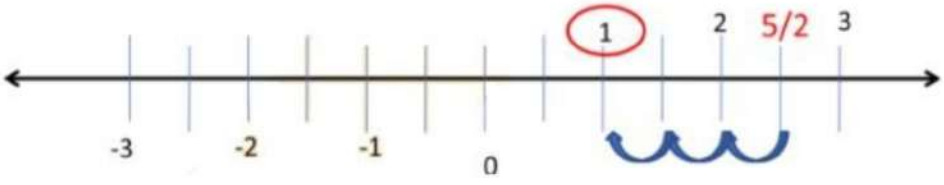
i) By number line

ii) By solving numerical values

Now let us draw a number line such that each gap represents $\frac{1}{2}$



Now, we mark $\frac{5}{2}$ and $\frac{3}{2}$ to subtract as each step is of $\frac{1}{2}$ we will move 3 steps towards left as we have to subtract both the numbers.



$\frac{5}{2} - \frac{3}{2} = 1$

So, our answer is 1.

Now let us have a look at another method

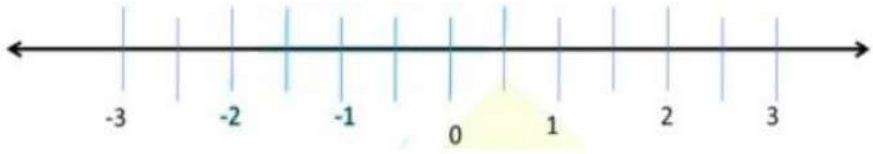
$\frac{5}{2} - \frac{3}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$

• Multiplication

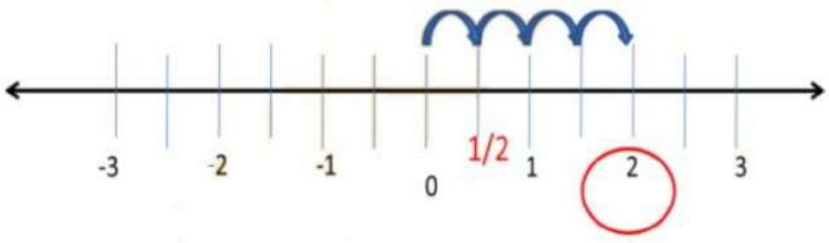
Let us multiply two numbers.
$\frac{1}{2} \times 4$
Let us have a look at the first question

We will solve it by two ways
i) By number line
ii) By solving numerical values

Now let us draw a number line such that each gap represents $\frac{1}{2}$.



Now, we mark $\frac{1}{2}$ and to multiply it by 4 we will move 4 steps towards the right of the gap $\frac{1}{2}$ as we have to multiply it by 4.



Now let us have a look at another method
$\frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$

• Division

Let us divide two numbers.

$$4 \div \frac{1}{2}$$

Let us have a look at the first question

We will do it in the same way as we used to do in a fractional way.

$$4 \div \frac{1}{2} = 4 \times 2 = 8$$