27. Specific Heat Capacities of Gases

Short Answer

1. Question

Does gas have just two specific heat capacities or more than two? Is the number of specific heat capacities of a gas countable?

Answer

No, the number of specific heat capacities of gas is infinite as it depends on the thermodynamic process followed by the gas.

The specific heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of that substance by 1 degree Celsius, or 1 Kelvin. It is denoted by C.

Gases are compressible substances. They have two well-known specific heat capacities: one at constant pressure(C_p)(isobaric process - constant pressure) and another at constant volume(C_v)(isochoric process - constant volume). However, gases can have many specific heat capacities depending on the other thermodynamic processes they follow, like adiabatic process, isothermal process, etc.

2. Question

Can we define specific heat capacity at constant temperature?

Answer

The specific heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of that substance by 1 degree Celsius, or 1 Kelvin. It is denoted by C.

Hence, $C = \frac{Q}{m \times dT}$, ... (i)

where

C = specific heat capacity

Q = heat required to raise the temperature by dT

m = molar mass

dT = change in temperature.

For constant temperature, dT = 0. Putting this value in (i), we get

$$C = \frac{Q}{m \times dT} = \infty$$

Hence, for a process at a constant temperature, the specific heat capacity is infinite.

3. Question

Can we define specific heat capacity for an adiabatic process?

Answer

The specific heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of that substance by 1 degree Celsius, or 1 Kelvin. It is denoted by C.

Hence, $C = \frac{Q}{m\,XdT}$, ... (i) where

C = specific heat capacity

Q = heat required to raise the temperature by dT

m = molar mass

dT = change in temperature.

For an adiabatic process, Q = 0. Substituting this value in (i), we get

C = 0.

Hence, for a process at a constant temperature, the specific heat capacity is zero.

4. Question

Does a solid also have two kinds of molar heat capacities C_P and C_V? If yes, do we have $C_P > C_V$? Is $C_P - C_V = R$?

Answer

Yes, a solid also has two kinds of molar heat capacities, C_p(at constant pressure) and C_v(at constant volume).

Solids are almost incompressible.

Hence, the values of C_p and C_v are such that $C_p > C_v$, but they are almost equal since the dependence on heat capacities is very less in the case of solids.

Since the values of C_p and C_v are not that different, C_p - C_v is much less than R.

5. Question

In a real gas the internal energy depends on temperature and also on volume. The energy increases when the gas expands isothermally. Looking into the derivation of

$C_P - C_V = R$, find whether $C_P - C_v$ will be more than R, less than R or equal to R for a real gas.

Answer

We know that, for an ideal gas,

$C_P - C_V = R$ … (i)

where

C_p = specific heat constant at constant pressure

C_v = specific heat constant at constant volume

R = universal gas constant

Multiplying by n x dT on both sides of (i), we get

$nC_PdT - nC_vdT = nRdT$

which gives

$(dQ)_P - (dQ)_v = nRdT$ …(ii)

Since $(dQ)_p = nC_pdT$ and $(dQ)_v = nC_vdT$ … (iii)

Where

n = number of moles

dT = change in temperature

$(dQ)_p$ = change in heat at constant pressure

$(dQ)_v$ = change in heat at constant volume

However, for a real gas, the internal energy depends on the temperature as well as the volume.

Hence, there will be an additional term on the right-hand side of (ii) which will indicate the change in the internal energy of the gas with volume at constant pressure. Let this term be u.

Hence, for a real gas, (ii) becomes :

$(dQ)_p - (dQ)_v = nRdT + u$ … (iv)

Again, dividing on both sides by ndT, we get

$C_p - C_v = R + \frac{u}{ndT}$ … (v),

which is greater than R.

Here,

C_p = specific heat constant at constant pressure

C_v = specific heat constant at constant volume

R = universal gas constant

n = number of moles

dT = change in temperature

Hence, from (v), we get $C_p - C_v > R$.

We conclude that for a real gas, $C_p - C_v > R$.

6. Question

Can a process on an ideal gas be both adiabatic and isothermal?

Answer

According to the first law of thermodynamics,

$dQ = dU + dW = nC_v dT + dW$ … (i),

where

dQ = heat supplied

dU = change in internal energy

dW = work done on the gas

n= number of moles

C_v = specific heat capacity at constant volume

dT = change in temperature

For an adiabatic process, dQ(heat supplied) = 0.

An adiabatic process occurs without the transfer of heat or mass of substances between the thermodynamic system and the surrounding.

For an isothermal process, dT(change in temperature) = 0

An Isothermal process is a change of system, in which the temperature remains constant $\Delta T=0$.

Putting these values in (i), we get

dW = 0,

which is not possible for either of the processes.

dW = 0 only in the case of a process where the volume is constant that is dV = 0,

since dW = PdV,

where P = pressure and dV = change in volume.

Hence, we conclude that a process cannot be both adiabatic and isothermal.

7. Question

Show that the slope of p –V diagram is greater for an adiabatic process as compared to an isothermal process.

Answer

For an isothermal process, the ideal gas equation is given as

PV = constant … (i),

Where

P = pressure

V = volume.

Differentiating on both sides of (i), we get

PdV + VdP = 0

On solving for $\frac{dp}{dv}$, we get

$\frac{dP}{dV} = -\frac{P}{V}$ … (ii)

For a graph of P versus V, dP/dV indicates the slope.

Hence, for an isothermal process, the slope of the p-V diagram is given by -P/V.

Now for an adiabatic process, the ideal gas equation is

PV^{γ}= constant … (iii),

where

P = pressure,

V = volume,

γ = ratio of specific heat capacities at constant pressure and constant volume.

Differentiating both sides of (ii), we get

$V^{\gamma}dP + \gamma V^{\gamma-1}PdV = 0$ which gives

$\frac{dP}{dV} = -\frac{\gamma P}{V}$... (iv)

Hence, for an adiabatic process, the slope of the p-V diagram is given by -γP/V.

Since $\gamma > 1$, we find that γP/V is greater than P/V, which concludes that slope of p-V diagram of an adiabatic process is steeper than that of an isothermal process(proved).

8. Question

Is a slow process always isothermal? Is a quick process always adiabatic?

Answer

An isothermal process is represented by the equation

PV = constant … (i),

where

P = pressure

V = volume.

To keep this product constant, a small change in V will only produce a small change in P and vice versa. Hence, an isothermal process is usually a slow process.

On the other hand, an adiabatic process is represented as

PV^{γ} = constant … (ii), where

P = pressure

V = volume

The γ= ratio of specific heat capacities at constant pressure to constant volume.

Now, $\gamma > 1$. Hence, the term V^{γ} will increase exponentially. Hence, to keep the product constant, a small change in V will cause a large change in P. Hence, an adiabatic process is usually a fast process.

9. Question

Can two states of an ideal gas be connected by an isothermal process as well as an adiabatic process?

Answer

An isothermal process is represented by

PV = constant … (i),

where P = pressure, V = volume.

This gives

$P_1 V_1 = P_2 V_2$ (ii),

Where

P_1, V_1 = initial values of pressure and volume

P_2, V_2 = final values of pressure and volume.

An adiabatic process is represented by

PV^γ = constant… (iii),

where

P = pressure,

V = volume,

γ = ratio of specific heat capacities at constant pressure and constant volume.

From (iii), we get

$P_1V_1^\gamma = P_2V_2^\gamma$… (iv),

Where P_1, V_1 = initial values of pressure and volume

P_2, V_2 = final values of pressure and volume.

Dividing (iv) by (ii), we get

$V_1^{\gamma-1} = V_2^{\gamma-1}$ … (v)

If the condition given by (v) is satisfied, then two states of an ideal gas can be connected by both an isothermal and an adiabatic process.

10. Question

The ratio C_P/C_V for gas is 1.29. What is the degree of freedom of the molecules of this gas?

Answer

Given:

$\gamma = \frac{C_P}{C_v}$ = 1.29

Formula used:

$\gamma = 1 + \frac{2}{f}$ … (i),

where

γ = ratio of molar heat capacities at constant pressure to constant volume

f = number of degrees of freedom

(i) becomes :

$$\gamma - 1 = \frac{2}{f}$$

$=> f = \frac{2}{\gamma - 1}$... (ii)

Substituting γ = 1.29 in (ii), we get

$f = \frac{2}{(1.29-1)}$ = 6.89 which is approximately equal to 7.

Thus, the number of degrees of freedom is approximately equal to 7.

Objective I

1. Question

Work done by a sample of an ideal gas in a process A is double the work done in another process B. The temperature rises through the same amount in the two processes. If C_A and C_B be the molar heat capacities for the two processes.

A. $C_A = C_B$

B. $C_A < C_B$

C. $C_A > C_B$

D. C_A and C_B cannot be defined

Answer

Q = nCdT ... (i),

where Q = work done by an ideal gas

n = number of moles

C = molar heat capacity

dT = rise in temperature.

For process A, let the value of C be C_A and for B, let it be C_B.

Since the work done by the gas in process A is twice that in B, and the rise in temperature is the same in both the cases, we get two equations:

$2Q = nC_A dT$ … (ii) and

$Q = nC_B dT$ … (iii),

Where

Q = work done in process B

n = number of moles of gas

dT = rise in temperature

Dividing (ii) by (iii), we get

$$2 = \frac{C_A}{C_B}$$

=> $C_A = 2C_B$

This proves that $C_A > C_B$.

2. Question

For a solid with a small expansion coefficient,

A. $C_P - C_V = R$

B. $C_P = C_V$

C. C_P is slightly greater than C_V

D. C_P is slightly less than C_V

Answer

For solids which have a small expansion coefficient, the work done on the solid is pretty small. Hence, the specific heat at constant pressure and at a constant volume only slightly different since the work done depends very little on the process. Hence, C_p is slightly greater than C_v but much less than R as in the case of ideal gases.

3. Question

The value of $C_P - C_V$ is 1.00 R for a gas sample in state A and is 1.08R in state B. Let P_A, P_B denote the pressures and T_A, and T_B denote the temperatures of the states A and B respectively. Most likely

A. $p_A < p_B$ and $T_A > T_B$

B. $p_A > p_B$ and $T_A < T_B$

C. $p_A = p_B$ and $T_A < T_B$

D. $p_A > p_B$ and $T_A = T_B$

Answer

For gas in state A, $C_p - C_v = R$. This represents an ideal gas. Now, for an ideal gas, we require very high temperature and very low pressure compared to a real gas

For gas in state B, $C_p - C_v = 1.08R$, which represents a real gas. Since gas A was ideal, its pressure must be much lower than that of B and temperature must be much higher than that of A.

Hence, we require the condition $p_A < p_B$ and $T_A > T_B$. This is given by option (a).

Options (b), (c) and (d) are incorrect because none of those satisfies the conditions for an ideal gas.

4. Question

Let C_V and C_P denote the molar heat capacities of an ideal gas at constant volume and constant pressure respectively. Which of the following is a universal constant?

A. C_P/C_V

B. $C_P\ C_V$

C. $C_P - C_V$

D. $C_P + C_V$

Answer

For an ideal gas, $C_p - C_v = R$.

Here, R is the universal gas constant.

Hence, the correct option is (c).

Option (a) is incorrect since $C_p/C_v = \gamma$, which differs for monoatomic, diatomic or polyatomic gases. Here γ = ratio of molar heat capacities at constant pressure and constant volume.

Option (b) is incorrect since $C_pC_v = \gamma C_v^2$, which is not a constant as γ varies.

Option (d) is incorrect since $C_p + C_v = (\gamma+1)C_v$, which is not a constant as γ varies.

5. Question

70 calories of heat is required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is

A. 30 calories

B. 50 calories

C. 70 calories

D. 90 calories

Answer

We know that, $Q = nC_pdT$… (i),

Where

Q = heat required to raise the temperature

n = number of moles

C_p = specific heat capacity at constant pressure

dT = rise in temperature.

In this first case,

Amount of heat required(Q) = 70 cal

Number of moles(n) = 2 mol

Rise in temperature(dT) = $(35-30)^0C = 5^0C$

Hence, from (i), we get $C_p = 7$ calmol^{-1} $^0C^{-1}$

Now, we know, $C_p - C_v = R$,

Where

C_p = specific heat capacity at constant pressure

C_v = specific heat capacity at constant volume

R = universal gas constant = 1.98 calmol^{-1} $^0C^{-1}$

Therefore, we get $C_v = C_p - R = (7-1.98)$ cal mol^{-1} $^0C^{-1}$

= 5.02 cal mol^{-1} $^0C^{-1}$

Now, in the new case, change in temperature dT = 5^0C as before. Number of moles(n) = 2, and C_v = 5.02 calmol^{-1} $^0C^{-1}$

Hence, amount of heat required to raise the temperature of the same gas through the same range

dQ = nCvdT = (2 x 5.02 x 5) cal = 50.2 cal which is approximately equal to 50 cal.

Hence, the correct option is (b).

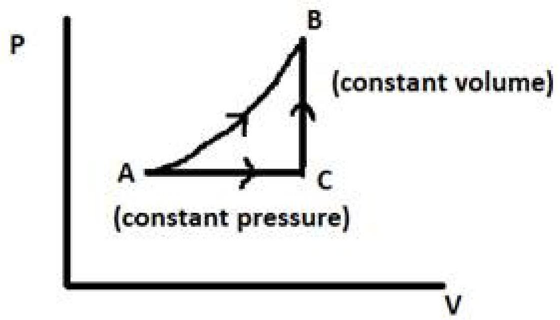
6. Question

The molar heat capacity for the process shown in the figure is

A. $C = C_p$B. $C = C_v$C. $C > C_v$D. $C = 0$

Answer

Let us consider the figure given below :



We consider the process AB to be composed of two processes, AC(at constant pressure) and CB(at constant volume), such that AB = AC + CB. C is the molar heat capacity of AB, C_p is the molar heat capacity of AC(constant pressure) and Cv is the molar heat capacity of CB(constant volume).

From the first law of thermodynamics, we know that Q = U + W … (i),

Where Q = heat supplied, U = change in internal energy, W = work done on the system.

Since the change in internal energy is independent of the path, it will have the same value for paths AB and ACB.

Hence, $U_{AB} = U_{ACB}$ … (ii), where U_{AB} = change in internal energy for path AB, U_{ACB} = change in internal energy for path ACB.

Now, the work done by a process is given by the area under the PV diagram. We can clearly see that the area under process AB is greater than the sum of areas under processes AC and CB.

Hence, $W_{AB} > W_{ACB}$ … (iii), where W_{AB} = work done for process following path AB, W_{ACB} = work done for process following path ACB.

Adding (ii) and (iii), we get :

$U_{AB} + W_{AB} > U_{ACB} + W_{ACB}$ … (iv)

But, from the first law of thermodynamics, Q = U + W, where Q = heat supplied, U = change in internal energy, W = work done on the system.

Hence, we get, $Q_{AB} > Q_{ACB}$ … (v),

We know, $Q_{AB} = nCdT$, $Q_{AC} = nC_pdT$, $Q_{CB} = nC_vdT$,

where Q_{AB} = heat supplied in process AB

Q_{AC} = heat supplied in process AC

Q_{CB} = heat supplied in process CB

n = number of moles

C = molar heat capacity

C_p = molar heat capacity at constant pressure

C_v = molar heat capacity at constant volume

dT = change in temperature.

Also, $Q_{ACB} = Q_{AC} + Q_{CB}$ … (vi), where

Q_{ACB} = heat supplied in process ACB

Q_{AC} = heat supplied in process AC

Q_{CB} = heat supplied in process CB

Hence, from (v), we get,

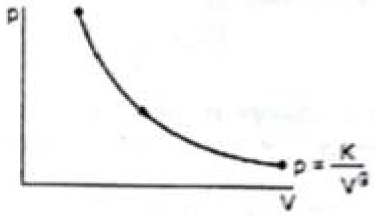
$nCdT > nC_pdT + nC_vdT$

Dividing by ndT on both sides :

$C > C_p + C_v \Rightarrow C > C_v$ (option c)

7. Question

The molar heat capacity for the process shown in the figure is



A. $C = C_P$

B. $C = C_V$

C. $C > C_V$

D. C = 0

Answer

Given : $p = \frac{K}{V^g}$ which gives us

$pV^g = K$, where p = pressure, V = volume, and g and K are constants. This represents an adiabatic process.

In an adiabatic process, Q(heat exchanged) = 0.

Now we know that Q = nCdT,

where n = number of moles, C = specific molar heat capacity and dT = rise in temperature.

Since Q = 0, we get C = 0.

Hence, the correct option is (d).

8. Question

In an isothermal process on an ideal gas, the pressure increases by 0.5%. The volume decreases by about

A. 0.25%

B. 0.5%

C. 0.7 %

D. 1%

Answer

Let the initial values of pressure and volume be P_1 and V_1, and let the final values of pressure and volume be P_2 and V_2.

Now for an isothermal process, PV = constant .. (i)

This gives us: $P_1V_1 = P_2V_2$.. (ii)

Since the pressure increases by 0.5%, the new pressure is

$$P_2 = P_1(1 + \frac{0.5}{100}) = 1.005P_1$$

Substituting this value in (ii) :

$P_1V_1 = 1.005P_1V_2$

=> $V_2 = \frac{V_1}{1.005} = 0.995\ V_1$.

Hence, the volume decreases by (1-0.995) = 0.005 = 0.5%

Hence, the correct option is (b).

9. Question

In an adiabatic process on gas with $\gamma = 1.4$, the pressure is increased by 0.5%. The volume decreases by about

A. 0.36%

B. 0.5%

C. 0.7%

D. 1%

Answer

For an adiabatic process, PV^{γ} = constant ... (i)

If P_1, V_1 represents the initial values of pressure and volume and P_2, V_2 represent the final values of pressure and volume,

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$... (ii)

Now, $\gamma = 1.4$ (given).

It is given that the pressure increases by 0.5%.

Hence, $P_2 = \left(1 + \frac{0.5}{100}\right)P_1 = 1.005P_1$.

Substituting in (ii) :

$P_1V_1^{\gamma} = 1.005 \text{ X } P_1V_2^{\gamma}$

=> $(V_1/V_2)^{\gamma} = 1.005$ ... (iii)

Taking log on both sides of (iii), we get

$$\gamma log\left(\frac{V_1}{V_2}\right) = log(1.005) = 0.002$$

$$=> log\left(\frac{V_1}{V_2}\right) = \frac{0.002}{1.4} = 0.0014 \text{ ... (iv)}$$

Taking inverse log on both sides of (iv), we get

$V_1/V_2 = 10^{0.0014} = 1.003$

=> $V_2 = V_1/1.003 = 0.997V_1$

Hence, the volume decreases by (1-0.997) = 0.003 which is approximately equal to 0.36%.

Hence, the correct option is (a).

10. Question

Two samples A and B are initially kept in the same state. Sample A is expanded through an adiabatic process and sample B through an isothermal process. The final volumes of the samples are the same. The final pressures in A and B are p_A and P_B respectively.

A. $P_A > P_B$

B. $P_A = P_B$

C. $P_A < P_B$

D. The relation between p_A and P_B cannot be deduced.

Answer

Adiabatic process is represented by : PV^γ = constant,

where P = pressure

V = volume

γ = ratio of specific heat capacities at constant pressure and constant volume

Therefore, we get $P_A = \text{constant}/V^\gamma$. .. (i)

Isothermal process is given by : PV = constant

Hence, we get P_B = constant/V ... (ii)

Since $\gamma > 1$, we get $p_A < p_B$. (proved)

11. Question

Let T_a and T_b be the final temperatures of the samples A and B respectively in the previous question.

A. $T_a < T_b$

B. $T_a = T_b$

C. $T_a > T_b$

D. The relation between T_a and T_b cannot be deduced.

Answer

Since sample B undergoes isothermal expansion, its temperature remains constant $= T_b$.

For an adiabatic process, since the heat supplied is 0, the internal energy will change by an amount $dU = nC_vdT$,

where dU = change in internal energy

n = number of moles

C_v = specific heat capacity at constant volume

dT = change in temperature

This change in internal energy will compensate for the constancy in heat.

Sample B is undergoing expansion through an isothermal process; its initial and final temperatures will be the same.

Sample A will expand at the cost of its internal energy.

Therefore, the final temperature will be less than the initial temperature,

since $dU < 0 \Rightarrow dT < 0$.

$T_b - T_a < 0$

$T_b > T_a$ or $T_a < T_b$

Hence, we get $T_a > T_b$.

12. Question

Let ΔW_a and ΔW_b be the work done by the systems A and B respectively in the previous question.

A. $\Delta W_a > \Delta W_b$

B. $\Delta W_a = \Delta W_b$

C. $\Delta W_a < \Delta W_b$

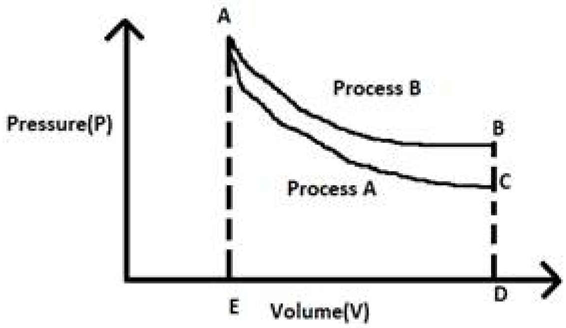
D. The relation between ΔW_a and ΔW_b cannot be deduced.

Answer

We know that for any given state, the slope of the p-V diagram of an adiabatic process is $-\gamma P/V$, while that of an isothermal process is $-P/V$.

Hence, the slope of an adiabatic process is more.

Now, the area under the curve of a p-V diagram gives the work done.



From this diagram, we can see that that area under the curve of process A(ACDE) which represents the adiabatic process with greater slope is less than that of the area under the curve of process B(ABDE), which represents the isotherm.

Hence, we can conclude that $\Delta W_a < \Delta W_b$

13. Question

The molar heat capacity of oxygen gas at STP is nearly 2.5R. As the temperature is increased, it gradually increases and approaches 3.5R. The most appropriate reason for this behaviour is that at high temperatures.

A. oxygen does not behave as an ideal gas

B. oxygen molecules dissociate in atoms

C. the molecules collide more frequently

D. molecular vibrations gradually become effective.

Answer

We know that the molar heat capacity of any gas depends on the degrees of freedom of the gas. On increasing the temperature of the gas, we allow more molecules to vibrate about their equilibrium positions, which in turn increases the degrees of freedom of the gas. For this reason, the molar heat capacity increases as the temperature of the gas are increased.

Hence, the correct answer is an option (d).

Objective II

1. Question

A gas kept in a container of finite conductivity is suddenly compressed. The process

A. must be very nearly adiabatic

B. must be very nearly isothermal

C. may be very nearly adiabatic

D. may be very nearly isothermal

Answer

Since the gas kept in the container is suddenly compressed, there is very little time for any heat to allow to enter or leave the container. Hence, it must be very nearly an adiabatic process. If the temperature remains constant during this process, then the process may also be isothermal. Option (a) is incorrect because it must be very nearly adiabatic but might not be completely depending on how

sudden the process is.

Option (b) is incorrect because if the compression is indeed done suddenly, then the temperature of the gas does not remain constant, and it may not be isothermal. It changes with the pressure and the volume of the gas according to the formulae:

i. $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

ii. PV^{γ} = constant

Where,

P1, V1, T1: initial values of pressure, volume and temperature

P2, V2, T2: final values of pressure, volume and temperature

γ: adiabatic index, varies from gas to gas

Therefore, the correct answers are option (c) and (d).

2. Question

Let Q and W denote the amount of heat given to an ideal gas and the work done by it in an isothermal process.

A. Q = 0

B. W = 0

C. Q ≠ W

D. Q = W

Answer

In an isothermal process, the temperature remains constant. The internal energy of an ideal gas is a state function that depends on temperature. Hence, change in internal energy is zero and from the first law of thermodynamics: $\Delta U = Q - W$, where ΔU = change in internal energy, Q = amount of heat given and W = work done by it.

Since $\Delta U = 0$ in this case, we get Q = W.

Options (a) and (b) are incorrect because we actually provide a finite amount of heat to the system, and hence work is also not zero.

Option (c) is incorrect because we just showed that Q = W.

Hence, the correct option is option (d).

3. Question

Let Q and W denote the amount of heat given to an ideal gas and the work done by it in an adiabatic process.

A. Q = 0

B. W = 0

C. Q = W

D. $Q \neq W$

Answer

In an adiabatic process, we assume that there is no heat exchange between the system and surroundings.

Hence, Q = 0. Also, U = W(when Q = 0), so $Q \neq W$ holds.

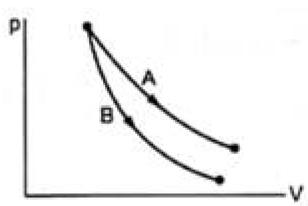
Option (b) is incorrect because due to change in temperature, there is still a change in internal energy and from the first law of thermodynamics, $\Delta U = W$. Hence $W \neq 0$.

Option (c) is incorrect because the heat exchange is zero, but the work done in general is not zero.

Hence, the correct answers are option (a) and (d).

4. Question

Consider the processes A and B shown in figure. It is possible that



A. both the processes are isothermal

B. both the processes are adiabatic

C. A is isothermal and B is adiabatic

D. A is adiabatic and B is isothermal

Answer

From the graph, we can see that the slope of process B is steeper than that of process A.

Now in an isothermal process, under constant temperature, Pressure(P) x Volume(V) = constant … (i) (according to Boyle's law)

Differentiating on both sides:

PdV + VdP = 0.

slope = $\frac{dP}{dV} = -\frac{P}{V}$ … (ii)

For an adiabatic process,

PV^{γ} = constant

Where,

γ is the ratio of specific heat of the gas at constant pressure and constant volume.

Differentiating the above question, we get

$V^{\gamma}dP + \gamma V^{\gamma-1}PdV = 0$

$$slope = \frac{dP}{dV} = -\frac{\gamma P}{V}$$

which is greater than that in case of the isothermal process.

Thus path (A) is for isothermal process while path (B) is for the adiabatic process.

Options (a) and (b) are incorrect since the slopes of the two paths Differ.

Option (d) is incorrect because the slope of path (A) is less than that of path (B).

Hence, the correct answer is option (c).

5. Question

Three identical adiabatic containers A, B and C contain helium, neon and oxygen respectively at equal pressure. The gases are pushed to half their original volumes.

A. The final temperatures in the three containers will be the same.

B. The final pressures in the three containers will be the same.

C. The pressures of helium and neon will be the same but that of oxygen will be different.

D. The temperatures of helium and neon will be the same but that of oxygen will be different.

Answer

Since this is an adiabatic process, Q(heat) = 0.

Hence, dU(change in internal energy) = dW(work done) = nC_vdT.

where n = no. of moles, C_v = specific heat of the gas at constant volume, and dT = change in temperature.

C_v is different for oxygen(diatomic gas) but same for helium and neon(monoatomic gases) so dT for oxygen is different.

W for adiabatic process = $(P_2V_2-P_1V_1)/(1-\gamma)$

In all cases, $V_2 = V_1/2$.

P_1, V_1, T_1 - initial pressure, volume and temperature

P_2, V_2, T_2 - final pressure, volume and temperature

V_1, P_1 are same for all gases and so is W. Only γ for oxygen is different compared to helium and neon since it is a diatomic gas while the rest are monoatomic.

Hence, options (c) and (d) are correct.

6. Question

A rigid container of negligible heat capacity contains one mole of an ideal gas. The temperature of the gas increases by 1°C it 3.0 cal of heat is added to it. The gas may be

A. helium

B. argon

C. oxygen

D. carbon dioxide

Answer

Let us use the formula,

Heat supplied to a gas, $dQ = nC_vdT$.

Where,

n is the number of mole of the ags

C_V is the specific heat of the gas at constant volume T is the temperature

Putting the values in the above formula, we get

dQ = 3.0 cal = (3.0 x 4.2)J = 12.6 J.

dT = 1°C, n = 1.

Therefore, C_V is approximately equal to 12.6 J/mol/°C ~ 1.5R = 3R/2

We know, $Cp = Cv + R = \frac{3R}{2} + R = \frac{5R}{2}$

Where C_p = specific heat at constant pressure

R = universal gas constant = 8.314 J/kg/mol

$\gamma = \frac{Cp}{Cv} = \frac{5}{3}$, which holds for monoatomic gases.

Helium and Argon are monoatomic and so options (a) and (b) hold.

7. Question

Four cylinders contain equal number of moles of argon, hydrogen, nitrogen and carbon dioxide at the same temperature. The energy is minimum in

A. argon

B. hydrogen

C. nitrogen

D. carbon dioxide

Answer

According to the law of equipartition of energy, the Kinetic energy associated with each degree of freedom of a molecule is kT/2,

KE= $\frac{f\,K\,T}{2}$

where k is Boltzmann constant,

T is absolute temperature.

f is the number of degrees of freedom

The number of degrees of freedom of a system is the minimum number of coordinates required to completely describe its position and orientation.

Now argon,

which is a monoatomic gas, can only translate along the x, y and z axes.

Hence, it has 3 degrees of freedom. Now, for a system,

total associated energy is:

Total energy of all the degrees of freedom, $E = \frac{fkT}{2}$

Where

E = total energy,

f = number of degrees of freedom

k = Boltzmann constant = 1.38 x $10^{-23} m^2\ kg\ s^{-2}\ K^{-1}$

T = absolute temperature (Kelvin)

For argon, f = 3. Therefore, $E = \frac{3kT}{2}$ which is minimum.

Options (b) and (c) are incorrect since they are diatomic and

have more than 3 degrees of freedom(translational and rotational degrees of freedom of 2 molecules).

Option (d) is incorrect since carbon dioxide is triatomic and has

more than 3 degrees of freedom(translational and rotational degrees of freedom of 3 molecules).

Hence, the correct answer is option (a).

Exercises

1. Question

A vessel containing one mole of a monatomic ideal gas (molecular weight = 20 g mol^{-1}) is moving on a floor at a speed of 50 ms^{-1}. The vessel is stopped suddenly. Assuming that the mechanical energy lost has gone into the internal energy of the gas, find the rise in its temperature.

Answer

Given:

number of moles, n = 1

Specific heat at constant temperature, C_v(monoatomic gas) = 3R/2 = 12.471 J/mol/K

R = universal gas constant = 8.314 J/mol/K

Initial velocity(v_i) = 50 m/s

Final velocity(v_f) = 0

Molecular weight(m) = 20 g/mol = 0.02 kg/mol

Formula Used:

i. Change in internal energy(dU) = nC_vdT,

Where,

n is the number of the moles of the gas,

C_v is the heat capacity at the constant volume

dT = rise in temperature.

ii. Mechanical energy lost, $E = -m(\frac{v_i^2 - v_f^2}{2})$

Where,

m is the molecular weight of the gas in kg

v_i is the initial velocity,

v_f is the final velocity,

equating equation (i) and (ii), we get

Putting the values in the above equation, we get

$$\frac{0.02X(2500-0)}{2} = 12.471XdT$$

=> dT = rise in temperature ~ 2 K (Answer).

2. Question

5g of a gas is contained in a rigid container and is heated from 15°C to 25°C. Specific heat capacity of the gas at constant volume is 0.172 cal $g^{-1}°C^{-1}$ and the mechanical equivalent of heat is 4.2 J cal^{-1}. Calculate the change in the internal energy of the gas.

Answer

Given:

Mass of the gas(m) = 5g

Change in temperature(dT) = 10°C

Specific heat at constant volume(C_v) = = 0.172 cal $g^{-1}°C^{-1}$

Mechanical equivalent of heat = 4.2 J cal^{-1}

Formula used:

Change in internal energy = mC_VdT

Where,

m = Mass of the gas

C_V = Specific heat at constant volume

dT = Change in temperature

Also, we know,

Heat(Joule)= Mechanical equivalent of heat× Heat(cal)

Putting the values in the above formula, we get

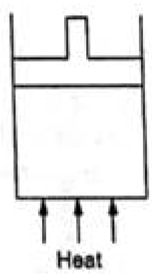
= (5x10x0.172) cal

= 8.6 cal = (8.6x4.2)J

= 36.12 J (Answer)

3. Question

Figure shows a cylindrical container containing oxygen (γ = 1.4) and closed by a 50 kg frictionless piston. The area of cross section is 100 cm^2, atmospheric pressure is 100 kPa and g is 10 ms^{-2}. The cylinder is slowly heated for some time. Find the amount of heat supplied to the gas if the piston moves out through a distance of 20 cm.



Answer

Given:

Mass of piston(m) = 50 kg

Area(A) = 100 cm^2 = $(100 \times 10^{-4}) m^2$ (since 1m = 100 cm)

Acceleration due to gravity, g = 10 ms^{-2}

Atmospheric pressure = 100 kPa

Distance through which it moves = 20 cm

$\gamma = 1.4$

Formula used:

Therefore, pressure exerted by piston = $\frac{Force}{area} = \frac{(m\,x\,g)}{area}$

= $((50 \times 10)/(100 \times 10^{-4}))$ Pa

= 50,000 Pa

Atmospheric pressure = 100 kPa = 1,00,000 Pa.

Therefore, Total pressure(P) = (50,000 + 1,00,000)Pa

=1,50,000Pa

Work done = Pressure x change in volume = P x dV

dV(change in volume) = distance moved by piston x Area

= ($20cm \times 100cm^2$)

= 2,000 cm^3 = 2,000 x 10^{-6} m^3 = 2 x 10^{-3} m^3

Therefore, Work = (1,50,000 x 2 x 10^{-3}) J = 300 J

Work done, W= PΔV =n R dT

We get,

$$dT = \frac{300}{nR}$$

Now, We calculate Q:

$dQ = nC_p dt = n \times C_p \times \frac{300}{nR} = 300 \times \frac{C_p}{R}$

Given: $\gamma = 1.4 = \frac{C_p}{C_v}$. Also, $C_p + C_V = R$. Solving these two equations, we get C_p = 7R/2, C_v = 5R/2.

Hence, $dQ = \frac{300X\frac{7R}{2}}{R} = \frac{300X7}{2}$ = 1050 J.

4. Question

The specific heat capacities of hydrogen at constant volume and at constant pressure are 2.4 cal g^{-1} °C^{-1} and 3.4 cal g^{-1} °C^{-1} respectively. The molecular weight of hydrogen is 2 g mol^{-1} and the gas constant, $R = 8.3 \times 10^7$ erg °$C^{-1}mol^{-1}$. Calculate the value of *J*

Answer

Given:

Specific heat at constant volume is $C_v = 2.4\ cal\ g^{-1}{}^oC^{-1}$

Specific heat at constant pressure is C_p=3.4 cal $g^{-1}{}^oC^{-1}$

Molecular mass of hydrogen is 2 g mol^{-1}

Gas constant, R=8.3× 10^7 gmol

Formula used:

$$(mXCp) - (mXCv) = R$$

Where:

m is the molecular weight of hydrogen

C_p is the specific heat at constant pressure

C_v is the specific heat at constant volume

R is the gas constant.

Putting the values in the above equation , we get

The gas constant R= (m X C_p) - (m X C_v) = 2 × (Cp-Cv)= 2×J

Now, 2× J=R

2× J=8.3 × 10^7 erg/ $mol^{-o}C$

Thus, J=4.15 × 10^7 erg/cal (Answer).

5. Question

The ratio of the molar heat capacities of an ideal gas is C_P/C_V = 7/6. Calculate the change in internal energy of 1.0 mole of the gas when its temperature is raised by 50 K

(a) keeping the pressure constant,

(b) keeping the volume constant and

(c) adiabatically.

Answer

Given:

n = number of moles = 1,

C_v = specific heat capacity at constant volume,

C_p = specific heat capacity at constant pressure

dT = change in temperature = 50K.

γ= Ratio of molar heat capacities = C_P/C_V = 7/6 => $C_v = 6C_p/7$.

(a) **Formula used:**

Pressure constant: Isobaric process. For an isobaric process,

change in internal energy $dU = nC_vdT$,

Where

n = number of moles,

C_v = specific heat at constant volume,

dT = rise in temperature

Also, C_p-C_v = R.

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

R = universal gas constant = 8.314 J/mol/K

Substituting: $C_p - 6C_p/7 = C_p/7 = R \Rightarrow C_p = 7R$.

Therefore $C_v = C_p - R = 6R$ = (6 X 8.314)J/mol/K

Therefore,

dU = 1 mol X (6 X 8.314)J/mol/K X 50K = 2494.2 J(Ans)

(b) Volume constant: Isochoric process, dV = 0(change in volume)

First law of thermodynamics gives us: dU = dQ - dW

Where dU = change in internal energy, dQ = change in heat,

dW = work done = Pressure x change in volume = PdV

Since dV = 0, dU = dQ.

Hence, $dU = nC_vdT$ since $dQ = nC_vdT$.

Where

n = Number of moles,

C_v = Specific heat at constant volume, dT =Change in temperature

Putting the values in the above formula, we get

Therefore, dU = 1 mol x (6 x 8.314) J/mol/K x 50K

= 2494.2 J(Ans)

(c) Adiabatic process: dQ(heat change) = 0. Therefore,

dU(change in internal energy) = dW(work done)

Since dQ = dU + dW.

For an adiabatic process, dW = dT/(γ-1).

dW = work done, dT = change in temperature

$\gamma = C_p/C_v = 7/6$, C_p = specific heat capacity at constant

pressure, C_v = specific heat capacity at constant volume

Therefore dU = - dW = - $\frac{nRdT}{\gamma - 1}$ = (8.314X50)/(7/6 - 1)

= 2494.2 J (Ans)

6. Question

A sample of air weighing 1.18g occupies 1.0×10^3 cm^3 when kept at 300K and 1.0×10^5 Pa. When 2.0 cal of heat is added to it at constant volume, its temperature increases by 1°C. Calculate the amount of heat needed to increase the temperature of air by 1°C at constant pressure if the mechanical equivalent of heat is 4.2×10^7 erg cal^{-1}. Assume that air behaves as an ideal gas.

Answer

Given:

Pressure, p= 1.0×10^5 Pa,

Temperature, T = 300K,

Universal gas constant, R = 8.314 J/kg/mol

V(volume) = 1.0×10^3 cm^3 = 0.001 m^3

Formula used:

1. Ideal gas equation: PV = nRT.

Where,

P = pressure,

V = volume,

n = number of moles,

R = universal gas constant = 8.314 J/kg/mol,

T = absolute temperature

2. Number of moles n = PV/RT = 100000x0.001/(8.314*300) = 0.04 mol

3. First law of thermodynamics: dQ = dU + dW = dU + PdV,

Where,

dQ = heat supplied,

dU = change in internal energy,

dW = PdV = work done, where P = pressure, dV = change in volume.

Since volume is constant, dV = 0 => dW = 0.

Hence, dQ = dU.

Heat(dQ) = 2 cal = $nC_v dT$ = 0.04 mol x C_v x 1K ,

Where n = number of moles, C_v = specific heat at constant volume, dT = rise in temperature.

=> C_v = 50 cal/mol/K. = (50 x 4.2 × 10^7)erg/cal x cal/mol/K = 2.1 x 10^9 erg/mol/K = 210 J/mol/K,

Since heat(J) = mechanical equivalent of heat x heat(cal) = 4.2 x heat(cal), and

1 J = 10^7 erg

We know, $C_p = (C_v+R)$,

Where C_p = specific heat at constant pressure, C_v = specific heat at constant volume, R = universal gas constant = 8.314 J/kg/mol

Therefore, C_p = (210+8.314) J/mol/K = 218.314 J/mol/K.

Therefore, heat required to raise the temperature by 1°C at constant pressure = $nC_p dT$,

Where n = number of moles, C_p = specific heat at constant pressure, dT = rise in temperature.

Hence, substituting, heat = (0.04 x 218.314 x 1) J = 8.737 J = (8.737/4.2) cal = 2.08 cal (since 1 J = 4.2 cal)(Ans)

7. Question

An ideal gas expands from 100 cm^3 to 200 cm^3 at a constant pressure of 2.0×10^5 Pa when 50J of heat is supplied to it. Calculate

(a) the change in internal energy of the gas.

(b) the number of moles in the gas if the initial temperature is 300K.

(c) the molar heat capacity C_P at constant pressure and

(d) the molar heat capacity C_V at constant volume.

Answer

Given:

(a) Pressure(P) = 2.0×10^5 Pa, dV(change in volume) = (200-100) cm^3 = $10^{-4} m^3$, since 1 m^3 = 10^6 cm^3

Heat(dQ) = 50 J.

Formula used:

Now we know, dQ = dU(change in internal energy) + dW(work)=

dU + PdV (first law of thermodynamics),

Where P = pressure, dV = change in volume.

=> dU = dQ - PdV = $(50 - (2.0 \times 10^5 \text{ x } 10^{-4}))$ J = 30 J (Ans)

(b) For constant pressure, from equation of state PV/T = constant,

Where P = pressure, V = volume, T = temperature.

Hence we get: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, where

V_1(initial volume) = 100 cm^3, V_2(final volume) = 200 cm^3, T_1 = 300K

$$=> T_2 = \frac{V_2 T_1}{V_1} = (300 \times \frac{200}{100})\,K = 600\,K.$$

Therefore, PdV = nRdT (for more than one mole),

Where P = pressure, dV = change in volume, n = number of moles, R = universal gas constant = 8.314 J/kg/mol, dT = change in temperature.

=> $2.0 \times 10^5 \text{ x } 10^{-4}$ = n x 8.314 x 300 (since T2-T1 = dT = 300K)

Therefore, n = 20/(R x 300) = 0.008 mol (Ans).

(c) dQ(heat) = 50 = $nC_p dT$ (at constant pressure), Where n = number of moles, C_p = specific heat at constant pressure, dT = rise in temperature.

=> 50 = 0.008 x C_p x 300

=> C_p = 20.83 J/mol/K. (Ans)

(d) At constant volume, dU(change in internal energy) = dQ(heat) = $nC_v dT$ (since work done dW = PdV = 0, where P = pressure, dV = change in volume), from first law of thermodynamics.

n= number of moles, C_v = specific heat capacity at constant volume, dT = change in temperature.

=> 30 = 0.008 x C_v x 300

=> C_v = 12.5 J/mol/K. (Ans)

8. Question

An amount Q of heat is added to a monatomic ideal gas in a process in which the gas performs a work Q/2 on its surrounding. Find the molar heat capacity for the process.

Answer

Given: Amount of heat added(dQ) = Q

Amount of work done(dW) = Q/2.

Formula used:

dQ(heat) = dU(internal energy) + dW(work done).

Here, heat = Q and Work = Q/2(given)

=> $U = Q - \frac{Q}{2} = \frac{Q}{2}$.

We can write U = $nC_v dT$ and Q = nCdT, where n = no of moles, C_v = specific heat capacity at constant volume(when dQ = dU), C = molar heat capacity and dT = change in temperature.

Therefore, $nCvdT = \frac{nCdT}{2}$ => C = $2C_v$.

For a monoatomic ideal gas, we know that C_v = (3R/2) J/kg/mol,

Where R = universal gas constant = 8.314 J/kg/mol

Therefore, C = 2*(3R/2) = 3R J/kg/mol. (Ans)

9. Question

An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation p = kV. Show that the molar heat capacity of the gas for the process is given by $C = C_v + \frac{R}{2}$.

Answer

Given:

P = kV …

Where,

P = pressure,

V = volume,

k = constant.

Formula used:

Equation of state of ideal gas:

PV = nRT = constant … (ii)

Where,

n is the number of moles of the gas,

R is the gas constant,

T is the temperature,

P = pressure, V^2

T = temperature.

From (i), multiplying by dV on both sides:

PdV = kVdV.

Integrating from V = V1 to V2, we get

$$\int dV = k\int_{V_1}^{V_2} V\,dV$$

$= k\ [\frac{V\ x\ V}{2}]$with lower limit V_1 and upper limit V_2

$= k\,\frac{[V_2^2 - V_1^2]}{2}$

Now, we know, PV = nRT - equation of state, Where P = pressure, V = volume, n = number of moles, R = universal gas constant, T = temperature

Hence we can write, $V_1 = nRT_1/P_1$. Since $P_1 = kV_1$, this becomes:

$kV_1^2 = nRT_1$. Similarly, $KV_2^2 = nRT_2$,

P_1, V_1, T_1 - Pressure, volume, temperature of first gas

P_2, V_2, T_2 - Pressure, volume, temperature of second gas

Therefore, subsituting, the above equation becomes:

$= \frac{k}{2} \times nR \frac{[T_2 - T_1]}{k} = \frac{nR}{2} dT \; (dT = T_2 - T_1)$ ... (iii)

Now, $V = \frac{nRT}{P} \Rightarrow V^2 = \frac{nRT}{k}$ (since P = kV) ... (iv)

But Q = U + ∫PdV (first law of thermodynamics), where Q = heat, U = change in internal energy, W = total work done = ∫PdV

=> $nCdT = nC_vdT + (nR/2)dT$

(since Q = nCdT and $U = nC_vdT$)

=> $C = C_v + nR/2$ (proved),

Where

n = number of moles,

C = specific heat capacity,

C_v = specific heat capacity at constant volume,

R = universal gas constant,

dT = rise in temperature.

10. Question

An ideal gas ($C_P/C_V = \gamma$) is taken through a process in which the pressure and the volume vary as $p = \alpha V^b$. Find the value of b for which the specific heat capacity in the process is zero.

Answer

Given:

$p = aV^b$

p = pressure, V = volume, a and b are constants.

Formula used:

We know, Q = U + ∫pdV from first law of thermodynamics,

where Q = change in heat, U = change in internal energy and ∫pdV = W = total work done, p = pressure, V = volume.

Since Q = nCdT, and U = $nC_v dT$, we get

$nCdT = nCvdT + \int_{V1}^{V2} aV^b dV$ ... (ii), n = no. of moles, C = specific

heat capacity, and C_v = specific heat capacity at constant volume,

dT = change in temperature,

Since specific heat capacity is 0(given),

$nCvdT + \frac{a}{b+1} X [V_2^{b+1} - V_2^{b+1}] = 0$ ...(iii)

(after integration of $\int aV^b dV$ from V_1 to V_2)

Now, from equation of state, PV = nRT,

Where

P = pressure,

V = volume,

n = number of moles,

R = universal gas constant,

T = temperature.

Substituting p = aV^b from (i):

aV^{b+1} = nRT

=> $V^{b+1} = nRT/a$ ... (iv)

Substituting (iv) in (iii),

$$nCvdT = -\frac{a}{b+1} X \frac{nR}{a} X (T2 - T1)$$

$nCvdT = -\frac{nR}{b+1} X dT$ (Since $(T_2 - T_1)$ = dT)

=> $b + 1 = -\frac{R}{Cv} = -\frac{Cp - Cv}{Cv} = -(\frac{Cp}{Cv} - 1) = -(\gamma - 1) = 1 - \gamma$

=> b = -γ (Ans)

11. Question

Two ideal gases have the same value of $C_P/C_V = \gamma$. What will be the value of this ratio for a mixture of the two gases in the ratio 1: 2?

Answer

We know Cp/Cv=γ, R=Cp-Cv,

where the molar heat capacity C, at constant pressure, is represented by Cp, at constant volume, the molar heat capacity C is represented by Cv and R is the universal gas constant.

Now,

$$Cv = \frac{R}{\gamma - 1}$$

For the first ideal gas,

$$Cv1 = \frac{R}{\gamma - 1}$$

$$Cp1 = \frac{\gamma R}{\gamma - 1}$$

Where C_{p1} and C_{V1} is the molar heat capacity at constant pressure and constant volume

Similarly, for the second ideal gas

$$Cv2 = \frac{R}{\gamma - 1}$$

$$Cp2 = \frac{\gamma R}{\gamma - 1}$$

Where C_{p2} and C_{V2} is the molar heat capacity at constant pressure and constant volume

Given,

$n1 : n2 = 1 : 2$

i.e

$dU_1 = nC_{V1}dT$

$dU_2 = 2nC_{V2}Dt$

When gas is mixed,

$$nC_{v1}dT + 2nC_{v2}dT = 3nC_v dT$$

$$C_v = \frac{C_{v1} + 2C_{v2}}{3}$$

$$C_v = \frac{\left(\frac{R}{\gamma - 1} + \frac{2R}{\gamma - 1}\right)}{3} \ (1)$$

Also,

$$C_p = \gamma C_v = \frac{\gamma R}{\gamma - 1} \ (2)$$

From (1) and (2)

$$\frac{C_p}{C_v} = \gamma$$

12. Question

A mixture contains 1 mole of helium ($C_P = 2.5$ R, $C_V = 1.5$R) and 1 mole of hydrogen ($C_P = 3.5$ R, $C_V = 2.5$ R). Calculate the values of C_P, C_V and γ for the mixture.

Answer

Given,

$C_{P1} = 2.5$ R, $C_{V1} = 1.5$R for helium

$C_{P2} = 3.5$ R, $C_{V2} = 2.5$ R for hydrogen

n1=n2=1

We know $dU = nC_v dT$

Where dU is the change in internal energy, n is the number of moles, Cv is the molar heat capacity at constant volume and dT is the change in temperature.

For the mixture,

$$(n1 + n2)C_v dT = n1C_{v1}dT + n2C_{v2}dT$$

$$C_v = \frac{n1C_{v1}dT + n2C_{v2}dT}{(n1 + n2)dT}$$

$$C_v = \frac{1.5RdT + 2.5dT}{(2)dT} = 2R$$

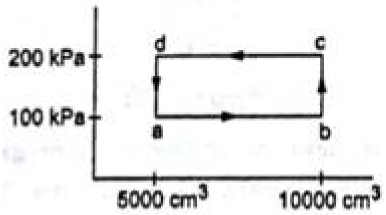
Also,

$$C_p = C_v + R = 3R$$

$$\gamma = \frac{C_p}{C_v} = 1.5$$

13. Question

Half mole of an ideal gas (γ =5/3) is taken through the cycle abcda as shown in figure. Take $R=\frac{25}{3}\ JK^{-1}\ mol^{-1}$.



(a) Find the temperature of the gas in the states a, b, c and d.

(b) Find the amount of heat supplied in the processes ab and bc.

(c) Find the amount of heat liberated in the processes cd and da.

Answer

Given, n=1/2, γ =5/3, R=25/3 J/Kmol

a) By ideal gas equation,

$PV = nRT$, where P, V and T are the pressure, volume and absolute temperature; n is the number of moles of gas; R is the ideal gas constant.

Here temperature at a, $Ta = \frac{PV}{nR} = \frac{5000 \times 10^{-6} \times 100 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 120\ K$

Here temperature at b,

$$Tb = \frac{PV}{nR} = \frac{10000 \times 10^{-6} \times 100 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 240\ K$$

Here temperature at c,

$$Tc = \frac{PV}{nR} = \frac{1000 \times 10^{-6} \times 200 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 480\ K$$

Here temperature at d,

$$Td = \frac{PV}{nR} = \frac{5000 \times 10^{-6} \times 200 \times 10^{3}}{\frac{1}{2} \times \frac{25}{3}} = 240\ K$$

b) Here ab is an isobaric process where heat supplied dQ can be expressed as

$$dQ = nCpdT = \frac{1}{2}\frac{R\gamma}{\gamma - 1}(Tb - Ta) = \frac{1}{2}\frac{\left(\frac{25}{3} \times \frac{5}{3}\right)}{\frac{5}{3} - 1}(240 - 120) = 1250\ J$$

Here bc is an isochoric process where heat supplied dQ is

$$dQ = nCvdT = \frac{1}{2}\frac{R}{\gamma - 1}(Tc - Tb) = \frac{1}{2}\frac{\frac{25}{3}}{\frac{5}{3} - 1} \times 240 = 1500\ J$$

c) Heat liberated in cd, isobaric process dQ is

$$dQ = -nCpdT = \frac{1}{2}\frac{R\gamma}{\gamma - 1}(Td - Tc) = -\frac{1}{2}\frac{\left(\frac{25}{3} \times \frac{5}{3}\right)}{\frac{5}{3} - 1}(-240) = 2500\ J$$

Heat liberated in da, isochoric process dQ is

$$dQ = -nCvdT = \frac{1}{2}\frac{R}{\gamma - 1}(Ta - Td) = -\frac{1}{2}\frac{\frac{25}{3}}{\frac{5}{3} - 1}(120 - 240) = 750\ J$$

14. Question

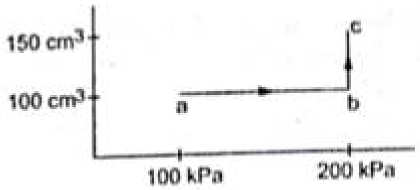
An ideal gas ($\gamma = 1.67$) is taken through the process abc shown in figure. The temperature at the point a is 300K. Calculate

(a) the temperature at b and c,

(b) the work done in the process,

(c) the amount of heat supplied in the path ab and in the path bc and

(d) the change in the internal energy of the gas in the process.



Answer

Let (P1, V1, T1), (P2, V2, T2), (P3, V3, T3) denote the pressure, volume and temperature at a, b and c respectively.

(a)For the process ab volume is constant

I.e. by ideal gas equation,

$$\frac{P1}{T1} = \frac{P2}{T2}$$

$$i.e. \frac{100}{300} = \frac{200}{T2}$$

$T2 = 600\ K$, temperature at b

For the process bc, pressure is constant.

By ideal gas equation,

$$\frac{V2}{T2} = \frac{V3}{T3}$$

$$I.e. \frac{100}{600} = \frac{150}{T3}$$

$T3 = 900\ K$,temperature at c

(b)Here process ab is isochoric i.e. W_{ab}=0

For process bc, P=200 kPa, change in volume is 50 cm^3 from b to c

$$Work\ done = PdV = 200 \times 10^3\ \times 50 \times 10^{-6} = 10\ J$$

(c) From the first law of thermodynamics,

$$dQ = dU + dW$$

Where dQ is the amount of heat supplied

As ab is isochoric process dW=0

$$dQ_{ab} = dU = nC_v dT$$

$$= \frac{PV}{RT} \times \frac{R}{\gamma - 1} dT$$

$$= \frac{200 \times 10^3 \times 150 \times 10^{-6}}{600 \times .67} \times 300 = 14.925$$

Here bc is an isobaric process where heat supplied dQ by first law of

Thermodynamics is

$$dQ_{bc} = dU = nC_p dT$$

$$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} dT$$

$$= \frac{200 \times 10^3 \times 150 \times 10^{-6}}{900 \times 8.3} \times \frac{1.67 \times 8.3}{.67} \times 300 = 24.925$$

(d) dQ = dU + W

Now, $dU = dQ - W$

$$= Heat\ supplied - Work\ done$$

$$= (24.925 + 14.925) - 10$$

$$= 29.85$$

15. Question

In Joly's differential steam calorimeter, 3g of an ideal gas is contained in a rigid closed sphere at 20°C. The sphere is heated by steam at 100°C and it is found that an extra 0.095 g of steam has condensed into water as the temperature of the gas becomes constant. Calculate the specific heat capacity of the gas in J g^{-1} K^{-1}. The latent heat of vaporization of water = 540 cal g^{-1}.

Answer

Here,

m1 = Mass of gas present = 3 g, θ1 = 20°C, θ2 = 100°C

m2 = Mass of steam condensed = 0.095 g, L = 540 Cal/g = 540 × 4.2 J/g

In Joly's differential steam calorimeter,

$$C_v = \frac{m2L}{m1(\theta 2 - \theta 1)}$$

$$Cv = \frac{0.095 \times 540 \times 4.2}{3 \times 80} = 0.89\, J/gK$$

16. Question

The volume of an ideal gas (γ = 1.5) is changed adiabatically from 4.00 litres to 3.00 litres. Find the ratio of

(a) the final pressure to the initial pressure and

(b) the final temperature to the initial temperature.

Answer

Here γ= 1.5, V1=4, V2=3.Let P1 and P2 be the initial and final pressure (a)Since it is an adiabatic process, So PV^{γ} = const.

$$\text{I.e.}\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma} = \left(\frac{4}{3}\right)^{1.5} = 1.54$$

(b)Also for an adiabatic process, $TV^{\gamma-1}$=constant

I.e.

$$T1V1^{\gamma-1} = T2V2^{\gamma-1}$$

$$\Rightarrow \frac{T2}{T1} = \left(\frac{V1}{V2}\right)^{\gamma-1}$$

$$= \left(\frac{4}{3}\right)^{1.5-1} = 1.154$$

17. Question

An ideal gas at pressure 2.5×10^5 Pa and temperature 300K occupies 100 cc. It is adiabatically compressed to half its original volume. Calculate

(a) the final pressure,

(b) the final temperature and

(c) the work done by the gas in the process. Take $\gamma = 1.5$.

Answer

Here given, $P1 = 2.5 \times 10^5$ Pa, V1 = 100 cc, T1 = 300 K, V2=50 cc

(a)Since it is an adiabatic process, So PV^{γ} = const.

$$\text{I.e.}\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma} = \left(\frac{100}{50}\right)^{1.5}$$

$$P2 = 2.5 \times 10^5 \times 2^{1.5} = 7.1 \times 10^5\ Pa$$

(b) Also for an adiabatic process, $TV^{\gamma-1}$=constant

I.e.

$$T1V1^{\gamma-1} = T2V2^{\gamma-1}$$

$$\Rightarrow \frac{T2}{T1} = \left(\frac{V1}{V2}\right)^{\gamma-1}$$

$$I.e. T2 = 300 \times 2^{0.5} = 424\ K$$

(c) Work done by the gas in the process

$$W = \frac{nR(T2 - T1)}{\gamma - 1} = \frac{P1V1(T2 - T1)}{T1(\gamma - 1)}$$

$$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}(424 - 300)}{300(1.5 - 1)} = 21\,J$$

18. Question

Air (γ = 1.4) is pumped at 2 atm pressure in a motor tyre at 20°C. If the tyre suddenly bursts, what would be the temperature of the air coming out of the tyre. Neglect any mixing with the atmospheric air.

Answer

Given $\gamma = 1.4$,

Initial pressure, P1=2 atm

Final pressure, P2=1 atm

Initial Temperature, T1=20°C=293 K

Here bursting of tire is an adiabatic process,

$$T1^{\gamma} P1^{1-\gamma} = T2^{\gamma} P2^{1-\gamma}$$

$$\Rightarrow T2^{\gamma} = \frac{T1^{\gamma} P1^{1-\gamma}}{P2^{1-\gamma}} = 293^{1.4} \times \left(\frac{2}{1}\right)^{-0.4}$$

$$\Rightarrow T2 = 240.3\,K$$

19. Question

A gas is enclosed in a cylindrical can fitted with a piston. The walls of the can and the piston are adiabatic. The initial pressure, volume and temperature of the gas are 100 kPa, 400 cm^3 and 300 K respectively. The ratio of the specific heat capacities of the gas is $C_P/C_V = 1.5$. Find the pressure and the temperature of the gas if it is

(a) suddenly compressed

(b) slowly compressed to 100 cm^3.

Answer

Given, P1 = 100 KPa = 10^5 Pa, V1 = 400 cm^3 , T1 = 300 K,

$C_P/C_V = 1.5$

(a)Suddenly compressed to V2 = 100 cm^3 I.e. it is an adiabatic process

∴ PV^{γ} = const.

I.e. $\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma} = \left(\frac{400}{100}\right)^{1.5}$

$P2 = 10^5 \times 4^{1.5} = 800\ KPa$

Also,

$T1P1^{1-\gamma} = T2P2^{1-\gamma}$

$\Rightarrow\ T2 = \frac{T1P1^{1-\gamma}}{P2^{1-\gamma}} = 300 \times \left(\frac{100}{400}\right)^{-0.5}$

$\Rightarrow T2 = 600\ K$

(b) Even though the container is slowly compressed the walls are adiabatic so heat transferred is 0.

Thus the values remain same

i.e. P2 = 800 KPa, T2 = 600 K.

20. Question

The initial pressure and volume of a given mass of a gas ($C_P/C_V = \gamma$) are P_0 and V_0. The gas can exchange heat with the surrounding.

(a) It is slowly compressed to a volume $V_0/2$ and then suddenly compressed to $V_0/4$. Find the final pressure.

(b) If the gas is suddenly compressed form the volume V_0 to $V_0/2$ and then slowly compressed to $V_0/4$, what will be the final pressure?

Answer

Given $C_P/C_V = \gamma$

Let $P1=P_0$ be the Initial Pressure, $V1=V_0$ be the Initial Volume

(a)Since the volume is slowly compressed, temperature remains constant I.e. Isothermal compression. Let P2 and $V2=V_0/2$ be the pressure and volume after slow compression

$\therefore P1V1 = P2V2$

$\Rightarrow P_0V_0 = P2V_0/2$

$\Rightarrow P2 = 2P_0$

When volume is suddenly compressed, it is an adiabatic process .Let P3 and V3 be the pressure and volume after sudden compression .i.e.

PV^{γ} = const.

$$\text{I.e.}\frac{P3}{P2} = \left(\frac{V2}{V3}\right)^{\gamma} = \left(\frac{V_0/2}{V_0/4}\right)^{\gamma} = (2)^{\gamma}$$

Substituting value of P2=2P_0

$$P3 = P_0 2^{\gamma+1}$$

(b) Since the volume is suddenly compressed, I.e. it is an adiabatic process Let P2 and V2=V_0/2 be the pressure and volume after sudden compression, then

PV^{γ} = const.

$$\text{I.e.}\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma} = \left(\frac{V_0}{V_0/2}\right)^{\gamma} = (2)^{\gamma}$$

Substituting value of P1=P_0

$$P2 = P_0 2^{\gamma}$$

Now, since the volume is slowly compressed, temperature remains constant I.e. Isothermal compression. Let P3 and V3=V_0/4 be the pressure and volume after slow compression.

$$\therefore P2V2 = P3V23$$

$$\Rightarrow P_0 \times 2^{\gamma}(V_0/2) = P3V_0/4$$

$$\Rightarrow P3 = P_0 2^{\gamma+1}$$

21. Question

Consider a given sample of an ideal gas ($C_P/C_V = \gamma$) having initial pressure P_0 and volume V_0.

(a) The gas is isothermally taken to a pressure $P_0/2$ and from there adiabatically to a pressure $P_0/4$. Find the final volume.

(b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $P_0/2$ and from there isothermally to a pressure $P_0/4$. Find the final volume.

Answer

Given $C_P/C_V = \gamma$, initial pressure P1=P_0 and initial volume V1= V_0

a) Since the gas is isothermally taken to pressure P2=P_0/2

$$\therefore P1V1 = P2V2$$

$$\Rightarrow P_0V_0 = P_0/2V2$$

$$\Rightarrow V2 = 2V_0$$

Let P3=$P_0/4$ and V3 be the pressure and volume after adiabatic compression.

Then,

PV^{γ} = const.

I.e. $\frac{P3}{P2} = \left(\frac{V2}{V3}\right)^{\gamma}$

$$\frac{P_0/4}{P_0/2} = \left(\frac{V2}{V3}\right)^{\gamma}$$

Substituting value of V2=2V_0

$$V3 = 2^{1/\gamma}(2)\,V_0 = 2^{(1+1/\gamma)}V_0$$

(b)Here P1=P_0, P2=$P_0/2$ and the process is adiabatic. Let V1=V_0 be the initial volume and V2 be the volume after process.

Then,

PV^{γ} = const.

I.e. $\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma}$

$$\frac{P_0/2}{P_0} = \left(\frac{V_0}{V2}\right)^{\gamma}$$

I.e. $V2 = V_0(2)^{1/\gamma}$

Let P3= $P_0/4$ and V3 be the pressure and volume after isothermal process Then,

$$P2V2 = P3V3$$

$$\Rightarrow \frac{P_0}{2}V_0(2)^{1/\gamma} = \frac{P_0}{4}V3$$

$$\Rightarrow V3 = 2^{(1+1/\gamma)}V_0$$

22. Question

A sample of an ideal gas (γ = 1.5) is compressed adiabatically from a volume of 150 cm^3 to 50 cm^3. The initial pressure and the initial temperature are 150 kPa and 300 K. Find

(a) the number of moles of the gas in the sample,

(b) the molar heat capacity at constant volume,

(c) the final pressure and temperature,

(d) the work done by the gas in the process and

(e) the change in internal energy of the gas.

Answer

Given P1 = 150 KPa = 150 × 10^3 Pa, V1 = 150 cm^3 ,V2=50 cm^3, T1 = 300 K

(a)By ideal gas equation,

$PV = nRT$, where P, V and T are the pressure, volume and absolute temperature; n is the number of moles of gas; R is the ideal gas constant

I.e.

$$n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = .009\ moles$$

(b)We know Cp/Cv=γ R=Cp-Cv,

where the molar heat capacity C, at constant pressure, is represented by Cp, at constant volume, the molar heat capacity C is represented by Cv

Now,

$$Cv = \frac{R}{\gamma - 1}$$

$$Cv = \frac{8.3}{0.5} = 16.6\,J$$

(c) Since the process is adiabatic,

PV^γ = const.

I.e.$\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^\gamma$

$$P2 = P1 \times \left(\frac{V1}{V2}\right)^\gamma$$

$$P2 = 150 \times 10^3 \times \left(\frac{150}{50}\right)^{1.5} = 780\ Kpa$$

Also, as the process is adiabatic,

$$T1^\gamma P1^{1-\gamma} = T2^\gamma P2^{1-\gamma}$$

$$\Rightarrow\ T2^\gamma = \frac{T1^\gamma P1^{1-\gamma}}{P2^{1-\gamma}} = 300^{1.5} \times \left(\frac{150}{780}\right)^{-0.5}$$

$$\Rightarrow T2 = 519.74\,K$$

(d) From the first law of thermodynamics,

$dQ = dU + dW$, where dQ is the amount of heat supplied which is zero in an adiabatic process.

i.e.

$dW = -dU$

$dW = -\mathrm{nCvdT}$, where n is the number of moles, Cv is the molar heat capacity at constant volume and dT is the change in temperature

$\mathrm{dW} = -0.009 \times 16.6 \times (520 - 300)$

$dW = -33\ J$

(e)Change in internal energy, dU is

$\mathrm{dU} = \mathrm{nCvdT}$, where n is the number of moles, Cv is the molar heat capacity at constant volume and dT is the change in temperature.

$\mathrm{dU} = 0.009 \times 16.6 \times (520 - 300)$

$dU = 33\ J$

23. Question

Three samples A, B and C of the same gas (γ = 1.5) have equal volumes and temperatures. The volume of each sample is doubled, the process being isothermal for A, adiabatic for B and isobaric for C. If the final pressure is equal for the three samples, find the ratio of the initial pressures.

Answer

Let V_A, VB, VC be the volume of three gases and T_A, TB, TCbe the temperature of A, B, C gas

Given, $T_A=T_B=T_C$, $V_A=V_B=V_C$

Here A is undergoing an isothermal process, where V1= V_A, V2=$2V_A$

Let P1= $P1_A$ and P2= $P2_A$ be the initial and final pressures,

Then,

$P1V1 = P2V2$

$\Rightarrow P1_A V_A = P2_A(2V_A)$

$\Rightarrow P2_A = P1_A/2$

Here B is adiabatic,

PV^{γ} = const, where $V1 = V_B$, $V2 = 2V_B$

Let $P1 = P1_B$ and $P2 = P2_B$ be the initial and final pressures,

$$\text{I.e.} \frac{P2}{P1} = \left(\frac{V_B}{2V_B}\right)^{\gamma}$$

$$\frac{P2_B}{P1_B} = \left(\frac{1}{2}\right)^{1.5}$$

$$P2_B = P1_B \left(\frac{1}{2}\right)^{1.5}$$

Here C is isobaric ,the pressure remains constant and equal to $P1_C$

Now, as the final pressures are equal for all the gases

$$\frac{P1_A}{2} = P1_B \left(\frac{1}{2}\right)^{1.5} = P1_C$$

$P1_B : P1_B : P1_C = 2 : 2^{1.5} : 1$, ratio of the initial pressures

24. Question

Two samples A and B of the same gas have equal volumes and pressures. The gas in sample A is expanded isothermally to double its volume and the gas in B is expanded adiabatically to double its volume. If the work done by the gas is the same for the two cases, show that γ satisfies the equation $1 - 2^{1-\gamma} = (\gamma - 1) \ln 2$.

Answer

Let P1 = Initial Pressure, V1 = Initial Volume, P2 = Final Pressure, V2 = Final Volume

Here A is expanded isothermally,

I.e. the work done,

$$W_A = nRT1 \ln\left(\frac{V2}{V1}\right)$$

Also, B is expanded adiabatically, i.e.

$$W_B = \frac{P1V1 - P2V2}{\gamma - 1}$$

Given $W_A = W_B$

i.e.

$$nRT1\ln\left(\frac{V2}{V1}\right) = \frac{P1V1 - P2V2}{\gamma - 1}\ (1)$$

In an adiabatic process,

PV^{γ} = const,

I.e. $\frac{P2}{P1} = \left(\frac{V1}{2V1}\right)^{\gamma}$

$$P2 = P1\left(\frac{1}{2}\right)^{\gamma}$$

Substituting in (1)

$$nRT1\ln\left(\frac{V2}{V1}\right) = \frac{P1V1 - P2V2}{\gamma - 1}$$

$$nRT1\ln(2) = \frac{P1V1(1 - \frac{1}{2^{\gamma}} \times 2)}{\gamma - 1}$$

We know, PV= nRT by ideal gas equation

i.e.

$$\ln(2) = \frac{(1 - \frac{1}{2^{\gamma}} \times 2)}{\gamma - 1}$$

$\ln 2(\gamma - 1) = 1 - 2^{1-\gamma}$, the required relation

25. Question

1 litre of an ideal gas (γ = 1.5) at 300 K is suddenly compressed to half its original volume.

(a) Find the ratio of the final pressure to the initial pressure.

(b) If the original pressure is 100 kPa, find the work done by the gas in the process.

(c) What is the change in internal energy?

(d) What is the final temperature?

(e) The gas is now cooled to 300 K keeping its pressure constant.

Calculate the work done during the process.

(f) The gas is now expanded isothermally to achieve its original volume of 1 litre. Calculate the work done by the gas.

(g) Calculate the total work done in the cycle.

Answer

Given γ = 1.5 T=300 K, initial volume V1=1 L, Final volume V2=1/2 L. Let P1 and P2 be the initial and final pressures

(a)Here the process is adiabatic since volume is changed suddenly,

i.e.

$$\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma}$$

$$P2 = P1 \times (2)^{\gamma}$$

$$\frac{P2}{P1} = (2)^{1.5}$$

(b)P1=100 KPa =10^5 Pa ,P2=$2^{1.5}(10^5)$ KPa

Work done in adiabatic process,

$$W = \frac{P1V1 - P2V2}{\gamma - 1}$$

$$W = \frac{10^5 \times 10^{-3} - 2^{1.5}(10^5)500 \times 10^{-3}}{1.5 - 1} = -82\,J$$

(c) Here dQ=0, as it an adiabatic process

By first law of thermodynamics,

$$dQ = dU + dW$$

i.e. $dU = -dW = 82\,J$

(d)For an adiabatic process, let TI and T2 be initial and final temperature

$TV^{\gamma-1}$=constant

I.e.

$$T1V1^{\gamma-1} = T2V2^{\gamma-1}$$

$$\Rightarrow \frac{T2}{T1} = \left(\frac{V1}{V2}\right)^{\gamma-1}$$

$$\Rightarrow T2 = T1\left(\frac{V1}{V2}\right)^{1.5-1} = 300 \times 2^{0.5}$$

$$\Rightarrow T2 = 424\,K$$

(e)Here the pressure is kept constant, i.e. isobaric

Work done in an isobaric process , $W = P \triangle V = nRdT$

Here, $n = \frac{PV}{RT} = \frac{(10^5 \times 10^{-3})}{R \times 300} = \frac{1}{3R}$

Work done, $W = nRdT = \frac{1}{3R} R(300 - 424) = -41.4\, J$

(f)Here the process is isothermal.

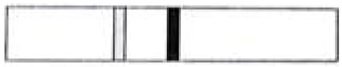
Work done, $W = nRTln\left(\frac{V2}{V1}\right) = \frac{1}{3R} \times R \times \ln(2)$

$W = 103\, J$

(g)Work done in the cycle,$W_{total} = -82 - 41.4 + 103 = -20.4\, J$

26. Question

Figure shows a cylindrical tube with adiabatic walls and fitted with an adiabatic separator. The separator can be slid into the tube by an external mechanism. An ideal gas (γ= 1.5) is injected in the two aides at equal pressures and temperatures. The separator remains in equilibrium at the middle. It is now slid to a position where it divides the tube in the ratio 1 : 3. Find the ratio of the temperatures in the two parts of the vessel.



Answer

Given:

The walls of the cylindrical tube and the separator are made with

adiabatic material. The separator can be slid in the tube by

external mechanism.

An ideal gas of $\gamma = 1.5$ is injected in the two aides of at equal pressure.

It is now slid to a position where it divides tube in the ratio 1:3.

The initial volume of the two aides are equal let's say V/2,

Where, the total volume of the tube is V.

Now say the, left part of tube has V/4 volume and the right side has 3V/4 volume so that the ratio between them is 1:3.

In adiabatic process, $PV^{\gamma} = K$ (K = non zero constant)

Where P is the pressure of the gas and V is the volume and $\gamma = \frac{C_p}{C_v}$

For ideal gas, $PV = nRT$

Where P is the pressure, V is the volume, T is the temperature of the gas and R is the gas constant and n is the number of moles of the gas.

Putting this in the adiabatic process condition we get,

$TV^{\gamma-1} = K'$ (K' is a non-zero constant)

Therefore, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\Rightarrow T_1\left(\frac{V}{2}\right)^{(1.5-1)} = T_2\left(\frac{V}{4}\right)^{(1.5-1)}$

$\Rightarrow T_1\left(\frac{V}{2}\right)^{(0.5)} = T_2\left(\frac{V}{4}\right)^{(0.5)}$

$\Rightarrow T_1\left(\frac{1}{2}\right)^{(0.5)} = T_2\left(\frac{1}{4}\right)^{(0.5)}$

$\Rightarrow T_1\left(\frac{1}{2}\right)^{(0.5)} = T_2\left(\frac{1}{2}\right)$

$\Rightarrow T_2 = \sqrt{2}T_1$

Again for the other part of the tube,

$\Rightarrow T_1'\left(\frac{V}{2}\right)^{(1.5-1)} = T_2'\left(\frac{3V}{4}\right)^{(1.5-1)}$

$\Rightarrow T_1'\left(\frac{V}{2}\right)^{(0.5)} = T_2'\left(\frac{3V}{4}\right)^{(0.5)}$

$\Rightarrow T_1'\left(\frac{1}{2}\right)^{(0.5)} = T_2'\left(\frac{V}{4}\right)^{(0.5)}$

$\Rightarrow T'_2 = \sqrt{\frac{2}{3}}T'_1$

As initially the gases were at the same pressure and volume, the temperatures would be the same as well.

Therefore, $T_1 = T_1'$

Therefore, , $T_2 = T_2' = \sqrt{2} : \sqrt{\frac{2}{3}} = \sqrt{3} : 1$

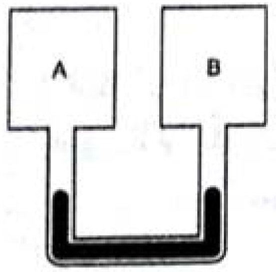
Therefore the ratio of the final temperatures will be $\sqrt{3} : 1$

27. Question

Figure shows two rigid vessels A and B, each of volume 200 cm^3 containing an ideal gas (C_V = 12.5 J K^{-1} mol^{-1}). The vessels are connected to a manometer tube containing mercury. The pressure in both the vessels is 75 cm of mercury and the temperature is 300 K.

(a) Find the number of moles of the gas in each vessel.

(b) 5.0 J of heat is supplied to the gas in the vessel A and 10 J to the gas in the vessel B. Assuming no appreciable transfer of heat from A to B calculate the difference in the heights of mercury in the two sides of the manometer. Gas constant R = 8.3 J K^{-1} mol^{-1}.



Answer

Given:

Gasses in both the vessels are at pressure of 75cm of mercury.

Therefore, $P = 0.75 \times 13600 \times 9.8\ Pa = 99960\ Pa$

The volume of the vessel is $200 cm^3 = 0.0002\ m^3$

The temperature of the gas is 300K

(a) For Ideal gasses,

$PV = nRT$ where P,V and T are the pressure, volume and temperature of the gas, n is the number of moles, and R is the gas constant.

$$\Rightarrow n = \frac{PV}{RT} = \frac{99960 \times 0.0002}{8.314 \times 300} = 0.008$$

Therefore, number of moles = 0.008

(b) The specific heat of the gas at constant volume is C_V = 12.5 J K^{-1} mol^{-1}

Therefore, $nC_V T = Q$

Where n is the number of moles, T is the rise in temperature, Q is the heat given.

Therefore, at constant volume, if we supply 5J and 10J heat to the vessels, the rise of temperature will be $\frac{5}{0.008 \times 12.5} = 50K$ and $\frac{10}{0.008 \times 12.5} = 100K$

So the change in pressure in the vessels will be governed by

$$P = \frac{nRT}{V}$$

So for the first vessel, Change in pressure

$$P = \frac{0.008 \times 8.314 \times 50}{0.0002} = 16628\ Pa$$

For the second vessel $P = \frac{0.008 \times 8.314 \times 100}{0.0002} = 33256 Pa$

Therefore, the difference of pressure of the two vessels is

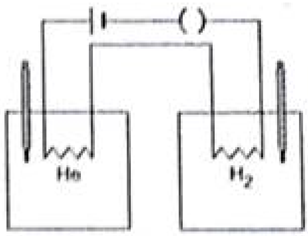
$$33256 - 16628 = 16628 Pa$$

Which is equivalent to $\frac{16628}{13600 \times 9.8} = 0.125m = 12.5cm$ of mercury.

Therefore, the height of the mercury in the manometer tube is 12.5cm

28. Question

Figure shows two vessels with adiabatic walls, one containing 0.1g of helium (γ = 1.67, M = 4 g mol^{-1}) and the other containing some amount of hydrogen (γ= 1.4, M = 2g mol^{-1}). Initially, the temperatures of the two gases are equal. The gases are electrically heated for some time during which equal amounts of heat are given to the two gases. It is found that the temperatures rise through the same amount in the two vessels. Calculate the mass of hydrogen.



Answer

Given:

Two vessels with adiabatic walls, one contains 0.1g of helium (γ = 1.67, M = 4 g mol^{-1}) and the other contains some amount of hydrogen (γ= 1.4, M = 2g mol^{-1})

The gasses are given the same amount of heat.

The temperature rises through the same amount.

0.1g of helium = 0.1/4 mole = 0.025mole

Let there be n moles of hydrogen in the other vessel.

$\gamma = \frac{C_p}{C_v}$ and $C_P - C_V = R$ so, $C_V = \frac{R}{\gamma - 1}$

As the vessels are of constant volume there will be no work done by the gasses. The heat supplied will totally be used to increase internal energy.

Therefore, $Q = nC_vT$ where Q is the heat supplied, n is the number of moles, C_V is the specific heat capacity of gas at constant volume, T is the change in temperature.

For helium, $Q = \frac{0.025 \times 8.314}{1.67^{-1}} T = 0.31\, T$

For hydrogen, $Q = \frac{n \times 8.314}{1.4^{-1}} T = 20.8\, nT$ we assume for both cases the rise of temperature is T.

As per question,

$0.31T = 20.8\, nT$

$\Rightarrow n = \frac{0.31}{20.8}$

$\Rightarrow$ n = 0.015

Again, Molar mass of hydrogen = 2g mol^{-1}

Therefore, 0.015 mole of hydrogen = $0.015 \times 2 = 0.03g$ hydrogen

Thus, there is 0.03g of hydrogen in the vessel.

29. Question

Two vessels A and B of equal volume V_0 are connected by a narrow tube which can be closed by a valve. The vessels are fitted with pistons which can be moved to change the volumes. Initially, the valve is open and the vessels contain an ideal gas ($C_P/C_V = \gamma$) at atmospheric pressure p_0 and atmospheric temperature T_0. The walls of the vessel A are diathermic and those of B are adiabatic. The valve is now closed and the pistons are slowly pulled out to increase the volumes of the vessels to double the original value.

(a) Find the temperatures and pressures in the two vessels.

(b) The valve is now opened for sufficient time so that the gases acquire a common temperature and pressure. Find the new values of the temperature and the pressure.

Answer

Given:

Two vessels A and B of equal volume V_0 are connected by a narrow tube which can be closed by a valve.

Vessels contain an ideal gas ($\frac{C_P}{C_V} = \gamma$) at atmospheric pressure p_0 and atmospheric temperature T_0.

The walls of the vessel A are diathermic and those of B are adiabatic.

The pistons are slowly pulled out to increase the volumes of the vessels to double the original value.

(a) As the pistons are moved slowly to increase the volume, the expansion of gas in the diathermic vessel will be an isothermic process thus the temperature will be fixed at T_0. P,V and T represent the pressure, volume and temperature of the gasses and subscripts 1 and 2 denote initial and final state respectively.

Thus,

$$P_1V_1 = P_2V_2$$

$$\Rightarrow P_0V = 2P_2V$$

$$\Rightarrow P_2 = \frac{P_0}{2}$$

For the adiabatic vessel,

$$P_1{V_1}^{\gamma} = P_2{V_2}^{\gamma}$$

$$\Rightarrow P_0V^{\gamma} = P_2(2V)^{\gamma}$$

$$\Rightarrow P_2 = \frac{P_0}{2^{\gamma}}$$

Again for ideal gasses, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\Rightarrow \frac{P_0V}{T_0} = \frac{2P_0V}{2^{\gamma}T_2}$$

$$\Rightarrow T_2 = \frac{T_0}{2^{\gamma-1}}$$

Thus the temperature and pressure in the diathermic vessel will T_0 and $P_0/2$ and in the adiabatic vessel, $\frac{T_0}{2^{\gamma-1}}$ and $\frac{P_0}{2^{\gamma}}$.

(b) When the valve is open, the temperature will remain T_0 throughout. Thus, there will be no change in temperature in the diathermic vessel so there will be change in pressure as well. For the gas in the diathermic vessel,

$P_1 = \frac{nRT_0}{4V}$ and for the adiabatic vessel $P_2 = \frac{nRT_0}{4V}$

Therefore $P_1 = P_2$

Again, $P_0 = P_1 + P_2$

Thus, $P_1 = P_2 = \frac{P_0}{2}$

Thus the final temperature, when the valve is open will be T_0 and the final pressure will be $\frac{P_0}{2}$.

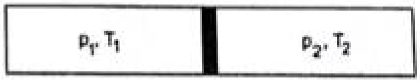
30. Question

Figure shows an adiabatic cylindrical tube of volume V_0 divided in two parts by a frictionless adiabatic separator. Initially, the separator is kept in the middle, an ideal gas at pressure p_1 and temperature T_1 is injected into the left part and another ideal gas at pressure p_2 and temperature T_2 is injected into the right part. $C_P/C_V = \gamma$ is the same for both the gases. The separator is slid slowly and is released at a position where it can stay in equilibrium. Find

(a) the volumes of the two parts,

(b) the heat given to the gas in the left part

(c) the final common pressure of the gases.



Answer

Given:

An adiabatic cylindrical tube of volume V_0 is divided in two parts by a frictionless adiabatic separator.

An ideal gas at pressure p_1 and temperature T_1 is injected into the left part and another ideal gas at pressure p_2 and temperature T_2 is injected into the right part.

(a) When the piston is slowly moved to the equilibrium position, one side increases in volume when the other side decreases.

The processes will be adiabatic,

For the left part,

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$ Where, subscript 1 and 2 represent the initial and the final state.

$\Rightarrow P_1(\frac{V_0}{2})^{\gamma} = PV_2^{\gamma}$..........(1)

And for the right part,

$P_2(\frac{V_0}{2})^{\gamma} = PV_1^{\gamma}$..........(2)

We are assuming P to be the common pressure.

Dividing (1) by (2) we get,

$$\frac{V_2{}^{\gamma}}{V_1{}^{\gamma}} = \frac{P_1}{P_2}$$

Again,

$$V_1 + V_2 = V_0$$

So, $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$

$$\Rightarrow \frac{V_0}{V_1} - 1 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$$

$$\Rightarrow V_1 = \frac{V_0}{1+\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}$$

Therefore, $V_2 = \frac{V_0\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}{1+\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}$

The final volume of the left and the right side will be $\frac{V_0}{1+\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}$ and $\frac{V_0\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}{1+\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}$ respectively.

(b) The heat given will be zero as the whole process is taking place in an adiabatic surrounding.

(c) So putting the above result in (1) we get,

$$P_1\left(\frac{V_0}{2}\right)^{\gamma} = P\left(\frac{V_0\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}{1+\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}}\right)^{\gamma}$$

$$\Rightarrow P = P_2\,\frac{\left(1+\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}\right)^{\gamma}}{2^{\gamma}} = \left(\frac{P_1{}^{\frac{1}{\gamma}}+P_2{}^{\frac{1}{\gamma}}}{2}\right)^{\gamma}$$

Thus the final common pressure of the gasses will be $\left(\frac{P_1{}^{\frac{1}{\gamma}}+P_2{}^{\frac{1}{\gamma}}}{2}\right)^{\gamma}$

31. Question

An adiabatic cylindrical tube of cross-sectional area 1 cm^2 is closed at one end and fitted with a piston at the other end. The tube contains 0.03g of an ideal gas. At 1 atm pressure and at the temperature of the surrounding, the length of the gas column is 40 cm. The piston is suddenly pulled out to double the length of the column. The pressure f the gas falls to 0.355 atm. Find the speed of sound in the gas at atmospheric temperature.

Answer

Given:

An adiabatic cylindrical tube of cross-sectional area 1 cm^2 is closed at one end and fitted with a piston at the other end.

The tube contains 0.03g of an ideal gas at 1 atm pressure and at the temperature of the surrounding.

The length of the gas column is 40 cm.

The piston is suddenly pulled out to double the length of the column and the pressure of the gas falls to 0.355 atm.

The expansion process of the gas is adiabatic, so $P_1 {V_1}^\gamma = P_2 {V_2}^\gamma$ Where, subscript 1 and 2 represent the initial and the final state.

When the length is increased to double the volume is also doubled.

So,

$$1 \times {V_1}^\gamma = P_2 (2V_1)^\gamma$$

$$\Rightarrow 1 = 0.355(2)^\gamma$$

$$\Rightarrow (2)^\gamma = 2.82$$

$$\Rightarrow \gamma = 1.49$$

The speed of sound in gas at atmospheric pressure is given as

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

where γ is the adiabatic constant, P is pressure and ρ is the volume density. The speed is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$v = \sqrt{\frac{1.49 \times 10^5}{\frac{m}{v}}}$$

$$v = \sqrt{\frac{1.49 \times 10^5}{\frac{0.03 \times 10^{-3}}{0.4 \times 10^{-4}}}} \approx 447\frac{m}{s}$$

32. Question

The speed of sound in hydrogen at 0°C is 1280 m s^{-1}. The density of hydrogen at STP is 0.089 kg m^{-3}. Calculate the molar heat capacities C_P and C_V of hydrogen.

Answer

Given:

The speed of sound in hydrogen at 0°C is 1280 m s^{-1}.

The density of hydrogen at STP is 0.089 kg m^{-3}.

At STP the pressure P is 1.013×10^5Pa.

So the speed of sound in hydrogen $v = \sqrt{\frac{\gamma P}{\rho}}$ where ρ is the density of the gas.

So, putting data in, we get, $\gamma = \frac{1280^2 \times 0.089}{1.013 \times 10^5} = 1.44$

So, $C_P = \frac{\gamma R}{\gamma - 1} = \frac{1.44 \times 8.314}{0.44} = 27.21\, J\, K^{-1}\, mol^{-1}$;

$$C_V = \frac{R}{\gamma - 1} = \frac{8.314}{0.44} = 18.89\, J\, K^{-1}\, mol^{-1}$$

33. Question

4.0 g of helium occupies 22400 cm^3 at STP. The specific heat capacity of helium at constant pressure is 5.0 cal K^{-1} mol^{-1}. Calculate the speed of sound in helium at STP.

Answer

Given:

4.0 g of helium occupies 22400 cm^3 at STP.

The specific heat capacity of helium at constant pressure is 5.0 cal K^{-1} mol^{-1} = 21J K^{-1} mol^{-1}

$$C_P = \frac{\gamma R}{\gamma - 1}$$

$$\Rightarrow 21\gamma - 21 = 8.314\gamma$$

$\Rightarrow \gamma = \frac{21}{12.686} = 1.65$

At STP the pressure P is 1.013×10^5Pa.

The velocity of sound will be $v = \sqrt{\frac{\gamma PV}{m}}$ where P is the pressure of the gas, V is the volume and M is the mass of the gas.

Thus putting the values given, we get,

$$v = \sqrt{\frac{1.65 \times 1.013 \times 10^5 \times 22400 \times 10^{-6}}{0.004}} = 969.05\frac{m}{s}$$

The speed of sound in helium is 969.05m/s.

34. Question

An ideal gas having density 1.7×10^{-3} g cm^{-3} at a pressure 1.5×10^5 Pa is filled in a Kundt tube. When the gas is resonated at a frequency of 3.0 kHz, nodes are formed at a separation of 6.0 cm. Calculate the molar heat capacities C_P and C_V of the gas.

Answer

Given:

An ideal gas having density 1.7×10^{-3} g cm^{-3} = 1.7 kg m^{-3} at a pressure 1.5×10^5 Pa is filled in a Kundt tube.

When the gas is resonated at a frequency of 3.0 kHz, nodes are formed at a separation of 6.0 cm.

the node separation is given by $\frac{l}{2}$ which is 6.0cm. Therefore,

$$l = 12cm = 0.12m$$

the frequency of the sound (f) is 3kHz

thus velocity of sound = $v = fl = 360\ m/s$

again,

$v = \sqrt{\frac{\gamma P}{\rho}}$ where P is the pressure and ρ is the density of the gas.

Thus, $\gamma = \frac{360^2 \times 1.7 \times 10^{-3}}{1.5 \times 10^5} = 1.4688$

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{0.4688} = 17.7\ J\ mol^{-1} K^{-1}$$

$$C_P = \frac{\gamma R}{\gamma - 1} = 1.4688 \times C_V = 1.4688 \times 17.07 = 26\, J\, mol^{-1}K^{-1}$$

35. Question

Standing waves of frequency 5.0 kHz are produced in a tube filled with oxygen at 300 K. The separation between the consecutive nodes is 3.3 cm. Calculate the specific heat capacities C_P and C_V of the gas

Answer

Given:

Standing waves of frequency 5.0 kHz are produced in a tube filled with oxygen at 300 K.

The separation between the consecutive nodes is 3.3 cm.

the node separation is given by $\frac{l}{2}$ which is 3.3cm. Therefore,

$$l = 6.6cm = 0.066m$$

the frequency of the sound is 5kHz

thus the velocity of sound will be $v = 5 \times 10^3 \times 0.066 = 330 \frac{m}{s}$

again,

$v = \sqrt{\gamma \frac{RT}{M}}$ where R is the gas constant, T is the temperature of the gas in Kelvin scale and M is the molar mass of the gas.

The molar mass of oxygen is $32 \frac{g}{mole} = \frac{0.032\, kg}{mole}$

Thus putting in the values in the above expression,

$$\gamma = \frac{330^2 \times 0.032}{8.314 \times 300} = 1.397$$

Again, $C_P = \frac{\gamma R}{\gamma - 1} = \frac{1.397 \times 8.314}{0.397} = 29.26\, J\, mol^{-1}K^{-1}$

$$C_v = \frac{R}{\gamma - 1} = \frac{8.314}{0.397} = 20.94\, J\, mol^{-1}K^{-1}$$

Therefore, the C_P and C_V for oxygen are $29.26\, J\, mol^{-1}K^{-1}$and $20.94\, J\, mol^{-1}K^{-1}$ respectively.