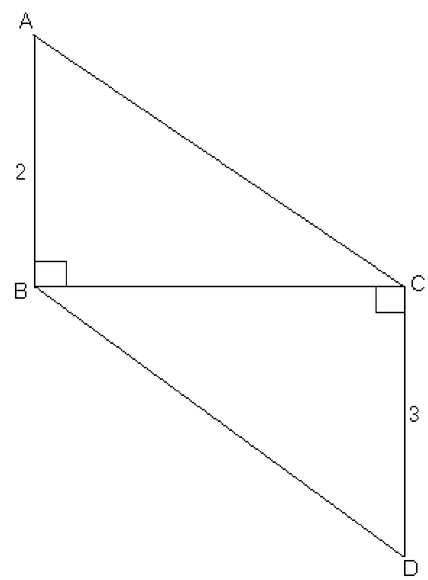
Maharashtra State Board
Class X Mathematics – Geometry
Board Paper – 2015

Time: 2 hours **Total Marks: 40**

Note:- (1) Solve all questions. Draw diagrams wherever necessary.
(2)Use of calculator is not allowed.
(3)Diagram is essential for writing the proof of the theorem.
(4)Marks of constructions should be distinct. They should not be rubbed off.

1. Solve any five sub-questions : **5**

i. In the following figure seg AB ⊥ seg BC, seg DC ⊥ seg BC. If AB = 2 and DC = 3, find $\dfrac{A(\triangle ABC)}{A(\triangle DCB)}$



ii. Find the slope and y-intercept of the line y = -2x + 3.

iii. In the following figure, in ΔABC, BC = 1, AC = 2, ∠B = 90°. Find the value of sin θ.

iv. Find the diagonal of a square whose side is 10 cm.

v. The volume of a cube is 1000 cm^3. Find the side of a cube.

vi. If two circles with radii 5 cm and 3 cm respectively touch internally, find the distance between their centres.

2. **Solve any four sub-questions :** **8**

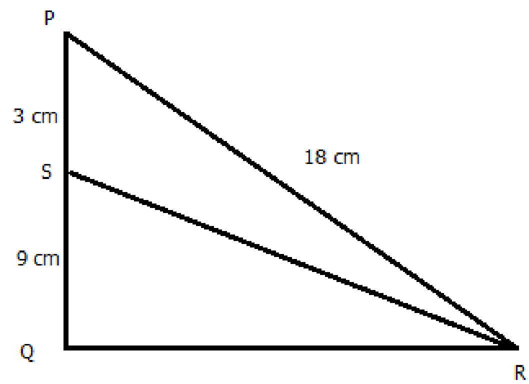
i. If $\sin \Theta = \frac{3}{5}$, where Θ is an acute angle, find the value of $\cos \Theta$.

ii. Draw $\angle ABC$ of measure $105°$ and bisect it.

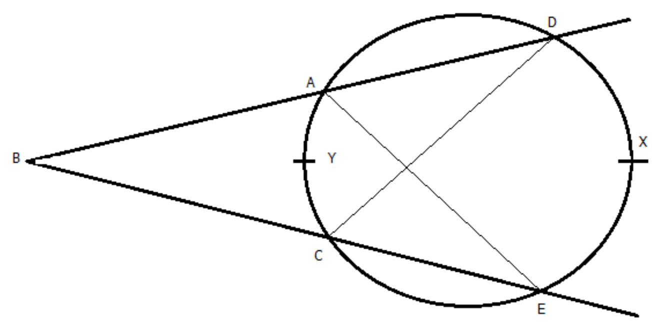
iii. Find the slope of the line passing through the points A(-2, 1) and B(0, 3).

iv. Find the area of the sector whose are length and radius are 8 cm and 3 cm respectively.

v. In the following figure, in Δ PQR, seg RS is the bisector of $\angle PRQ$. PS = 3, SQ = 9, PR = 18. Find QR.



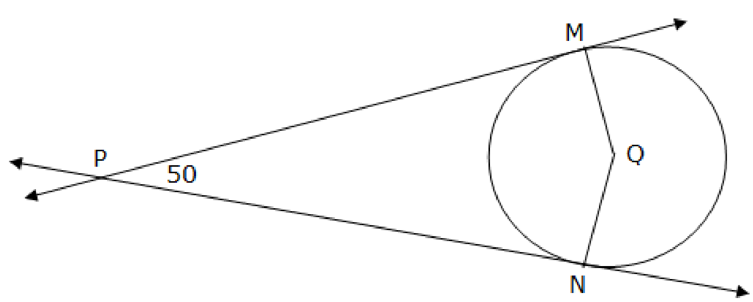
vi. In the following figure, if m(are DXE) = 90° and m(are AYC) = 30°. Find $\angle DBE$.



3. **Solve any three sub-questions :** **9**

i. In the following figure, Q is the centre of a circle and PM, PN are tangent segments to the circle. If $\angle MPN = 50°$, find $\angle MQN$.

ii.



Draw the tangents to the circle from the point L with radius 2.7 cm. Point 'L' is at a distance 6.9 cm from the centre 'M'.

iii. The ratio of the areas of two triangles with the common base is 14 : 9. Height of the larger triangle is 7 cm, then find the corresponding height of the smaller triangle.

iv. Two building are in front of each other on either side of a road of width 10 metres. From the top of the first building which is 40 metres high, the angle of elevation to the top of the second is 45°. What is the height of the second building?

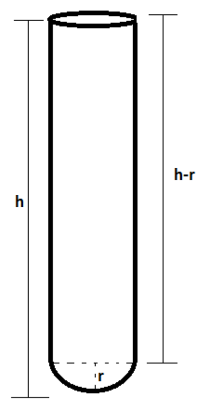
v. Find the volume and surface area of a sphere of radius 2.1 cm. $\left(\pi = \frac{22}{7}\right)$

4. Solve any two sub-questions : **8**

i. Prove that 'the opposite angles of a cyclic quadrilateral are supplementary'.

ii. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta.\cos^2\theta$.

iii. A test tube has diameter 20 mm and height is 15 cm. The lower portion is a hemisphere. Find the capacity of the test tube. ($\pi = 3.14$)



5. Solve any two sub-questions : **10**

i. Prove that the angle bisector of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

ii. Write down the equation of a line whose slope is $\frac{3}{2}$ and which passes through point P, where P divides the line segment AB joining A(-2, 6) and B(3, -4) in the ratio 2 : 3.

iii. $\Delta RST \sim \Delta UAY$, In ΔRST, RS = 6 cm, $\angle S = 50°$, ST = 7.5 cm. The corresponding sides of ΔRST and ΔUAY are in the ratio 5 : 4. Construct Δ UAY.

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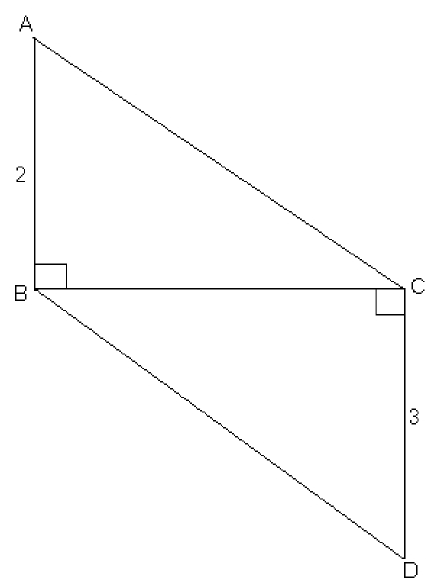
1.

i. In the following figure ΔABC and ΔDCB have a comman base BC.

$$\therefore \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{AB}{DC}$$

$$\left(\begin{array}{l}\because \text{The ratio of areas of two triangles with the same base} \\ \text{is equal to the ratio of their corresponding heights.}\end{array}\right)$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{2}{3}$$



ii. $y = -2x + 3$

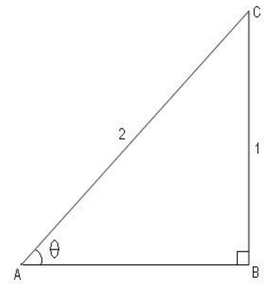
Comparing the equation with,

$y = mx + c$, we get

$\text{slope} = m = -2$

$\text{y-intercept} = c = 3$

iii. In ΔABC, $m\angle B = 90°$,



$\therefore \sin\theta = \dfrac{\text{Opposite side}}{\text{Hypotenuse}}$

$\therefore \sin\theta = \dfrac{1}{2}$

iv. Side of a square = 10 cm.

Diagonal of a square $= \sqrt{2} \times$ side of a square

$= \sqrt{2} \times 10$

$= 10\sqrt{2}$

$\therefore$ The diagonal of a square is $10\sqrt{2}$ cm

2.

i. $\sin \boldsymbol{\theta} = \dfrac{3}{5}$

We know $\sin \boldsymbol{\theta} = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{p}{h}$

$\therefore \dfrac{p}{h} = \dfrac{3}{5}$ $\quad [\because \text{Opposite} = \text{Perpendicular} = p]$

p = 3k, h = 5k

Let the adjacent (base) side be b.

Thus, b = $\sqrt{(5k)^2 - (3k)^2} = 4k$

$\cos \boldsymbol{\theta} = \dfrac{4k}{5k} = \dfrac{4}{5}$

ii. Steps for construction:

(a) Draw BC = 5 cm (any)

(b) Place the centre of the protractor on B along the base line BC

(c) Using protractor construct $m\angle ABC = 105°$.

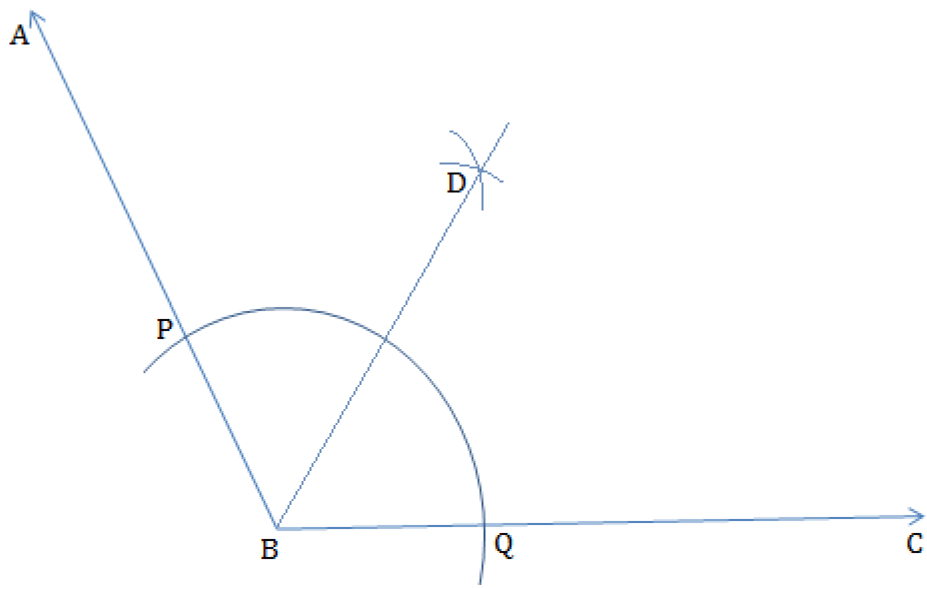
(d)Draw an arc with centre at B and any fixed radius.

(e) Mark the points of intersection of the arc with the arms as P and Q.

(f) Keeping the same radius and with centres P and Q, draw arcs.

(g) Mark the point of intersection of the arcs as D

(h) Join BD. BD is the angle bisector of $\angle ABC$.



iii. Slope of a line passing through 2 points (x_1, y_1) and (x_1, y_1) = $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

Slope of a line passing through 2 points (-2, 1) and (0, 3) = $\left(\frac{3-1}{0+2}\right) = \frac{2}{2} = 1$

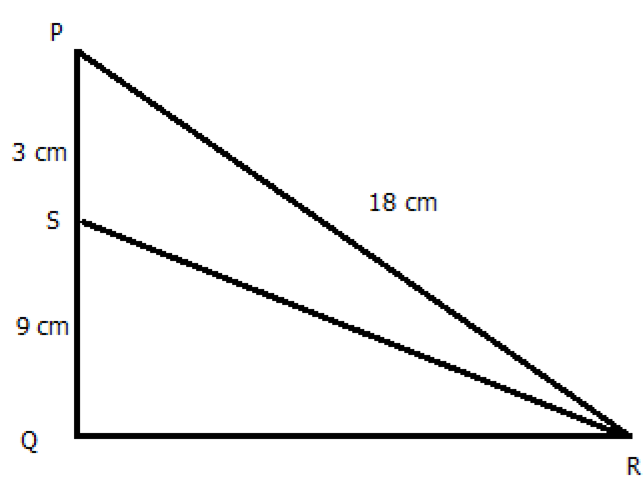
iv. Length of arc, S = 8 cm
Radius of circle, r = 3 cm

S= r$\boldsymbol{\theta} \Rightarrow \boldsymbol{\theta}$ = $\frac{S}{r} = \frac{8}{3}$ radians

$\frac{8}{3}\text{radians} = \frac{180}{\pi} \times \frac{8}{3} = \left(\frac{480}{\pi}\right)^{\circ}$

Area of the sector = $\frac{\theta}{360} \times \pi r^2 = \frac{480}{360 \times \pi} \times \pi r^2 = \frac{4}{3} \times 9 = 12 \text{ cm}^2$

v.



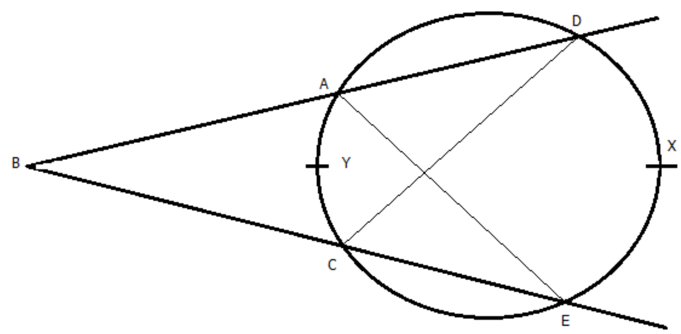
SR is the bisector of $\angle R$.

$$\frac{RP}{PS}=\frac{QR}{SQ}$$

$$\Rightarrow \frac{18}{3}=\frac{QR}{9}$$

$$\therefore QR=54 \text{ cm}$$

vi.



By inscribed angle theorem,

$$m\angle AEB=\frac{1}{2}\times m\angle AYC=\frac{1}{2}\times 30^\circ=15^\circ....(1)$$

$$m\angle EAD=\frac{1}{2}\times m\angle DXE=\frac{1}{2}\times 90^\circ=45^\circ......(2)$$

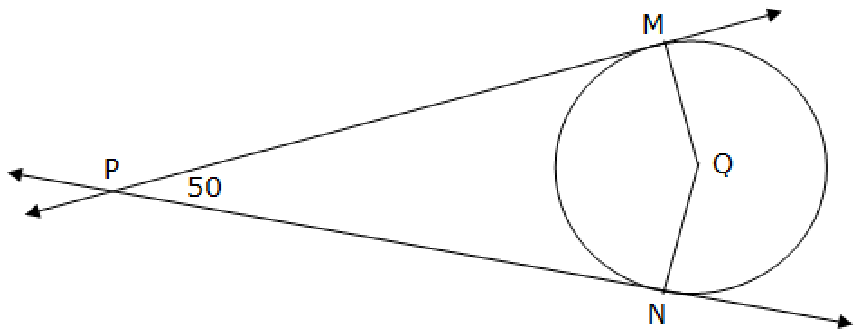
$$\angle DBE+\angle AEB=\angle EAD$$

$$\Rightarrow m\angle DBE+15^\circ=45^\circ$$

$$\Rightarrow m\angle DBE=45^\circ-15^\circ=30^\circ$$

3.

i.



Seg PM and seg PN are tangents to the circle and seg QM and seg QN are the radii from the points of contacts.

$m\angle PMQ = m\angle PNQ = 90°$ ... (Tangent is perpendicular to the radius) ... (1)

The sum of the measures of the angles of a quadrilateral is 360°.

$m\angle MPN + m\angle PMQ + m\angle MQN + m\angle PNQ = 360°$

$50° + 90° + m\angle MQN + 90° = 360°$

$230° + m\angle MQN = 360°$

$m\angle MQN = 360° - 230° = 130°$ ... [From (1)]

ii. Steps of construction:

Construct a circle with centre M and radius 2.7 cm.

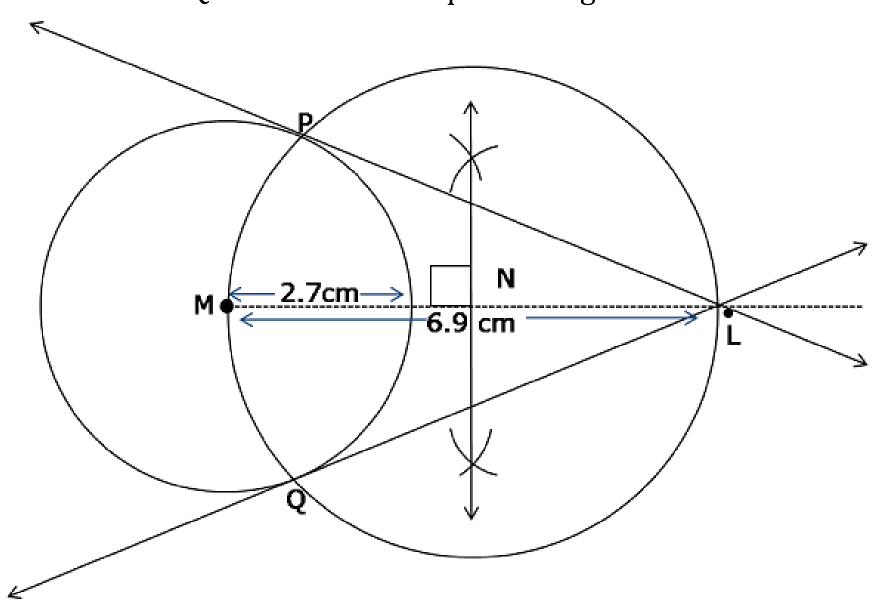
Take point L such that ML = 6.9 cm.

Obtain midpoint N of segment ML.

Draw a circle with centre N and radius NM.

Let P and Q be the points of intersection of these two circles.

Draw lines LP and LQ which are the required tangents.



iii. Let the height of the larger triangle be h_1 and that of the smaller triangle be h_2.
The ratio of the areas of two triangles with a common base is equal to the ratio of their corresponding heights.

$$\frac{\text{Area(larger Triangle)}}{\text{Area(smallerTriangle)}}=\frac{h_1}{h_2}$$

$$\frac{14}{9}=\frac{7}{h_2}$$

$$14\times h_2=9\times 7$$

$$\therefore h_2=\frac{9\times 7}{14}=\frac{9}{2}$$

$$\therefore h_2=4.5\text{ cm}$$

The corresponding height of the smaller triangle is 4.5 cm

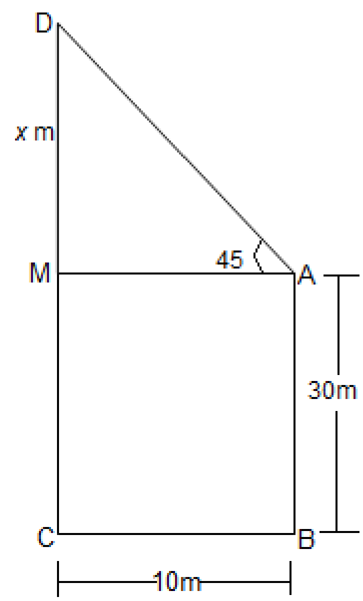
iv. Let AB and CD represent two buildings. AB = 30 m, BC is the width of the road.
BC = 10 m
m∠MAD = 45° ---- (angle of elevation)
ABCM is a rectangle.
AM = BC = 10 m ---(1)
AB = MC = 30 m ---(2)
Let MD = x,
Then in right angled ΔAMC,

$$\tan\angle MAD=\tan 45^\circ=\frac{MD}{MA}$$

$$\therefore 1=\frac{x}{10}$$

$$\therefore x=10$$

Now,
CD = CM + MD = 30 + 10 = 40 m.
Thus the height of the second building is 40 m.



v. Radius of the sphere = 2.1 cm.

Surface Area of Sphere = $4\pi r^2 = 4\times\frac{22}{7}\times(2.1)\times(2.1)=55.44\text{ cm}^2$

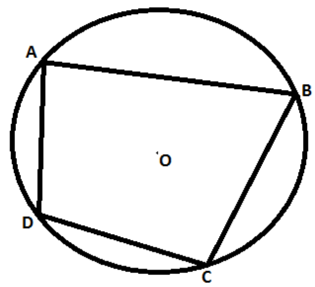
Volume of sphere = $\frac{4}{3}\pi r^3=\frac{4}{3}\times\frac{22}{7}\times(2.1)(2.1)(2.1)=38.308\text{ cm}^3$

4.

i. Given: □ABCD is cyclic quadrilateral

To prove: $\angle BAD + \angle BCD = 180^\circ$

and $\angle ABC + \angle ADC = 180^\circ$



Proof :

Arc BCD is intercepted by the inscribed $\angle BAD$.

$\therefore \angle BAD = \frac{1}{2} m(\text{arc BCD})$.....(1)

(Inscribed angle theorem)

Arc BAD is intercepted by the inscribed $\angle BCD$.

$\therefore \angle BCD = \frac{1}{2} m(\text{arc DAB})$.....(2)

(Inscribed angle theorem)

From (1) and (2) we get

$\angle BAD + \angle BCD = \frac{1}{2}\left[m(\text{arc BCD}) + m(\text{arc DAB})\right]$

$= \frac{1}{2} \times 360^\circ$

$= 180^\circ$

Again, as the sum of the measures of angles of a quadrilateral is 360°.

$\therefore \angle ADC + \angle ABC = 360^\circ - \left[\angle BAD + \angle BCD\right]$

$= 360^\circ - 180^\circ$

$= 180^\circ$

Hence the opposite angles of a cyclic quadrilateral are supplementary.

ii. $LHS = \sin^6\theta + \cos^6\theta$

$$= \left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3$$

$$= \left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cdot\cos^2\theta\right)$$

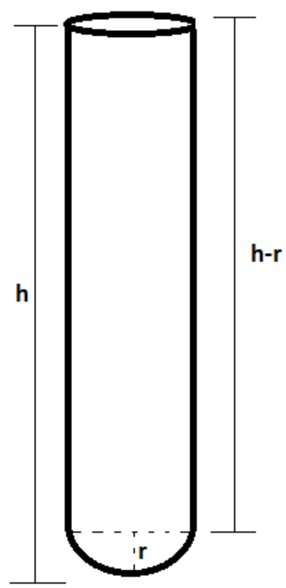
$$= (1)\left[\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cdot\cos^2\theta - \sin^2\theta\cdot\cos^2\theta\right]$$

$$= (1)\left[(1)^2 - 3\sin^2\theta\cdot\cos^2\theta\right]$$

$$= 1 - 3\sin^2\theta\cdot\cos^2\theta$$

$$= RHS$$

iii.



Radius of test tube (r) = $\frac{20}{2}$ = 10 mm = 1 cm

Height of test tube = 15 cm

Upper portion of test tube is cylinder and

lower portion of test tube is hemisphere.

Height of cylinder (H)= h-r = $15 - 1 = 14\ cm$

Volume of test tube = Volume of cylinder +Volume of hemisphere

$$= \pi r^2 H + \frac{2}{3}\pi r^3$$

$$= (3.14)(1)^2(14) + \frac{2}{3}(3.14)(1)^3$$

$$= 43.96 + 2.09$$

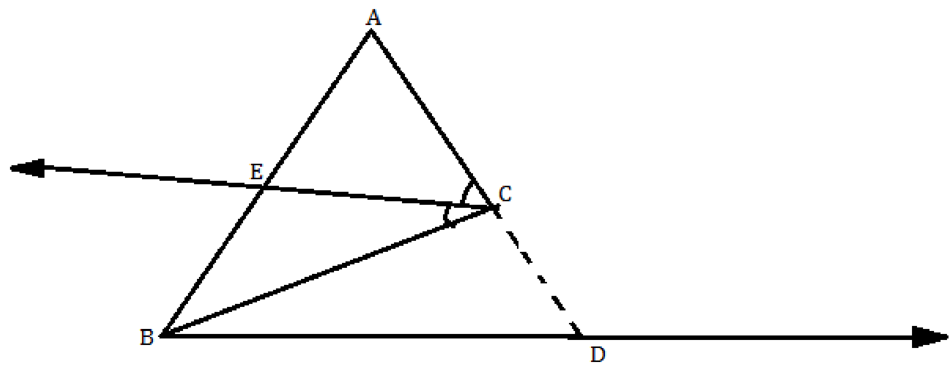
$$= 46.05\ cm^3$$

Capacity of test tube is $46.05\ cm^3$.

5.

i. Consider ΔABC,

Observe the following figure.



CE bisects $\angle ACB$.

Draw a line parallel to ray CE passing through the point B.

Extend AC so as to intersect it at D.

Line CE is parallel to line BD and AD is the transversal.

$\therefore \angle ACE = \angle CDB$ [corresponding angles] ....(1)

Now consider BC as the transversal.

$\therefore \angle ECB = \angle CBD$ [alternate angles] ....(2)

But $\angle ACE = \angle ECB$ [given] ....(3)

$\therefore \angle CBD = \angle CDB$ [from (1), (2) and (3)

In ΔCBD, side CB = side CD [sides opposite to equal angles]

$\therefore CB = CD$ ....(4)

Now in ΔABD, seg $EC \parallel$ side BD [construction]

$\therefore \frac{AE}{EB} = \frac{AC}{CD}$ [B.P.T] ....(5)

$\therefore \frac{AE}{EB} = \frac{AC}{CB}$ [from equations (4) and (5)]

Thus, the angle bisector of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

ii. Suppose that P(x, y) divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n.

Then the co-ordinates of P are given by the formula,

$x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$

$\Rightarrow x = \frac{2(3)+3(-2)}{2+3}$ and $y = \frac{2(-4)+3(6)}{2+3}$

$\Rightarrow x = 0$ and $y = \frac{-8+18}{5}$

$\Rightarrow x = 0$ and $y = \frac{10}{5}$

$\Rightarrow x = 0$ and $y = 2$

Thus $P(x, y) \equiv P(0, 2)$

Now we need to find the equation of the line

whose slope is $m = \frac{3}{2}$ and passing though the point $P(x_1, y_1) \equiv P(0,2)$

is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{3}{2}(x - 0)$

$\Rightarrow 2(y-2) = 3(x-0)$

$\Rightarrow 2y - 4 = 3x$

$\Rightarrow 3x - 2y + 4 = 0$

iii. Given that $\Delta RST \sim \Delta UAY$.

In ΔRST, RS = 6 cm, $m\angle S = 50°$, ST = 7.5 cm.

Given that the corresponding sides of ΔRST and ΔUAY are in the ratio 5 : 4.

$\therefore \frac{RS}{UA} = \frac{ST}{AY} = \frac{RT}{UY} = \frac{5}{4}$;

$\angle S = \angle A = 50^\circ$

$\therefore \frac{RS}{UA} = \frac{5}{4}$

$\therefore \frac{6}{UA} = \frac{5}{4}$

$\therefore \frac{6 \times 4}{5} = UA$

$\therefore UA = 4.8$ cm

Similarly,

$\frac{ST}{AY} = \frac{5}{4}$;

$\therefore \frac{7.5}{AY} = \frac{5}{4}$

$\therefore \frac{7.5 \times 4}{5} = AY$

$\therefore AY = 6 \text{ cm}$

Therefore, In $\triangle UAY$, UA = 4.8 cm, AY = 6 cm and $m\angle A = 50^{\circ}$