CBSE Class 12 - Mathematics
Sample Paper 09

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. From the matrix equation AB = AC we can conclude B = C, provided
   a. A is symmetric matrix
   b. A is singular matrix
   c. A is square matrix
   d. A is non-singular matrix
2. If $\Delta = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$, then equals

a. –abc

b. 2abc

c. 0

d. None of these

3. The derivative of f(x) = | x | at x = 0 is

a. 1

b. – 1

c. All of these

d. None of these

4. If x = at^2 ,y = 2 a t, then $\frac{d^2y}{dx^2}$ is equal to

a. None of these

b. 0

c. $\frac{1}{t^2}$

d. $-\frac{1}{2a\ t^3}$

5. Determine order and degree (if defined) of $\frac{d^4y}{dx^4}$ +sin(y’’’) = 0

a. 2, degree undefined

b. 1, degree undefined

c. 4, degree undefined

d. 3, degree undefined

6. $\cos 2\theta$ is not equal to

a. $1 - 2\sin^2\theta$

b. $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

c. $2\cos^2\theta - 1$

d. $\frac{1+\tan^2\theta}{1-\tan^2\theta}$

7. If P(A) = $\frac{1}{2}$, P(B) = 0, then P(A|B) is

a. 0

b. not defined

c. $\frac{1}{2}$

d. 1

8. $\int_{-2}^{2} x\,|x|\,dx$is equal to

a. -2

b. $\frac{8}{3}$

c. $\frac{-8}{3}$

d. 0

9. Cartesian equation of a plane that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1$and $A_2x + B_2y + C_2z + D_2 = 0$ is

a. $(A_1x + B_1y - C_1z + D_1) + \lambda\,(A_2x + B_2y + C_2z + D_2) = 0.$

b. $(-A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0.$

c. $(A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0.$

d. $(A_1x - B_1y + C_1z + D_1) + \lambda\,(A_2x + B_2y + C_2z + D_2) = 0.$

10. Coinitial Vectors are

a. Two or more pseudo vectors having the same initial point

b. Two or more vectors having the same final point

c. Two or more vectors having the same initial point

d. Two or more force vectors having the same initial point

11. Fill in the blanks:

    A function is called an onto function, if its range is equal to ________.

12. Fill in the blanks:

    If A and B be two events and P(A | B) = P (A), then A is ________ of B.

13. Fill in the blanks:

    If A is a symmetric matrix, then A^3 is a ________ matrix.

14. Fill in the blanks:

    The indefinite integral of $x^2 + 7$ is ________.

OR

Fill in the blanks:

The indefinite integral of $x^2 + 7$ is ________.

15. Fill in the blanks:

    The intermediate solutions of constraints must be checked by substituting them back into ________ equations.

OR

Fill in the blanks:

A problem which seeks to maximise or minimise a function is called an ________ problem.

16. Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

17. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

18. Evaluate $\int_0^2 \sqrt{4-x^2}dx.$

OR

If $f(a+b-x) = f(x)$ then $\int_a^b (x)f(x)dx =$

19. Find the area of the region enclosed by the lines y=x,x=e,the curve y= $\frac{1}{x}$ and the positive x-axis.

20. Find a unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Section B

21. Consider f : N $\rightarrow$ N , g : N$\rightarrow$ N and h :N$\rightarrow$R defined as f (x) = 2x, g (x) = 3y + 4 and h (z) = sin z for all x, y, z $\in$ N. Show that ho (gof) = (hog) of.

22. Differentiate $\sin^2$x w.r. to $e^{\cos x}$

OR

Find $\frac{dy}{dx}$ when x and y are connected by the relation given $\sin(xy) + \frac{x}{y} = x^2 - y$

23. Find $\vec{a} \times \vec{b}$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

24. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

OR

Find the slope of the normal to the curve x = 1 - a$\sin\theta$, y = b$\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

25. Find distance of a point (2,5,-3) from the plane $\vec{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$

26. Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.

Section C

27. Find the value of the following: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

28. Find $\frac{dy}{dx}$, $y=(\sin x-\cos x)^{(\sin x-\cos x)}$

OR

If x cos (a+ y) =cos y, then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$. Hence, show that $\sin a\frac{d^2y}{dx^2}+\sin 2(a+y)\frac{dy}{dx}=0.$

29. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the probability distribution of the random variable X and hence find the mean of the distribution.

30. Solve the following problem graphically minimize or maximize Z = 3x + 9y
Subject to the constraints :
$x+3y\leq 60$
$x+y\geq 10$
$x\leq y$
$x\geq 0, y\geq 0$

31. Solve the following differential equation.
$\frac{dy}{dx}+y=\cos x-\sin x$

OR

Solve the following differential equation.
$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x; x\neq 0$

32. Find $\int\frac{\log|x|}{(x+1)^2}dx.$

Section D

33. Solve by matrix method

x – y + z = 4

$2x + y - 3z = 0$

$x + y + z = 2$

OR

If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A + I)$, where I is 3×3 unit matrix.

34. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

35. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts.

OR

Using integration, find the area of the region bounded by the lines 3x - y - 3 = 0, 2x + y - 12 = 0, x - 2y - 1 = 0.

36. Find the vector equation of the plane passing through the intersection of planes $\overline{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\overrightarrow{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5$ and the point (1,1,1).

Solution

Section A

1. (d) A is non-singular matrix

   Explanation:

   Here, only non- singular matrices obey cancellation laws.

2. (c) 0

   Explanation:

   Because, the determinant of a skew symmetric matrix of odd order is always zero and of even order is a non zero perfect square.

3. (d) None of these

   Explanation:

   $f'(x) = \frac{d}{dx}(|x|) = \frac{x}{|x|}$,which does not exist at x = 0 .

4. (d)

   $-\frac{1}{2a\ t^3}$

   Explanation:

   $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t} \ldots\ldots.(1) \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = -\frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{t^2}.\frac{1}{2at} = -\frac{1}{2at^3}$$

5. (c) 4, degree undefined

   Explanation:

   Order = 4, degree not defined , because the function y’’’ present in the angle of sine function.

6. (d)

$\frac{1+\tan^2\theta}{1-\tan^2\theta}$

Explanation:

$\cos 2\theta$ is equals to $cos(\theta+\theta) = cos\theta.cos\theta - sin\theta.sin\theta = \cos^2\theta - \sin^2\theta$

$= \frac{cos^2\theta - sin^2\theta}{cos^2\theta + sin^2\theta} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

7. (b) not defined

   Explanation:

   We know that :
   $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{0}$

   which is not defined

8. (d) 0

   Explanation:

   f(x) = x|x| is an odd function as :

   $f(-x) = (-x)\,|-x| = -x\,|x| = -f(x)$

   $\Rightarrow \int\limits_{-2}^{2} x\,|x|\,dx = 0$

9. (c)

   $(A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2) = 0.$

   Explanation:

   In Cartesian coordinate system :
   Cartesian equation of a plane that passes through the intersection of two given plane $(A_1x + B_1y + C_1z + D_1) + (A_2x + B_2y + C_2z + D_2)$ = 0 is given by :

   $(A_1x + B_1y - C_1z + D_1) + \lambda\,(A_2x + B_2y + C_2z + D_2) = 0.$

10. (c) Two or more vectors having the same initial point

    Explanation:

Two vectors whose initial point is same are called co- initial vectors.

11. codomain

12. Independent

13. symmetric

14. $\frac{1}{3}x^3 + 7x + c$

    OR

    $\frac{1}{3}x^3 + 7x + c$

15. constraint

OR

    optimisation

16. According to the question,we have to evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

    Now, $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix} = 2[35 - (20)]$
    $= 2 \times 15 = 30$

17. According to the question, equation of line is
    $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$
    It can be rewritten as
    $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$
    Here, Direction ratios of the line are (-2, 6, -3).
    :. Direction cosines of the line are
    $\frac{-2}{\sqrt{(-2)^2+6^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+6^2+(-3)^2}}$
    $\frac{-3}{\sqrt{(-2)^2+6^2+(-3)^2}}$ i.e. $\frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}$ and $\frac{-3}{\sqrt{49}}$
    Thus, Direction cosines of line are $\left(-\frac{2}{7}, \frac{6}{7}, -\frac{3}{7}\right)$

18. According to the question , $I=\int_0^2\sqrt{4-x^2}dx$

$=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$
$\left[\because\int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c\right]$
$=0+2\sin^{-1}1-0-0$

$=2\sin^{-1}\left(\sin\frac{\pi}{2}\right)$

$=2\times\frac{\pi}{2}=\pi[\because\sin^{-1}(\sin\ x)=x]$

OR

$I=\int_a^b(x).f(x)dx$ ...(1)

$I=\int_a^b(a+b-x).f(a+b-x)dx$

$I=\int_a^b(a+b-x).f(x)dx\ [\because f(a+b-x)=f(x)]$

$=\int_a^b[(a+b).\,f(x)-xf(x)]dx$

$=\int_a^b(a+b)f(x)dx-\int_a^b xf(x)dx$

$I=(a+b)\int_a^b f(x)dx-I$

$I=\frac{(a+b)}{2}\int_a^b f(x)dx$

19. The point of intersection of y = x and y = 1/x is (1,1) .Also the meeting point of x = e and y = 1/x is (e, 1/e). The limits on the x axis for the required shaded region extends from 1 to e split as follows.

0-1 below the line y = x and 1-e below the curve y = 1/x. Hence,the required area = $\int_0^1 xdx+\int_1^e\frac{1}{x}\ dx$

$=\frac{1}{2}+1$

$=\frac{3}{2}$ square units.

20. Given vector $\vec{a}$ = $2\hat{i}$ - $3\hat{j}$ + $6\hat{k}$
$\Rightarrow|\vec{a}|$ = $\sqrt{4+9+36}$ = 7

$\therefore \hat{a}$ = $\frac{\vec{a}}{|a|}$ = $\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

$\Rightarrow \hat{a}$ = Unit vector in direction of $\vec{a}$

= $\frac{2}{7}\hat{i}$ - $\frac{3}{7}\hat{j}$ + $\frac{6}{7}\hat{k}$

Section B

21. We have,

ho(gof) (x) = h(gof(x)) = h(g(f(x)))

= h(g(2x)) = h(3(2x) + 4)

= h(6x + 4) = sin (6x + 4) $\forall$x $\in$N

((hog(of) (x) = (hog) (f(x)) = (hog) (2x)

= h(g(2x)) = h(3(2x) + 4)

= h(6x + 4) = sin(6x + 4) $\forall$x $\in$N

This shows, ho(gof) = (hog) of.

22. u = $\sin^2$x

$\frac{du}{dx} = 2\sin x.\cos x$

v = $e^{\cos x}$

$\frac{dv}{dx} = e^{\cos x}(-\sin x)$

$\frac{du}{dv} = \frac{2\sin x.\cos x}{-e^{\cos x}.\sin x}$

$= -\frac{2\cos x}{e^{\cos x}}$

OR

We have, $\sin(xy) + \frac{x}{y} = x^2 - y$

On differentiating both sides w.r.t. x, we get

$\frac{d}{dx}(\sin xy) + \frac{d}{dx}\left(\frac{x}{y}\right) = \frac{d}{dx}x^2 - \frac{d}{dx}y$

$\Rightarrow \cos xy.\frac{d}{dx}(xy) + \frac{y\frac{d}{dx}x - x.\frac{d}{dx}y}{y^2} = 2x - \frac{dy}{dx}$

$$\Rightarrow \cos xy\left[x.\frac{d}{dx}y+y\frac{d}{dx}.x\right]+\frac{y-x\frac{dy}{dx}}{y^2}=2x-\frac{dy}{dx}$$

$$\Rightarrow x\cos xy\frac{dy}{dx}+y\cos xy+\frac{y}{y^2}-\frac{x}{y^2}\frac{dy}{dx}=2x-\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left[x\cos xy-\frac{x}{y^2}+1\right]=2x-y\cos xy-\frac{1}{y}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{xy^2cosxy-x+y^2}{y^2}\right]=\left[\frac{2xy-y^2cosxy-1}{y}\right]$$

$$=\frac{\left(2xy-y^2\cos xy-1\right)y}{\left(xy^2\cos xy-x+y^2\right)}$$

23. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 3\\ 3 & 5 & -2\end{vmatrix}$

$$=\hat{i}(-2-15)-\hat{j}(-4-9)+\hat{k}(10-3)$$

$$=-17\hat{i}+13\hat{j}+7\hat{k}$$

24. Let r be the radius of the sphere.

$\therefore$ Surface area of the sphere (S) $=4\pi r^2$

$\therefore \frac{dS}{dr}=8\pi r$

$\Rightarrow dS=8\pi r\ dr$

$\Rightarrow dS=8\pi r\,\Delta r$

According to question r = 9 m and Δr =.03 m

$=2.16\pi$ square meters

OR

Given: Equations of the curves are x = 1 - asin θ and y = bcos$^2\theta$

$\therefore \frac{dx}{d\theta}=0-a\cos\theta$ and $\frac{dy}{d\theta}=b\frac{d}{d\theta}(\cos\theta)^2$

$\Rightarrow \frac{dx}{d\theta} = -a\cos\theta$ and $\frac{dy}{d\theta} = b.2\cos\theta\frac{d}{d\theta}\cos\theta = -2b\cos\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2b\cos\theta\sin\theta}{-a\cos\theta}$

$= \frac{2b}{a}\sin\theta$

$\therefore$ Slope of the tangent at $\theta = \frac{\pi}{2}$

$= \frac{2b}{a}\sin\frac{\pi}{2} = \frac{2b}{a}$

And Slope of the normal at $\theta = \frac{\pi}{2}$

$= \frac{-1}{m} = \frac{-1}{\frac{2b}{a}}$

$= \frac{-a}{2b}$

25. $\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

$\overrightarrow{N} = 6\hat{i} - 3\hat{j} + 2\hat{k}, d = 4$

$Required\ distance = \frac{\left|\vec{a}.\vec{N} - d\right|}{\left|\overrightarrow{N}\right|} \quad \left[\because \vec{r}.\vec{N} = d\right]$

$= \frac{\left|\left(2\hat{i}+5\hat{j}-3\hat{k}\right).\left(6\hat{i}-3\hat{j}+2\hat{k}\right)-4\right|}{\left|6\hat{i}-3\hat{j}+2\hat{k}\right|}$

$= \frac{|12-15-6-4|}{\sqrt{36+9+4}} = \frac{13}{7}$

26. On a throw of three dice, we have sample space [n(S)] = 6^3 = 216

Let $E_1$is the event when the sum of numbers on the dice was six and E_2 is the event when three two's occurs.

E^1 = {(1,1,4),(1,2,3),(1,3,2),(1,4,1) ,(2,1,3),(2,2,2),(2,3,1),(3,1,2),(3,2,1),(4,1,1)}

$\Rightarrow n(E_1) = 10$ and $E_2 = \{2, 2, 2\}$

$\Rightarrow n(E_2) = 1$

Also, $(E_1 \cap E_2) = 1$

$$\therefore P\left(\frac{E_2}{E_1}\right) = \frac{P\cdot(E_1 \cap E_2)}{P(E_1)} = \frac{\frac{1}{216}}{\frac{10}{216}} = \frac{1}{10}$$

Section C

27. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\sin^{-1}\sin\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\times\frac{1}{2}\right]$

$= \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$

28. $y = (\sin x - \cos x)^{\sin x - \cos x}$

Taking log both sides

$\log y = \log(\sin x - \cos x)^{\sin x - \cos x}$

$\log y = (\sin x - \cos x).\log(\sin x - \cos x)$

Differentiate both sides w.r.t. x

$\frac{1}{y}\cdot\frac{dy}{dx} = (\sin x - \cos x).\frac{1}{(\sin x - \cos x)}(\cos x + \sin x)$
$+\log(\sin x - \cos x).(\cos x + \sin x)$

$\frac{dy}{dx} = y\left[((\cos x + \sin x)) + \log((\sin x - \cos x)).(\cos x + \sin x)\right]$

OR

Given, x cos(a + y) = cos y

$\Rightarrow \quad x = \frac{\cos y}{\cos(a+y)}$

On differentiating both sides w.r.t y, we get

$\frac{dx}{dy} = \frac{\cos(a+y)\frac{d}{dy}\cos y - \cos y\frac{d}{dy}\cos(a+y)}{\cos^2(a+y)}$ [by using quotient rule of derivative]

$= \frac{\cos(a+y)\times(-\sin y) + \cos y \times \sin(a+y)}{\cos^2(a+y)}$

$= \frac{\sin(a+y)\cos y - \cos(a+y)\sin y}{\cos^2(a+y)}$

$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y-y)}{\cos^2(a+y)} = \frac{\sin a}{\cos^2(a+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.........(i)

Again, on differentiating both sides of Eq.(i) w.r.t x, we get

$\frac{d^2y}{dx^2} = \frac{1}{\sin a}\frac{d}{dx}\cos^2(a+y)$

$= \frac{1}{\sin a} \times \frac{d}{dy}\cos^2(a+y) \times \frac{dy}{dx}$

$= \frac{1}{\sin a} \times 2\cos(a+y)[-\sin(a+y)] \times \frac{dy}{dx}$

$= -\frac{2\sin(a+y)\cos(a+y)}{\sin a} \times \frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{\sin 2(a+y)}{\sin a}\frac{dy}{dx}$

$\therefore \quad \sin a\frac{d^2y}{dx^2} + \sin(2a+y)\frac{dy}{dx} = 0$

29. The first six positive integers are 1, 2, 3, 4, 5 and 6.

We can select the two positive numbers in 6 $\times$ 5 = 30 ways.

Out of this 2 numbers are selected at random and X denotes the larger of two numbers.

Since X is the larger of the two numbers, X can take values 2, 3, 4, 5 or 6.

P(X = 2) = P(larger number is 2) = {(1,2) and (2,1)} = $\frac{2}{30}$

P(X = 3) = P(larger number is 3) = {(1,3), (3,1), (2,3), (3,2)} = $\frac{4}{30}$

P(X = 4) = P(larger number is 4) = {(1,4), (4,1), (2,4), (4,2), (3,4), (4,3)} = $\frac{6}{30}$

P(X = 5) = P(larger number is 5) = {(1,5), (5, 1), (2,5), (5,2), (3,5), (5,3), (4,5), (5.4)} = $\frac{8}{30}$

P(X = 6) = P(larger number is 6) = {(1,6), (6,1), (2,6), (6,2), (3,6), (6,3), (4,6), (6,4), (5,6), (6,5)} = $\frac{10}{30}$

Thus, the probability distribution of X is

X	2	3	4	5	6
P(X)	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

Mean [E(X)] = 2 $\times$ $\frac{2}{30}$ + 3 $\times$ $\frac{4}{30}$ + 4 $\times$ $\frac{6}{30}$ + 5 $\times$ $\frac{8}{30}$ + 6 $\times$ $\frac{10}{30}$ = $\frac{14}{3}$

30. The linear inequations or constraints

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$ and objective functions is max or min $(Z) = 3x + 9y$

Reducing the above inequations into equations and finding their point of intersection i.e.,

$x + 3y = 60$ ... (i)

$x + y = 10$ ... (ii)

$x = y$ ... (iii)

$x = 0, y = 0$... (iv)

Equations	Point of Intersection
(i) and (ii)	$x = -15, y = 25$
	Point is $\Rightarrow (-15, 25)$
(i) and (iii)	x = 15 $\Rightarrow$ y = 15
	Point is $\Rightarrow (15, 15)$
(ii) and (iii)	$x = 5, y = 5$
	Point is $(5, 5)$
(i) and (iv)	when x = 0 $\Rightarrow$ y = 20,
	Point is$(0, 20)$
	when y = 0 $\Rightarrow$ x = 60,
	Point is $(60, 0)$
(ii) and (iv)	when x = 0 $\Rightarrow$ y = 10,
	Point is $(0, 10)$
	when y = 0 $\Rightarrow$ x = 10,
	Point is $(10, 0)$

Now for feasible region,

For $x + 3y \leq 60$, putting x = 0 and y = 0, we have

$0 + 0 \geq 10$ i.e., Not true

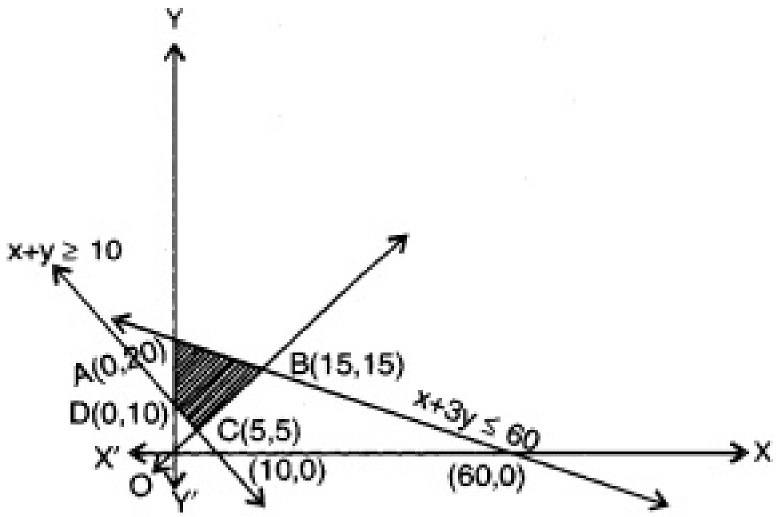
$\Rightarrow$ The shaded region will be away from origin.

Also, we have $x \geq 0$, and $y \geq 0$ indicates that the shaded part will exist in first quadrant only. Here feasible region will be ABCDA, having corner points as $A(0,20), B(15,15), C(5,5)$ and $D(0,10)$.

For optimal point substituting the value of all corner points in objective function $Z=3x+9y$

Corner points	Z	
$A(0,20)$	180	Maximum
$B(15,15)$	180	
$C(5,5)$	60	Maximum
$D(0,10)$	90	

So that the minimum value of Z is 60 at C (5, 5) of the feasible region and the maximum value at A (0, 20) and B(15, 15) is Z = 180.



31. Given differential equation is

$$\frac{dy}{dx}+y=\cos x-\sin x$$

which is a linear differential equation of the form

$$\frac{dy}{dx}+Py=Q$$

Here, P = 1 and Q = cos x - sinx

$$\therefore \quad \text{IF}=e^{\int P dx}=e^{\int 1 dx}=e^{x}$$

We know that the solution of linear differential equation is given by

$$y \times \text{IF}=\int(Q \times \text{ IF }) dx+C$$

$\therefore \quad ye^x = \int e^x(\cos x - \sin x)dx + C$

$\Rightarrow \quad ye^x = e^x \cos x + C$

$[\because \int e^x\left(f(x) + f'(x)\right)dx = e^x f(x) + C$ where $f(x) = \cos x; f'(x) = -\sin x]$

Therefore,on dividing both sides by e^x, we get,

y = cos x + Ce^{-x}

which is the required solution.

OR

Given differential equation is

$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

which is a homogeneous differential equation because $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$.

put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow x\cos v\left[v + x\frac{dv}{dx}\right] = vx\cos v + x$

$vx\cos v + x^2\cos v\frac{dv}{dx} = vx\cos v + x$

$\Rightarrow \quad x^2\cos v\frac{dv}{dx} = x$

$\Rightarrow \quad \cos v dv = \frac{dx}{x}$

On integrating both sides, we get

$\int \cos v dv = \int \frac{dx}{x}$

$\Rightarrow \quad \sin v = \log|x| + C$

$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = \log|x| + c\left[\text{ put } v = \frac{y}{x}\right]$

This is the required solution of given differential equation.

32. Let $I = \int \frac{\log|x|}{(x+1)^2}dx = \int \log|x| \cdot \frac{1}{(x+1)^2}\,dx$

On applying integration by parts, we get,

$I = \log|x| \cdot \int \frac{dx}{(x+1)^2} - \int \frac{d}{dx}(\log|x|) \cdot \left(\int \frac{dx}{(x+1)^2}\right)dx$

$\left[\int \underset{I}{u}.\underset{II}{v}\,dx = u\int v dx - \int\left(\frac{d}{dx}(u)\int v dx\right)dx\right]$

$= \log|x|\frac{(-1)}{x+1} + \int \frac{1}{x(x+1)}dx$

$= \frac{-\log|x|}{x+1} + I_1$ (say)..........(i)

Consider, $I_1 = \int \frac{dx}{x(x+1)}$

Now, by using partial fraction method,

Let $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$\Rightarrow$ 1 = A(x + 1) + Bx

Therefore,on putting x = 0, we get A = 1

and again putting x = -1, we get B= -1

$\therefore \quad I_1 = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx = \int \frac{1}{x} dx - \int \frac{dx}{x+1}$

= log |x| - log |x + 1| + C ...(ii)

Therefore, from Eqs. (i) and (ii), we get

$I = \frac{-\log|x|}{x+1} + \log|x| - \log|x+1| + C$

$= \frac{-\log|x|}{x+1} + \log\left|\frac{x}{x+1}\right| + C$

$\left[\because \log m - \log n = \log \frac{m}{n}\right]$

Section D

33. $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 10 \neq 0$

Also, $\text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(adJA)$

$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

System of equation can be written is

$X = A^{-1}B$

$$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

x=2. y = -1. z = 1

OR

We have, $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$\therefore A^2 = A.A$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$\therefore A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \text{...(i)}$$

Now, $A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

And $A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix}$ ...(ii)

Thus, we see that A^2 + A = A(A + I) [using Eqs. (i) and (ii)]

34. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\implies \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$

$$\implies \frac{dy}{dx} = \frac{\frac{-2x}{a^2}}{\frac{-2y}{b^2}}$$

$$= \frac{-2x}{a^2} \times \frac{b^2}{-2y}$$

$$= \frac{x_o}{y_o} \cdot \frac{b^2}{a^2}$$

Equation is,

$$y - y_1 = \frac{dy}{dx}(x - x_1) \Rightarrow y - y_0 = \frac{x_o}{y_o}\frac{b^2}{a^2}(x - x_0)$$

$$\implies yy_o a^2 - y_o{}^2 a^2 = xx_o b^2 - x_o{}^2 b^2$$

$$\implies x_o{}^2 b^2 - y_o{}^2 a^2 = xx_o b^2 - yy_o b^2$$

Dividing by a^2b^2

$$\frac{x_o{}^2 b^2}{a^2b^2} - \frac{y_o{}^2 a^2}{a^2b^2} = \frac{xx_o b^2}{a^2b^2} - \frac{yy_o a^2}{a^2b^2}$$

$$\implies \frac{x_o{}^2}{a^2} - \frac{y_o{}^2}{b^2} = \frac{xx_o}{a^2} - \frac{yy_o}{b^2}$$

$$1 = \frac{xx_o}{a^2} - \frac{yy_o}{b^2}$$

35. The given curves are $y^2 = 4x$ and $x^2 = 4y$

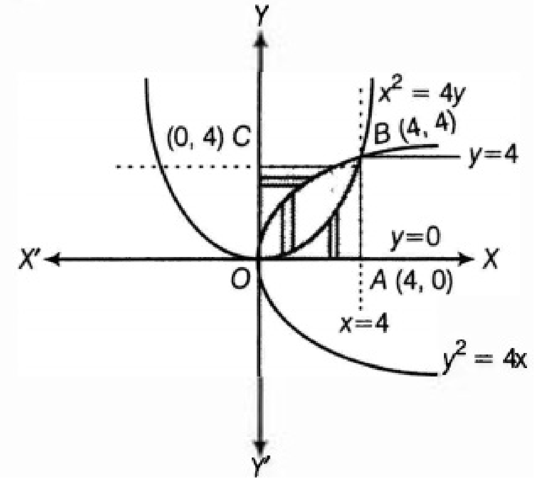
Let OABC be the square whose sides are represented by following equations

Equation of OA is $y = 0$

Equation of AB is $x = 4$

Equation of BC is $y = 4$

Equation of CO is $x = 0$



On solving equations $y^2 = 4x$ and $x^2 = 4y$, we get A(0, 0) and B(4, 4) as their points of intersection.

The Area bounded by these curves

$= \int_0^4 \left[y_{(\text{ parabola } y^2=4x)} - y_{(\text{ parabola } x^2=4y)}\right] dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$

$= \left[2 \cdot \frac{2}{3} x^{3/2} - \frac{x^3}{12}\right]_0^4$

$= \left[\frac{4}{3} x^{3/2} - \frac{x_3}{12}\right]_0^4$

$= \frac{4}{3} \cdot (4)^{3/2} - \frac{64}{12}$

$= \frac{4}{3} \cdot \left(2^2\right)^{3/2} - \frac{64}{12}$

$= \frac{4}{3} \cdot (2)^3 - \frac{64}{12}$

$= \frac{32}{3} - \frac{16}{3}$

$= \frac{16}{3}$ sq units

Hence, area bounded by curves $y^2 = 4x$ and x = 4y is $\frac{16}{3}$ sq units ......(i)

Area bounded by curve $x^2 = 4y$ and the lines x = 0, x = 4 and X-axis

$= \int_0^4 y_{(\text{ parabola } x^2=4y)} dx$

$= \int_0^4 \frac{x^2}{4} dx$

$= \left[\frac{x^3}{12}\right]_0^4$

$= \frac{64}{12}$

$= \frac{16}{3}$ sq units ....(ii)

The area bounded by curve $y^2 = 4x$, the lies y = 0, y = 4 and Y-axis

$= \int_0^4 x_{(\text{ parabola } y^2=4x)} dy$

$= \int_0^4 \frac{y^2}{4} dy$

$= \left[\frac{y^3}{12}\right]_0^4$

$= \frac{64}{12}$

= $\frac{16}{3}$ sq units ....(iii)

From Equations. (i), (ii) and (iii), area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of square into three equal parts.

OR

Given lines are

3x - y - 3 = 0 ...(i)

2x + y - 12 = 0 ...(ii)

x - 2y - 1 = 0 ...(iii)

For intersecting point of (i) and (ii),

Adding (i) and (ii),

$\Rightarrow$ 3x - y - 3 + 2x + y - 12 = 0

$\Rightarrow$ 5x - 15 = 0

$\Rightarrow$ x = 3

Putting x = 3 in (i), we get

9 - y - 3 = 0

$\Rightarrow$ y = 6

Intersecting point of (i) and (ii) is (3, 6).

For intersecting point of (ii) and (iii),

Multiply (iii) by 2, and (ii) - (iii)

$\Rightarrow$ 5y - 10 = 0

$\Rightarrow$ y = 2

Putting y = 2 in (ii) we get

2x + 2 - 12 = 0

$\Rightarrow$ x = 5

Intersecting point of (ii) and (iii) is (5, 2).

For Intersecting point of (i) and (iii),

Multiply (i) by 3 and (i) - (iii)

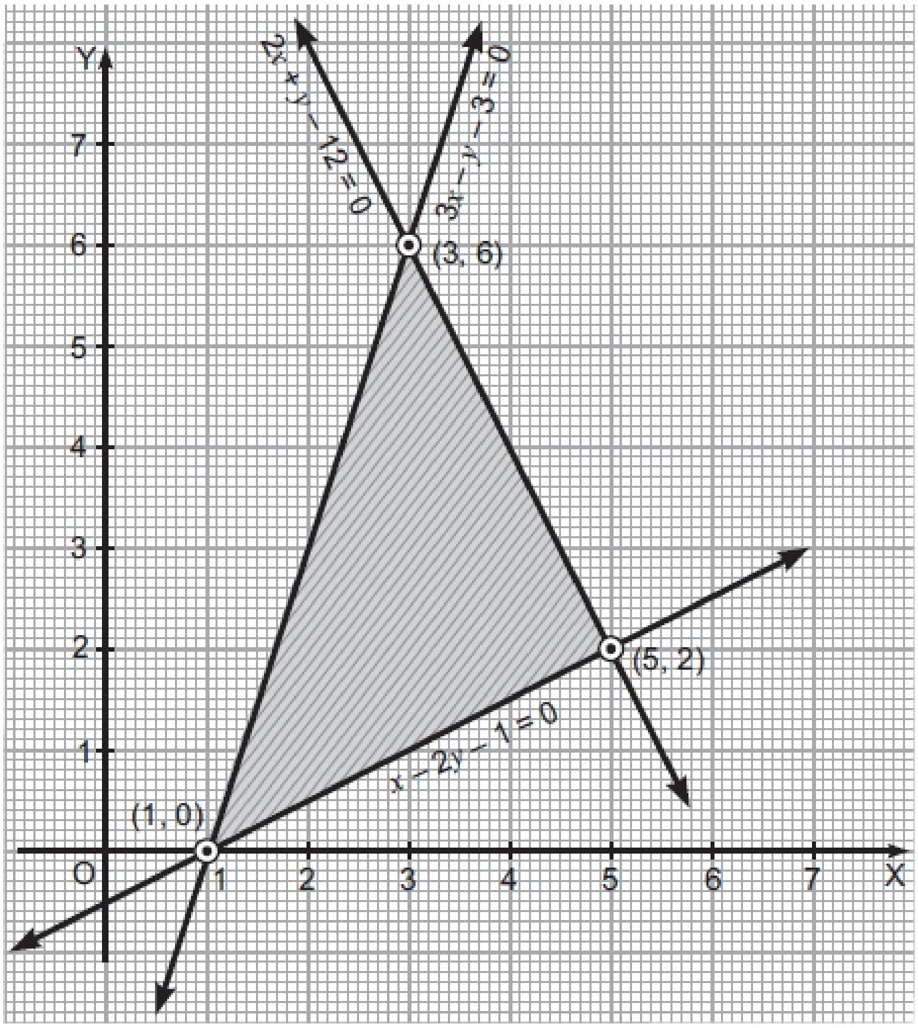
$\Rightarrow$ 3x - y - 3 - 3x + 6y + 3 = 0

$\Rightarrow$ 5y = 0

$\Rightarrow$ y = 0

Putting y = 0 in (i), we get 3x - 3 = 0 $\Rightarrow$ x = 1

Intersecting point (i) and (iii) is (1, 0).



Shaded region is the required region.

Required Area

= $\int_1^3 (3x-3)dx$ + $\int_3^5 (-2x+12)dx$ - $\int_1^5 \frac{x-1}{2}dx$

= $3\int_1^3 xdx - 3\int_1^3 dx - 2\int_3^5 xdx$ + $12\int_3^5 dx - \frac{1}{2}\int_1^5 xdx + \frac{1}{2}\int_1^5 dx$

= $3\left[\frac{x^2}{2}\right]_1^3 - 3[x]_1^3 - 2\left[\frac{x^2}{2}\right]_3^5$ + $12[x]_3^5 - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^5 + \frac{1}{2}[x]_1^5$

= $\frac{3}{2}$ (9 - 1) -3(3 - 1) - (25 - 9) + 12(5 - 3) - $\frac{1}{4}$ (25 - 1) + $\frac{1}{2}$ (5 - 1)

= 12 - 6 - 16 + 24 - 6 + 2

= 10 sq. unit

36. $\overrightarrow{n}_1 = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{n}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$d_1 = 6$, d_2 = -5

Using the relation

$\overrightarrow{r} \cdot (\overrightarrow{n_1} + \lambda \overrightarrow{n_2}) = d_1 + \lambda d_2$

$\overrightarrow{r} \cdot \left[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}\right] = 6 - 5\lambda$...... (1)

taking $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$(x\hat{i} + y\hat{j} + z\hat{k}). \left[(1+2\lambda)i + (1+3\lambda)j + (1+4\lambda)\hat{k}\right] = 6 - 5\lambda$

$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z = 6 - 5\lambda$

(x + y + z - 6) + λ(2x + 3y + 4y + 4z + 5) = 0 ..... (2)

plane passes through the point (1, 1, 1)

$\lambda = \frac{3}{14}$

Put λ in eq (1),

$\overrightarrow{r} \cdot \left[\left(1 + \frac{3}{7}\right)\hat{i} + \left(1 + \frac{9}{14}\right)\hat{j} + \left(1 + \frac{6}{7}\right)\hat{k}\right] = 6 - \frac{15}{14}$

$\overrightarrow{r} \cdot \left(\frac{10}{7}\hat{i} + \frac{23}{14}\hat{j} + \frac{13}{7}\hat{k}\right) = \frac{69}{14}$

$\overrightarrow{r} \cdot \left(20\hat{i} + 23\hat{j} + 26\hat{k}\right) = 69$