Sample Question Paper - 28
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Find the roots of the quadratic equation $\frac{11}{3+x} = 4(3-x)$

2. A circus tent is cylindrical to a certain height and conical above it. If its diameter is 105 m and its slant height is 40 m, then find the curved surface area of cone.

3. If 18, *a, b*, –3 are consecutive terms of an A.P., then find *a, b.*

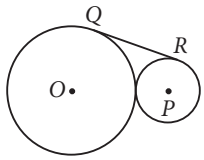
OR

The first and the last terms of an A.P. are 8 and 65 respectively. If the sum of all its terms is 730, then find its common difference.

4. If the median of the data: 6, 7, $x - 2$, x, 17, 20 written in ascending order, is 16, then find the value of x.

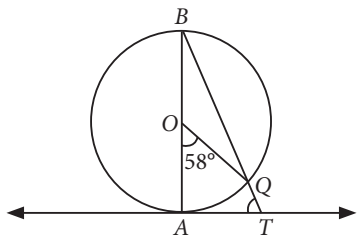
5. The number obtained by adding 12 to a natural number is 160 times of the multiplicative inverse of the natural number. Find the number.

6. Two circles with centres *O* and *P*, and radii 8 cm and 4 cm touch each other externally. Find the length of their common tangent *QR*.



OR

In the given figure, *AB* is a diameter of a circle with centre *O*, *AT* is a tangent and $\angle AOQ = 58°$, then find $\angle ATQ$.



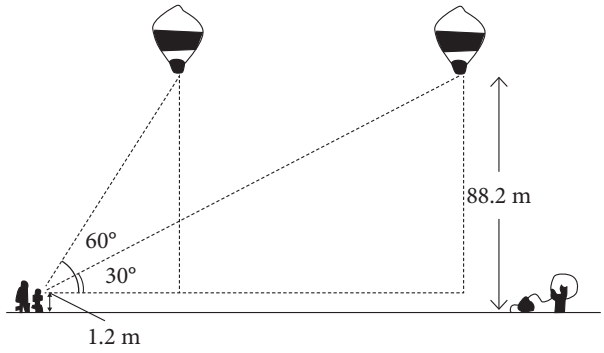
SECTION - B

7. Draw two tangents at the end points of the diameter of a circle of radius 3.5 cm. Are these tangents parallel?

8. The angles of depression of the top and bottom of a 8 m tall building from the top of a multi storied building are 30° and 45°, respectively. Find the height of the multi storied building and the distance between the two buildings.

OR

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces 30°. Find the distance travelled by the balloon during the interval.



9. If the median of the following distribution is 46, find the missing frequencies p and q.

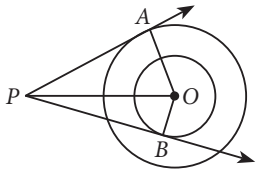
Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	**Total**
Frequency	12	30	p	65	q	25	18	230

10. The mean of the following frequency distribution is 25.2. Find the missing frequency x.

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency	8	x	10	11	9

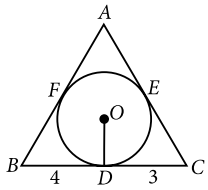
SECTION - C

11. In the given figure, if O is the centre of two concentric circles with $OA = 6$ cm, $OB = 3$ cm and $AP = 10$ cm, then find the length of BP and OP.



OR

In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21$ cm^2, then find the lengths of sides AB and AC.



12. A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. [Use $\sqrt{3}$ = 1.732]

Case Study - 1

13. Rohan's class teacher arranged a historical trip to Tajmahal. She explained it is made of a number of solid shapes and the tomb is the central focus of entire complex. The most spectacular feature is marble dome that surmounts the tomb. Height and length of dome is approximate equal to 35 m and make more noticeable by the cylindrical drum it sits on, which is approx 7 m high. The shape of dome is emphasised by four smaller domes placed at its corner, which replicate the main dome. It is surrounded by four minarets each 40 m high.

(i) Find curved surface area of 4 small domes, if its base radius is about 5 m.

(ii) Find the volume of cylindrical drum on which high dome is situated, if its radius is 4 m.

Case Study - 2

14. Uses of quadratic equations in daily life :

In real life problem, if a state government wants to open gym in every society and for that purpose they plan to distribute some rectangular plots each of area equal to 560 m^2. The dimensions of the rectangular plots are such that their breadth are 8 m shorter than their length.

(i) Form the quadratic equation from above given information.

(ii) Find the length and breadth of the plot.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. We have, $\frac{11}{3+x} = 4(3-x)$

$\Rightarrow$ $4(3 + x)(3 - x) = 11 \Rightarrow 4(9 - x^2) = 11$

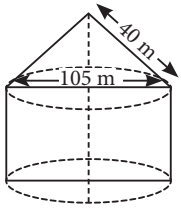
$\Rightarrow$ $36 - 4x^2 = 11 \Rightarrow 4x^2 = 25$

$\Rightarrow$ $x^2 = \frac{25}{4} = \left(\frac{5}{2}\right)^2$ $\therefore$ $x = \pm\frac{5}{2}$

2. Curved surface area of cone = πrl

$= \pi\left(\frac{105}{2}\right)40$

$= \frac{22}{7} \times 2100 = 6600 \text{ m}^2$



3. We have, 18, a, b, -3 are in A.P.

$\therefore$ $a - 18 = b - a = -3 - b$

So, $a - 18 = b - a \Rightarrow 2a - b = 18$ ...(i)

and $b - a = -3 - b \Rightarrow a - 2b = 3$ ...(ii)

Solving (i) and (ii), we get $a = 11$ and $b = 4$.

OR

Let a and d be the first term and common difference respectively of the A.P.

Given, $a = 8$ and $l = 65 = a + (n - 1)d$

$\Rightarrow$ $65 = 8 + (n - 1)d \Rightarrow 57 = (n - 1)d$ ...(i)

Also, $S_n = 730$ (Given) $\Rightarrow$ $\frac{n}{2}(a + l) = 730$

$\Rightarrow$ $n[8 + 65] = 1460 \Rightarrow n = \frac{1460}{73} = 20$

Putting value of n in (i), we get $57 = (20 - 1)d$

$\Rightarrow$ $57 = 19d \Rightarrow d = 3$

4. Here, $n = 6$, which is even,

$\therefore$ Median $= \frac{1}{2}\left(\left(\frac{6}{2}\right)^{\text{th}} \text{term} + \left(\frac{6}{2}+1\right)^{\text{th}} \text{term}\right)$

$\Rightarrow$ $16 = \frac{1}{2}$ (3^{rd} term + 4^{th} term)

$\Rightarrow$ $32 = x - 2 + x$

$\Rightarrow$ $2x = 34 \Rightarrow x = 17$

5. Let the natural number be x.

According to question,

$x + 12 = \frac{160}{x}$

$\Rightarrow$ $x^2 + 12x = 160 \Rightarrow x^2 + 12x - 160 = 0$

$\Rightarrow$ $x^2 + 20x - 8x - 160 = 0$

$\Rightarrow$ $x(x + 20) - 8(x + 20) = 0$

$\Rightarrow$ $(x - 8)(x + 20) = 0 \Rightarrow x = 8$ or $x = -20$

Since, $x = -20$ is not a natural number.

So, required natural number is 8.

6. Join O to P and Q. Join P to R.

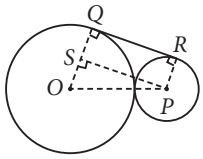
Draw $SP \perp OQ$.

Now $SP = QR$, as they are opposite sides of rectangle $PRQS$.

$OP = 8$ cm + 4 cm = 12 cm

$OS = 8$ cm $- 4$ cm = 4 cm

Now, in ΔOSP, $OP^2 = OS^2 + SP^2$



$\Rightarrow$ $SP = \sqrt{OP^2 - OS^2} = \sqrt{12^2 - 4^2}$ cm $= 8\sqrt{2}$ cm

$\therefore$ $QR = 8\sqrt{2}$ cm.

OR

Given, $\angle AOQ = 58°$

$\angle ABQ = \frac{1}{2} \angle AOQ$ ($\because$ Angle subtended by an arc at the centre is double the angle subtended by the same arc at any point of circle)

$\Rightarrow$ $\angle ABQ = \frac{1}{2} \times 58° = 29°$

Now, $\angle BAT = 90°$ ($\because$ $OA \perp AT$)

In ΔABT,

$\angle ABT + \angle BAT + \angle ATB = 180°$

(Angle sum property of triangle)

$\Rightarrow$ $29° + 90° + \angle ATB = 180°$

$\Rightarrow$ $\angle ATB = 180° - 90° - 29° = 61°$

7. Steps of construction :

Step-I : Draw a circle with centre O and radius 3.5 cm.

Step-II : Draw the diameter POQ.

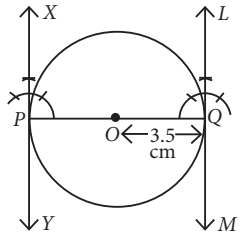
Step-III : Construct perpendiculars on PQ at both end points P and Q.

Thus, XPY and LQM are the two tangents at P and Q to the circle with centre O.

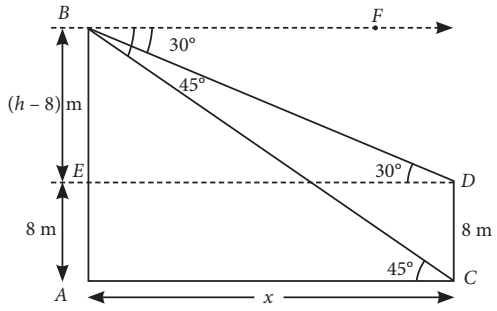
Since $\angle XPQ + \angle PQL$

$= 90° + 90° = 180°$

$\therefore$ $XPY \parallel LQM$



8. Let AB and CD be the two buildings.



Let the height of the multi-storied building be h m and the distance between the two buildings be x m.

$AE = CD = 8$ m [Given]

$BE = AB - AE = (h - 8)$ m

and $AC = DE = x$ m

Now, ln ΔACB,

$$\tan 45° = \frac{AB}{AC}$$

$$\Rightarrow \quad 1 = \frac{h}{x} \Rightarrow x = h \qquad ...(i)$$

In ΔBDE, $\tan 30° = \frac{BE}{ED}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-8}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(h-8) \qquad ...(ii)$$

From (i) and (ii), we get

$$h = \sqrt{3}h - 8\sqrt{3} \Rightarrow \sqrt{3}h - h = 8\sqrt{3}$$

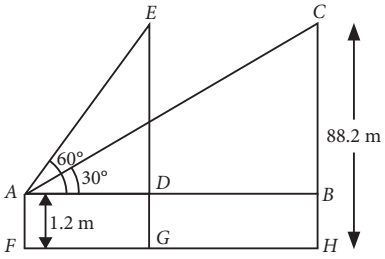
$$\Rightarrow \quad h(\sqrt{3}-1) = 8\sqrt{3}$$

$$\Rightarrow \quad h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow h = (12 + 4\sqrt{3}) \text{ m}$$

Thus, distance between the two buildings

$= (12 + 4\sqrt{3})$ m

OR



From the given figure, the angle of elevation for the first position of the balloon *i.e.*, $\angle EAD = 60°$ and for second position *i.e.*, $\angle BAC = 30°$.

Also, $ED = CB = 88.2 - 1.2 = 87$ m

Let $AD = x$ m and $AB = y$ m

Then in ΔADE, $\tan 60° = \frac{DE}{AD}$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{x} \Rightarrow x = \frac{87}{\sqrt{3}} \qquad ...(i)$$

In ΔABC, $\tan 30° = \frac{BC}{AB}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{y} \quad \Rightarrow \quad y = 87\sqrt{3} \qquad ...(ii)$$

Subtracting (i) from (ii), we get

$$y - x = 87\sqrt{3} - \frac{87}{\sqrt{3}} = 87\left(\frac{3-1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad y - x = 58\sqrt{3} \text{ m}$$

Hence, the distance travelled by the balloon BD *i.e.*, $y - x = 58\sqrt{3}$ m.

9. The frequency distribution table for the given data can be drawn as :

Class	**Frequency (f_i)**	**Cumulative frequency (*c.f.*)**
10-20	12	12
20-30	30	42
30-40	p	$42 + p$
40-50	65	$107 + p$
50-60	q	$107 + p + q$
60-70	25	$132 + p + q$
70-80	18	$150 + p + q$
Total	230	

Since, median is 46, so 40-50 is median class.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$\Rightarrow \quad 46 = 40 + \left(\frac{\frac{230}{2} - (42+p)}{65}\right) \times 10$$

$$\Rightarrow \quad 6 = \frac{(115 - 42 - p)}{65} \times 10$$

$\Rightarrow \quad 39 = 73 - p \Rightarrow p = 34$

Also, $150 + p + q = 230$

$\Rightarrow \quad 150 + 34 + q = 230 \Rightarrow q = 230 - 184 \Rightarrow q = 46$

$\therefore$ Missing frequencies p and q are 34 and 46 respectively.

10. The frequency distribution table from the given data can be drawn as :

Class-interval	**Frequency (f_i)**	**Class Marks (x_i)**	$f_i x_i$
0-10	8	5	40
10-20	x	15	$15x$
20-30	10	25	250
30-40	11	35	385
40-50	9	45	405
Total	$38 + x$		$1080 + 15x$

It is given that mean = 25.2

$$\Rightarrow \quad \frac{\sum f_i x_i}{\Sigma f_i} = 25.2 \Rightarrow \frac{15x + 1080}{x + 38} = \frac{252}{10}$$

$\Rightarrow$ $10 \times (15x + 1080) = 252 \times (x + 38)$

$\Rightarrow$ $150\,x + 10800 = 252\,x + 9576$

$\Rightarrow$ $252\,x - 150\,x = 10800 - 9576$

$\Rightarrow$ $102\,x = 1224 \Rightarrow x = 12$

11. In ΔAPO, $OA \perp AP$

$\therefore$ $OP^2 = PA^2 + OA^2$ (By Pythagoras theorem) ...(i)

In ΔBPO, $OB \perp BP$

$\therefore$ $OP^2 = BP^2 + OB^2$ (By Pythagoras theorem) ...(ii)

From (i) and (ii), we get

$PA^2 + OA^2 = BP^2 + OB^2$

$\Rightarrow$ $10^2 + 6^2 = BP^2 + 3^2$

$\Rightarrow$ $100 + 36 = BP^2 + 9$

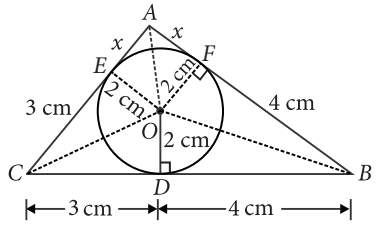
$\Rightarrow$ $BP^2 = 136 - 9 = 127 \Rightarrow BP = \sqrt{127}$ cm

Now, from (ii) $OP^2 = 127 + 3^2 = 127 + 9 = 136$ cm

$\therefore$ $OP = \sqrt{136} = 2\sqrt{34}$ cm

OR

Let E and F be the points where the tangents AC and AB touches the circle respectively. Join OE and OF.



Now, radius is perpendicular to tangent at the point of contact.

So, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$.

Join OA, OB and OC.

Since, tangents drawn from an external point to a circle are equal.

$\therefore$ $BD = BF = 4$ cm, $CD = CE = 3$ cm and

$AE = AF = x$ cm (say)

Now, area of ΔABC = area of ΔAOB + area of ΔBOC + area of ΔAOC

$$\Rightarrow \quad 21 = \frac{1}{2} \times AB \times OF + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OE$$

$$\Rightarrow \quad 21 = \frac{1}{2} \times (x + 4) \times 2 + \frac{1}{2} \times 7 \times 2 + \frac{1}{2} \times (x + 3) \times 2$$

$\Rightarrow$ $21 = x + 4 + 7 + 3 + x$

$\Rightarrow$ $2x + 14 = 21 \Rightarrow 2x = 7$

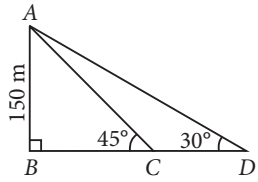
$$\Rightarrow \quad x = \frac{7}{2} = 3.5$$

$\therefore$ $AB = 3.5 + 4 = 7.5$ cm and $AC = 3.5 + 3 = 6.5$ cm

12. Let AB be the lighthouse of height 150 m.

Let initially boat is at C and after 2 minutes it reaches at D.

In right ΔABC, $\frac{AB}{BC} = \tan 45°$



$$\Rightarrow \quad \frac{150}{BC} = 1 \Rightarrow BC = 150 \text{ m}$$

In right ΔABD, $\frac{AB}{BD} = \tan 30°$

$$\Rightarrow \quad \frac{150}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 150\sqrt{3} \text{ m}$$

$\therefore$ Distance covered in 2 minutes is $CD = BD - BC$

$= 150\sqrt{3} - 150 = 150(\sqrt{3} - 1)$m

$$\therefore \quad \text{Speed of boat} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{150(\sqrt{3} - 1)}{2}$$

$= 75 \times (1.732 - 1) = 75 \times 0.732 = 54.9$ m/minutes

13. (i) Curved surface area of 4 small domes

$= 4 \times 2\pi r^2$

$$= 4 \times 2 \times \frac{22}{7} \times 5 \times 5 = 628.5 \text{ m}^2$$

(ii) Volume of cylindrical drum = $\pi r^2 h$

$$= \frac{22}{7} \times 4 \times 4 \times 7 \qquad [\because \text{ radius} = 4 \text{ m, height} = 7 \text{ m}]$$

$= 352$ m^3

14. (i) Suppose length of the plot = x metres and breadth of the plot = $(x - 8)$ metres.

Area of rectangular plot = Length × Breadth

$= x \times (x - 8)\text{m}^2 = (x^2 - 8x)\text{m}^2$

Given, area of the plot = 560 m^2

$\Rightarrow$ $x^2 - 8x = 560$ or $x^2 - 8x - 560 = 0$

(ii) The quadratic equation is $x^2 - 8x - 560 = 0$

Compare the quadratic equation with $ax^2 + bx + c = 0$

$\Rightarrow a = 1, b = -8, c = -560$.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot (-560)}}{2}$$

$$\Rightarrow \quad x = \frac{8 \pm \sqrt{64 + 2240}}{2}$$

$$\Rightarrow \quad x = \frac{8 \pm \sqrt{2304}}{2}$$

$$\Rightarrow \quad x = \frac{8 \pm 48}{2}$$

$\Rightarrow$ $x = 28$ or $x = -20$ (rejected)

So length is $x = 28$ m and breadth is $x - 8 = 20$ m.