HOTS (Higher Order Thinking Skills)

Que 1. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Sol. Given: A Δ ABC in which AD is a median.

To prove: AB + AC > 2AD

Construction: Produce AD to E such that

AD = DE. Join EC

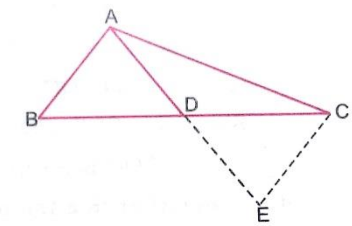

Fig. 7.35

Proof: In triangles ADB and EDC, we have

AD = DE (By construction)

BD = DC (∴ AD is the median)

and, ∠ADB = ∠EDC (Vertically opposite angles)

∴ Δ ADB ≅ Δ EDC (SAS congruence criterion)

⇒ AB = EC (CPCT) ...(i)

In Δ AFC, we have

AC + EC > AE ...(ii)

[As sum of the two sides of a triangle is greater than the third side]

Also, AE = 2AD (by construction) ...(iii)

Using (i) and (iii) in (ii), we get

AC + AB>2AD

Que 2. ABC is a triangle with ∠B = 2∠C.D is a point on BC such that AD bisects ∠BAC and AD = CD. Prove that ∠BAC = 72^0.

Sol. Given, In ΔABC, ∠B = 2∠C, AD = CD

And AD bisects ∠BAC.

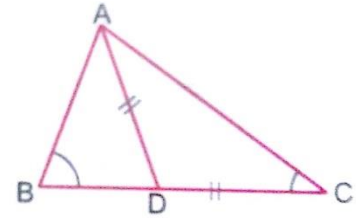

Fig. 7.36

Since AD = CD ⇒ ∠C = ∠DAC

But ∠B = 2∠C ⇒ ∠B = 2∠DAC

⇒ ∠B = ∠A = x (say) [∴ AD in bisector of ∠BAC]

Now, $\angle A + \angle B + \angle C = 180^0$ [Angle Sum Property]

$$x + x + \frac{\angle B}{2} = 180^0$$

$\Rightarrow$ $2x + \frac{x}{2} = 180^0$ $\Rightarrow$ $\frac{4x+x}{2} = 180^0$

$\Rightarrow$ $\frac{5x}{2} = 180^0$ $\Rightarrow$ $x = \frac{180^0 \times 2}{5}$

$\Rightarrow$ $\angle A = 72^0$ $\Rightarrow$ $\angle BAC = 72^0$

Que 3. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle.

Sol. Given: ΔOAB is an equilateral triangle

To prove: ΔCOD is an isosceles triangle

Since ΔAOB is an equilateral triangle

$\therefore$ $\angle OAB = \angle OBA = 60^0$ ...(i)

Also, $\angle DAB = \angle CBA = 90^0$ ...(ii) ($\because$ ABCD is a square)

Subtracting (i) from (ii), we get

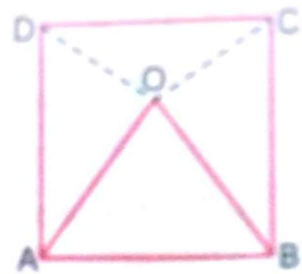

Fig. 7.37

$\angle DAB - \angle OAB = \angle CBA - \angle OBA = 90^0 - 60^0$

i.e., $\angle DAO = \angle CBO = 30^0$

Now, in ΔAOD and ΔBOC

AO = BO (given)

$\angle DAO = \angle CBO$ (proved above)

AD = BC (ABCD is a square)

$\therefore$ ΔAOD ≅ ΔBOC (By SAS congruence)

$\Rightarrow$ DO = OC (CPCT)

Since, in ΔCOD, CO = OD

$\therefore$ ΔCOD is an isosceles triangle.

Que 4. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.

Sol. Let ΔABC be a triangle in which AC is longest side.

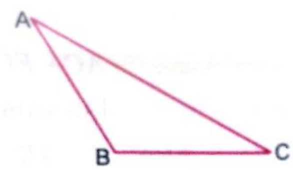

Fig. 7.38

$\Rightarrow$ $\angle B$ is largest angle

$\Rightarrow$ $\angle B > \angle A$ ...(i)

And $\angle B > \angle C$ ...(ii)

Adding (i) and (ii), we get

$\Rightarrow$ $\angle B + \angle B > \angle A + \angle C$

$\Rightarrow$ $2\angle B > \angle A + \angle C$

$\Rightarrow$ $2\angle B + \angle B > \angle A + \angle B + \angle C$

$\Rightarrow$ $3\angle B > 180^0$ $\Rightarrow$ $\angle B > 60^0$

$\Rightarrow$ $\angle B > \frac{2}{3}$ x right angle. [**Note:** $60^0 = \frac{2}{3} \times 90^0$]