CGPET 2016

Solved Paper

Question 1

Consider a thin uniform square sheet made of a rigid material. If its side is a, mass m and moment of inertia I about one of its diagonals, then

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Options:

A. $I > \frac{ma^2}{12}$

B. $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

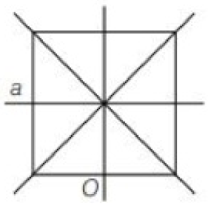
C. $I = \frac{ma^2}{12}$

D. $I = \frac{ma^2}{24}$

Answer: C

Solution:

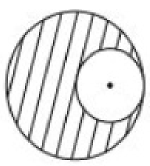
Solution:
In a uniform square plate due to symmetry, moment of inertia about all the axis passing through the center and lying in the plane of the plate is same.



i Diagonal = I parallel to side
$I = \frac{ma^2}{12}$

Question 2

From a solid sphere of mass M and radius R, aspherical portion of radius $\frac{R}{2}$ is removed, asshown in figure. Taking gravitationalpotential $V = 0$ at $r = a$. The potential at thecenter of the cavity thus formed is($G =$ gravitational constant)



Options:

A. $\frac{-2\,GM}{3R}$

B. $\frac{-2\,GM}{R}$

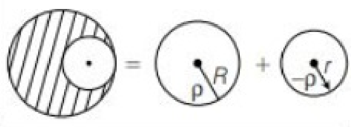
C. $\frac{-GM}{2R}$

D. $\frac{-GM}{R}$

Answer: D

Solution:

Solution:



It is given, V = 0 at $r = \infty$

Gravitational potential at internal point of a solid sphere at a distance r, is given by the following formula

$V = -\frac{GM}{R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$

Here,

$r = \frac{R}{2}$

$V_1 = -\frac{GM}{R}\left[\frac{3}{2} - \frac{R^2}{8R^2}\right]$

$= -\frac{GM}{R}[\frac{12-1}{8}]$

$= -\frac{11}{8}\frac{GM}{R}$

Because of sphere removed-

$V_2 = \frac{3}{2}.\frac{\frac{GM}{8}}{\frac{R}{2}} = \frac{3}{2} \times \frac{2\,GM}{8R}$

$\frac{3}{8}\frac{GM}{R}$

The net potential at the center of the cavity

$V = V_1 + V_2$

$= -\frac{11}{8}\frac{GM}{R} + \frac{3}{8}\frac{GM}{R}$

$= \frac{GM}{R}[\frac{3}{8} - \frac{11}{8}]$

$-\frac{GM}{R}$

Question 3

A very long (length L) cylindrical galaxy ismade of uniformly distributed mass and hasradius $R(R << L)$. A star outside the galaxy isorbiting the galaxy in a plane perpendicularto the galaxy and passing through its center.If the time period of star is T and its distancefrom the galaxy's axis is r, then

Options:

A. $T^2 \propto r^3$

B. $T \propto r^2$

C. $T \propto r$

D. $T \propto \sqrt{r}$

Answer: C

Solution:

Solution:
Length of the galaxy $= L$
Mass is uni-formally distributed.
Distance of star from axis of galaxy $= r$
Star is orbiting the galaxy. So, centripetal force must equal to the force experience by gravitation of galaxy
$\therefore F_c = \frac{2GM}{L \cdot r} m$
$F_c = \frac{mv^2}{r}$ [$\because v = \omega r = m\omega^2 r$]
$m\omega^2 r = \frac{2\,GMm}{L \cdot r}$
but, $\omega = \frac{2\pi}{T}$
where, T = time period
$m[\frac{2\pi}{T}]^2 \cdot r = \frac{2\,GMm}{L \cdot r}$
$\therefore T \propto r$

Question 4

A pendulum made of a uniform wire of cross-sectional area A has time period T. When anadditional mass M is added to its bob, thetime period changes to T_M . If the Young'smodulus of the material of the wire is Y , then $\frac{1}{Y}$ is equal to ($g =$ gravitational acceleration)

Options:

A. $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

B. $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

C. $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

D. $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

Answer: C

Solution:

Solution:
The time period of a pendulum-
$T = 2\pi\frac{\sqrt{l}}{g}$ ...(i)
When additional mass M is added to its bob, thelength of the wire will increase, let it is Δl then newtime period is,

$T_M = 2\pi\frac{\sqrt{l\Delta l}}{g}$ ...(ii)

Young's modulus of wire-

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F\,.\,l}{\Delta l\,.\,A}$$

$\therefore Y\,.\,\Delta l A = F\,.\,l$

$\Delta l = \frac{F\,.\,l}{AY}$ {F = Mg]

$\Delta l = \frac{Mgl}{AY}$

Time Period-

$$T_M = 2\pi\sqrt{\frac{l + Mg\frac{l}{AY}}{g}}$$ ...(iii)

Dividing Eq. (iii) by Eq. (i), we get-

$$\left(\frac{T_M}{T}\right) = \frac{2\pi\sqrt{\frac{l + \frac{Mgl}{A\,.\,Y}}{g}}}{2\pi\frac{\sqrt{l}}{g}}$$

$$\left(\frac{T_M}{T}\right)^2 = \frac{l + \frac{Mg}{AY}}{l} = 1 + \frac{Mg}{AY}$$

$$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

$$= \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

Question 5

One end of a horizontal thick copper wire oflength 2L and radius 2R is welded to an end ofanother horizontal thin copper wire of lengthL and radius R. When the arrangement isstretched by applying forces at two ends, theratio of the elongation in the thin wire to thatin the thick wire is

Options:

A. 0.25

B. 0.50

C. 2.00

D. 4.00

Answer: C

Solution:

Solution:

Let the Young's modulus of copper wire is Y. So,

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F\,.\,L}{A\,.\,\Delta l}$$ (i)

When the arrangement is stretched by applying forces at two ends, the same force will experienced by the wires.

From Eq. (i), $\Delta l = \frac{F\,.\,L}{A\,.\,Y}$

If the increase in length of the wires are Δl_1 and Δl_2, then

$\Delta l_1 = \frac{F\,.\,L}{A\,.\,Y}$

$\Delta l_2 = \frac{F . L'}{A' . Y}$ [$\because A = \pi R^2$]
$\frac{\Delta l_1}{\Delta l_2} = \frac{L}{R^2} \times \frac{(2R^2)}{2L} = 2$ [$\therefore A' = \pi (2R)^2$]

Question 6

A body initially at $80°$C cools to $64°$C in 5 minand to $52°$C in 10 min. The temperature ofthis body after 15 min will be(assuming heat loss by radiation only)

Options:

A. 40°C

B. 43°C

C. 41°C

D. 39°C

Answer: B

Solution:

Solution:
Let the temperature of surroundings is θ_0
Making use of Newton's law of cooling,
$\frac{\log(64-\theta_0)}{\log(80-\theta_0)} = -\frac{k}{m \times s} \times 5$ ...(i)
$\log\left(\frac{64-\theta_0}{52-\theta_0}\right) = \frac{k}{m \times s} \times 5$
In next 10 min, the temperature becomes 52°.
$\frac{\log(52-\theta_0)}{\log(64-\theta_0)} = -\frac{k}{m \times s} \times 10$ ...(ii)
On dividing Eq. (ii) by Eq. (i), we get
$\log\left(\frac{52-\theta_0}{64-\theta_0}\right) = 2\log\left(\frac{64-\theta_0}{80-\theta_0}\right)$
$(52-\theta_0) \times (80-\theta_0) = (64-\theta_0)^2$
or,
$52 \times 80 - 132\theta_0 + \theta_0^2 = (64)^2 - 128\theta_0 + \theta_0^2$
$4\theta_0 = 4160 - 4096 = 64$
$\theta_0 = \frac{64}{4} = 16°C$
$\log\left(\frac{\theta-\theta_0}{80-\theta_0}\right) = \log(\frac{64-16}{80-16})^2$
where, θ is the temperature of the body after 15 min.
$\frac{\theta-16}{80-16} = \frac{27}{64}$

$\theta = 43°C$

Question 7

A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with anotherwave to form stationary wave such that thepoint $x = 0$ is a node. The equation for theother wave is

Options:

A. $a\sin(kx+\omega t)$

B. $-a\cos(\omega l+kx)$

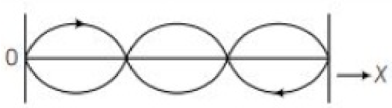
C. $-a\cos(kx-\omega t)$

D. $-a\sin(kx-\omega t)$

Answer: B

Solution:

Solution:



At $x=0$, the equation for other wave will be
$-\cos(\omega t-kx)$
Because the wave will propagate in the opposite direction with phase difference of π
Negative sign will be used as the boundary is rigid.or
$y=y_1+y_2$
$y=a[\cos(kx-\omega t)-\cos(kx-\omega t)]$
At, $x=0, y=0$
$\therefore y_2$ must be $[-a\cos(kx+\omega t)]$

Question 8

A particle moves on Y-axis according to equation $y = y_0 \sin^2 \omega t$, the motion is simple harmonic

Options:

A. with amplitude y_0

B. with amplitude $2y_0$

C. with time period $\frac{2\pi}{\omega}$

D. with time period $\frac{\pi}{\omega}$

Answer: D

Solution:

Solution:
$y = y_0 \sin^2 \omega t$
$= \frac{1}{2} y_0 (1-\cos 2\omega t)$
Clearly above mention has a time period,
$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Question 9

A sonometer wire under tension of 64 N vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near sonometer hears one beat per second. The speed with which the tuning fork moved is (speed of sound in air is $330 \mathrm{m}/s$, vibrating portion of sonometer wire has length 10 cm and mass 1 gm)

Options:

A. $0.117 \mathrm{m/s}$

B. $0.752 \mathrm{m/s}$

C. $0.342 \mathrm{m/s}$

D. $0.435 \mathrm{m/s}$

Answer: B

Solution:

Solution:
According to Doppler's principle due to motion of source,
$\frac{n'}{n} = \frac{v}{v + v_s}$
$\Rightarrow \frac{n}{n'} = \frac{v + v_s}{v} = 1 + \frac{v_s}{v}$
$n - n' = n'(\frac{v_s}{v})$
$n' = (n - n')\frac{v}{v_s}$
$n(\frac{v}{v + v_s}) = (n - n').\frac{v}{v_s}$ ....(i)
Here, $v = 330 \mathrm{m/s}$
$v_s = ?$
$n = \frac{1}{2l}\sqrt{\frac{T}{M}} = \frac{1}{2 \times 10^{-1}}\sqrt{\frac{64}{10^{-2}}}$
$= \frac{1}{2 \times 10^{-1}} \times \frac{8}{10^{-1}} = \frac{800}{2} = 400$
Given, $n - n' = 1$(beat)
So, substitution of values in Eq. (i), we get
$v_s = 0.752 \mathrm{m/s}$

Question 10

Consider interference between two sources of intensities I and $4I$. The intensity at point where the phase difference is π, is

Options:

A. $1I$

B. 4I

C. 51I

D. 31I

Answer: A

Solution:

Solution:
When phase difference is π, the difference will be destructive
$I=(a_1-a_2)^2$
$I \propto a^2$
$a_1 \propto \sqrt{I_1}=\sqrt{I}$
$a_2 \propto \sqrt{I_2}=2\sqrt{I}$
$I=(\sqrt{I}-2\sqrt{I})^2$
$=(\sqrt{I})^2[1-2]^2$
$I=I$

Question 11

If the frequency of light in a photoelectric experiment is doubled, the stopping potential will become

Options:

A. doubled

B. halved

C. more than double

D. less than double

Answer: C

Solution:

Solution:
Einstein's photoelectric equation,
$hv=eV_0+W$
$W=hv_0$(work function)
$eV_0=hv-W_0$
$V_0=$ stopping potential
$W_0=$ work function of metal surface
According to question, if the frequency of light is doubled $i.e.\ 2v$ then stopping potential,
$eV'_0=2hv-W_0$
$\therefore V'_0>2V_0$

Question 12

Who is the first scientist to measure the specific charge of an electron?

Options:

A. Millikan

B. Rutherford

C. Thomson

D. Weinbridge

Answer: C

Solution:

Solution:
Specific charge $= \frac{q_e}{m_e}$ where, m_e is the mass of electron.
Thomson first determined the specific charge for charge particles like proton, electron, helium etc.

Question 13

The wavelength will be minimum when electron transits from n = ... to n = ...

Options:

A. $n = 5$ to $n = 4$

B. $n = 4$ to $n = 3$

C. $n = 3$ to $n = 2$

D. $n = 2$ to $n = 1$

Answer: D

Solution:

Solution:
The wavelength of the photon will be minimum when electron jumps from $n = 2$ to $n = 1$ because these are the closest energy levels.

Question 14

The number of active nuclei in two radioactive substances are in the ratio of 2:3 initially. If their half lifes are one hour and two hours respectively, then the ratio of active nuclei after 6 hours is in the ratio of

Options:

A. $1:1$

B. $1:12$

C. $4:3$

D. $12:1$

Answer: B

Solution:

Solution:
Let m_1 and m_2 are undacayed substances

$m_1 m_{01} = \frac{(\frac{1}{2})^1}{6} = (\frac{1}{2})^6$

$\frac{m_2}{m_{02}} = \frac{(\frac{1}{2})^6}{2} = (\frac{1}{2})^3$

$\therefore \frac{m_1 / m_{01}}{m_2 / m_{02}} = \frac{(\frac{1}{2})^2}{(\frac{1}{2})^3}$

$\frac{m_1}{m_2} \times \frac{m_{02}}{m_{01}} = (\frac{1}{2})^3$

$\frac{m_1}{m_2} = (\frac{1}{2})^3 \times \frac{m_{01}}{m_{02}}$

$= (\frac{1}{2})^3 \times \frac{2}{3}$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} = \frac{1}{12} = 1:12$

Question 15

Different wavelengths are coming out of a coolidge tube in an X-ray experiment. The possible one that is not present of the following is

Options:

A. $25\,\text{pm}$ (picometer)

B. $50\,\text{pm}$

C. $75\,\text{pm}$

D. $100\,\text{pm}$

Answer: A

Solution:

Solution:
The wavelength of X- rays in specturn is of the order of
$1Å - 100Å$
$\therefore 25\,\text{pm} = 25 \times 10^{-12}\text{m}$
$= 0.25 \times 10^{-10}\text{m}$
$= 0.25Å$

So, it will be the possible wavelength which will not present in coolidge tube.

Question 16

A point charge is producing electric field and in this electric field, an electric dipole is placed. Which of the following is most appropriate answer?

Options:

A. The net electric force on dipole must be zero

B. Net electric force on dipole may be zero

C. Torque on the dipole due to electric field must be zero

D. Torque on the dipole due to electric field may be zero

Answer: D

Solution:

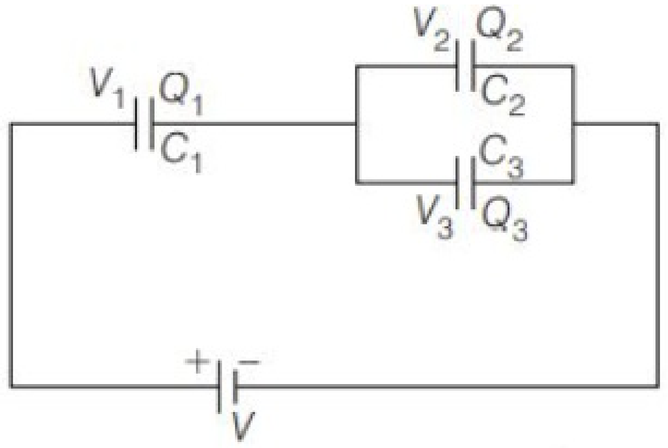
Solution:

The electric field due to point charge, F = $\frac{k \cdot q}{r^2}$

When the dipole is placed in this electric field, each charge of the dipole will experience the force in different directions. So, these forces will produce a torque.

Question 17

Three capacitors C_1, C_2, C_3 are connected to a battery of V volt as shown in figure. The charges and potentials are shown in figure. Then, the correct answer is



Options:

A. $Q_1 = Q_2 = Q_3, V_1 = V_2 = V_3 = V$

B. $Q_1 = Q_2 + Q_3, V = V_1 + V_2 + V_3$

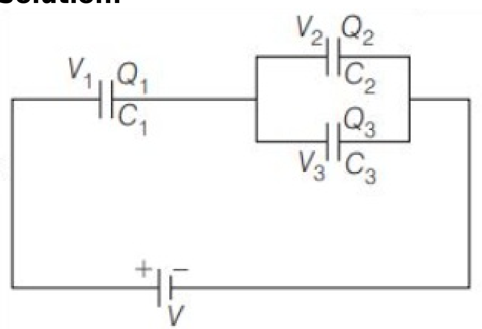
C. $Q_1 = Q_2 + Q_3, V = V_1 + V_2$

D. $Q_2 = Q_3, V_2 = V_3$

Answer: C

Solution:

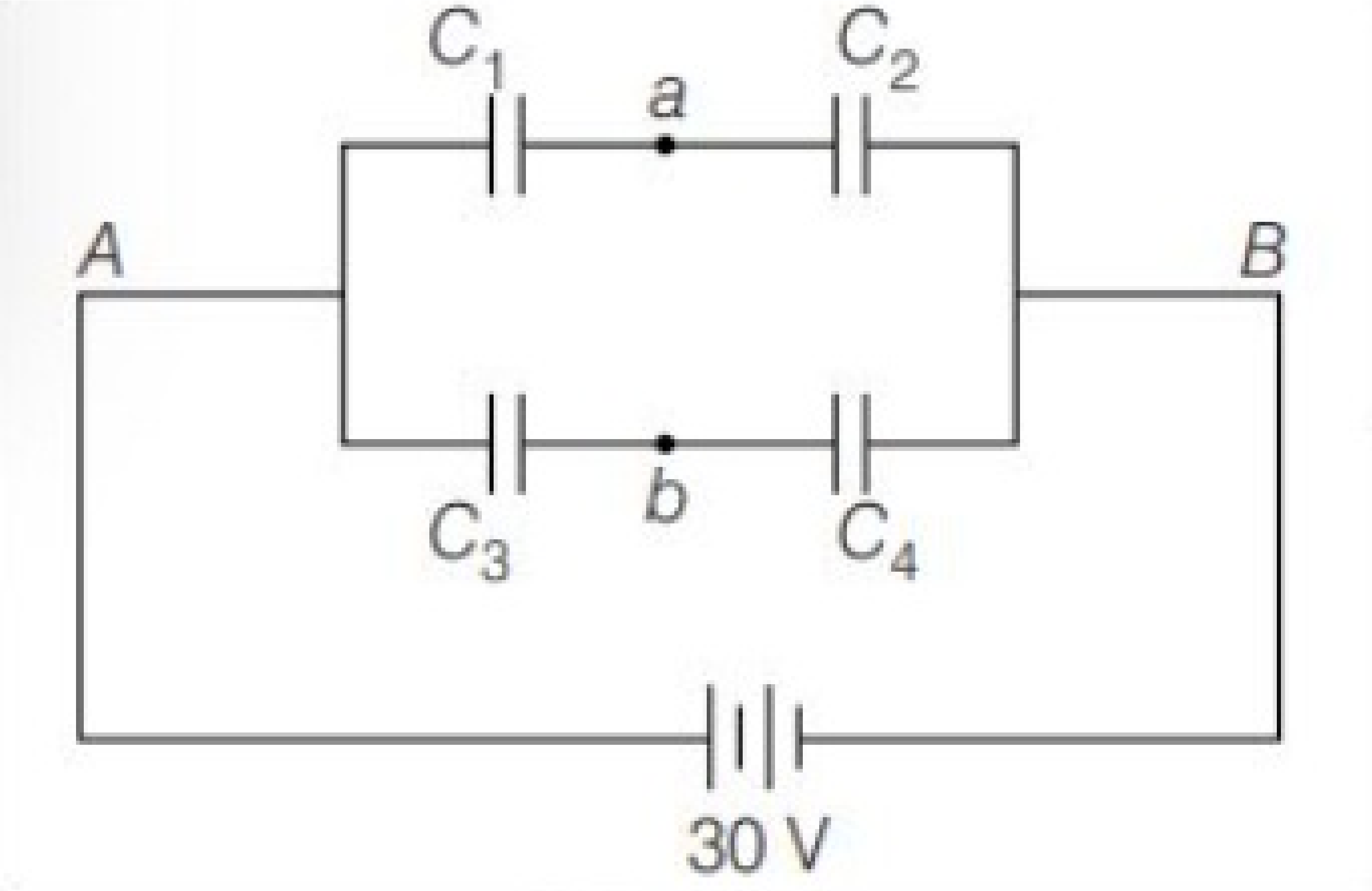
Solution:



According to Kirchhoff's first law,
$Q_1 = Q_2 + Q_3$
As C_2 and C_3 are in parallel, $V_2 = V_3$ and according to Kirchhoff's second law
$V = V_1 + V_2 = V_1 + V_3$

Question 18

Four capacitors with capacitance $C_1 = 1\mu F$, $C_2 = 1.5\mu F$, $C_3 = 2.5\mu F$ and $C_4 = 0.5\mu F$ are connected as shown in figure to a 30V source. The potential difference between points a and b is



Options:

A. 5V

B. −9V

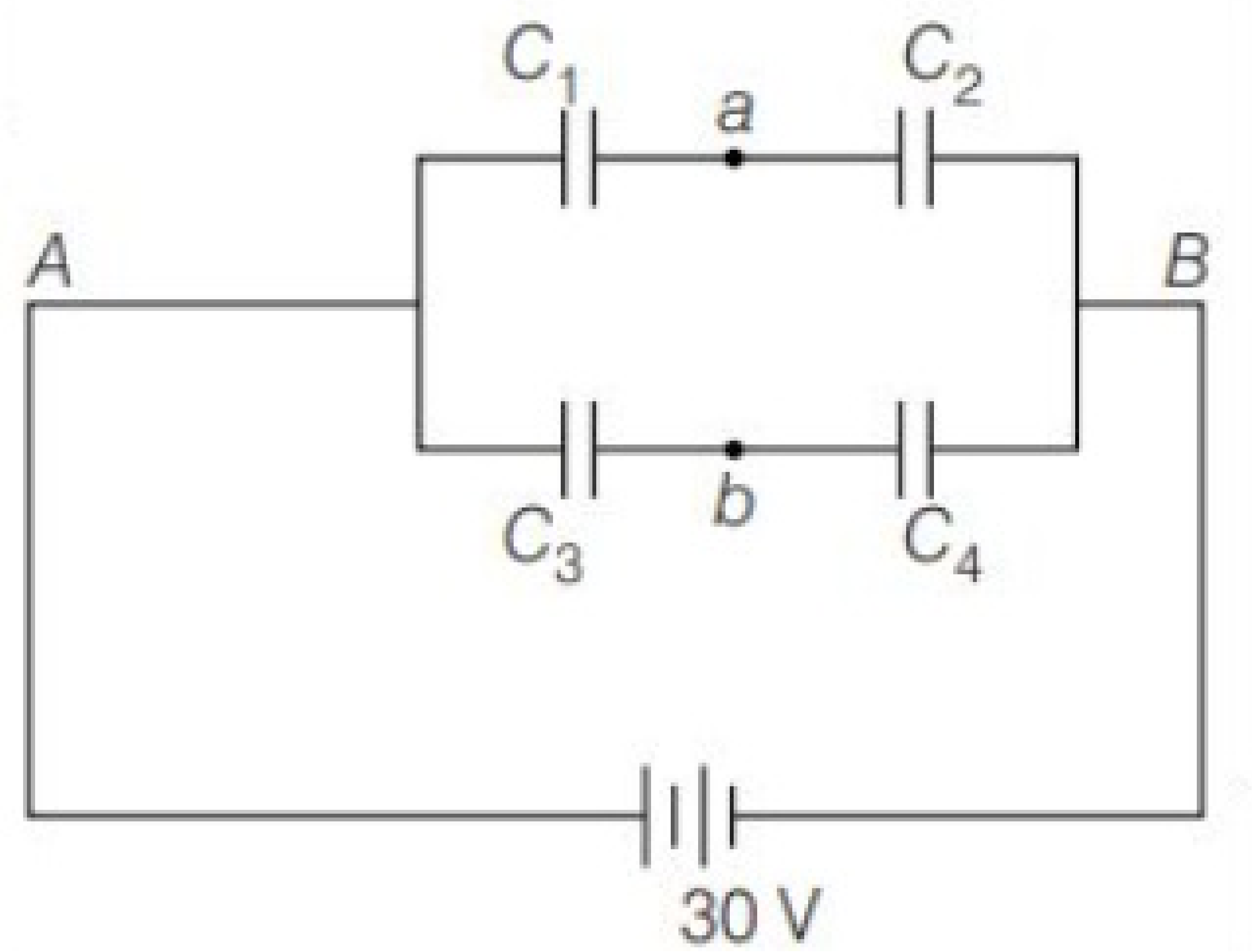
C. 10V

D. $-13V$

Answer: D

Solution:

Solution:



$C_1 = 1\mu F$
$C_2 = 1.5\mu F$
$C_3 = 2.5\mu F$
$C_4 = 0.5\mu F$
Equivalent capacitance of C_1 and C_2,
$\frac{1}{C_{eqI}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.0} + \frac{1}{1.5}\mu F$
$= 1 + \frac{2}{3} = \frac{5}{3}$
$C_{eqI} = \frac{3}{5}\mu F$
Equvalent capacitance of C_3 and C_4,
$\frac{1}{C_{eqII}} = \frac{1}{2.5} + \frac{1}{0.5}$
$\frac{1}{C_{eqII}} = \frac{2}{5} + 2 = \frac{12}{5}$
$C_{eqII} = \frac{5}{12}\mu F$
Total capacitance of circuit,
$C_{eq} = C_{eqI} + C_{eqII}$
$= \frac{3}{5} + \frac{5}{12} = \frac{36 + 25}{60}$
$= \frac{61}{60}\mu F$
Total charge given by the cell,
$Q_{total} = C_{eq} \times V$
$= \frac{61}{60} \times 30$
$= \frac{61}{2}\mu C$
This charge is distributed in upper (I) and lower (II0 branches of the circuit.
$\frac{Q_1}{C_{eqI}} = \frac{Q_2}{C_{eqII}}$
$\frac{Q_1}{\frac{3}{5}} = \frac{Q_2}{\frac{5}{12}}$

$\frac{5Q_1}{3} = \frac{12Q_2}{5}$
$Q_1 + Q_2 = \frac{61}{2}$
$\therefore Q_1 + \frac{25}{36}Q_1 = \frac{61}{2}$
$\frac{36+25}{36}Q_1 = \frac{61}{2}$
$\frac{61}{36}Q_1 = \frac{61}{2}$
$Q_1 = 18\mu C$
$Q_2 = \frac{61}{2} - 18$
$= \frac{61-36}{2} = \frac{25}{2}\mu C$
If $V_1 =$ potential difference across capacitor C_1
$V_1 = \frac{Q_1}{C_1} = \frac{18}{1} = 18V$
$V_2 = \frac{Q_2}{Q_3} = \frac{\frac{25}{2}}{2.5} = \frac{\frac{25}{2}}{\frac{5}{2}} = 5V$
So, we have,
$V_A - V_a = 18V$
$V_A - V_b = 5V$
$(V_A - V_b) - (V_A - V_a) = 5 - 18$
$V_a - V_b = -13V$

Question 19

A 600 pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. Electrostatic energy lost in the process is

Options:

A. 6×10^{-6}J

B. 3×10^{-6}J

C. 6×10^{-9}J

D. 3×10^{-9}J

Answer: A

Solution:

Solution:
When the capacitor of 600 pF is charged by 200V supply, the electrostatic energy stored $= 1.2CV^2$
$= \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2$
$= \frac{1}{2} \times 600 \times 10^{-12} \times 200 \times 200$
$= 12 \times 10^{-6}$J
When it is connected to 600 pF uncharged capacitor, the electrostatic energy will be equally divided in the both the capacitors. So, the energy loss in first capacitor
$= \frac{1}{2} \times 12 \times 10^{-6}$J $= 6 \times 10^{-6}$J

Question 20

A wire of resistance 5Ω is drawn out so that its length is increased by twice its original length. Its new resistance is

Options:

A. 5Ω

B. 15Ω

C. 25Ω

D. 45Ω

Answer: D

Solution:

Solution:

$R = \frac{\rho l}{A}$

$R' = \frac{\rho . l'}{A'}$

$l' = l + 2l = 3l$

Since, volume of wire remains unchanged on increasing length, hence

$A . l = A' \times l' = A' \times 3l$

$A' = \frac{A}{3}$

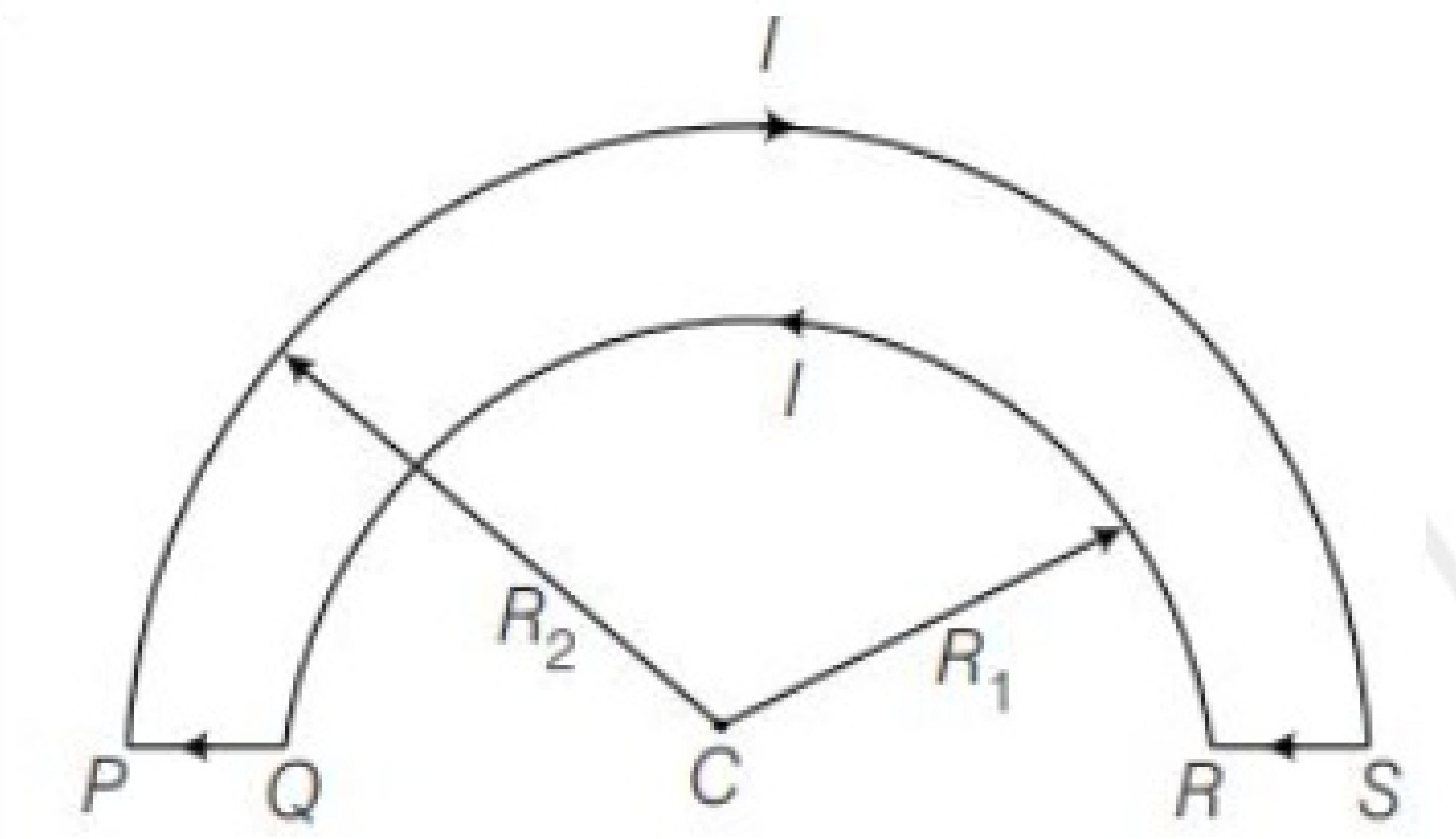
$R' = \rho \times \frac{3.l}{\frac{A}{3}} = \frac{\rho . l}{A}.9$

$R = \frac{\rho l}{A} = 5\Omega$

$R' = 5 \times 9 = 45\Omega$

Question 21

Two wire loops $PQRSP$ formed by joining two semicircle wires of radii R_1 and R_2 carrying a current I as shown in figure. The magnetic field at centre C is



Options:

A. $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

B. $\frac{\mu_0 I}{4}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

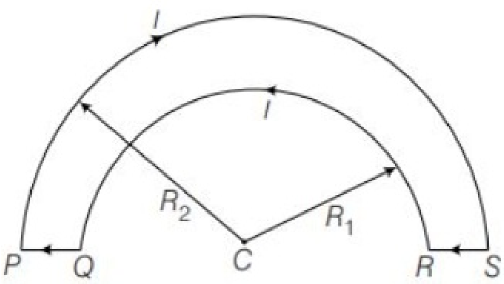
C. $\frac{\mu_0 I}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

D. $\frac{\mu_0 I}{2}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

Answer: A

Solution:

Solution:



Magnetic field at C due to semicircle of radius R_1

$B_1 = \frac{\mu_0 I}{4R_1}$ [outside of paper]

$B_2 = \frac{\mu_0 I}{4R_2}$ [inside of paper]

PQ and RS are in the direction of point C, so field will be zero. So, $B_{PQ} = B_{RS} = 0$

The resultant field at C,

$B = B_1 + B_2$

$B = \frac{\mu_0 I}{4R_1} - \frac{\mu_0 I}{4R_2}$

$= \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Question 22

A galvanometer of resistance 240Ω allows only 4% of the main current after connecting a shunt resistance. The value of the shunt resistance is

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Options:

A. 5Ω

B. 8Ω

C. 10Ω

D. 20Ω

Answer: C

Solution:

Solution:
Shunt resistance,

$S = \frac{I_g}{I - I_g} . G$

Given,

$I_g = I \times \frac{4}{100} = \frac{I}{25}$

$G = 240\Omega$

$S = \frac{\frac{I}{25}}{I - \frac{I}{25}} \times 240$

$= \frac{I \times 25}{24I \times 25} \times 240$

$= 10\Omega$

Question 23

Two very long straight wires P and Q carry currents of 10A and 20A respectively and are at 20 cm apart. If a third wire, R of length 15 cm having a current of 10A is placed in middle between them, the direction of current in all the three wires is the same. How much force will act on R ?

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Options:

A. 3.0×10^{-5}N towards Q

B. 3.0×10^{-5}N towards P

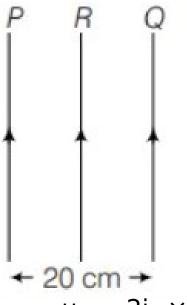
C. 3.0×10^{-7}N towards Q

D. 3.0×10^{-7}N towards P

Answer: A

Solution:

Solution:
The wires P and Q carry current in same direction, therefore they attract each other. The force on R due to P is towards the wire P and is given by,



$F_{RP} = \frac{\mu_0}{4\pi} \times \frac{2i_P \times i_R}{r_{PR}} \times I$

$= \frac{10^{-7} \times 2 \times 10 \times 10}{0.10} \times 0.15\text{N}$

$P_{RP} = 3 \times 10^{-5}\text{N}$ [towards left]

Similarly, wires Q and R attract each other as they also carry the current in the same direction. The force on R due to current in Q is towards right hand side. Therefore the force on R, due to Q, is given by

$F_{QR} = \frac{\mu_0}{4\pi} \cdot \frac{2i_Q i_R}{r_{QR}} I = \frac{10^{-7} \times 2 \times 20 \times 10 \times 0.15}{0.10}$

$F_{QR} = 6 \times 10^{-5}\text{N}$ [towards right]

∴ Net force on R is

$F = 6 \times 10^{-5} - 3 \times 10^{-5}\text{N}$

$= 3 \times 10^{-5}\text{N}$ [towards right (Q)]

Question 24

A transformer with efficiency 80% works at 4 kW and 100V. If the secondary voltage is 200V, then the primary and secondary currents are respectively

Options:

A. 40A, 16A

B. 16A, 40A

C. 20A, 40A

D. 40A, 20A

Answer: A

Solution:

Solution:
$\eta = 80\%$ output power.
$P = 4kW = 4 \times 10^3 W$
$V_1 = 100V, I_1 = ?$
$V_2 = 200V, I_2 = ?$
Power $P = V_1 \times I_1$
$I_1 = \frac{P}{V_1} = \frac{4 \times 10^3}{100}$
$= 40A$
$I_2 = \frac{P_0}{V_2} = \frac{4 \times 10^3}{200} \times \frac{80}{100} = 16A$
Alternate method
Here, $\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_2 I_2}{V_1 I_1}$
$\frac{80}{100} = \frac{200 \times I_2}{100 \times 40}$
$I_2 = \frac{20 \times 8}{10} = 16A$

Question 25

In series LR circuit $X_L = 3R$, now a capacitor with $X_C = R$ is added in series. The ratio of new to old power factor is

Options:

A. 1

B. 2

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

Answer: D

Solution:

Solution:
Power factor = Resistance/Impendance
Let the resistance is R,
$Z_{RL} = \sqrt{R^2 + X_L^2}$
$= \sqrt{R^2 + (3R)^2} = \sqrt{R^2 + 9R^2}$
$= R\sqrt{10}$
when, $X_C = R$ is added the impedance of the circuit,
$Z_{RLC} = \sqrt{R^2 + (X_L - X_C)^2}$
$= \sqrt{R^2 + (3R - R)^2}$
$= \sqrt{R^2 + 4R^2} = R\sqrt{5}$
$\cos\phi = \frac{R}{R\sqrt{10}}$
$\cos\phi' = \frac{R}{R\sqrt{5}}$
$\frac{\cos\phi'}{\cos\phi} = \sqrt{2}$

Question 26

A ball of mass $M = \mathbf{0.5}\,\text{kg}$ is attached to one end of a string of length $L = \mathbf{0.5}\,\text{m}$. The ball rotates circularly in horizontal plane along vertical axis. If maximum tension which can be applied on string is $\mathbf{324}\,\text{N}$, what will be the maximum expected angular velocity of the ball?

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Options:

A. $9\,\text{rad} / s$

B. $18\,\text{rad} / s$

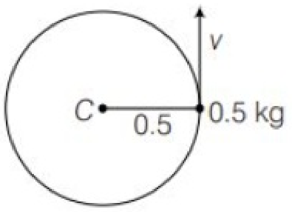
C. $27\,\text{rad} / s$

D. $36\,\text{rad} / s$

Answer: D

Solution:

Solution:
The tension produced in string must be equal to the centripetal force, when the particle moves in a circular path.



$\therefore \frac{mv^2}{r} = T$

Here, T = 324N

r = 0.5m

m = 0.5 kg

Let the angular velocity is ω, then $v = \omega r$,

$\frac{m\omega^2 r^2}{r} = T$

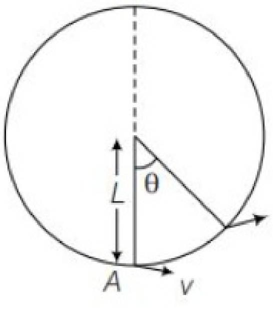
$\omega = \sqrt{\frac{T}{m} \cdot r}$

$= \sqrt{\frac{324}{0.5 \times 0.5}}$

$= \frac{1}{0.5}\sqrt{324} = \frac{18}{0.5} = 36\,\text{rad} / \text{s}$

Question 27

A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies



Options:

A. $\theta = \frac{\pi}{4}$

B. $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

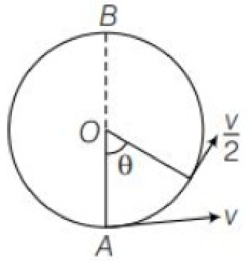
C. $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

D. $\frac{3\pi}{4} < \theta < \pi$

Answer: D

Solution:

Solution:
The bob is moving in a vertical plane, so at highest point velocity must be



$v = \sqrt{5gr}$
$= \sqrt{5gL}$ ...(I)
From Equation,
$v^2 = u^2 = 2gh$ ...(ii)
$(\frac{v}{2})^2 = v^2 - 2gh$
$\frac{v^2}{4} = v^2 - 2gh$
$h = L(1 - \cos\theta)$ ....(iii)
Solving Eqs. (i), (ii) and (iii), we get
$\cos\theta = -\frac{7}{8}$
$\theta = \cos^{-1}(\frac{7}{8}) \simeq 151°$
$\therefore \frac{3\pi}{4} < \theta < \pi$ must be correct.

Question 28

A block of mass $M = 10$ kg rests on a horizontal table. The coefficient of friction between the block and table is 0.05, when hit by a bullet of mass 50g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of 2m on the table. If a freely falling object were to acquire speed $\frac{V}{10}$ after being dropped from height H, then neglecting energy losses and taking $g = 10\text{ms}^{-2}$, the value of H is close to

Options:

A. 0.2 km

B. 0.3 km

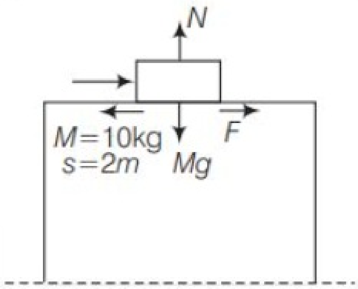
C. 0.4 km

D. 0.5 km

Answer: C

Solution:

Solution:
Given,
Mas of block = 10kg
Mass of bullet = 50kg
$= 50 \times 10^{-3}$ kg
$= 5 \times 10^{-2}$ kg
m = 50
v = ?



Speed of bullet = V
Distance covered by the block = s = 2m
Coefficient of friction between the block and the table μ = 0.05
Thus,
$F = F_f, N = Mg$
$\Rightarrow Ma = \mu N =$ acceleration of block
$\Rightarrow Ma = \mu Mg = \mu N$
$\Rightarrow a = 0.05 \times 10 = 0.05 / s^2$
Now, for speed of block
$v_{block}^2 = u^2 + 2as$
$= 0 + 2as = 2 \times 0.5 \times 2 = 2m / s^2$
$\Rightarrow v_{block} = \sqrt{2}$
From law of conservation of momentum
$mv = Mv_{block}$
$\Rightarrow v = \dfrac{10 \times \sqrt{2}}{50 \times 10^{-3}}$
$= \dfrac{\sqrt{2}}{5} \times 10^3 = 2\sqrt{2} \times 10^2$
Now for freely falling body
Final velocity $= \dfrac{v}{10}$
Initial velocity u = 0
Using formula $v^2 = u^2 = 2gH$
$\Rightarrow (\dfrac{v}{10})^2 = -2gH$
$\Rightarrow \left[\dfrac{2\sqrt{2} \times 10^2}{10}\right]^2 = -2gH$
$\Rightarrow -2gH = (20\sqrt{2})^2 = 400 \times 2 = 800$
$\Rightarrow H = \dfrac{800}{2} \times 10(=)40M = 40 \times 10^{-3}$ km
$= 0.04$ km

Question 29

A particle of mass m is attached to one end of a massless spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its

equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5u_0$, it collides elastically with a rigid wall. After this collision,

Options:

A. the speed of the particle when it returns to its equilibrium position is u_0. The time at which particle passes through equilibrium for second time is, $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

B. the time at which the particle passes through the equilibrium for the first time is $t = \pi\sqrt{\frac{m}{k}}$

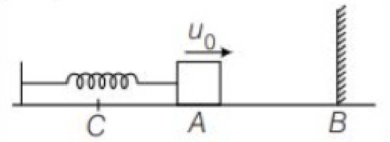
C. the time at which maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

D. the time at which minimum compression of the spring occurs is $t = \frac{3\pi}{4}\sqrt{\frac{m}{k}}$

Answer: A

Solution:

Solution:
Velocity of particle is instant t is,
$u = u_0 \cos \omega t$
when u = 0.5, then



$0.5u_0 = u_0 \cos \omega t$

$\cos \omega t = \frac{1}{2} = \cos 60°$

$\omega t = \frac{\pi}{3}$

$\frac{2\pi}{T} \times t = \frac{\pi}{3}$

$\Rightarrow t = \frac{T}{6}$

Now, time to reach equilibrium position IInd time is,

$t = t_{AB} + t_{BA} + t_{AC} + t_{CA}$

$= \frac{T}{6} + \frac{T}{6} + \frac{T}{4} + \frac{T}{4} = \frac{5}{6}T$

$= \frac{5}{3}\pi\sqrt{\frac{m}{k}}$

Question 30

The work done on the particle of mass M by a force, is given as

$$k\left[\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{i} + \frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{j}\right]$$

(k being constant of appropriate dimensions). When the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the xy-plane is

Options:

A. $2k\frac{\pi}{a}$

B. $k\frac{\pi}{a}$

C. $k\frac{\pi}{2a}$

D. 0

Answer: D

Solution:

Solution:
The force acting on the particle,

$$F = k\left[x\frac{\hat{i}}{(x^2+y^2)^{\frac{3}{2}}} + y\frac{\hat{j}}{(x^2+y^2)^{\frac{3}{2}}}\right]$$

The work done,

$dW = F \,.\, dx$

$$dW = k\left[x\frac{\hat{i}}{(x^2+y^2)^{\frac{3}{2}}} + y\frac{\hat{j}}{(x^2+y^2)^{\frac{3}{2}}}\right]dx$$

$$= k\left[\frac{x\,.\,dx}{(x^2+y^2)^{\frac{3}{2}}} + \frac{y\,.\,dy}{(x_y^{22})^{\frac{3}{2}}}\right]$$

$$= k\left[\frac{xdx+ydy}{(x^2+y^2)^{\frac{3}{2}}}\right]$$

Let $x^2+y^2=r^2$

$xdx+ydy=r^2$

$dW = k\,.\,\frac{r\,.\,dr}{r^3} = \frac{k}{r^2}\,.\,dr$

$W = [-\frac{k}{r}]_{r_1}^{r_2}$

Now, $r_1 = a, r_2 = a$

$\therefore W = [-\frac{k}{r} + \frac{k}{r}] = 0$

$W = 0$

Question 31

A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement is

Options:

A. net elongation of spring is $\frac{4\pi R^3\rho g}{3k}$ and the light sphere is completely submerged

B. net elongation of spring is $\frac{8\pi R^3 \rho g}{3k}$

C. the light sphere is partially submerged

D. net elongation of spring is $\frac{2\pi R^3 \rho g}{3k}$

Answer: A

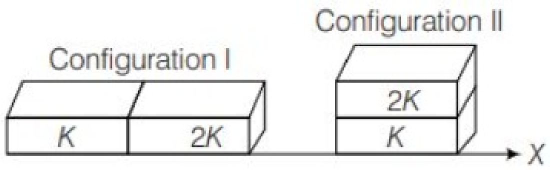
Solution:

Solution:
Force on each sphere is F.
F = weight of the fluid displaced
$= \frac{4}{3}\pi R^3 \rho . g = kx$
$\therefore x = \frac{4\pi R^3 \rho g}{3k}$

Question 32

Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or II as shown in figure. One of the blocks has thermal conductivity K and the other $2K$. The temperature difference between the ends along the X-axis is the same in both the configurations. It takes $9s$ to transport a certain amount of heat from the hot end to the cold end in configuration I. The time to transport the same amount of heat in the configuration II is

Configuration II is



Options:

A. 2.0s

B. 3.0s

C. 4.5s

D. 6.0s

Answer: A

Solution:

Solution:
Thermal resistance of the combination are
$R_I = R_1 + R_2$
$= \frac{l}{K + 1A} + \frac{l}{K_2A}$

$$= \frac{l}{A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$R_1 = \frac{l}{A}\left(\frac{K_2 + K_1}{K_1 K_2}\right)$$

R_{II} (Parallel combination)

$$R_{II} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{\frac{l}{K_1 A} \times \frac{l}{K_2 A}}{\frac{l}{A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)}$$

$$\frac{l}{A(K_1 + K_2)}$$

So, the heat flow rates are

Case I

$$I = \frac{Q}{t_1} = \frac{\Delta Q}{\frac{l}{A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)} \text{ .....(i)}$$

$$\frac{Q}{t_1} = \frac{\Delta Q}{\frac{l}{A}(3\frac{K}{3K^2})} = \frac{\Delta Q}{\frac{l}{A}(\frac{3}{2K})}$$

Case II

$$\frac{Q}{t_2} = \frac{\Delta Q}{\frac{l}{A}\left(\frac{1}{K_1 + K_2}\right)} = \frac{\Delta Q}{\frac{l}{A}(\frac{1}{3K})} \text{ .....(ii)}$$

Divide Eq. (i) and Eq. (ii), we get

$$\frac{\left(\frac{Q}{t_1}\right)}{\left(\frac{Q}{t_2}\right)} = \frac{\frac{\Delta Q}{\frac{l}{A}(\frac{3}{2K})}}{\frac{\Delta Q}{\frac{l}{A}(\frac{1}{3K})}} = \frac{2}{9}$$

$$\frac{t_2}{t_1} = \frac{2}{9}$$

$$t = 9 \times \frac{2}{9} = 2s$$

Question 33

A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways
(i) Sequentially keeping in contact with2 reservoirs such that each reservoir supplies same amount of heat
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat
In both the cases, body is brought from initialtemperature 100° C to final temperature 200° C.Entropy change of the body in two casesrespectively is

Options:

A. In 2, In 2

B. 2 8 In 2, In 2

C. In 2, In 2

D. In 2, In 2

Answer: B

Solution:

Solution:
Change in entropy due to condition is
$\Delta S = C \log \frac{T_2}{T_1}$
C= Heat capacity
Now, C = N1J / °C
$T_2 = 200°C$
$T_1 = 100°C$
In case I
$\Delta S_1 = 1 \times 2 \times \log \frac{200}{100} = 2 \log 2$
In case II
$\Delta S_2 = 1 \times 8 \times \log \frac{200}{100} = 8 \log 2$
In first case, number of reservoir is 2.
In second case, number of reservoir is 8.

Question 34

The initial pressure and volume of a gas are p_1 and V_1, the gas after expansion attains final volume V_2. Let W_1, W_2 and W_3 are the corresponding work done by the gas under isothermal, adiabatic and isobaric processes respectively then

Options:

A. $W_1 = W_2 = W_3$

B. $W_2 > W_1 > W_3$

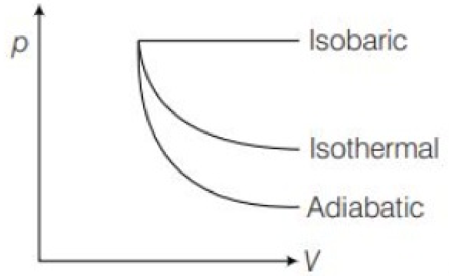
C. $W_3 > W_1 > W_2$

D. $W_3 > W_2 > W_1$

Answer: C

Solution:

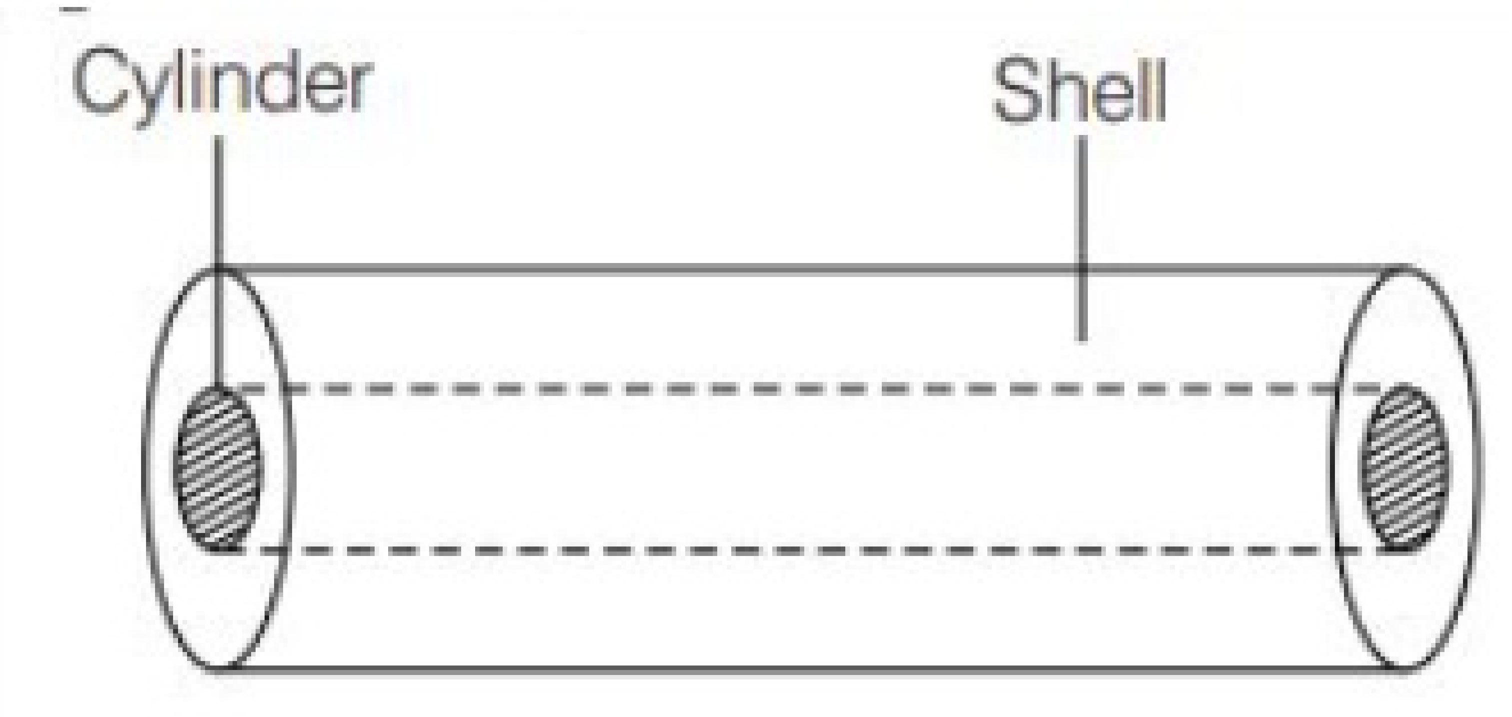
Solution:



Work done = area under the curve
∴ Wisobaric is largest and Wadiabatic is smallest
∴ $W_3 > W_1 > W_2$

Question 35

A composite bar consists of a cylinder of radius R and thermal conductivity K_1 fitted inside a cylindrical shell of internal radius R and external radius $2R$. If the thermal conductivity of shell is K_2, then the equivalent thermal conductivity of the composite bar is



Options:

A. $K_1 + K_2$

B. $\frac{K_1 + 3K_2}{4}$

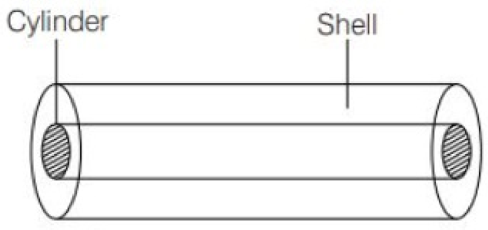
C. $K_1 + 3K_2$

D. $\frac{K_2 + 3K_1}{4}$

Answer: B

Solution:

Solution:



Equivalent thermal resistance of cylinder and shell is,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K4\pi R^2}{l} = \frac{K_1\pi R^2}{l} + \frac{K_2 3\pi R^2}{l}$$

$$K = \frac{K_1 + 3K_2}{4}$$

Question 36

In Young's double slit experiment, interference is produced due to slits distance d metre apart. The fringe pattern is observed in a screen distant D metre from the slit. If λ in metre, denotes the wavelength of light, the number of fringes per metre of the screen is

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Options:

A. $\frac{d}{\lambda D}$

B. $\frac{D}{\lambda d}$

C. $\frac{\lambda d}{D}$

D. $\frac{\lambda D}{2d}$

Answer: A

Solution:

Solution:
The fringe width,
$\beta = \frac{\lambda D}{d}$
λ = wavelength of light
D = distance of screen from slits
d = distance between slits
Let there are n number of fringes in one metre.
$\therefore \frac{\lambda D}{d} \times n = 1$
$n = \frac{1}{\frac{\lambda D}{d}} = \frac{d}{\lambda D}$

Question 37

A glass prism of angle 72° and refractive index 1.66 is immersed in a liquid of refractive index 1.33. The angle of minimum deviation for a parallel beam of light passing through the prism is (use, $sin\ 47^{\circ}11' = 0.7336$ and $sin\ 36^{\circ} = 0.5871$)

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Options:

A. $11^{\circ}11'$

B. $22^{\circ}22'$

C. $47^{\circ}11'$

D. $44^{\circ}44'$

Answer: B

Solution:

Solution:
Prism

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \frac{1.66}{1.33} = \frac{\sin\left(\frac{72^\circ}{2} + \frac{\delta_m}{2}\right)}{\sin\left(\frac{72^\circ}{2}\right)}$$

$$\sin\left(\frac{72^\circ}{2} + \frac{\delta_m}{2}\right) = \sin 36^\circ \times \frac{1.66}{1.33}$$

$$= 0.5871 \times \frac{1.66}{1.33} = 0.7336$$

$$\Rightarrow \sin\left(\frac{72^\circ}{2} + \frac{\delta_m}{2}\right) = \sin 47^\circ 11'$$

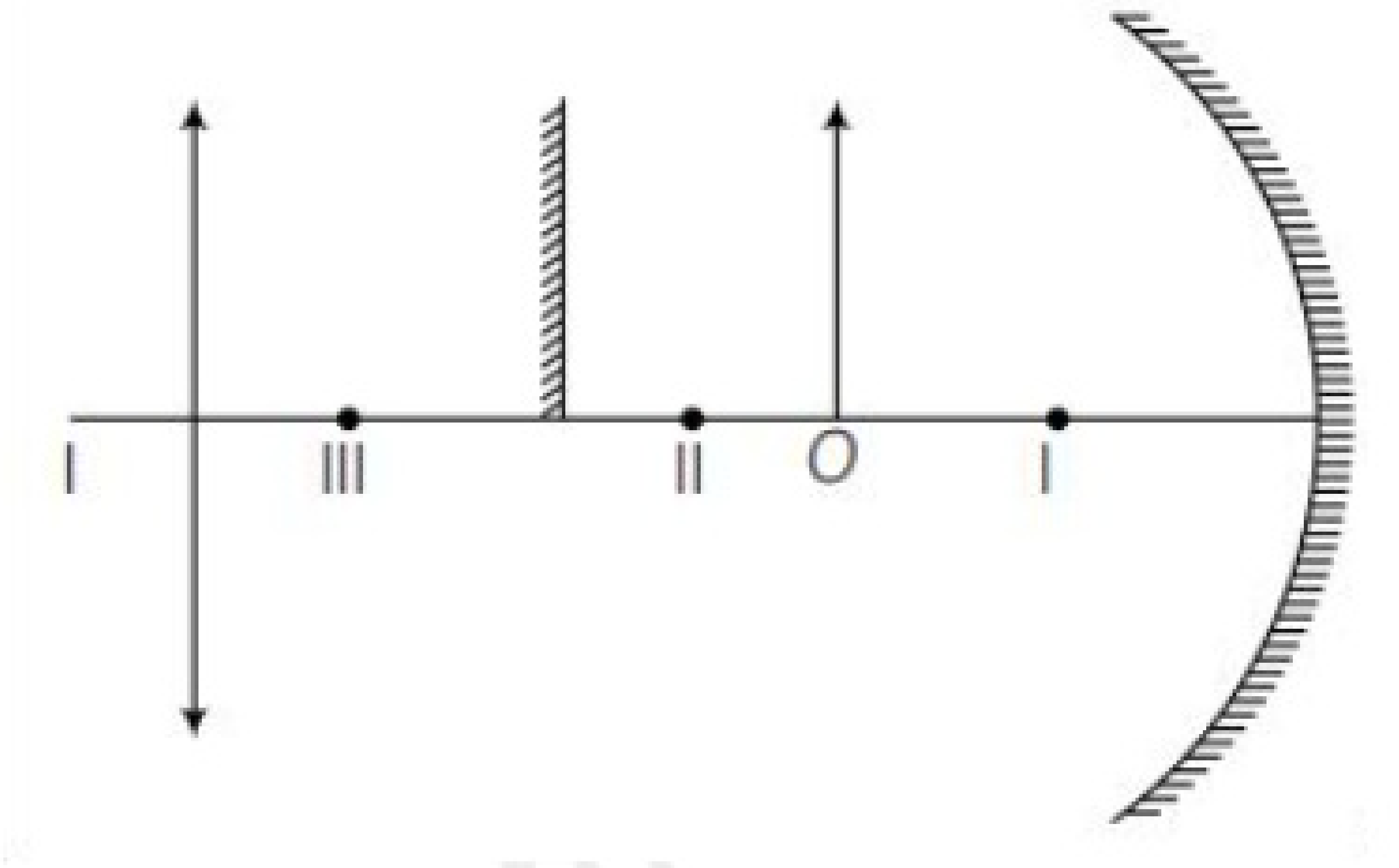
$$36^\circ + \frac{\delta_m}{2} = 47^\circ 11'$$

$$\frac{\delta_m}{2} = 11^\circ 11$$

$$\delta_m = 2 \times (11^\circ 11') = 22^\circ 22'$$

Question 38

An object O is placed between a concave mirror and a plane mirror so that the first image in both the mirrors coincide at I. What should be the position of centre of curvature of concave mirror?



Options:

A. I

B. II

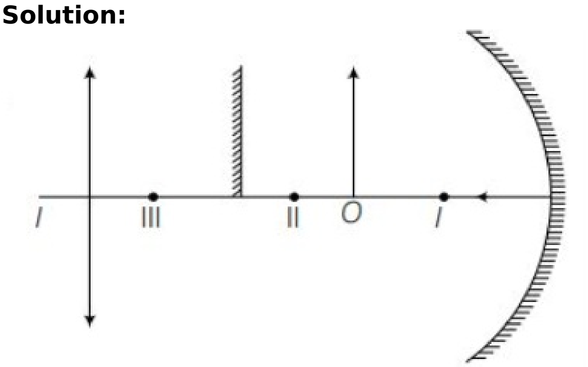
C. III

D. O

Answer: B

Solution:

Solution:



As a real and inverted image of O is obtained by concave mirror.
∴ I must be the focal point.
And hence, II must be the centre of curvature.

Question 39

The magnification (of positive sign) of an object placed in front of a

convex lens of focal length 20 cm, is same in value but of negative sign when the object is moved a distance of 20 cm. Find the magnification.

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Options:

A. 1.5

B. 2.0

C. 2.5

D. 3.0

Answer: B

Solution:

Solution:
Case I
Magnification,
$m = \frac{f}{f - u}$
$f = +f$
$u = -u$
Case II
m is negative.
$\therefore m = -(\frac{f}{f - (u + 20)})$
Because object is shifted back by 20 cm
$\frac{f}{f - u} = -\frac{f}{f - (u + 20)}$
$20 - u = -20 + (u + 20)$
$40 = 2u + 20$
$u = 10$
$\therefore m = \frac{f}{f - u} = \frac{20}{20 - 10} = 2.0$

Question 40

In a telescope, the focal length of objective is 60 cm and eye piece is 10 cm. When it is focussed on an object parallel rays come out of eyepiece. If the angle subtended at objective by object is 3°, then what is the angular width of image?

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Options:

A. 24°

B. 18°

C. 12°

D. 6°

Answer: B

Solution:

Solution:
Final image is at infinity (∞)

$\therefore m = \text{magnification} = \frac{f_o}{f_e}$

$= \frac{60}{10} = 6 = \frac{\beta}{\alpha}$

β = angular width of the image

$\beta = \alpha \times 6$

$= 3° \times 6$

$= 18°$

Question 41

Diffusion current in a p - n junction is greater than the drift current in magnitude, if the junction is

Options:

A. reverse biased

B. unbiased

C. forward biased

D. None of the above

Answer: C

Solution:

Solution:
Diffusion current is greater than the drift current in magnitude

$I_{diff} > I_{drift}$

It should be forward biased.

Question 42

When a semiconductor device is connected to a battery through a resistance, some current flows through it. Now, if the battery is reversed, the current becomes almost zero the device may be

Options:

A. pure semiconductor

B. $p-n$ junction

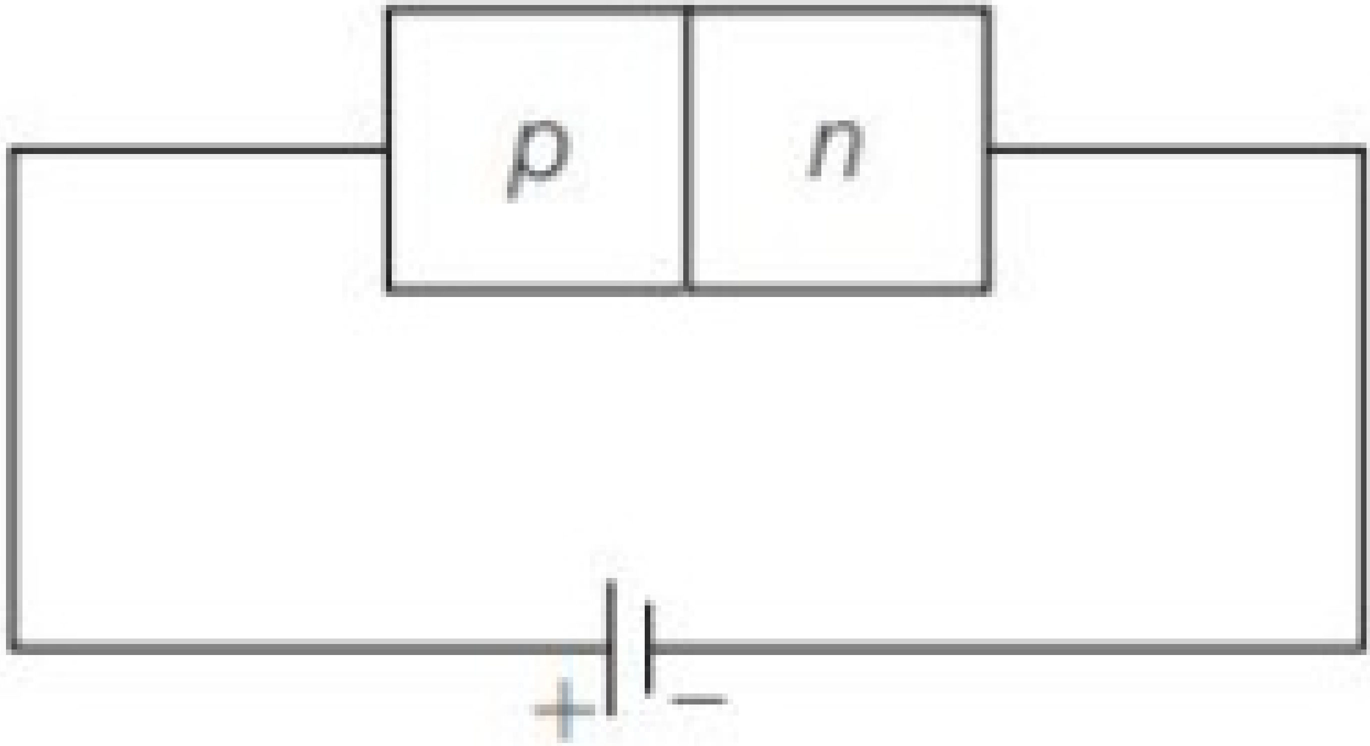
C. p-type semiconductor

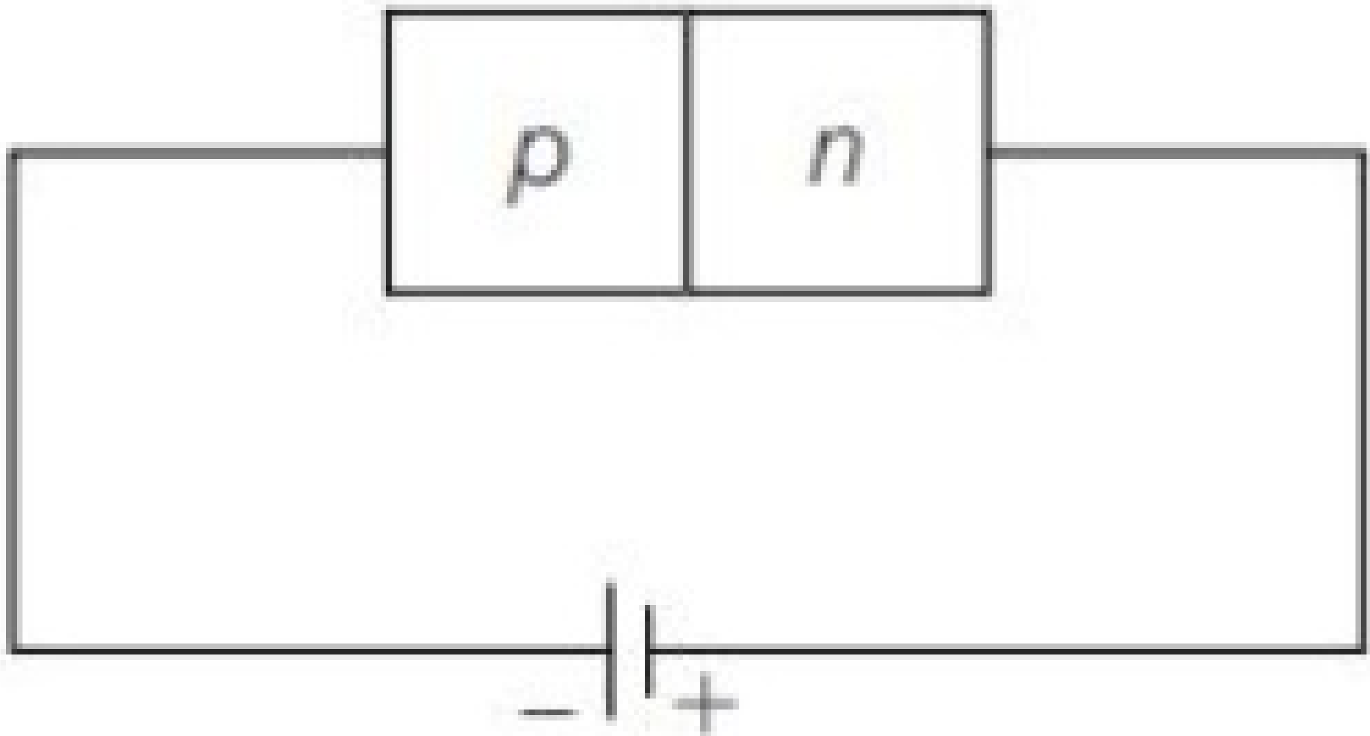
D. n-type semiconductor

Answer: B

Solution:

Solution:
In forward bias, some current flows but when it is reverse biased charge carriers n (electrons) and p (holes) will drift away from the junction.It should be $\mathrm{p}-\mathrm{n}$ junction.





Question 43

If a dip circle is positioned at 45° to magnetic meridian, the apparent dip angle is 30°, then what is actual dip angle?

Options:

A. $\tan^{-1}(\frac{1}{\sqrt{3}})$

B. $\tan^{-1}(\frac{2}{\sqrt{3}})$

C. $\tan^{-1}(\frac{1}{\sqrt{6}})$

D. $\tan^{-1}(\frac{\sqrt{3}}{2})$

Answer: C

Solution:

Solution:

tan (Actual dip) = sin β × tan (apparent dip)

$\tan\alpha = \sin 45^\circ \times \tan 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{6}}$

$\alpha = \tan^{-1}(\frac{1}{\sqrt{6}})$

Question 44

The work done in rotating a magnet of magnetic moment M in a magnetic field through 90° is x times that of work done in rotating the same through 60° in same situation. Then, the value of x is

Options:

A. 2

B. $\frac{1}{2}$

C. 4

D. $\frac{1}{4}$

Answer: A

Solution:

Solution:
Work done,
$W = MB(\cos\theta_1 - \cos\theta_2)$
In first case, $\theta_1 = 0$ and $\theta_2 = 90°$
$\therefore W_1 = MB(\cos 0° - \cos 90°) = MB$
In second case, $\theta_1 = 0°, \theta_2 = 60$
$\therefore W_2 = MB(\cos 0° - \cos 60°)$
$= MB(1 - \frac{1}{2}) = \frac{MB}{2}$
Given $W_1 = xW_2$
$x = 2$

Question 45

Two positive charges of $2\mu C$ and $1\mu C$ are kept at a distance of one metre. The value of electric field at the centre of the line joining the charges in newton/coulomb is

Options:

A. 3.6×10^4

B. 1.8×10^4

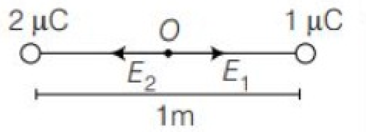
C. 10.8×10^4

D. 5.6×10^4

Answer: A

Solution:

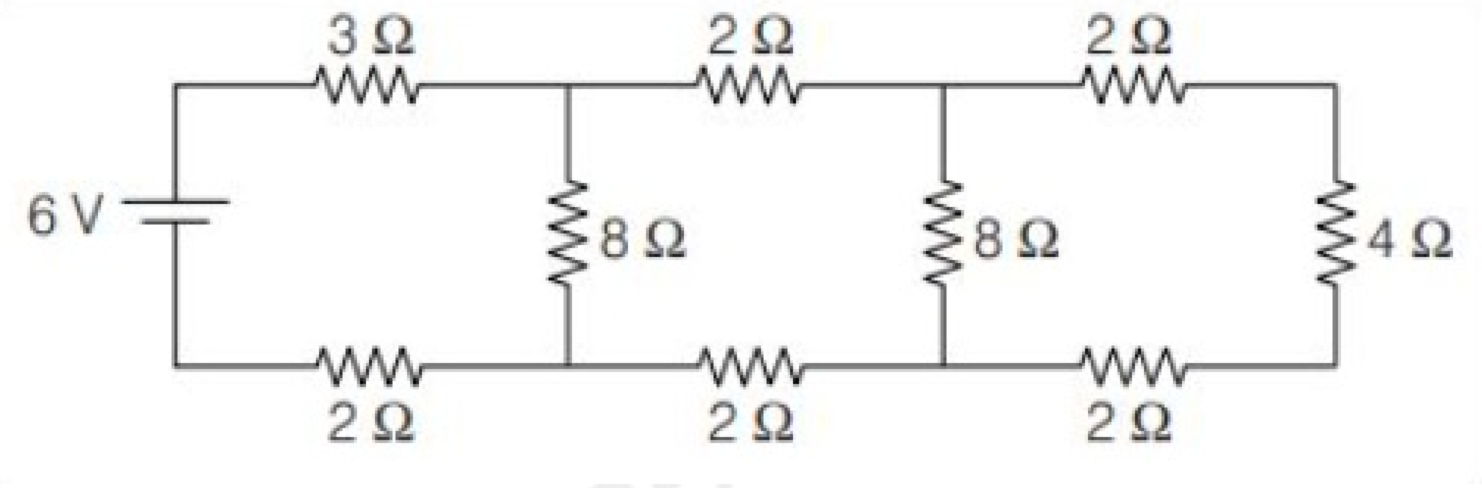
Solution:



Electric field,
$E = \frac{Kq}{r^2}$
$E_1 = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.5)^2}$
$E_2 = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{(0.5)^2}$
Resultant electric field at O is given by,
$E = E_1 - E_2$
$= \frac{E_1 = 9 \times 10^9 \times 2 \times 10^{-6}}{(0.5)^2} - \frac{E_2 = 9 \times 10^9 \times 1 \times 10^{-6}}{(0.5)^2}$
$= \frac{9 \times 10^9 \times 10^{-6}}{(0.5)^2}[2 - 1]$
$= \frac{9 \times 10^3}{0.25}$
$= 3.6 \times 10^4 N / C$

Question 46

In the circuit shown in figure, the current through 4Ω resistance is



Options:

A. 1.67Ω

B. 0.167Ω

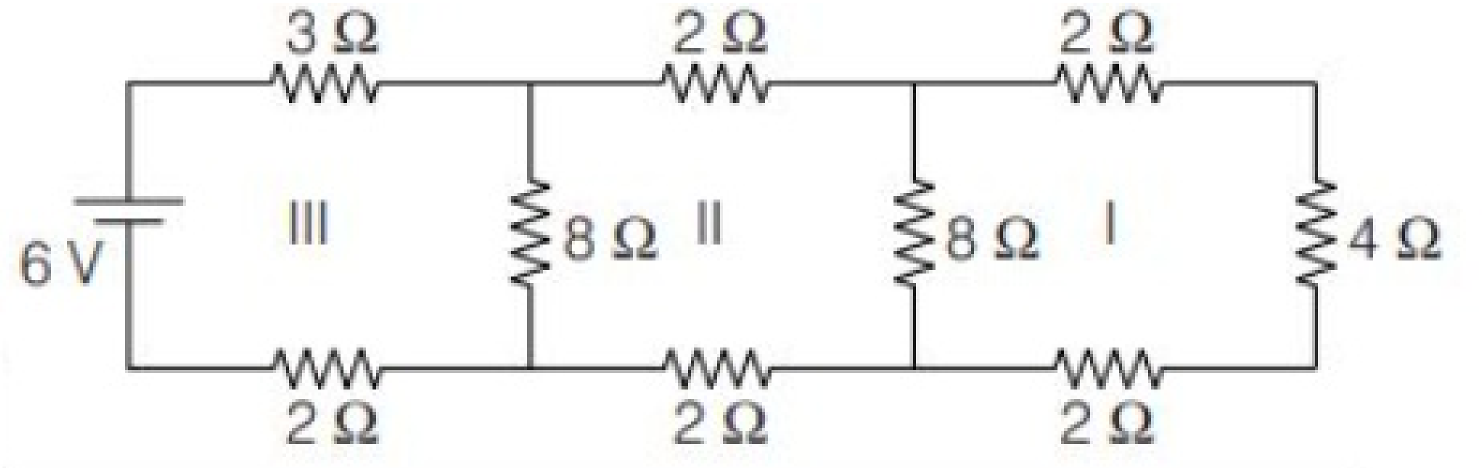
C. 2.37Ω

D. 0.237Ω

Answer: B

Solution:

Solution:



Equivalent resistance of loop I,
$= 2 + 4 + 3 = 8\Omega$
$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{8 \times 8}{8 + 8} = \frac{8 \times 8}{16}$
$= 4\Omega$
Equivalent resistance of loop II,
$= 2 + 4 + 2 = 8\Omega$
$R'_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{8 \times 8}{8 + 8}$
$= 4\Omega$
Equivalent resistance loop III,
$= 3 + 2 + 4 = 9\Omega$
Maximum current, $i = \frac{6}{9} = \frac{2}{3}A$
$= 0.66A$
This current is divided into two equal parts.

Current in first 2 Ω resistor $= \frac{1}{3}$A

The current in 4Ω resistor $= \frac{1}{3 \times 2}$A

$= \frac{1}{6}$A = 0.167A

Question 47

Amount of heat produced per second in calories when a bulb of $50\text{W}, 200\text{V}$ glows (assuming that only 20% of the electric energy is converted into light) $(J = 4.2\text{J}/\text{cal})$

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Options:

A. $40\,\text{cal}/s$

B. $28\,\text{cal}/s$

C. $18.22\,\text{cal}/s$

D. $9.52\,\text{cal}/s$

Answer: D

Solution:

Solution:
Power of bulb = 50 W
Voltage = 200 V
20% is converted into light,
J = $4.2\text{J}/\text{cal}$.
So, amount of heat produced per second will be
$\frac{80}{100}$ of the total power

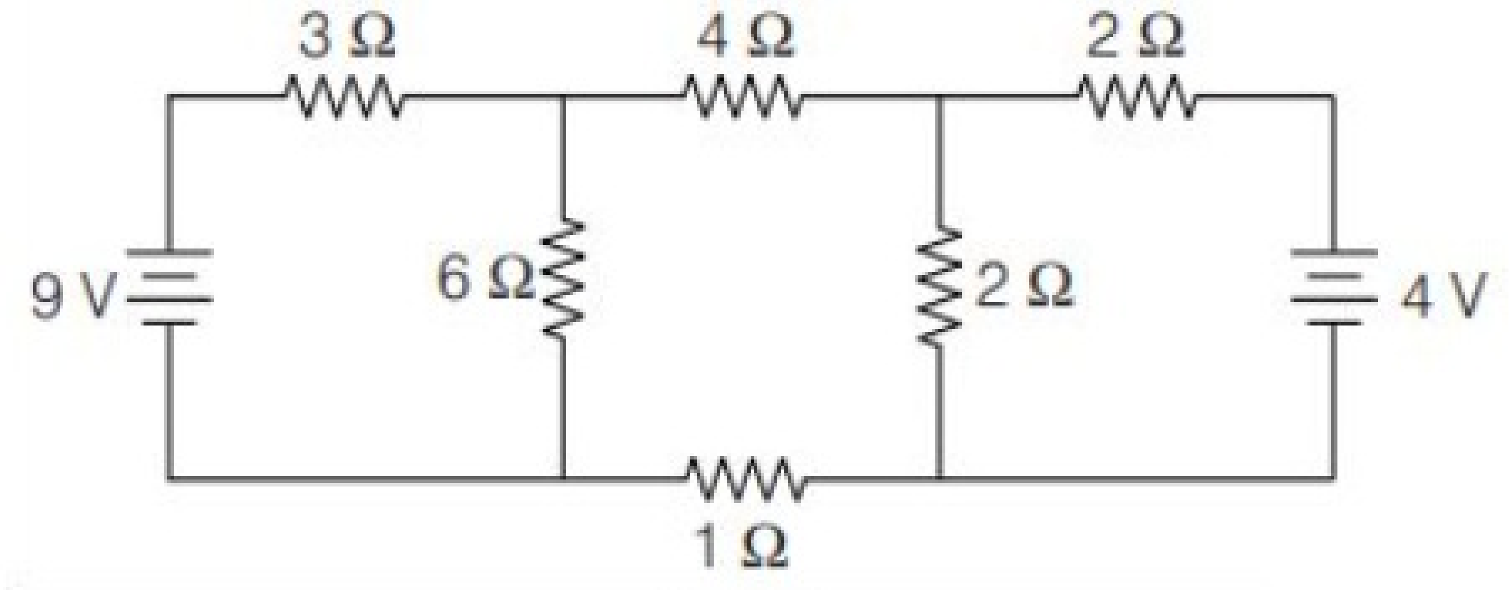
$= 50 \times \frac{80}{100}$J

$= 50 \times \frac{80}{100} \times \frac{1}{4.2}\,\text{cal}/\text{s}$

$= \frac{40}{4.2} = 9.52\,\text{cal}/\text{s}$

Question 48

For the circuit shown in figure, the voltage across 4Ω resistance is



Options:

A. 5V

B. 4V

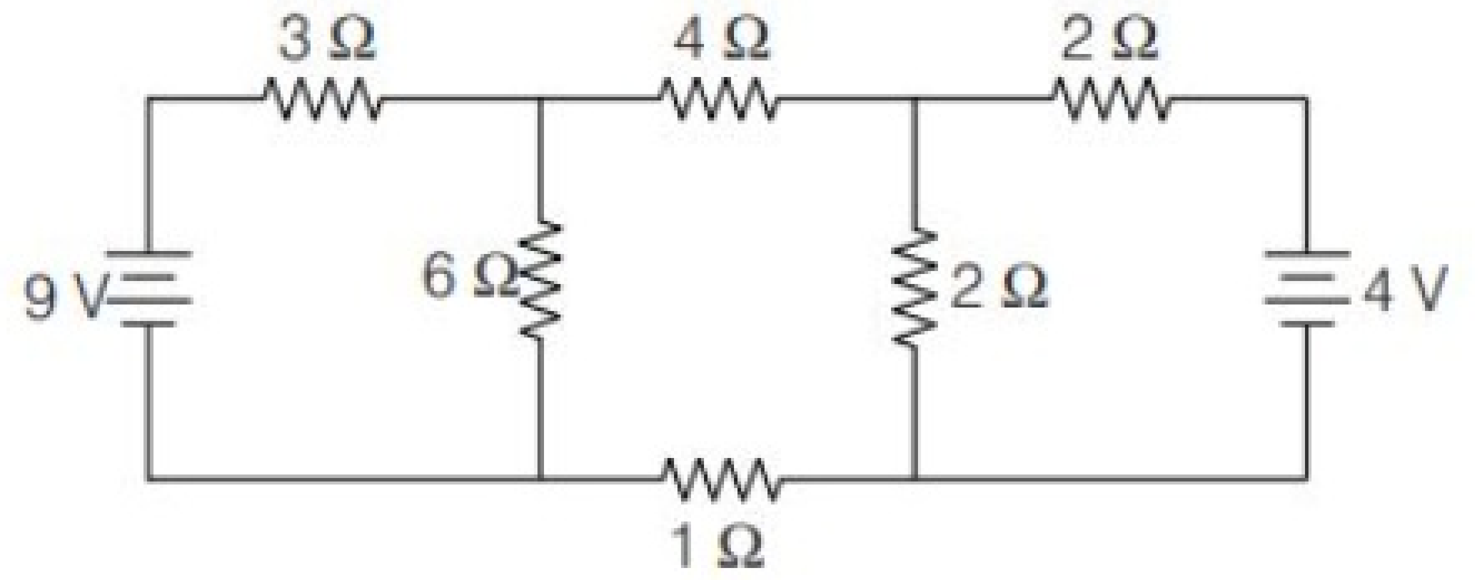
C. 2V

D. 1V

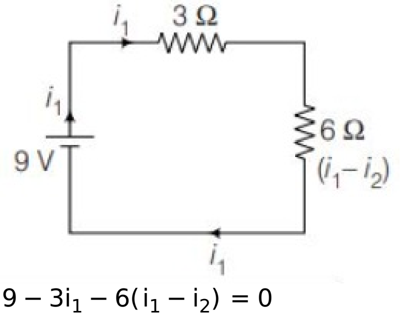
Answer: B

Solution:

Solution:

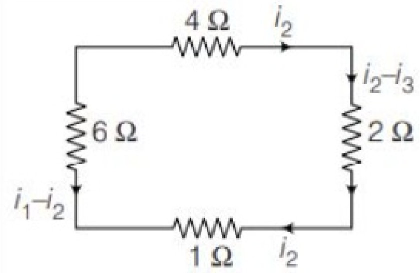


We have 3 loops
In loop I,

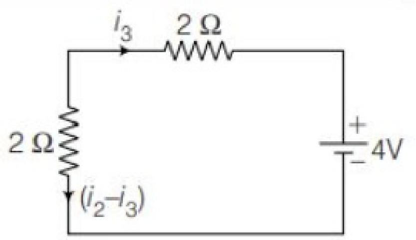


$9 - 3i_1 - 6(i_1 - i_2) = 0$

$9 = 9i_1 - 6i_2$ ......(i)
In loop II,



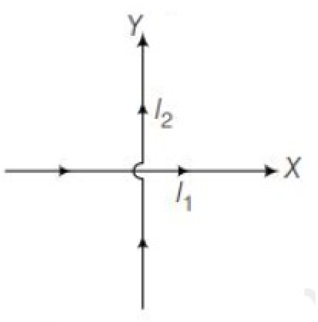
$-4i_2 - (i_2 - i_3)x^2 - i_2 + 6(i_1 - i_2) = 0$
$6i_1 - 13i_2 + 2i_3 = 0$ .....(ii)
In loop III,



$-2i_3 - 4 + 2(i_2 - i_3) = 0$
$\Rightarrow -2i_3 + 2i_2 - 2i_3 = 4$
$\Rightarrow 4i_3 - 2i_2 = 4$ ....(iii)
Substituting for i_1 and i_3 in EQ,(ii) from Eq. (i) and (iii), we get
$i_1 = 9 + 6\frac{i_2}{9}$
$i_3 = (-4) + 2i + \frac{2}{4}$
$6(1 + \frac{2}{3}i_2) - 13i_2 + 2(1 + \frac{1}{2}i_2) = 0$
$\Rightarrow 6 + 4i_2 - 13i_2 + 2 + i_2 = 0$
$\Rightarrow 8 - 8i_2 = 0$
$\therefore i_2 = 1A$
Thus, the potental drop across 4Ω resistor,
$= i_2 \times R = 1 \times 4 = 4V$

Question 49

Two long straight conductors with currents I_1 and I_2 are placed along X-axis and Y-axis as shown in figure. The equation of locus of zero magnetic induction is



Options:

A. $y = x$

B. $y = \frac{I_2 x}{I_1}$

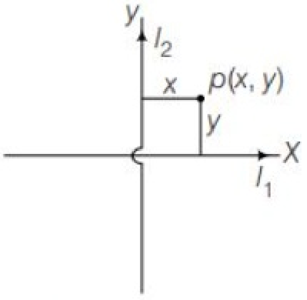
C. $y = \frac{I_1}{I_2}x$

D. $y = I_1 I_2 x$

Answer: C

Solution:

Solution:



Direction of magnetic fields are in opposite directions in Ist and IIIrd quadrants.
Let $P(x, y)$ is a point at which magnetic field is zero then $B_1 = B_2$

$\frac{\mu_0 I_1}{2\pi y} = \frac{\mu_0 I_2}{2\pi x}$

$\frac{I_1}{I_2} x = y$ is the locus of all points where $B_{net} = 0$

Question 50

What is the magnetic field induction at the centre of a coil bent in the form of a square of side $2a$, carrying current I ?

Options:

A. $\frac{\mu_0 I}{\pi a}$

B. $\frac{\sqrt{2}\mu_0 I}{\pi a}$

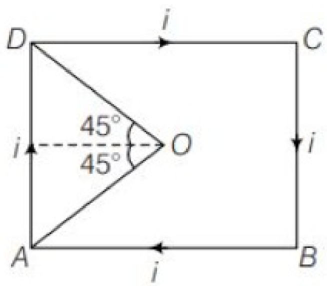
C. $\frac{2\sqrt{2}\mu_0 I}{\pi a}$

D. $\frac{4 \cdot \mu_0 I}{\pi \text{a}}$

Answer: B

Solution:

Solution:



Let O be the centre of square $ABCD$ of side $2a$ carrying current i ampere. The magnetic field due to infinite length of wire AD is given by

$B_1 = \frac{\mu_0 i}{4\pi a}(\sin\alpha + \sin\beta)$

$(\text{Here}, \alpha = \beta = 45°)$

$\frac{\mu_0 i}{4\pi a}(\sin 45° + \sin 45°)$

$= \frac{\mu_0 i}{4\pi a}(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})$

$= \frac{\sqrt{2}\mu_0 i}{4\pi a}$

So, by the symmetry the magnetic field at O due to each side will be $\frac{\sqrt{2}.\mu_0 i}{4\pi a}$

So, Net magnetic field at the centre O of current carrying square is,

$B = 4B_1$

$= \frac{4 \times \sqrt{2}\mu_0 i}{4\pi(a)}$

$= \sqrt{2}\frac{\mu_0 i}{\pi a}$

Question 51

The length of unit cell edge of a body-centred cubic metal crystal is 352 pm. The radius of metal atom is

Options:

A. 162.4 pm

B. 152.4 pm

C. 142.4 pm

D. 156.4 pm

Answer: B

Solution:

Solution:

∵ For bcc - crystal,

$\sqrt{3}a = 4r$

$\therefore r = \frac{\sqrt{3}.a}{4} = \frac{1.732 \times 352}{4}$

$r = 152.4 \text{ pm}$

Question 52

The half-life of a radioactive element depends upon

Options:

A. amount of element

B. temperature

C. pressure

D. independent of all the above

Answer: D

Solution:

Solution:
All radioactive nuclides decay by a first order process and its half-life is independent of amount of substance, temperature and pressure.

Question 53

The element $^{232}_{90}$ Th belongs to thorium series. Which of the following will act as the end product of the series?

Options:

A. $_{82}Pb^{208}$

B. $_{82}Bi^{209}$

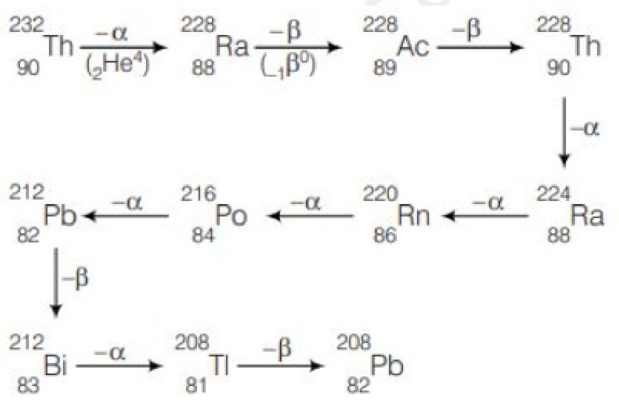
C. $_{82}Pb^{206}$

D. $_{82}Pb^{207}$

Answer: A

Solution:

Solution:
$_{82}Pb^{208}$ will be the end product of thorium series.
$_{90}Th^{232} \xrightarrow{6\alpha, 4\beta} {}_{82}Pb^{208}$
It is shown as follows



Question 54

Consider the following reversible reactions,

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)(K_1)$ ....(i)
$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)(K_2)$.....(ii)
$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g)(K_3)$ .....(iii)

The equilibrium constant for the reaction

$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$

will be

Options:

A. $K_1K_2K_3$

B. $\frac{K_1K_2}{K_3}$

C. $\frac{K_1K_3^3}{K_2}$

D. $\frac{K_2K_3^3}{K_1}$

Answer: D

Solution:

Solution:
Step 1 On reversing Eq. (i) we have
$2NH_3(g) \leftrightharpoons N_2 + 3h_2, K'_1(new) = \frac{1}{K_1}$ ......(iv)
Step 2 Multiply Eq. (iii) by 3, we have
$3H + 2 + \frac{3}{2}O_2 \leftrightharpoons 3H_2O, K_3^3$ ....(v)
Step 3 Add Eq. (ii), Eq. (iv) and Eq. (v), we get
$2NH_3(g) \leftrightharpoons N_2 + 3H_2(g), \frac{1}{K_1}$
$N_2(g) + O_2(g) \leftrightharpoons 2NO(g), K_2$
$3H_2(g) + \frac{3}{2}O_2(g) \leftrightharpoons 3H_2O(g), K_3^3$
$2NH_3(g) + \frac{5}{2}O_2(g) \leftrightharpoons 2NO(g) + 3H_2O(g)$
and new equlibrium constant $= \frac{K_2K_3^3}{K_1}$

Question 55

The number of moles of sodium acetate to be added to 0.1M acetic acid for the buffer to have a pH = 4.7 is [pK_a. for acetic acid is 4.7]

Options:

A. 0.2M

B. 0.4M

C. 0.1M

D. None of these

Answer: C

Solution:

Solution:

$\because$ pH = $pK_a + \log\frac{[\text{Salt}]}{[\text{Acid}]}$

Given, pH = 4.7

$pK_a = 4.7$

$\therefore 4.7 = 4.7 + \log\frac{[\text{Salt}]}{0.1}$

$\therefore \log[CH_3COONa] - \log(0.1) = 0$

$\log[CH_3COONa] - \log(10^{-1}) = 0$

$\log[CH_3COONa] + \log 10 = 0$

$\log[CH_3COONa] = 1.0000$

$\therefore$ On taking antilog on both the sides,

$[CH_3COONa] = 0.1M$

Question 56

Saturated solution of KNO_3 is used to make salt bridge because

Options:

A. KNO_3 is highly soluble in water

B. velocity of K^+ is greater than that of NO_3^-

C. velocity of NO_3^- is greater than that of K^+

D. velocity of K^+ and NO_3^- are almost equal

Answer: D

Solution:

Solution:

Cations and anions of the salt used in the salt bridge must have their comparable ionic radii in order to have same ionic speed so that they reach to the respective electrode compartments simultaneously. Hence, KNO_3 is used to make salt bridge because velocity of K^+ and NO_3^- ions are almost equal.

Question 57

Syneresis is a process in which

Options:

A. spontaneous outcome of internal liquid without disturbing gel structure takes place

B. sol particles absorb light

C. aggregation of molecules to form micelle

D. separation of soluble impurities from sols take place

Answer: A

Solution:

Solution:
Syneresis is a process in which spontaneous outcome of internal liquid without disturbing gel structure takes place.

Question 58

Among the following statements, the incorrect one is

Options:

A. Calamine and siderite are carbonates

B. Argentite and cuprite are oxides

C. Zinc blende and pyrites are sulphides

D. Malachite and azurite are ores of copper

Answer: B

Solution:

Solution:
(a) Calamine $(ZnCO_3)$ and siderite $(FeCO_3)$ both are carbonates.
(b) Argentite $= Ag_2S$
Culprite $= Cu_2O$
(c) Zinc blende $= ZnS$
and pyrites $= FeS_2$ are sulphide ares.
(d) Malachite and azurite $[Cu(OH)_2 . 2CuCO_3]$ are ores of copper.
Hence, option (b) is incorrect.

Question 59

Microcosmic salt when heated strongly, a transparent bead is formed which is used in the identification of

Options:

A. ZnO

B. Al_2O_3

C. MgO

D. SiO_2

Answer: D

Solution:

Solution:
Microcosmic salt when heated strongly is used to identify SiO_2.

Question 60

In nitroprusside ion, the iron and NO as Fe^{2+} and NO^+ rather than Fe^{3+} and NO. These forms can be differentiated by

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Options:

A. estimating the concentration of iron

B. measuring the concentration of CN^-

C. measuring the solid state magnetic moment

D. thermally decomposing the compound

Answer: C

Solution:

Solution:
In nitroprusside ion, the iron and NO are present as Fe^{2+} and NO^+ which are differentiated by measuring the solid state magnetic moment.

Question 61

Which of the following compounds gives metal on heating?

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Options:

A. $AgNO_3$

B. $Ca(NO_3)_2$

C. $Ni(NO_3)_2$

D. $Pb(NO_3)_2$

Answer: A

Solution:

Solution:
$AgNO_3$ on heating gives silvermetal. It is stable upto 650K and starts decomposing above this temperature.
$3AgNO_3 \xrightarrow{700K} \underset{(Metal)}{3Ag} + N_2 + NO + 4O_2$

Question 62

Which of the following compounds gives PELIGOT's salt with chromyl chloride?

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Options:

A. Borax

B. Oxone

C. Sylvine

D. Trona

Answer: C

Solution:

Solution:
Chromyl chloride when reacts with PELIGOT salt (a potassium compound) give 'sylvine' (compound similar to KCl).

Question 63

Which of the following compound liberate Cl_2 gas when react with I_2 ?

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Options:

A. Potassium permanganate

B. Potassium chlorate

C. Hypo

D. Potassium dichromate

Answer: B

Solution:

Solution:
$KClO_3$ solution liberates Cl_2 gas with I_2.

$2KClO_3 + I_2 \longrightarrow 2KIO_3 + Cl_2$

Question 64

Which of the following reactions give oxygen gas?
1. $KMnO_4 + 2KF + H_2O_2 \, 50\% \, HF \longrightarrow$
2. $F_2 + NaOH \longrightarrow$
3. $XeF_2 + 2H_2O \longrightarrow$
4. $Hg_2CrO_4 \longrightarrow$

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Options:

A. 1, 2, 3

B. 1, 3, 4

C. 2, 3, 4

D. 1, 2, 4

Answer: A

Solution:

Solution:
Reaction (1), (2) and (3) give oxygen.
$(1) \, 2KMnO_4 + 2H_2O_2 + 2KF \longrightarrow 2KMnO_2 + 3O_2 + 2H_2O + 2KOH + F_2$
$(2) \, 2F_2 + 4NaOH \longrightarrow 4NaF + O_2(g) + 2H_2O$
$(3) \, 2XeF_2 + 2H_2O \longrightarrow 2Xe + 4HF + O_2(g)$

Question 65

Which of the following set of combination are correct?
1. Fenton reagent : Alcohol → Aldehyde
2. Ziegler-Natta catalyst : Polymerisation
3. Adam catalyst : Oxidation
4. Fe/Mo **catalyst : Synthesis of** NH_3

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Options:

A. 1 and 3

B. 1 and 2

C. 2 and 3

D. 2 and 4

Answer: B

Solution:

Solution:
Fenton reagent $(FeSO_4 + H_2O_2)$ is used to convert alcohol to aldehyde while Ziegler-Natta catalyst $[TiCl_3 + Al(Et)_3]$ is used in polymerisation.

Question 66

Which compound does not form 2-methyl but-2-ene on heating with alc. KOH ?

Options:

A. 1-bromo-2 methyl propane

B. 2-bromo-3-methyl butane

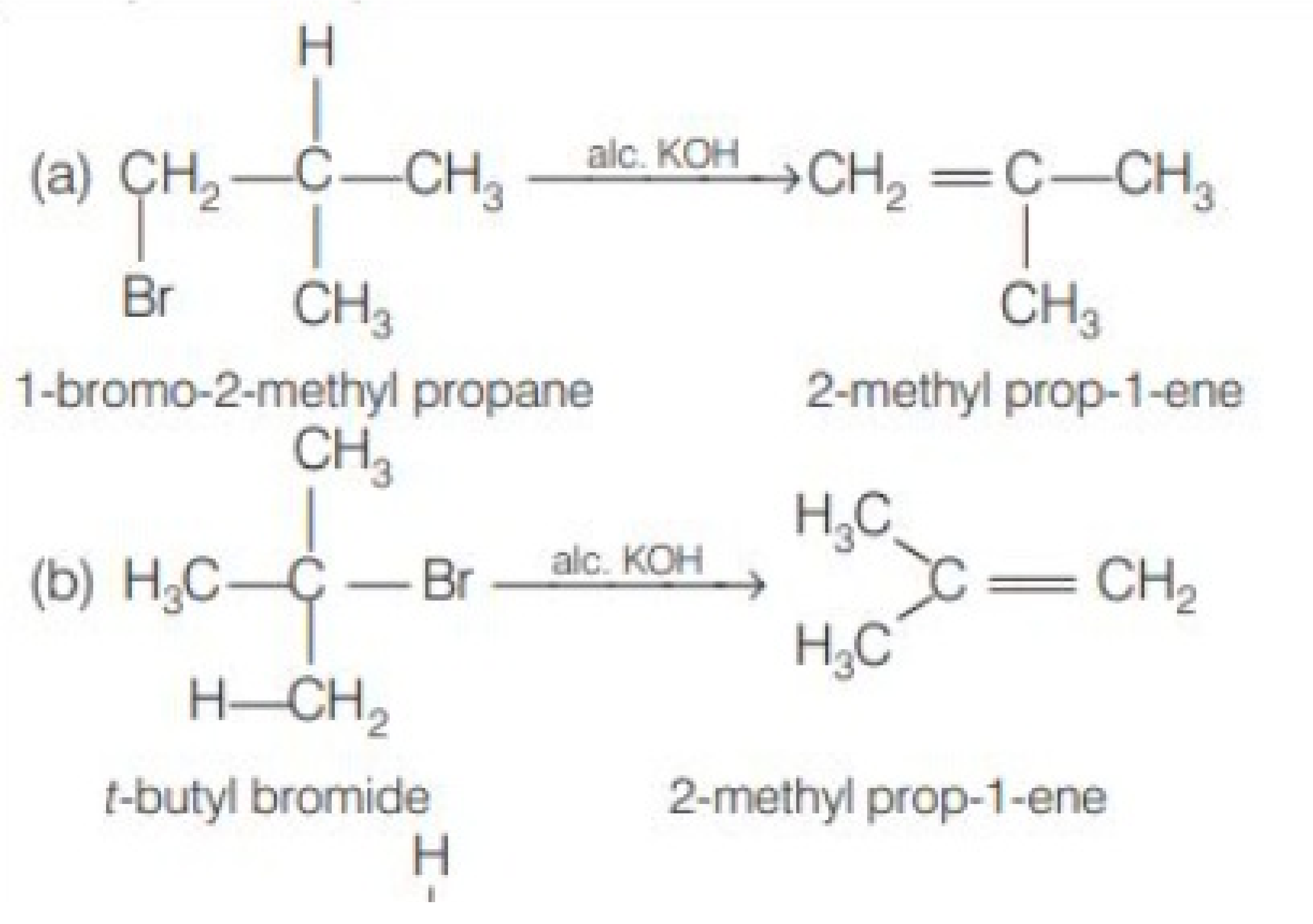
C. 2-bromo-2-methyl butane

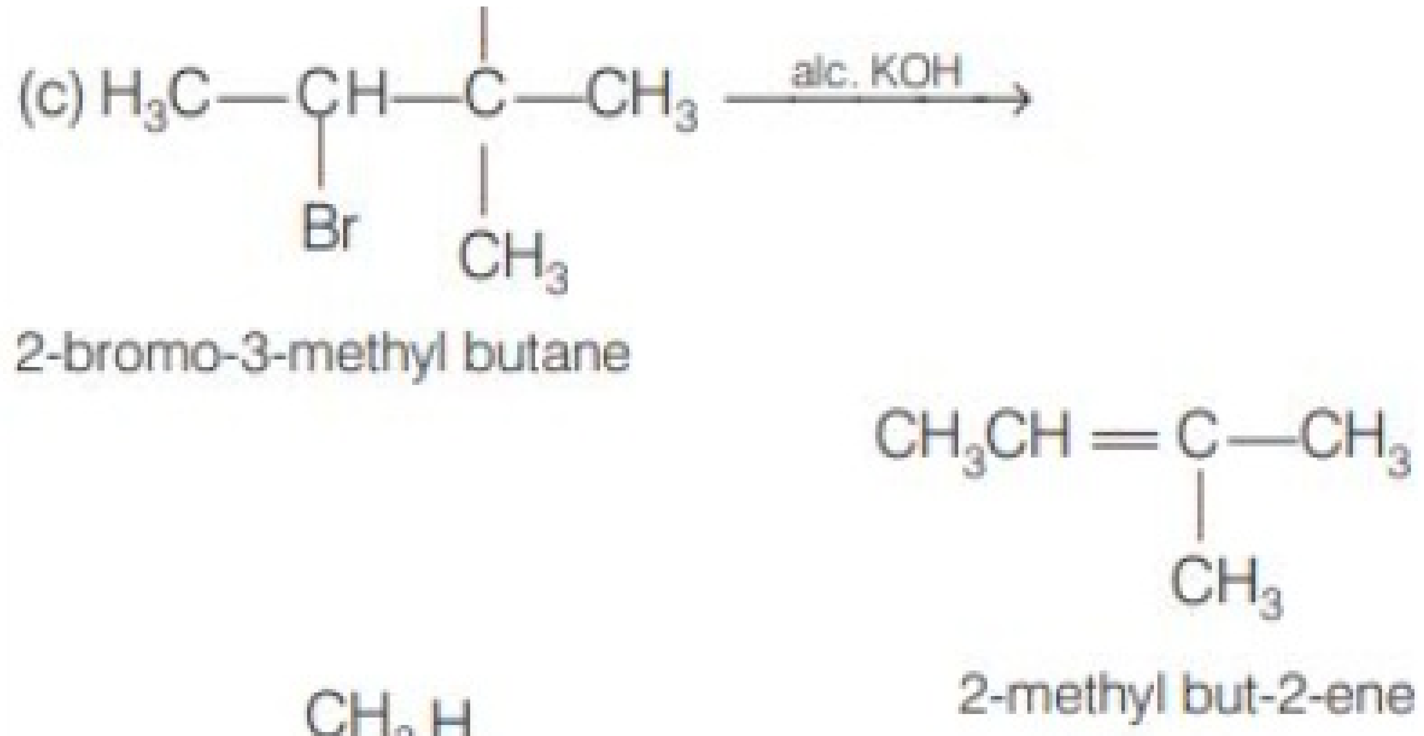
D. None of the above

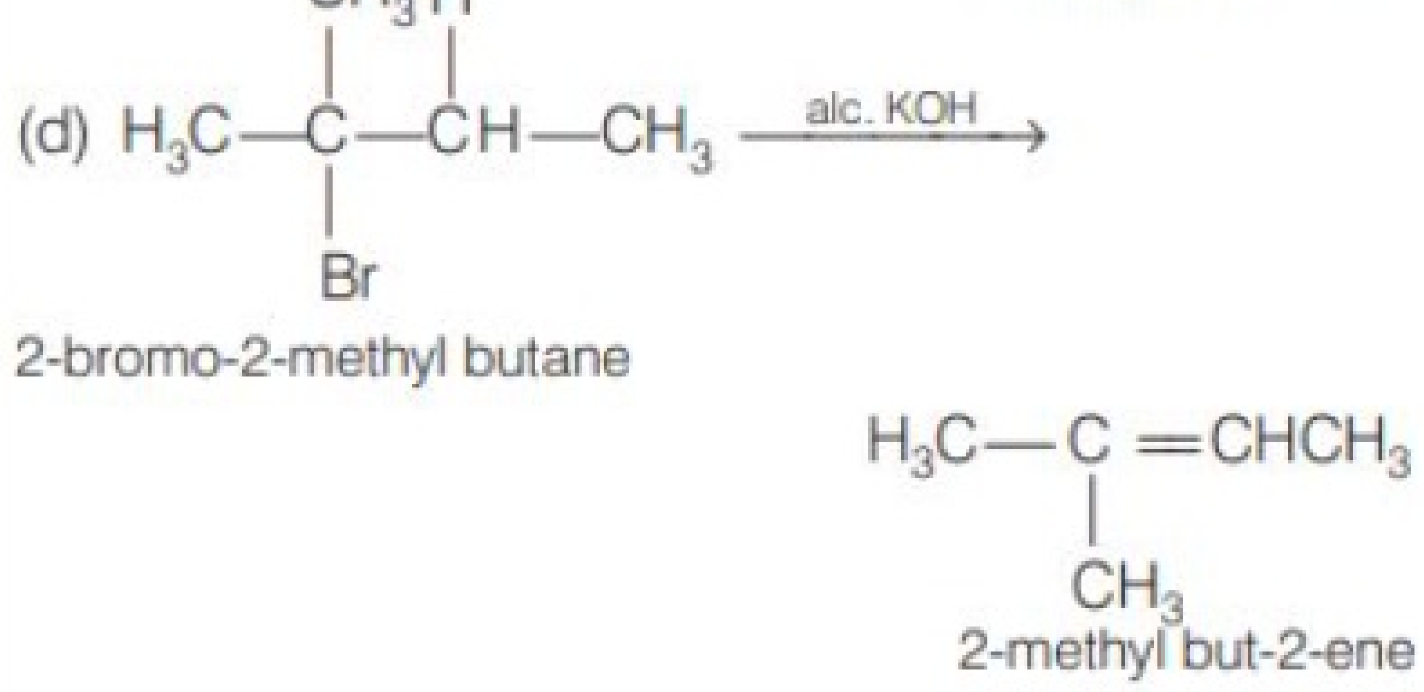
Answer: A

Solution:

Solution:







Question 67

In the reaction,
Acetylen

$$\xrightarrow[NH_4Cl]{CuCl} H_2C=CH-C\equiv CH \xrightarrow[\text{Catalyst}]{\text{Lindlor}} A$$

The final product A **is**

Options:

A. butene

B. butyne-2

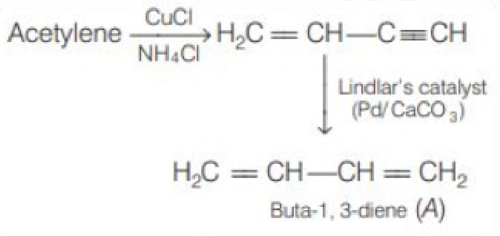
C. butyne-1

D. buta-1, 3-diene

Answer: D

Solution:

Solution:
The Lindlar's catalyst is a palladium catalyst $(Pd / CaCO_3)$ deliberately poisoned with lead. It is used to reduce alkynes to cis alkenes.



Question 68

The olefin which on ozonolysis gives CH_3CH_2CHO and CH_3CHO is

Options:

A. 2-butene

B. 1-pentene

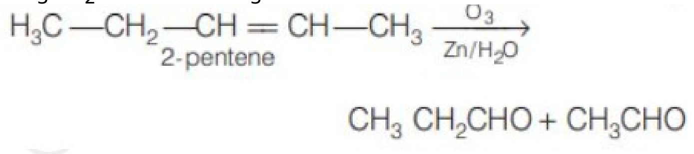
C. 1-butene

D. 2-pentene

Answer: D

Solution:

Solution:
The olefin is 2-pentene which on ozonolysis gives
CH_3CH_2CHO and CH_3CHO as

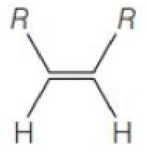


Question 69

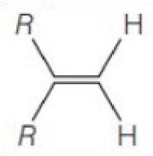
Which one of the following alkenes will react faster with H_2 under catalytic hydrogenation conditions?

Options:

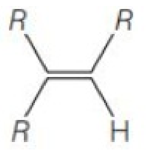
A.



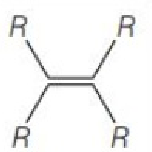
B.



C.



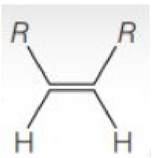
D.



Answer: A

Solution:

Solution:
As



is least stable thus, reacts faster with H_2 under catalytic hydrogenation due to two R - groups in the same plane. So, more surface area provides to hydrogenation reaction and alkenes will react faster with H_2.

Question 70

Drugs are the chemicals with

Options:

A. low molecular masses

B. high molecular masses

C. low atomic mass

D. high atomic mass

Answer: B

Solution:

Solution:
Drugs are the chemicals with high molecular masses. These are mostly polymers of various elements.

Question 71

Which of the following compounds cannot be identified by carbylamine test?

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Options:

A. $CHCl_3$

B. $C_6H_5 - NH - C_6H_5$

C. $C_6H_5NH_2$

D. $CH_3CH_2NH_2$

Answer: B

Solution:

Solution:
$C_6H_5 - NH - C_6H_5$ cannot be identified by carbylamine test as it is used to test primary amines or $CHCl_3$.

Question 72

The hard plastic covers of telephones are made from polymers of

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Options:

A. acrylonitrile

B. styrene

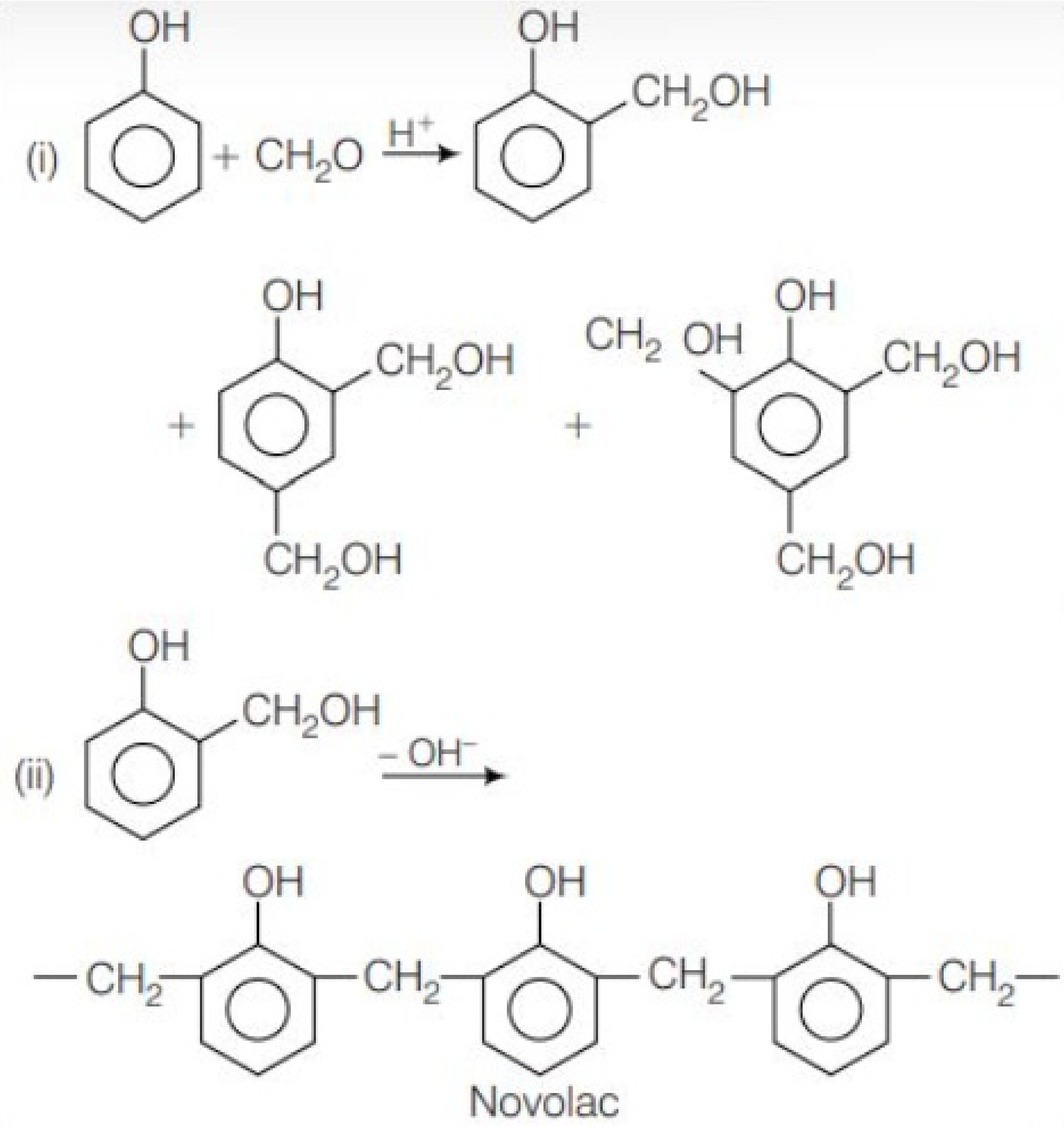
C. fluoromethane

D. phenol and formaldehyde

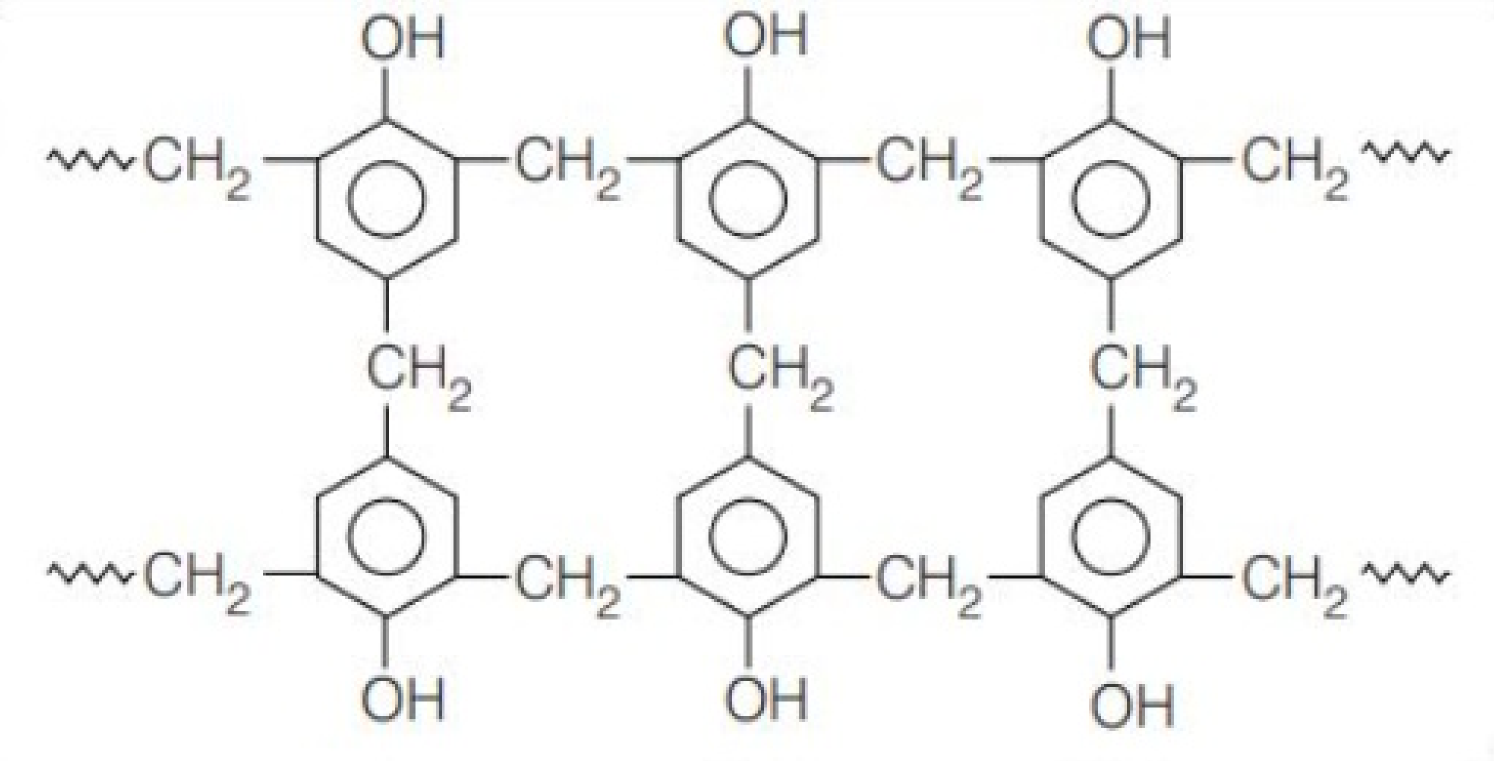
Answer: D

Solution:

Solution:
Phenol-formaldehyde is used in making covers of telephones. It is made by the reaction of phenol and Formaldehyde.
Step I Formation of novolac



Step II On heating above structure, it gives network of phenol and formaldehyde, also known as bakelite.



Question 73

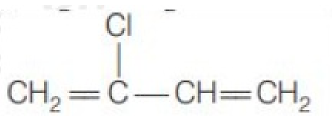
Which one of the following monomers gives the polymer neoprene on polymerisation?

Options:

A. $CH_2 = CH - Cl$

B. $CCl_2 = CCl_2$

C.

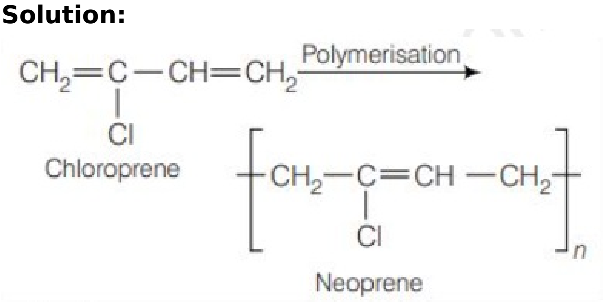


D. $CF_2 = CF_2$

Answer: C

Solution:

Solution:



Neoprene is a type of synthetic rubber.

Question 74

During the process of digestion, the proteins present in food materials are hydrolysed to amino acids.

Proteins $\xrightarrow{\text{Enzyme }(A)}$ Polypeptides $\xrightarrow{\text{Enzyme }(B)}$ Amino acids

The two enzymes A and B involved in the process are respectively

Options:

A. amylase and maltase

B. diastase and lipase

C. pepsin and trypsin

D. invertase and zymase

Answer: C

Solution:

Solution:
On the basis of secondary structure, most of the long polypeptide chains get coiled (or folded) to produce a three-dimensional structure. α-helix is an example of this.
When these structures form tertiary structure called fibrous protein and if has long thin chains (threads), are called globular protein.
Hence, coiled form of fibrous protein is known as globular protein.

Question 75

If radius of first Bohr's orbit of hydrogen atom is ' x ', then the de-Broglie wavelength of electron in 3rd orbit is nearly

Options:

A. $2\pi x$

B. $6\pi x$

C. $9x$

D. $\frac{x}{3}$

Answer: C

Solution:

Solution:
Proteins $\xrightarrow{\text{Pepsin(A)}}$ Polypeptides $\xrightarrow{\text{Trypsin(B)}}$ Amino acids

----

Question 76

If radius of first Bohr's orbit of hydrogen atom is ' x ', then the de-Broglie wavelength of electron in 3rd orbit is nearly

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Options:

A. $2\pi x$

B. $6\pi x$

C. $9x$

D. $\frac{x}{3}$

Answer: B

Solution:

Solution:

As, $r_n = r_0 \frac{n^2}{Z} (\because Z = 1, r_0 = x)$

$r_3 = 3^2 . x = 9x$

Also, $mvr = n\frac{nh}{2\pi}$

$mv = \frac{nh}{2\pi r_3}$

$= \frac{3h}{2\pi 9x} = \frac{h}{6\pi x}$

$\therefore \lambda = \frac{h}{mv} = \frac{h \times 6\pi x}{h} = 6\pi x$

Question 77

Quantum numbers $l = 2$ and $m = 0$ represent the orbital

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Options:

A. d_{xy}

B. $d_{x^2-y^2}$

C. d_{z^2}

D. d_{xz}

Answer: C

Solution:

Solution:
For l = 2
$m = 0 (m = +2, +1, 0, -1, -2)$
but, m = 0 is considerd as d_{z^2}.
$\therefore$ m = 0 represents d_{z^2} orbital.

Question 78

Which of the following statements is false?
1. Non-bonding pairs occupy more space than bonding pairs.
2. The bonding orbitals in trigonal bipyramidal molecule are described as sp^3d hybrids.
3. $SnCl_2$ has linear shape.
4. PCl_4^+ and $AlCl_4^-$ are isoelectronic.

Options:

A. 1

B. 2

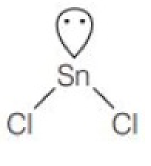
C. 3

D. 4

Answer: C

Solution:

Solution:
As Sn has one lp and two bp in its structure of $SnCl_2$. It is of bent structure.



Question 79

1% (W / V) solutions of KCl is dissociated to the extent of 82%. The osmotic pressure at 300K will be

Options:

A. 3.2 atm

B. 5.824 atm

C. 4.0 atm

D. 6.0 atm

Answer: D

Solution:

Solution:

$\because \pi = i \frac{W}{M \times V} . RT$

and $i = \frac{100 - \alpha + n\alpha}{100}$

$i = \frac{100 - 82 + 2 \times 82}{100} = 1.82$

$\therefore \pi = \frac{1.82 \times 1 \times 0.08 \times 300 \times 1000}{74. \times 100}$

$\pi = 6.0$ atm

Question 80

Match List I with List II and select the correct answer using the given codes.

List I (Crystal system)	List II (Examples)
A. Cubic	1. TiO_2
B. Tetragonal	2. Graphite
C. Hexagonal	3. $K_2Cr_2O_7$
D. Triclinic	4. ZnS

Options:

A. 2 3 4 1

B. 1 4 3 2

C. 3 2 1 4

D. 4 1 2 3

Answer: D

Solution:

Solution:

(A) $\longrightarrow$ 4, ZnS (cubic)
(B) $\longrightarrow$ 1, TiO_2 (Tetragonal)
(C) $\longrightarrow$ 2, Graphite (Hexagonal)
(D) $\longrightarrow$ 3, $K_2Cr_2O_7$ (Triclinic)

Question 81

Match List I with List II and select the correct answer using the codes.

List I	List II
A. Spontaneous process	1. $\Delta H < 0$
B. Exothermic process	2. Heat of reaction
C. Enthalpy at constant pressure	3. $\Delta G < 0$
D. Cyclic process	4. $\Delta U = 0, \Delta H = 0$

Options:

A. 4 3 1 2

B. 3 1 2 4

C. 1 3 4 2

D. 1 2 3 4

Answer: B

Solution:

Solution:
(A) $\longrightarrow 3, \Delta G = -ve$
(B) $\longrightarrow 1, \Delta H = -ve$
(C) $\longrightarrow 2$, \Heat of reaction
(D) $\longrightarrow 4, \Delta U = 0, \Delta H = 0$

Question 82

Standard enthalpies of formation of O_3, CO_2, NH_3 and HI are $142.2, -393.2, -46.2$ and 25.9 kJ mol^{-1} respectively. Decreasing order of their stability is

Options:

A. $HI > NH_3 > CO_2 > O_3$

B. $NH_3 > CO_2 > HI > O_3$

C. $CO_2 > NH_3 > HI > O_3$

D. $O_3 > HI > NH_3 > CO_2$

Answer: C

Solution:

Solution:
More be the negative value of standard enthalpy of formation, more the species will be stable.
Since,
$O_3 = +142.2$ kJ / mol
$CO_2 = -393.2$ kJ / mol
$NH_3 = -46.2$ kJ / mol
$HI = +25.9$ kJ / mol
Hence, correct order is
$CO_2 > NH_3 > HI > O_3$

Question 83

A hydrogenation reaction is carried out at 500 K. If the same reaction is carried out in the presence of a catalyst at the same rate, the temperature required is 400 K. If the catalyst lowers the activation barrier by 40 kJ mol^{-1}, the activation energy of the reaction will be

Options:

A. 100 kJ mol^{-1}

B. 200 kJ mol^{-1}

C. 300 kJ mol^{-1}

D. 175 kJ mol^{-1}

Answer: B

Solution:

Solution:
Given, let $E_a =$ Original activation energy
Thus,
$E'_a = E_a - 40$ (lower activation energy)
∵ Rate are same

$$\frac{E_a}{R \times 500} = \frac{E'_a}{R \times 400}$$

$$\frac{E_a}{500} = \frac{E_a - 40}{400}$$

$400E_a = 500(E_a - 40)$
$400E_a = 500E_a - 20000$
$\therefore 100E_a = 20000$

$$\therefore E_a = \frac{20000}{100}$$

$= 200$ kJ / mol

Question 84

A drop of solution (volume $0.05\ \mathrm{mL}$) contains 3×10^{-6} mole H^{+}. If the rate constant of disappearance of H^{+} is $1.0 \times 10^{7}\ \mathrm{mol\ L}^{-1}\ s^{-1}$, how long would it take for H^{+} in drop to disappear?

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Options:

A. $4 \times 10^{-6} s$

B. $1 \times 10^{-7} s$

C. $3 \times 10^{-6} s$

D. $6 \times 10^{-9} s$

Answer: D

Solution:

Solution:

Concentration of drop $= \frac{n}{V} = \frac{3 \times 10^{-6}}{0.05} \times 1000$

Concentration $= 0.06\ \mathrm{mol\ L}^{-1}$

$= 6 \times 10^{-2}\ \mathrm{mol\ L}^{-1}$

$\therefore$ Rate of disappearance $= \frac{\Delta \text{Concentration}}{\text{Time}}$

$\therefore$ Time $= \frac{\text{Concentration}}{\text{Rate}} = \frac{6 \times 10^{-2}}{10^{7}}$

$= 6 \times 10 - 9\mathrm{s}$

Question 85

One Faraday of charge passes through solution of $\mathrm{AgNO_3}$ and $\mathrm{CuSO_4}$ connected in series and the concentration of two solutions being in the ratio $1:2$. The ratio of amount of Ag and Cu deposited on Pt electrode is

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Options:

A. $107.9:63.54$

B. $54:31.77$

C. $107.9:31.77$

D. $54:63.54$

Answer: C

Solution:

Solution:

$\because \frac{W_{Ag} \times n}{M_{Ag}} = \frac{W_{Cu} \times n}{M_{Cu}}$
$\frac{W_{Ag} \times 1}{107.9} = \frac{W_{Cu} \times 2}{63.54}$
$\frac{W_{Ag}}{W_{Cu}} = \frac{107.9}{31.77}$
$\therefore$ Ratio is $107.9 : 31.77$

Question 86

First and second ionisation enthalpies of Mg are 737.76 and $1450.73 J mol^{-1}$ respectively. The energy required to convert all the atoms of magnesium to magnesium ions present in $24 g$ of magnesium vapours is

Options:

A. $24 kJ$

B. $2.188 kJ$

C. $12 kJ$

D. $4.253 kJ$

Answer: B

Solution:

Solution:
$\because$ First ionisation enthalpy for 1 mole of Mg
$= 737.76 J / mol^{-1}$
and second ionisation enthalpy for 1 mole of Mg
$= 1450.73 J / mole^{-1}$
Total energy required $= 737.76 + 1450.73$
$= 2188.49 J / mole^{-1}$
Also, 24g of Mg $= 1$ mole of Mg
$\therefore$ Energy required $= 2188.49 J = 2.188 kJ$

Question 87

First ionisation potentials of nitrogen and oxygen in eV respectively are

Options:

A. $14.5, 13.6$

B. $13.6, 14.5$

C. $13.6, 13.6$

D. $14.6, 14.6$

Answer: A

Solution:

Solution:
∵ Nitrogen has half filled configuration, thus its ionisation enthalpy will always be greater than that of oxygen.
First ionisation potential of nitrogen $= 14.5 \text{ eV}$
First ionisation potential of oxygen $= 13.6 \text{ eV}$

Question 88

In the following, the element with the highest electropositivity is

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Options:

A. copper

B. silver

C. gold

D. caesium

Answer: D

Solution:

Solution:
Caesium is the most electropositive element because of largest size as
Ionisation enthalpy $\propto \frac{1}{\text{Atomic size}}$ or Electropositive character $\propto$ Atomic size

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Question 89

The metal X is prepared by the electrolysis of fused chloride. It reacts with hydrogen to form a colourless solid from which hydrogen is released on treatment with water. The metal is

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Options:

A. Al

B. Ca

C. Cu

D. Zn

Answer: B

Solution:

Solution:
The metal ' X ' is Ca . When it undergoes electrolysis, it gives $H_2(g)$ because $Ca^{2+}(aq)$ has higher discharge potential than hydrogen.

Question 90

Mark the wrong statement. When Na_2S is added to sodium nitroprusside solution,

Options:

A. beautiful violet colour is produced

B. a complex $[Fe(CN)_5NOS]^{4-}$ is formed

C. a complex $[Fe(CN)_5NOS]^{2-}$ is formed

D. a complex $Na_4[Fe(CN)_5NOS]$ is formed

Answer: B

Solution:

Solution:
When sodium sulphide is added to sodium nitroprusside, a purple or violet colour is solution observed.

$$Na_2S + Na_2[Fe(CN)_5NO] \longrightarrow \underset{\text{(Purple colour)}}{Na_4[Fe(CN)_5NOS]}$$

or $\underset{\text{(Complex ion)}}{[Fe(CN)_5NOS]^{4-}}$

Question 91

Identify the pair of complex showing geometrical isomerism and diamagnetic.
1. $[PdCl_2(PPh_3)_2]; [Pt(NH_3)_2ClBr]$
2. $[PdCl_2(PPh_3)_2]; [NiCl_2(PPh_3)_2]$
3. $[Ni(CO)_4]; [Co(NH_3)_6]Cl_3$

Options:

A. 1

B. 1,2 and 3

C. 2 and 3

D. 3 and 4

Answer: A

Solution:

Solution:
Only (1) is the correct answer, as $[PdCl_2(PPh_3)_2]$ and $[Pt(NH_3)_2ClBr]$ show geometrical isomerism due to dsp^2 hybridisation and two distinct ligands and is diamagnetic in nature due to the absence of unpaired electron.

Question 92

Which of the following complex compounds gives 5 isomers? ($M=$ metal ; $a, b, c, d, e, f=$ ligands)

Options:

A. Ma_4b_2

B. $Ma_2b_2c_2$

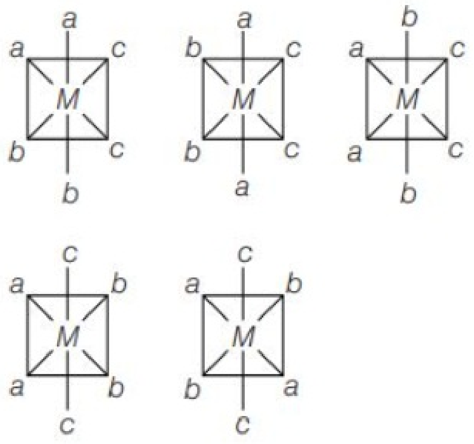
C. Mabcdef

D. Ma_3b_3

Answer: B

Solution:

Solution:
The complex compound gives five isomers is $Ma_2b_2c_2$.



Question 93

Which is incorrectly matched?

Options:

A. Golden spangles : $PbCrO_4$

B. $Cu_2S + FeS$: Matte

C. ZnO : Pompholyx

D. 10 vol $H_2O_2 : 3.035\,\%$

Answer: C

Solution:

Solution:

Question 94

The degree of unsaturation of C_6H_6 and butyne-2 respectively are

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Options:

A. 6 and 3

B. 6 and 2

C. 2 and 6

D. 4 and 2

Answer: D

Solution:

Solution:
The degree of unsaturation is 4 or C_6H_6 and 2 for $C_4H_6(\text{butyne}-2)$ as
Formula for unsaturation
$= \frac{2C_n + 2 - H}{2}$
For C_6H_6
(i) degree of unsaturation $= \frac{2 \times 6 + 2 - 6}{2}$
$= \frac{12 + 2 - 6}{2}$
$= \frac{14 - 6}{2}$
$= \frac{8}{2} = 4$
(ii) degree of unsaturation for $(C_4H_6)(\text{Butyne} - 2)$
$= \frac{2 \times 4 + 2 - 6}{2}$
$= \frac{8 + 2 - 6}{2}$
$= \frac{10 - 6}{2}$
$= \frac{4}{2} = 2$

Question 95

Compounds (A) gives two mol acetone and glyoxal on ozonolysis?

Options:

A. 2, 5-dimethyl hexyne

B. 2, 5-dimethyl hex-3-ene

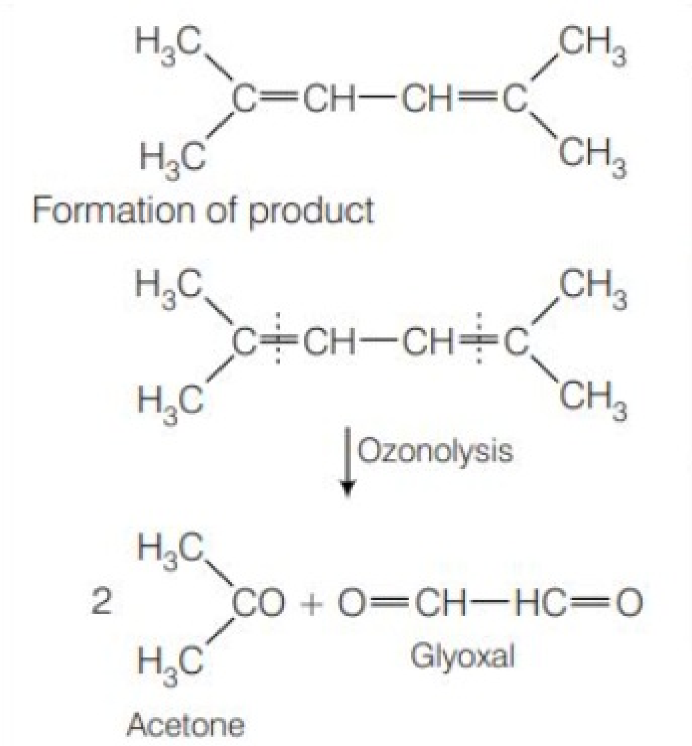
C. 2, 5-dimethyl hex-2-yne

D. 2, 5-dimethyl hexa-2, 4-diene

Answer: D

Solution:

Solution:
2, 5 - dimethyl hexa-2, 4 - diene



Question 96

Which of the following is a nacrotic analgesic?

Options:

A. Paracetamol

B. Aspirin

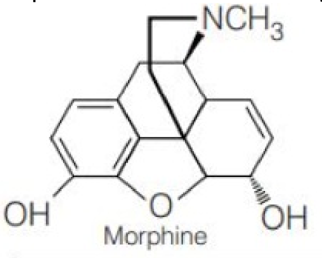
C. Morphine

D. None of the above

Answer: C

Solution:

Solution:
Morphine is a narcotic analgesic.



Question 97

Which among the following is not an antibiotic?

Options:

A. Erythromycin

B. Oxytocin

C. Penicillin

D. Tetracycline

Answer: B

Solution:

Solution:
Oxytocin is not an antibiotic. It is used for milk extraction.

Question 98

$(CH_3)_2CO \xrightarrow[HCl]{NaCN} A \xrightarrow{H_3O^+} B$

In the above sequence of reactions, A and B respectively are

Options:

A. $(CH_3)_2C(OH)CN, (CH_3)_2CHCOOH$

B. $(CH_3)_2C(OH)CN, (CH_3)_2C(OH)_2$

C. $(CH_3)_2C(OH)CN, (CH_3)_2C(OH)COOH$

D. $(CH_3)_2C(OH)CN, (CH_3)_2C = O$

Answer: C

Solution:

Solution:
(i) $(CH_3)_2CO \xrightarrow[HCl]{NaCN} \underset{\text{Cyanohydrin}(A)}{(CH_3)_2C(OH)CN}$
(ii) $(CH_3)_2C(OH)(CN) \longrightarrow CH^-$ part on hydrolysis give $-COOH$.
Thus, $(CH_3)_2C(OH)COOH(B)$.

Question 99

Which of the following is the most reactive towards ring nitration?

Options:

A. Benzene

B. Toluene

C. m-xylene

D. Mesitylene

Answer: D

Solution:

Solution:
As mesitylene contains $3-CH_3$ groups (electron donating groups) thus, is most reactive towards ring nitration.

Question 100

In the following reaction, $C_6H_5NO_2 \xrightarrow{Sn/HCl} X \xrightarrow{C_6H_5COCl} Y + HCl$ the product Y is

Options:

A. azobenzene

B. acetanilide

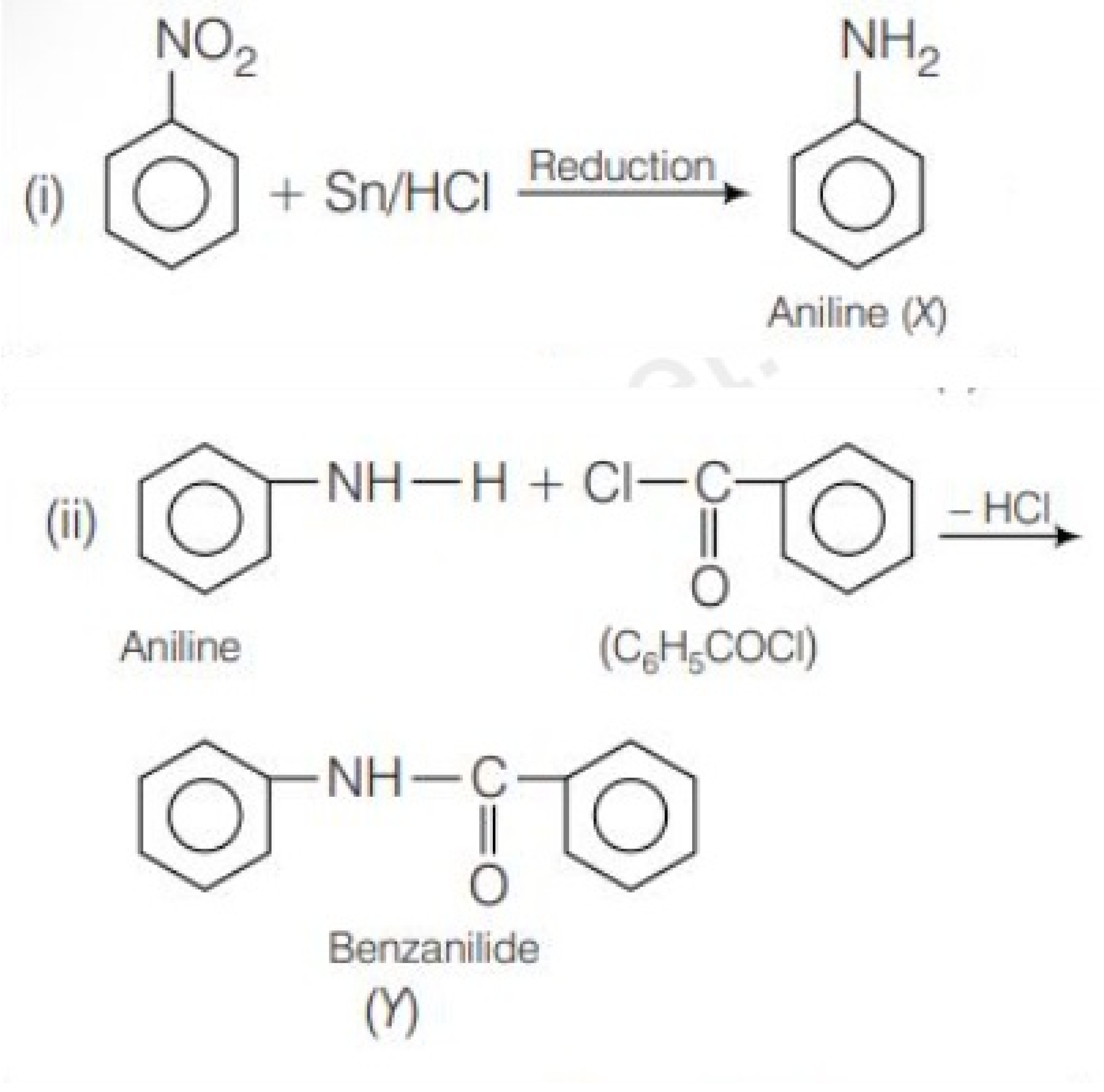
C. benzanilide

D. hydrazobenzene

Answer: C

Solution:

Solution:



Question 101

In how many ways can 5 keys be put in a ring?

Options:

A. 4!

B. 5 !

C. $\frac{4!}{2}$

D. $\frac{5!}{2}$

Answer: A

Solution:

Solution:
To arrange n objects in a ring,
number of ways $=(n-1)!$
$\therefore$ 5 keys to be put in a ring $=(5-1)!=4!$ ways

Question 102

The value of $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$ is

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Options:

A. 232

B. 352

C. 452

D. 532

Answer: B

Solution:

Solution:
$(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$
$=[(\sqrt{5})^5+{}^5C_1(\sqrt{5})^4(1)+{}^5C_2(\sqrt{5})^3(1)+{}^5C_3(\sqrt{5})^2(1)$
$+{}^5C_4(\sqrt{5})^1(1)+{}^5C_5(\sqrt{5})^0(1)]-[(\sqrt{5})^5-{}^5C_1(\sqrt{5})^4(1)$
$+{}^5C_2(\sqrt{5})^3(1)-{}^5C_3(\sqrt{5})^2(1)+{}^5C_4(\sqrt{5})^1(1)-{}^5C_5(\sqrt{5})^0(1)]$
$=2[{}^5C_1(\sqrt{5})^4]+2[{}^5C_3(\sqrt{5})^2]+2({}^5C_5)$
$=2\times 5\times 25+2\times 10\times 5+2$
$=250+100+2=352$

Question 103

The sum of the series $\frac{1}{2\cdot 3}+\frac{1}{4\cdot 5}+\frac{1}{6\cdot 7}+...$ is

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Options:

A. $\log\left(\frac{e}{2}\right)$

B. $\log(\frac{2}{e})$

C. $\frac{e}{2}$

D. $\frac{2}{e}$

Answer: A

Solution:

Solution:

Given series is $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}$

$\therefore a_n = \frac{1}{(\text{nth term of } 2,4,6....)(\text{nth term of } 3,5,7....)}$

$= \frac{1}{(2n)(2n+1)}$

$= (\frac{1}{2n}-\frac{1}{2n+1})$ [by using partial fraction]

Thus, $a_k = (\frac{1}{2n}-\frac{1}{2n+1})$

On putting $k = 1,2,3,...$ we get $a_1 = (\frac{1}{2}-\frac{1}{3})$

$a_2 = (\frac{1}{4}-\frac{1}{5})$

$a_3(\frac{1}{6}-\frac{1}{7})$

$a_n = (\frac{1}{2n}-\frac{1}{2n+1})$

$\therefore$

$$\sum_{i=1}^{n} a_k$$

$= (\frac{1}{2}-\frac{1}{3})+(\frac{1}{4}-\frac{1}{5})+(\frac{1}{6}-\frac{1}{7})+.....+(\frac{1}{2n}-\frac{1}{2n+1})$

$= -(-\frac{1}{2}+\frac{1}{3}+-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}+...)$

$= 1-(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}+...)$

$= \log e - \log 2 = \log(\frac{e}{2})$

Question 104

The maximum value of $3\cos\theta + 4 sin\ \theta$ is

Options:

A. 3

B. 4

C. 5

D. None of these

Answer: C

Solution:

Solution:

$\because$ Maximum value of $a\cos\theta + b\sin\theta = \sqrt{a^2+b^2}$
$\therefore$ Maximum value of $3\cos\theta + 4\sin\theta = \sqrt{3^2+4^2}$
$= \sqrt{25} = 5$

Question 105

The period of $sin\ \theta - \sqrt{3}\cos\theta$ is

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: D

Solution:

Solution:
Let $f(\theta) = \sin\theta - \sqrt{3}\cos\theta$
Then, $f(2\pi \pm \theta) = \sin(2\pi + \theta) - \sqrt{3}\cos(2\pi + \theta)$
$= \sin\theta - \sqrt{3}\cos\theta = f(\theta)$
i.e. $f(T + \theta) = f(\theta)$
Hence, the period of $f(\theta)$ is 2π

Question 106

If the tangent at the point $(2\sec\theta, 3\tan\theta)$ of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to $3x - y + 4 = 0$, then the value of θ is

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: C

Solution:

Solution:
Given equation of the hyperbola is
$\frac{x^2}{4} - \frac{y^2}{9} = 1$
On differentiating w.r.t.x , we get
$\frac{2x}{4} - \frac{2yy'}{9} = 0$
$\Rightarrow y' = \frac{2x}{4} \times \frac{9}{2y} = \frac{9x}{4y}$
$\therefore m_1 = \frac{9 \times 2\sec\theta}{4 \times 3\tan\theta} = \frac{3}{2}\operatorname{cosec}\theta$
and given line is $3x - y + 4 = 0$
$\therefore m_2 = \frac{-3}{-1} = 3$
Since, both the lines are parallel.
$\therefore m_1 = m_2$
$\Rightarrow \frac{3}{2}\operatorname{cosec}\theta = 3$
$\Rightarrow \operatorname{cosec}\theta = 2$
$\Rightarrow \theta = \operatorname{cosec}^{-1}(2)$
$\Rightarrow \theta = \frac{\pi}{6}$

Question 107

If $\cos\theta = \frac{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}{sin^2\alpha + sin^2\beta + sin^2\gamma}$, where α, β, γ are the angles made by a line with the positive directions of the axes of reference, then the measure of θ is

Options:

A. 60°

B. 90°

C. 30°

D. 45°

Answer: A

Solution:

Solution:
Given, $\cos\theta = \frac{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}{\sin^2\alpha + \sin^2\beta + \sin^2\gamma}$ ......(i)
where, α, β, γ are the angles made by a line with positive direction of the axes of reference.
We know that, $l^2 + m^2 + n^2 = 1$
$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \ldots\ldots(ii) [l = \cos\alpha, m = \cos\beta \text{ and } n = \cos\gamma]$
$\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$
$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$
$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ ....(iii)
$\cos\theta = \frac{1}{2}$ [from Eqs. (ii) and (iii)]
$\Rightarrow \theta = \cos^{-1}(\frac{1}{2})$
$\Rightarrow \theta = \frac{\pi}{3} = 60^\circ$

Question 108

The equation of the plane through intersection of planes $x + 2y + 3z = 4$ and $2x + y - z = -5$ and perpendicular to the plane $5x + 3y + 6z = -8$ is

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Options:

A. $23x + 14y - 9z = -48$

B. $51x + 15y - 50z = -173$

C. $7x - 2y + 3z = -81$

D. None of the above

Answer: B

Solution:

Solution:
Given planes $P_1 : x + 2y + 3z - 4 = 0$
$P_2 : 2x + y - z + 5 = 0$
The equation pf plane through intersection of P_1 and P_2 is
$P_{1\lambda} P_2 = 0$
$\Rightarrow (x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$
$\Rightarrow x(1 + 2\lambda) + y(2 + \lambda) + z(3 - \lambda) + (5\lambda - 4) = 0$ ....(i)
Since, the plane is perpendicular to
$5x + 3y + 6z + 8 = 0$
$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$
$\Rightarrow (1 + 2\lambda)(5) + (2 + \lambda)(3) + (3 - \lambda)(6) = 0$
$\Rightarrow 5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$
$\Rightarrow 7\lambda + 29 = 0$
$\therefore \lambda = \frac{-29}{7}$
Now, equation of the plane is
$(x + 2y + 3z - 4) + (\frac{-29}{7})(2x + y - z + 5) = 0$
$\Rightarrow 7x + 14y + 21z - 28 - 58x - 29y + 29z - 145 = 0$
$\Rightarrow -51x - 15y + 50z - 173 = 0$
$\Rightarrow 51x + 15y - 50z = -173$

Question 109

If l, m, n are the direction cosines of a line, then the maximum value of lmn is

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Options:

A. $\frac{1}{3\sqrt{3}}$

B. $\frac{1}{5\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{2}}$

Answer: A

Solution:

Solution:
∵ l, m, n are the direction cosines of a line.
∴ $l^2 + m^2 + n^2 = 1$
⇒ AM ≥ GM
⇒ $\frac{l^2 + m^2 + n^2}{3} \geq (l^2 \cdot m^2, n^2)^{\frac{1}{3}}$
⇒ $\frac{l^2 + m^2 + n^2}{3} \geq (lmn)^{\frac{2}{3}}$
⇒ $lmn \leq (\frac{1}{3})^{\frac{3}{2}}$
⇒ $lmn \leq \frac{1}{3\sqrt{3}}$

Question 110

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is d, then $[d]$, where $[\cdot]$ is the greatest integer function, is equal to

Options:

A. 0

B. 1

C. 2

D. 3

Answer: A

Solution:

Solution:
Given lines are
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$
and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$
∴ Shortest distance is given by

$$d = \frac{\begin{bmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}}{\sqrt{(15-16)^2 + (12-10)^2 + (8-9)^2}}$$

$$= \frac{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}}{\sqrt{1+4+5}}$$

$\Rightarrow d = \frac{1(15-16) - 2(10-12) + 8 - 9)}{\sqrt{6}}$

$= \frac{-1+4-2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$

$\therefore [d] = 0$

Question 111

Which of the following function is inverse of itself?

Options:

A. $f(x) = \frac{1-x}{1+x}$

B. $g(x) = 5^{\log x}$

C. $h(x) = 2^{x(x-1)}$

D. None of the above

Answer: A

Solution:

Solution:

$fof(x) = f(f(x))$

$f(\frac{1-x}{1+x}) = \frac{1-(\frac{1-x}{1+x})}{1+(\frac{1-x}{1+x})} = x, \forall x$

Hence, f is inverse of itself.

Question 112

Let $f: R \to R$ be a differentiable function and $f(1) = 4$. Then, the value of $\lim_{x \to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

Options:

A. $8f'(1)$

B. $4f'(1)$

C. $2f'(1)$

D. $f'(1)$

Answer: A

Solution:

Solution:
Given, $f: R \to R$ and $f(1) = 4$

Let $I = \lim_{x \to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$

$= \frac{\lim_{x \to 1} \int_4^{f(x)} 2t dt}{\lim_{x \to 1} (x-1)}$

$= \lim_{x \to 1} \frac{2f'(x)f(x)}{1}$ [using L' Hospital's rule]

$= 2f(1)f'(1)$

$= 2 \times 4f'(1) = 8f'$

Question 113

If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = [f(\frac{x}{2})]^2 + [g(\frac{x}{2})]^2$ and given that $F(5) = 5$, then the value of $F(10)$ is

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Options:

A. 15

B. 0

C. 5

D. 10

Answer: C

Solution:

Solution:
Given, $f''(x) = -f(x)$ and $g(x) = f'(x)$

$\Rightarrow g'(x) = -f(x)$ and $g(x) = f'(x)$ ....(i)

Now, $F(x) = [f(\frac{x}{2})]^2 + [g(\frac{x}{2})]^2$

$\Rightarrow F'(x) = f(\frac{x}{2})f'(\frac{x}{2}) + g(\frac{x}{2})g'(\frac{x}{2})$

$\Rightarrow F'(x) = f(\frac{x}{2})g(\frac{x}{2}) - g(\frac{x}{2})f(\frac{x}{2}) = 0$[from Eq. (i)]

i.e. $F(x) =$ Contant

$\therefore F(10) = F(5) = 5$

Question 114

The value of $\lim_{x \to 0} \frac{\sin x^4 - x^4 \cos x^4 + x^{20}}{x^4(e^{2x^4} - 1 - 2x^4)}$ is equal to

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Options:

A. 0

B. $-\frac{1}{6}$

C. $\frac{1}{6}$

D. Does not exist

Answer: C

Solution:

Solution:

$$\lim_{x \to 0} \frac{\sin x^4 - x^4 \cos x^4 + x^{20}}{x^4(e^{2x^4} - 1 - 2x^4)}$$

Using L'Hospital Rule

$$= \lim_{x \to 0} \frac{4x^7 \sin x^4 + 20x^{19}}{8e^{2x^4}x^7 - 16x^7 + 4e^{2x^4} - 4x^3}$$

$$= \lim_{x \to 0} \frac{4x^3(x^4 \sin x^4 + 5x^{16})}{4x^3(2e^{2x^4}x^4 - 4x^4 + e^{2x^4} - 1)}$$

Using L'Hospital Rule

$$= \lim_{x \to 0} \frac{4x^3 \sin^4 + 4x^7 \cos x^4 + 80x^{15}}{16e^{2x^4}x^7 + 16e^{2x^4}x^3 - 16x^3}$$

$$= \lim_{x \to 0} \frac{4x^3(\sin x^4 + x^4 \cos x^4 + 20x^{12})}{4x^3(4e^{2x^4} + 4e^{2x^4} - 4)}$$

Using L'Hospital Rule

$$= \lim_{x \to 0} \frac{4x^3 \cos x^4 + 4x^3 \cos x^4 - 4x^7 \sin x^4 + 240x^{11}}{32e^{2x^4}.x^7 + 48e^{2x^4}.x^3}$$

$$= \lim_{x \to 0} \frac{4x^3(2 \cos x^4 - x^4 \sin x^4 + 60x^{11})}{4x^3(8e^{2x^4}.x^4 + 12e^{2x^4})}$$

$$= \lim_{x \to 0} \frac{2 \cos x^4 - x^4 \sin x^4 + 60x^{11}}{e^{2x^4}(8x^4 + 12)}$$

$$= \lim_{x \to 0} \frac{2 \cos(0)^4 - (0)^4 \sin(0)^4 + 60(0)^{11}}{e^{2(0)^2}[8(0)^4 + 12]}$$

$$= \frac{2}{12} = \frac{1}{6}$$

Question 115

A function g defined for all real x > 0 satisfies $g(1) = 1, g'(x^2) = x^3$ for all x > 0, then the value of $g(4)$ is

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Options:

A. $\frac{13}{3}$

B. 3

C. $\frac{67}{5}$

D. None of these

Answer: C

Solution:

Solution:

Given, $g(1) = 1, g'(x^2) = x^3, \forall x > 0$

Now, $g'(x^2) = x^3$
Put $x^2 = y$

$\therefore g'(y) = (y)^{\frac{3}{2}}\left[\because x^3 = x^2 \cdot x = y \cdot \sqrt{y} = y^{\frac{3}{2}}\right]$

$\Rightarrow \int g'(y)\, dy = \int (y)^{\frac{3}{2}} dy$

$\Rightarrow g(y) = \dfrac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1}$

$\Rightarrow g(y) = \frac{2}{5} y^{\frac{5}{2}} + C$

or $g(x)^2 = \frac{2}{5} x^5 + C [\because x^2 = y] \ldots.(i)$

Now, $g(1) = \frac{2}{5} + C$

$\Rightarrow C = 1 - \frac{2}{5} [\because g(1) = 1]$

$\Rightarrow C = \frac{3}{5}$

From Eq. (i), $g(x^2) = \frac{2}{5} x^5 + \frac{3}{5}$

On taking $x = 2$, we get

$g(4) = \frac{2}{5} . 2^5 + \frac{3}{5} = \frac{2.32 + 3}{5}$

$= \frac{67}{5}$

Question 116

The area bounded by the curves $y = x^2$, $y = -x^2$ and $y^2 = 4x - 3$ is k, then the value of $6k$ is

Options:

A. 2

B. 3

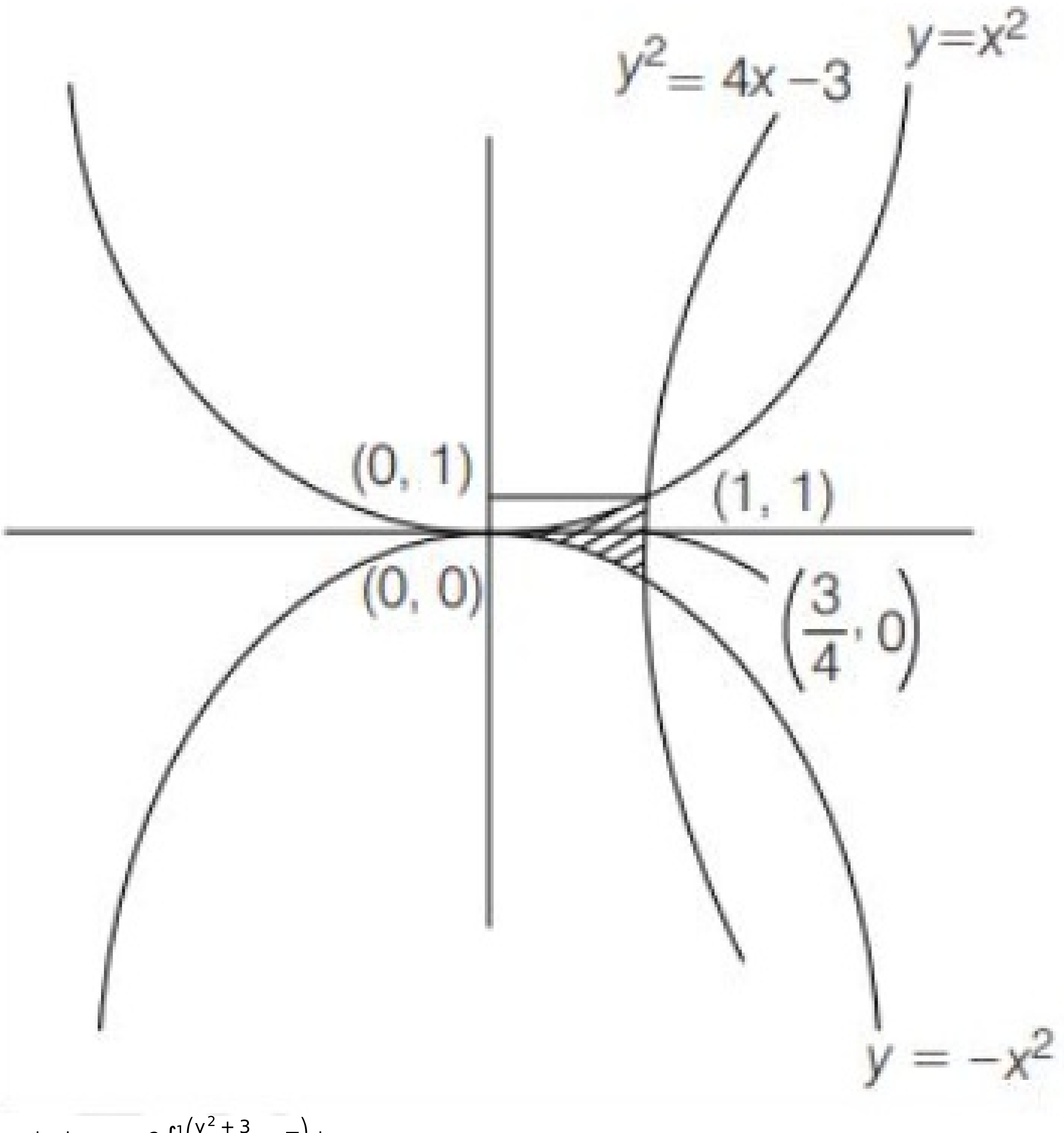
C. 0

D. 4

Answer: A

Solution:

Solution:



$\therefore$ Required area $= 2\int_0^1 \left(\frac{y^2+3}{4} - \sqrt{y}\right) dy$

$= 2\left[\frac{y^3}{12} + \frac{3y}{4} - \frac{2y^{\frac{3}{2}}}{3}\right]_0^1$

$= [\frac{1}{12} + \frac{3}{4} - \frac{2}{3}] = 2[\frac{1+9-8}{12}]$

$= \frac{2 \times 2}{12} = \frac{1}{3}$

$\therefore$ k $= \frac{1}{3}$

Now, $6k = 6 \times \frac{1}{3} = 2$

Question 117

The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ is

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Options:

A. 1

B. 2

C. 3

D. None of the above

Answer: A

Solution:

Solution:
We have,
$\sqrt{1-y^2}+\sqrt{1-x^2}=a(x-y)$
Put $x=\sin A$ and $y=\sin B$, we get
$\cos A+\cos B=a(\sin A-\sin B)$
$\Rightarrow \cot\left(\frac{A-B}{2}\right)=a$
$\Rightarrow A-B=2\cot^{-1}a$
$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$
On differentiating w.r.t.x, we get
$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\cdot\frac{dy}{dx}=0$
$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{1-x^2}$
Clearly, it is differential equation of first order and first degree.

Question 118

The solution of the differential equation $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$ is

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Options:

A. $y=C(x+a)(1-ay)$

B. $y=C(x+a)(1+ay)$

C. $y=C(x-a)(1-ay)$

D. None of the above

Answer: A

Solution:

Solution:
We have, $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$

$\Rightarrow y - ay^2 = (a+x)\frac{dy}{dx}$
$\Rightarrow \frac{dx}{a+x} = \frac{1}{y - ay^2}dy$
$\Rightarrow \int\frac{1}{a+x}dx = \int\frac{1}{y-ay^2}dy$
$\Rightarrow \int\frac{1}{a+x}dx = \int\frac{1}{y(1-ay)}dy$
$\Rightarrow \int\frac{1}{a+x}dx = \int(\frac{1}{y} + \frac{a}{1-ay})\,dy$
$\Rightarrow \log(a+x) + \log C = \log y - \log(1-ay)$
$\Rightarrow (a+x)C = \frac{y}{1-ay}$
$\Rightarrow C(x+a)(1-ay) = y$

Question 119

The solution of differential equation $(2y-1)\,dx - (2x+3)\,dy = 0$ will be

Options:

A. $\frac{2x-1}{2y+3} = C$

B. $\frac{2y+1}{2x-3} = C$

C. $\frac{2x+3}{2y-1} = C$

D. $\frac{2x-1}{2y-1} = C$

Answer: C

Solution:

Solution:
We have,
$(2y-1)\,dx - (2x+3)\,dy = 0$
$\Rightarrow (2y-1)\,dx = (2x+3)\,dy$
$\Rightarrow (\frac{1}{2x+3})\,dx = (\frac{1}{2y-1})\,dy$
$\Rightarrow \int\frac{1}{2x+3}dx = \int\frac{2}{2y-1}dy$
$\Rightarrow \log(2x+3) = \log(2y-1) + \log C$
$\Rightarrow \frac{2x+3}{2y-1} = C$

Question 120

The solution of the differential equation $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$ will be

Options:

A. $y \sin y = x^2 \log x + C$

B. $y \sin y = x^2 + C$

C. $y \sin y = x^2 + \log x + C$

D. $y \sin y = x \log x + C$

Answer: A

Solution:

Solution:
We have,

$$\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y}$$

$$\Rightarrow \int (\sin y + y \cos y)\, dy = 2 \int x \log x\, dx + \int x\, dx$$

$$\Rightarrow y \sin y = x^2 \log x + C$$

Question 121

For $n = 4$, using trapezoidal rule, the value of $\int_0^2 \frac{dx}{1+x}$ will be

Options:

A. 1.1125

B. 1.1176

C. 1.1180

D. None of these

Answer: A

Solution:

Solution:
Here, $n = 4$ and $a = 0, b = 2$

$$\therefore \Delta x = 2 - \frac{0}{4} = \frac{1}{2} = 0.5$$

Now, $y_0 = f(a) = f(0) = \frac{1}{1+0} = 1$

$$y_1 = f(a + \Delta x) = f(0 + 0.5)$$

$$= f(0.5) = \frac{1}{1+0.5} = \frac{1}{1.5} = 0.6666$$

$$y^2 = f(a + 2\Delta x) = f(0 + 2 \times 0.5)$$

$$= f(1) = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

$$y_3 = f(a + 3\Delta x) = f(0 + 3 \times 0.5)$$

$$= f(1.5) = \frac{1}{1+1.5} = \frac{1}{2.5} = 0.4$$

$$y_4 = f(b) = f(1) = \frac{1}{1+2} = \frac{1}{3} = 0.3333$$

$$\int_0^2 \frac{1}{1+x} dx = 0.5[\frac{1}{2} + 0.6666 + 0.5 + 0.4 + \frac{0.3333}{2}]$$

$$= 0.5[0.5 + 0.6666 + 0.5 + 0.4 + 0.16665]$$

$$= 0.5 \times 2.23325 = 1.116625(\text{approx})$$

Question 122

The value of $\int_0^6 \frac{dx}{1+x^2}$ by choosing six sub-intervals and by using Simpson's Rule will be

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Options:

A. 1.3562

B. 1.3662

C. 1.3456

D. 1.2662

Answer: B

Solution:

Solution:
Here, n = 6

$\because \Delta x = 6 - \frac{0}{6} = 1$

Now, $y(0) = f(0) = \frac{1}{1+0} = 1$

$y(1) = f(0 + \Delta x) = \frac{1}{1+1} = 0.5$

$y(2) = f(0 + 2\Delta x) = \frac{1}{1+4} = \frac{1}{5} = 0.2$

$y(3) = f(0 + 3\Delta x) = \frac{1}{1+9} = \frac{1}{10} = 0.1$

$y(4) = f(0 + 4\Delta x) = \frac{1}{1+16} = \frac{1}{17} = 0.058$

$y(5) = f(0 + 4\Delta x) = \frac{1}{1+25} = \frac{1}{26} = 0.038$

$y(6) = f(0 + 6\Delta x) = \frac{1}{1+36} = \frac{1}{37} = 0.027$

$\therefore \int_0^6 \frac{dx}{1+x^2}$

$= \frac{1}{3}[1 + 4(0.5 + 0.1 + 0.038) + 2(0.2 + 0.058) + 0.027]$

$= \frac{1}{3}[1 + 2.552 + 0.516 + 0.027]$

$= \frac{1}{3} \times 4.095 = 1.365 = 1.3662$

Question 123

ISP stands for

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Options:

A. Instructions Set Processor

B. Information Standard Processing

C. Interchange Standard Protocol

D. Interrupt Service Procedure

Answer: A

Solution:

Solution:
ISP stands for Instruction set processor.

Question 124

The 8-bit encoding format used to store data in a computer is known as

Options:

A. ASCII

B. EBCDIC

C. ANCI

D. USCII

Answer: B

Solution:

Solution:
The 8-bit encoding format used to store data in a computer is known as USCII.

Question 125

The small extremely fast RAM's are called as

Options:

A. cache

B. heaps

C. accumulators

D. stacks

Answer: A

Solution:

Solution:

The small extrenely fast RAM's are called as cache.

Question 126

The value of $i^2 + i^4 + i^6 + \ldots (2n+1)$ terms is

Options:

A. -1

B. 1

C. $-i$

D. i

Answer: A

Solution:

Solution:
$i^2 + i^4 + i^6 + \ldots (2n+1)$ terms
$= i^2[1-(i^2)^{2n+1}] = i^2(\frac{1+1}{1+1}) = -1$

Question 127

The argument of the complex number $-1 + i\sqrt{3}$ is

Options:

A. 45°

B. 60°

C. 120°

D. 150°

Answer: C

Solution:

Solution:
Given,
$z = -1 + i\sqrt{3}$
Now,
$\tan\alpha = \frac{|b|}{|a|} = \frac{\sqrt{3}}{1} = \sqrt{3}$
$\Rightarrow \alpha = \tan^{-1}\sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$
Clearly the point representing z lies in the second quadrant.

$\therefore \theta = \pi - \alpha \Rightarrow \theta = \pi - \frac{\pi}{3}$
$\Rightarrow \theta - \frac{2\pi}{3} \Rightarrow \theta = 120°$

Question 128

If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the AM of a and b, then n is equal to

Options:

A. -1

B. 0

C. 1

D. None of these

Answer: B

Solution:

Solution:
Given, AM of a and b = $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$
$\therefore \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$
$\Rightarrow 2(a^{n+1}+b^{n+1)} = (a+b)(a^n+b^n)$
$\Rightarrow 2a^{n+1}+2b^{n+1} = a.a^n + a.b^n + b.a^n + b.b^n$
$\Rightarrow 2a^{n+1}+2b^{n+1} = a^{n+1} + ab^n + ba^n + b^{n+1}$
$\Rightarrow a^{n+1}+b^{n+1} = ab^n + ba^n$
$\Rightarrow a^{n+1} - ba^n = ab^n - b^{n+1}$
$\Rightarrow a^n(a-b) = b^n(a-b)$
$\Rightarrow a^n = b^n$
$\Rightarrow (\frac{a}{b})^n = 1$
$\therefore n = 0$

Question 129

$x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} \ldots \infty$ equal to

Options:

A. 0

B. 1

C. ∞

D. x

Answer: D

Solution:

Solution:

$\frac{x^1}{2} \cdot \frac{x^1}{4} \cdot \frac{x^1}{8} \cdot \cdot \cdot \cdot \infty$

$= \frac{1}{x^2} + \frac{1}{4} + \frac{1}{8} + ... = x^{\frac{\frac{1}{2}}{1 - \frac{1}{2}}}$

$= x^{\frac{\frac{1}{2}}{\frac{1}{2}}} = x$

Question 130

If the roots of the equation $x^2 + px + q = 0$ differ by 1 , then

Options:

A. $p^2 = 4q + 1$

B. $p^2 = 4q$

C. $p^2 = 4q - 1$

D. $p^2 = -4q$

Answer: A

Solution:

Solution:

Given equation is $x^2 + px + q = 0$
Let the roots of given wquation be α and β
Then, $\alpha = \beta = 1$
Bur $\alpha + \beta = -p$
and $\alpha\beta = q$
Hence, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$\Rightarrow (1)^2 = (-p)^2 - 4q$
$\Rightarrow 1 = p^2 - 4q$
$\therefore p^2 = 4q + 1$

Question 131

If $a = 2, b = 3$ and $c = 5$ in ΔABC, then $\angle C$ is to equal

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. None of these

Answer: D

Solution:

Solution:
Given, $a = 2, b = 3$ and $c = 5$
The, $\cos C = \frac{a^2 + b^2 + c^2}{2ab}$
$\Rightarrow \cos C = \frac{(2)^2 + (3)^2 - (5)^2}{2 \times 2 \times 3} = \frac{4 + 9 - 25}{12}$
$\Rightarrow \cos C = \frac{-12}{12}$
$\Rightarrow \cos C = -1$
$\Rightarrow \angle C = \cos^{-1}(-1)$
$\therefore \angle C = \pi$

Question 132

The value of $sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{2}{3}$ is

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Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{6}$

D. None of these

Answer: D

Solution:

Solution:
$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{2}{3}$
$= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{2 \times \frac{2}{3}}{1 + (\frac{2}{3})^2}\left[\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1 + x^2}\right]$
$= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{12}{13}$
$= \sin^{-1}[\frac{4}{5}\sqrt{1 - \frac{144}{169}} + \frac{12}{13}\sqrt{1 - \frac{16}{25}}]$
$[\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1 - y^2} + y\sqrt{1 - x^2})]$
$= \sin^{-1}[\frac{4}{5}\sqrt{\frac{25}{169}} + \frac{12}{13}\sqrt{\frac{9}{25}}]$
$= \sin^{-1}[\frac{4}{5} \times \frac{5}{13} + \frac{12}{13} \times \frac{3}{5}]$

$= \sin^{-1}[\frac{20}{65} + \frac{36}{65}] = \sin^{-1}(\frac{56}{65})$

Question 133

The centroid of the triangle formed by the lines $x + y = 1, 2x + 3y = 6$ and $4x - y = -4$ lies in the quadrant

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Options:

A. I

B. II

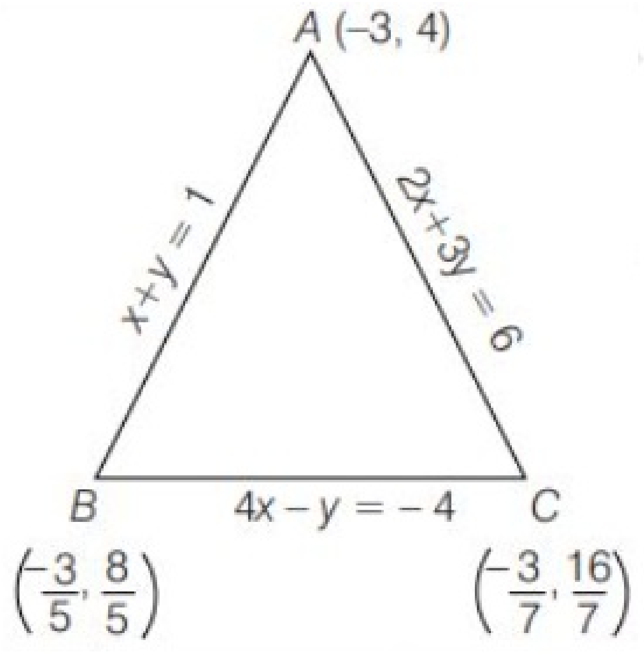
C. III

D. IV

Answer: B

Solution:

Solution:
Given, $x + y = 1 \ldots (i)$
$2x + 3y = 6 \ldots (ii)$
and $4x - y = -4 \ldots (iii)$
On solving Eqs. (i) and (ii), we get
$x = -3$ and $y = 4$
Now, solving Eqs. (ii) and (iii), we get
$x = -\frac{3}{7}$ and $y = \frac{16}{7}$
and solving Eqs. (i) and (iii), we get
$x = -\frac{3}{5}$ and $y = \frac{8}{5}$



∴ Centroid of ΔABC

$$= \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

$$= \left[\frac{(-3) + (-\frac{3}{5}) + (-\frac{3}{7})}{3}, \frac{4 + \frac{8}{5} + \frac{16}{7}}{3}\right]$$

$$= [-\frac{141}{105}, \frac{276}{105}] \equiv (-x, y)$$

Hence, the centroid lies in II quadrant.

Question 134

If an equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$, the length of its each side is

Options:

A. $\sqrt{2}a$

B. $\sqrt{3}a$

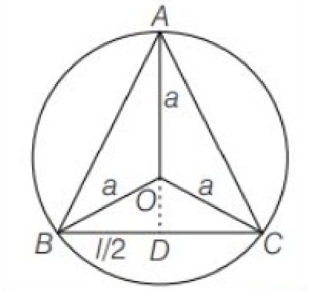
C. $\frac{\sqrt{3}}{2}a$

D. $\frac{1}{\sqrt{3}}a$

Answer: B

Solution:

Solution:
Given equation of the circle is
$x^2 + y^2 = a^2$
where, radius $= a$
Let ABC be the inscribed equilateral triangle and length of its each sides be l.



Area of $\Delta ABC = \frac{\sqrt{3}}{4}l^2$
Also, $OD = \sqrt{a^2 - \frac{l^2}{4}}$ [from ΔODB]
$\therefore$ Area of $\Delta OBC = \frac{1}{2} \times l \times \sqrt{a^2 - \frac{l^2}{4}}$
Now, Area of $\Delta ABC = 3 \times$ Area of ΔOBC
$\Rightarrow \frac{\sqrt{3}}{4}l^2 = 3 \times \frac{l}{2}\sqrt{a^2 - \frac{l^2}{4}}$
$\Rightarrow \sqrt{3}l = 3\sqrt{4a^2 - l^2}$
$\Rightarrow 3l^2 = 9(4a^2 - l^2)$
$\Rightarrow l^2 = 12a^2 - 3l^2$
$\Rightarrow 4l^2 = 12a^2$
$\Rightarrow l^2 = 3a^2$
$\Rightarrow l = \sqrt{3}a$

Question 135

If the vertex is $(3, 0)$ and the extremities of the latusrectum are $(4, 3)$ and $(4, -3)$, then the equation of the parabola is

Options:

A. $y^2 = 4(x - 3)$

B. $x^2 = 4(y - 3)$

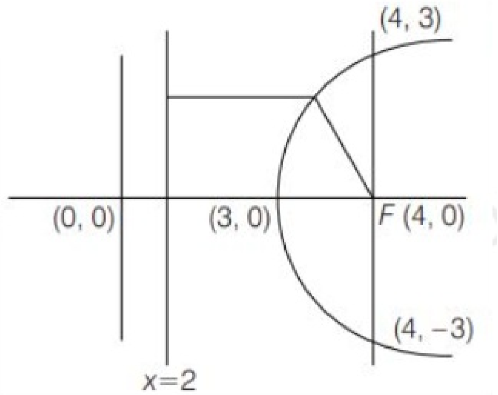
C. $y^2 = -4(x + 3)$

D. $x^2 = -4(y+3)$

Answer: A

Solution:

Solution:



Equation of the parabola is

$\left|\frac{x-2}{\sqrt{1+0}}\right|^2 = (x-4)^2 + y^2$

$\Rightarrow (x-2)^2 = (x-4)^2 + y^2$

$\Rightarrow x^2 + 4 - 4x = x^2 + 16 - 8x + y^2$

$\Rightarrow y^2 = 4x - 12$

$\Rightarrow y^2 = 4(x-3)$

Question 136

If G and G' are respectively centroid of $\triangle ABC$ and $\Delta A'B'C'$, then $AA' + BB' + CC'$ is equal to

Options:

A. $2GG'$

B. 3GG'

C. $\frac{2}{3}GG'$

D. $\frac{1}{3}GG'$

Answer: B

Solution:

Solution:

Let a, b, c be the position vectors of A, B and C, respectively.

Then, the position vector of G is $\frac{a+b+c}{3}$

Let the position vectors of A', B' and C' be a', b' and c', respectively.

Then, the position of G is $\frac{a'+b'+c'}{3}$

$\therefore AA' + BB' + CC' = (a'-a) + (b'-b) + (c'-c)$

$\Rightarrow AA' + BB' + CC' = (a'+b'+c') - (a+b+c)$

$= 3\left(\frac{a'+b'+c'}{3} - \frac{a+b+c}{3}\right)$

$= 3GG'$

Question 137

If $a = 3\hat{i} - 4\hat{j} + 5\hat{k}$, $b = \hat{i} + \hat{j} + \hat{k}$ and $c = -2\hat{i} + 3\hat{j} - 5\hat{k}$, and if [] is the least integer function, then [a + b + / * * / c] is equal to

Options:

A. 1

B. 2

C. 3

D. 0

Answer: C

Solution:

Solution:
Given, $a = 3\hat{i} - 4\hat{j} + 5\hat{k}$, $b = \hat{i} + \hat{j} + \hat{k}$ and $c = -2\hat{i} + 3\hat{j} - 5\hat{k}$
$\therefore a + b + c = 3\hat{i} - 4\hat{j} + 5\hat{k} + \hat{i} + \hat{j} + \hat{k} - 2\hat{i} + 3\hat{j} - 5\hat{k}$
$= 2\hat{i} + 0\hat{j} + \hat{k}$
$\Rightarrow |a + b + c| = \sqrt{(2)^2 + (0)^2 + (1)^2}$
$= \sqrt{4 + 0 + 1} = \sqrt{5} = 2.236$
$[a + b + c] = [2.236] = 3$

Question 138

If $a = -\hat{i} + \hat{j} + \hat{k}$ and $b = 2\hat{i} + \hat{k}$, then the vector satisfying the following conditions
(i) it is coplanar with a and b,
(ii) it is perpendicular to b and
(iii) $a \cdot c = 7$, is

Options:

A. $-\hat{i} + 2\hat{j} + 2\hat{k}$

B. $-\frac{3}{2}\hat{i} + \frac{5}{2}\hat{j} + 3\hat{k}$

C. $-3\hat{i} + 5\hat{j} + 6\hat{k}$

D. $-6\hat{i} + \hat{k}$

Answer: B

Solution:

Solution:
Let r be a vector coplanar with a and b are perpendicular to b.
Then, $r = b \times (a \times b)$
$\Rightarrow r = (b . b)a - (b . a)b$
$\Rightarrow r = 5a + b$
$r = -3\hat{i} + 5\hat{j} + 6\hat{k}$
Let $c = \lambda r$, then
$a . c = 7$
$\Rightarrow \lambda = \frac{1}{2}$
Hence, $c = -\frac{3}{2}\hat{i} + \frac{5}{2}\hat{j} + 3\hat{k}$

Question 139

If the vectors $b = \left(\tan\alpha, -1, 2\sqrt{\sin\frac{\alpha}{2}}\right)$ and $c = \left(\tan\alpha, \tan\alpha, -\frac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and a vector $a = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the Z-axis, then the value of α is

Options:

A. $(4n+2)\pi + \tan^{-1}2$

B. $(4n+2)\pi - \tan^{-1}2$

C. $(4n+1)\pi + \tan^{-1}2$

D. $(4n+1)\pi - \tan^{-1}2$

Answer: D

Solution:

Solution:
Since, the vector $a = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the Z - axis. Therefore, its z - component is negative.
i.e. $\sin 2\alpha < 0$
$\therefore -1 \le \sin 2\alpha < 0$....(i)
Since, b and c are orthogonal.
$\therefore b . c = 0$
$\Rightarrow \tan^2\alpha - \tan\alpha - 6 = 0$
$\Rightarrow (\tan\alpha - 3)(\tan\alpha + 2) = 0$
$\Rightarrow \tan\alpha = 3, -2$
$\therefore \tan\alpha = 3$
Then, $\sin 2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$
$= \frac{6}{10} > 0$, which is a contradiction to ......(i)
$\therefore \tan\alpha = 3$ is not possible.
Thus, $\tan\alpha = -2$ and for this value of $\tan\alpha$, we get
$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{4}{3}$
Since, $\sin 2\alpha < 0$ and $\tan 2\alpha > 0$

Therefore, 2α is in third quadrant.
Also, $\sqrt{\sin\frac{\alpha}{2}}$ is meaningful, if $0 < \sin\frac{\alpha}{2} < 1$
when these conditions are satisfied, α is given by
$\alpha = (4n+1)\pi - \tan^{-1}2$.

Question 140

If $f: R \rightarrow R$ be the signum function and $g: R \rightarrow R$ be the greatest integer function, then $\sin\{\pi((fog)(\frac{1}{2}))\}$ is equal to

Options:

A. 1

B. $\frac{\sqrt{3}}{2}$

C. 0

D. $\frac{1}{\sqrt{2}}$

Answer: C

Solution:

Solution:
We have,
$f(x) = \text{sgn}(x)$
and $g(x) = [x]$
Now, $fog(\frac{1}{2}) = f(g(\frac{1}{2})) = f([\frac{1}{2}])$
$= f(0) [\because [0.5] = 0]$
$= \text{Sgn}(0) = 0 [\because \text{Sgn}(0) = 0]$
Now, $\sin[\pi\{fog(\frac{1}{2})\}] = \sin(\pi \times 0) = \sin 0° = 0$

Question 141

**A particle moving on a curve has the position at a time t is given by $x = f'(t)\sin t + f'(t)\cos t$
$y = f'(t)\cos t - f'(t)\sin t$, where f is a twice differentiable function. Then, the velocity of the particle at time t is**

Options:

A. $f'(t) + f''(t)$

B. $f't) - f''(t)$

C. $f'(t) + f'''(t)$

D. $f'(t) - f''(t)$

Answer: C

Solution:

Solution:
Given that,
$x = f'(t)\sin t + f''(t)\cos t$
and $y = f'(t)\cos t - f''(t)\sin t$
$\therefore V_x = \frac{dx}{dt} = [f''(t)\sin t + f'(t)\cos t] + [f'''(y)\cos t - f''(t)\sin t]$
$= f'(t)\cos t + f'''(t)\cos t$
and $V_y = \frac{dy}{dt} = [f''(t)\cos t - f'(t)\sin t] - [f'''(t)\sin t + f''(t)\cos t]$
$= -[f'(t)\sin t + f'''(t)\sin t]$
we know that,
$|V| = \sqrt{(V_x)^2 + (V_y)^2}$
$= \sqrt{[f'(t)\cos t + f'''(t)\cos t]^2 + \{-[f'(t)\sin t + f'''(t)\sin t]^2\}}$
$= \sqrt{[f'(t)]^2[\cos^2 t + \sin^2 t] + [f'''(t)^2[\cos^2 t + \sin^2 t] + 2f'(t)f'''(t)}$
$(\cos^t + \sin^2 t)$
$= \sqrt{[f'(t)]^2 + [f'''(t)]^2 + 2f'(t)f'''(t)}$
$= \sqrt{[f'(t) + f'''(t)]^2}$
$= f'(t) + f'''(t)$

Question 142

$f(x)$ and $g(x)$ are differentiable in the interval $[0, 1]$ such that $f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2$, then Rolle's theorem is applicable for which of the following in $[0, 1]$?

Options:

A. $f(x) - g(x)$

B. $f(x) - 2g(x)$

C. $f(x) + 3g(x)$

D. None of the above

Answer: B

Solution:

Solution:
Let $\phi(x) = f(x) - 2g(x), x \in [0, 1]$
Clearly, $\phi(x)$ is continuous on [0, 1] and differentiable on $(0, 1)$, as $f(x)$ and $g(x)$ are differentiable on $[0, 1]$.
Also, $\phi(0) = f(0) - 2g(0) = 2 - 0 = 2$
and $\phi(1) = f(1) - 2g(1) = 6 - 2 \times 2 = 2$
$\because \phi(0) = \phi(1)$
Thus, $\phi(x)$ satisfies all the three conditions of Rolle's theorem.
Therefore, there exists a point $x \in (0, 1)$ such that
$\phi'(x) = 0 \Rightarrow f'(x) - 2g'(x) = 0$
$\therefore f'(x) = 2g'(x)$

Question 143

If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A \cos 4x + B$, then the value of A is

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Options:

A. $\frac{1}{2}$

B. $\frac{1}{8}$

C. $-\frac{1}{8}$

D. $\frac{1}{4}$

Answer: C

Solution:

Solution:
Let
$I = \int \frac{1 + \cos 4x}{\cot x - \tan x} dx$
$\Rightarrow I = \int \frac{2\cos^2 x \cdot \sin x \cos x}{\cos^2 x - \sin^2 x} dx$
$\Rightarrow I = \int \sin 2x \cos 2x \, dx$
$\Rightarrow I = \frac{1}{2} \int \sin 4x \, dx$
$\Rightarrow I = -\frac{1}{8} \cos 4x + B$
On comparing with the given integral, we get
$A = -\frac{1}{8}$

Question 144

I $f(x) = \sqrt{x}, g(x) = e^x - 1$ and $\int f \circ g(x) dx = A f o g(x) + B \tan^{-1}(f \circ g(x)) + C$, then the value of $A + B$ is

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Options:

A. 1

B. 2

C. 3

D. None of these

Answer: D

Solution:

Solution:
We have, $f(x) = \sqrt{x}$
and $g(x) = e^x - 1$
$\therefore fog(x) = f\{g(x)\} = f\{e^x - 1\}$
$\Rightarrow fog(x) = \sqrt{e^x - 1}$ ....(i)
Let $I = \int fog(x)\,dx$
$= \int \sqrt{e^x - 1}\,dx$ [from Eq. (i)]
$= \int \frac{e^x - 1}{\sqrt{e^x - 1}}\,dx$
$= \int \frac{e^x}{\sqrt{e^x - 1}}\,dx - \int \frac{1}{\sqrt{e^x - 1}}$ ....(ii)
Consider $I_1 = \int \frac{e^x}{\sqrt{e^x - 1}}\,dx$
and $I_2 = \int \frac{1}{\sqrt{e^x - 1}}\,dx$
Now, $I_1 = \int \frac{e^x}{\sqrt{e^x - 1}}\,dx$
Put $e^x - 1 = t$
$\Rightarrow e^x\,dx = dt$
$\therefore I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C_1 = 2\sqrt{e^x - 1} + C_1$
and $I_2 = \int \frac{1}{\sqrt{e^x - 1}}\,dx$
Put $e^x - 1 = z^2$
$\Rightarrow e^x\,dx = 2z\,dz \Rightarrow dx = \frac{2z}{z^2 + 1}\,dz$
$\therefore I_2 = \int \frac{1}{z} \cdot \frac{2z}{z^2 + 1}\,dz = 2\int \frac{1}{z^2 + 1}\,dz$
$= 2tab^{-1}z + C_2 = 2\tan^{-1}\sqrt{e^x - 1} + C_2$
$\because I = I_1 - I_2$ [from Eq. (ii)]
$\therefore I = 2\sqrt{e^x - 1} + C_1 - 2\tan^{-1}\sqrt{e^x - 1} - C_2$
$= 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + C$[where, $C = C_1 - C_2$]
$= 2\,fog(x) - 2\tan^{-1} fog(x) + C$[$\because fog(x) = \sqrt{e^x - 1}$]
Now, comparing with the given integral, we get
$A = 2$ and $B = -2$
Hence, $A + B = 2 + (-2) = 2 - 2 = 0$

Question 145

The value of $\int_0^1 \tan^{-1}\left(\frac{2x - 1}{1 + x - x^2}\right) dx$ is

Options:

A. 0

B. 1

C. -1

D. None of these

Answer: A

Solution:

Solution:
Let
$I = \int_0^1 \tan^{-1}\{\frac{2x - 1}{1 + x - x^2}\}\,dx$
$= \int_0^1 \tan^{-1}\{\frac{x + x - 1}{1 - x(x - 1)}\}\,dx$
$= \int_0^1 [\tan^{-1}x + \tan^{-1}(x - 1)]\,dx$

$= \int_0^1 \tan^{-1}x\, dx + \int_0^1 \tan^{-1}(1 - x - 1)\, dx$
$[\because \int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx]$
$= \int_0^1 \tan^{-1}x\, dx + \int_0^1 \tan^{-1}(-x)\, dx$
$= \int_0^1 \tan^{-1}x\, dx - \int_0^1 \tan^{-1}x\, dx = 0$

Question 146

If A and B are mutually exclusive events, then $P(A/B)$ is equal to

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Options:

A. 0

B. 1

C. $\frac{P(A \cap B)}{P(A)}$

D. $\frac{P(A \cap B)}{P(B)}$

Answer: A

Solution:

Solution:
Given, A and B are mutually exclisive events.
Then, $P(A \bigcap B) = 0$
Now, $P(\frac{A}{B}) = P\frac{(A \bigcap B)}{P(B)} = \frac{0}{P(B)} = 0$

Question 147

If $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then

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Options:

A. $P(A \cup B)^C = P(A^C)P(B^C)$

B. $P(\frac{A}{B}) = P(A)$

C. Both (a) and (b) are true

D. None of the above

Answer: C

Solution:

Solution:
We know that,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
But it is given that,
$P(A \cup B) = P(A) + P(B) - P(A)P(B)$
$\therefore P(A \cap B) = P(A)P(B)$
$\Rightarrow$ A and B are independent events.
$\therefore P(A \cup B)^C = P(A^C \cap B^C)$
$= P(A^C)P(B^C)$
and $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)}$
$= P(A)$

Question 148

If both the coefficients of regression between x and y are 0.8 and 0.2, then the coefficient of correlation between them will be

Options:

A. 0.4

B. 0.6

C. 0.3

D. 0.5

Answer: A

Solution:

Solution:
Given, $b_{yx} = 0.8$ and $b_{xy} = 0.2$
$\therefore r = \sqrt{b_{yx} . b_{xy}}$
$= \sqrt{0.8 \times 0.2} = 0.4$

Question 149

If the angle between two lines of regression is θ, then the value of θ will be

Options:

A. $\tan^{-1}\left[\frac{b_{yx} - \frac{1}{b_{xy}}}{1 + \frac{b_{xy}}{b_{yx}}}\right]$

B. $\tan^{-1}\left[\frac{b_{yx} - b_{xy} - 1}{b_{xy} + b_{yx}}\right]$

C. $\tan^{-1}\left[\frac{b_{xy}-\frac{1}{b_{yx}}}{1+\frac{b_{xy}}{b_{yx}}}\right]$

D. $\tan^{-1}\left[\frac{b_{yx}-b_{xy}}{1+b_{yx}\cdot b_{xy}}\right]$

Answer: C

Solution:

Solution:
Equation of regression line of y on x is
$y-\bar{y}=r.\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \Rightarrow y-\bar{y}=b_{yx}(x-\bar{x})$
∴ Slope of regression line y on $M_1 = x$ is b_{yx}.
Now, equation of regression line of x is y is
$x-\bar{x}=b_{xy}(y-\bar{y})$
Slope of regression line x on $M_2 = y$ is $\frac{1}{b_{xy}}$
If the angle between two lines is θ, then
$\tan\theta=\pm\frac{M_1-M_2}{1+M_1M_2}=\left[\frac{b_{yx}-\frac{1}{b_{xy}}}{1+\frac{b_{yx}}{b_{xy}}}\right]$
or $\tan\theta=\left[\frac{b_{xy}-\frac{1}{b_{yx}}}{1+\frac{b_{xy}}{b_{yx}}}\right] \Rightarrow \theta=\tan^{-1}\left[\frac{b_{xy}-\frac{1}{b_{yx}}}{1+\frac{b_{xy}}{b_{yx}}}\right]$

Question 150

The positive root of equation $x^3-2x-5=0$ lies in the interval

Options:

A. $(0,1)$

B. $(1,2)$

C. $(2,3)$

D. $(3,4)$

Answer: C

Solution:

Solution:
By Newton Raphson Formula,
$X_{n+1}=X_n-\frac{f(x_n)}{f'(x_n)}$

$\therefore X_{n+1} = X_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$

$n = 0, 1, 2, \ .....$

Let $x_0 = 2$ ....(i)

Then, $f(x_0) = f(2) = 2^3 - 2(2) - 5 = -1$

and $f'(x_0) = f'(2) = 3(2)^2 - 2 = 10$

By putting $n = 0$ in Eq. (i),

$x_1 = 2 - (\frac{-1}{10}) = 2.1$

$\Rightarrow f(x_1) = f(2.1) = (2.1)^3 - 2(2.1)5 = 0.061$

$f'(x_1) = f'(2.1) = 3(2.1)^2 - 2 = 1.23$

$x_2 = 2.1 - \frac{0.061}{11.23} = 2.094$

So, the positive roots of the equation lies in interval (2, 3).
