Long Answer Questions-I (PYQ)

[4 Mark]

Q.1. Find the inverse of $A = \begin{pmatrix} 3 & -1 \\ -4 & 1 \end{pmatrix}$ using elementary transformations.

Ans.

Given, $A = \begin{bmatrix} 3 & -1 \\ -4 & 1 \end{bmatrix}$

For elementary row operations, we write

$$A = IA \quad \Rightarrow \quad \begin{bmatrix} 3 & -1 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -1/2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} A \qquad \left[\text{Applying } R_1 \to R_1 + \tfrac{1}{2} R_2\right]$$

$$\begin{bmatrix} 1 & -1/2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 4 & 3 \end{bmatrix} A \qquad \left[\text{Applying } R_2 \to R_2 + 4R_1\right]$$

$$\begin{bmatrix} 1 & -1/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ -4 & -3 \end{bmatrix} A \qquad \left[\text{Applying } R_2 \to -R_2\right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix} A \qquad \left[\text{Applying } R_1 \to R_1 + \tfrac{1}{2} R_2\right]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix} \quad \Rightarrow \quad A^{-1} = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$$

Q.2. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Ans.

Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

For finding the inverse, using elementary row operation we write

$$A = IA$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$ and $R_3 \rightarrow R_3 - 2R_1$, we get

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \quad \left[\text{Applying } R_1 \rightarrow R_1 - \tfrac{1}{3}R_2\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -7/9 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/3 & 1/9 & 0, \ , A \\ -2 & 0 & 1 \end{bmatrix} \quad \left[\text{Applying } R_2 \rightarrow \tfrac{1}{9}R_2\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/3 & 1/9 & 0 \\ -1/3 & 5/9 & 1 \end{bmatrix} A \quad [\text{Applying } R_3 \rightarrow R_3 + 5R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -1/3 & 5/9 & 1 \end{bmatrix} A \quad [\text{Applying } R_1 \rightarrow R_1 - 3R_3,\ R_2 \rightarrow R_2 + 7R_3]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A \quad [\text{Applying } R_3 \to 9R_3]$$

$$\Rightarrow \quad I = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$

Q.3. Using elementary row operations (transformations), find the inverse of the following matrix:

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix}$$

Ans.

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

For elementary row operations, we proceed as $A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\text{Applying } R_1 \leftrightarrow R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad [\text{Applying } R_3 \to R_3 - 3R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad [\text{Applying } R_1 \to R_1 - 2R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \text{ [Applying } R_3 \to R_3 + 5R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \text{ [Applying } R_1 \to R_1 + R_3]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A \text{ [Applying } R_2 \to R_2 - 2R_3]$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

Q.4. Find the adjoint of the matrix $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ **and hence, show that A. (adj A) = $|A|I_3$.**

Ans.

Here, $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$\therefore \quad |A| = -1(1-4) + 2(2+4) - 2(-4-2) = 3 + 12 + 12 = 27$

For *adj A*

$$A_{11} = (1-4) = -3 \; A_{21} = -(-2-4) = 6 \; A_{12} = -(2+4) = -6$$

$$A_{22} = (-1+4) = 3 \; A_{13} = (-4-2) = -6 \; A_{23} = -(2+4) = -6$$

$$A_{31} = (4+2) = 6 \; A_{32} = -(2+4) = -6 \; A_{33} = (-1+4) = 3$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\text{Again, } A.\text{adj } A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+12+12 & -6-6+12 & -6+12-6 \\ -6-6+12 & 12+3+12 & 12-6-6 \\ -6+12-6 & 12-6-6 & 12+12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= |A| I_3 \; = \text{RHS.}$

Q.5. If $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$ find A^{-1}, using elementary row transformations.

Ans.

For using elementary row transformations, we take

$A = IA$, where I is identity matrix of order 3×3.

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying, $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying, $R_2 \to R_2 - 3R_3$, we get

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying, $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix}$

Q.6. Find the inverse of the following matrix using elementary operations :

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Ans.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Let $\quad A = IA$

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\text{Applying } R_2 \to R_2 + R_1]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\text{Applying } R_1 \to R_1 + R_3,\ R_2 \to R_2 + 2R_3]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \quad [\text{Applying } R_3 \to R_3 + 2R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \quad [\text{Applying } R_1 \to R_1 + R_3]$$

Hence, $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Q.7. **If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.**

Ans.

Given $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$ and $A'=\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}$

$|A'| = 1(-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$

Hence, $(A')^{-1}$ will exist.

$A_{11}=\begin{vmatrix}-1 & 2\\ 4 & 1\end{vmatrix}=-1-8=-9;$ $\quad A_{12}=-\begin{vmatrix}-2 & 2\\ 3 & 1\end{vmatrix}=-(-2-6)=8$

$A_{13}=\begin{vmatrix}-2 & -1\\ 3 & 4\end{vmatrix}=-8+3=-5;$ $\quad A_{21}=-\begin{vmatrix}0 & -2\\ 4 & 1\end{vmatrix}=-(0+8)=-8$

$A_{22}=\begin{vmatrix}1 & -2\\ 3 & 1\end{vmatrix}=1+6=7;$ $\quad A_{23}=-\begin{vmatrix}1 & 0\\ 3 & 4\end{vmatrix}=-(4-0)=-4$

$A_{31}=\begin{vmatrix}0 & -2\\ -1 & 2\end{vmatrix}=0-2=-2;$ $\quad A_{32}=-\begin{vmatrix}1 & -2\\ -2 & 2\end{vmatrix}=-(2-4)=2$

$A_{33}=\begin{vmatrix}1 & 0\\ -2 & -1\end{vmatrix}=-1-0=-1$

$$\text{adj }(A')=\begin{bmatrix}-9 & 8 & -5\\ -8 & 7 & -4\\ -2 & 2 & -1\end{bmatrix}^T=\begin{bmatrix}-9 & -8 & -2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}$$

$$(A')^{-1}=\frac{1}{1}\begin{bmatrix}-9 & -8 & -2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}=\begin{bmatrix}-9 & -8 & -2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}$$

Q.8. The management committee of a residential colony decided to award some of its members (say *x*) for honesty, some (say *y*) for helping others and some others (say *z*) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for co-operation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category.

Ans.

According to question, the system of equations is

$x + y + z = 12; \quad 2x + 3y + 3z = 33 \quad \text{and} \quad x - 2y + z = 0$

The above system of linear equation can be written in matrix form as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 1(3+6) - 1(2-3) + 1(-4-3) = 9 + 1 - 7 = 3$

$A_{11} = 9; \quad A_{12} = 1; \quad A_{13} = -7$

$A_{21} = -3; \quad A_{22} = 0; \quad A_{23} = 3$

$A_{31} = 0; \quad A_{32} = -1; \quad A_{33} = 1$

$$\text{Adj } A = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}' = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

Putting the value of X, A^{-1} and B in $X = A^{-1} B$, we get

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 108 - 99 \\ 12 + 0 + 0 \\ -84 + 99 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$\Rightarrow \quad x = 3, \; y = 4, \; x = 5$

Number of awards for honesty = 3

Number of awards for helping others = 4

Number of awards for supervising = 5.

Q.9. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method.

Ans.

Let monthly incomes of Ayran and Babban be ₹ $3x$ and ₹ $4x$ while their monthly expenditure be ₹ $5y$ and ₹ $7y$ respectively.

According to question

$$3x - 5y = 15000 \qquad ...(i)$$

$$4x - 7y = 15000 \qquad ...(ii)$$

Above system of linear equations (i) and (ii) may be written in matrix form as follows

$AX = B$ where

$$A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$\because \quad AX = B \quad \Rightarrow X = A^{-1}B \qquad ...(iii)$

Now $\quad |A| = -21 + 20 = -1$

$$\text{adj } A = \begin{bmatrix} -7 & -4 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{-1}\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix} = \begin{bmatrix} 105000 - 75000 \\ 60000 - 45000 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix} \quad \Rightarrow \quad x = 30000, y = 15000$$

Hence monthly income of Aryan = ₹ 90,000

and monthly income of Babban = ₹ 1,20,000

Q.10. A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately.

Ans.

Let the charges of English and Hindi typing per page be ₹ x and ₹ y respectively. The information given in the question can be written in matrix form as

$$\Rightarrow \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 145 \\ 180 \end{bmatrix} \Rightarrow \begin{bmatrix} 10x + 3y \\ 3x + 10y \end{bmatrix} = \begin{bmatrix} 145 \\ 180 \end{bmatrix}$$

Equating, we get

$10x + 3y = 145$ ...(i)

$3x + 10y = 180$ ...(ii)

Applying (i) × 10 and 3 × (ii), we get

$$\begin{array}{r} 100x + 30y = 1450 \\ -9x \pm 30y = -540 \\ \hline 91x = 910 \end{array}$$

$$\Rightarrow \quad x = \tfrac{910}{91} = 10$$

Putting $x = 10$ in (i), we get

$$\therefore \quad \begin{aligned} y &= \tfrac{145-100}{3} \\ y &= \tfrac{45}{3} = 15 \end{aligned}$$

Hence, charges for English and Hindi typing are ₹10 and ₹15 respectively.

Q.11. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2. Using matrices, find the dimensions of the plot.

Ans.

Let length and breadth of rectangular plot be x and y.

According to question

$(x – 50)$ $(y + 50) = xy$ $\Rightarrow 50x – 50y = 2500$ $\Rightarrow$ $x – y = 50$ ...(i)

And $(x – 10)$ $(y – 20) = xy – 5300$ $\Rightarrow 20x + 10y = 5500$ $\Rightarrow$ $2x + y =$ 550 ...(ii)

Linear equations (i) and (ii) may be written in matrix form as follows

$\because \quad AX = B;$ where

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$\because \quad AX = B \quad \Rightarrow \; X = A^{-1}B \quad ...(iii)$

$$|A| = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} = 1 + 2 = 3 \neq 0$$

Now,

$$\text{Adj } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}' = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\therefore \; A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

From (iii)

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 600 \\ 450 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$\Rightarrow \quad x$ = Length = 200 m and y = Breadth = 150 m.

Q.12. A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types.

Ans. Let ₹ x and ₹ y be the fees for rich and poor children respectively.

According to question

$5x + 20y = 9000$

$25x + 5y = 26000$

Above system of equation is written in matrix form as

$AX = B$, where $A = \begin{bmatrix} 5 & 20 \\ 25 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 5 & 20 \\ 25 & 5 \end{vmatrix} = 25 - 500 = -475 \neq 0$

Again, $\text{adj } A = \begin{bmatrix} 5 & -25 \\ -20 & 5 \end{bmatrix}^T = \begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{-475} \begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}$

$\because \quad AX = B \qquad \Rightarrow \quad X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{475} \begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix} \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = -\frac{1}{475} \begin{bmatrix} 45000 - 520000 \\ -225000 + 130000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{475} \begin{bmatrix} -475000 \\ -95000 \end{bmatrix} \qquad \Rightarrow \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1000 \\ 200 \end{bmatrix}$$

$\Rightarrow \; x = 1000$ and $y = 200$

Hence, fee for rich children = ₹ 1000.

Fee for poor children = ₹ 200.

Q.13. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less. Using matrix method, find the number of children and the amount distributed by Seema.

Ans. Let the number of children be ₹ x and the amount donated by Seema to each child be ₹ y.

$\therefore$ From question

$(x - 8)(y + 10) = xy$ and $(x + 16)(y - 10) = xy$

$\Rightarrow \quad xy + 10x - 8y - 80 = xy$ and $xy - 10x + 16y - 160 = xy$

$\Rightarrow \quad 5x - 4y = 40 \qquad \qquad ...(i)$

and $5x - 8y = -80 \qquad \qquad ...(ii)$

Equation (*i*) and (*ii*) may be written in matrix form as

$AX = B \quad \Rightarrow \quad X = A^{-1}B \quad ...(iii)$

Where $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$

Now, $|A| = -40 + 20 = -20$

$$\text{Adj } A = \begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

Putting A^{-1} in (iii), we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -320 & -320 \\ -200 & -400 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -640 \\ -600 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$\Rightarrow \quad x = 32, \; y = 30$

Q.14. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust.

Ans.

Let ₹ x and ₹ y be the amount of money invested in 1st and 2nd bond respectively.

According to information given in question

$$x \times \frac{10}{100} + y \times \frac{12}{100} = 2800$$
$$x \times \frac{12}{100} + y \times \frac{10}{100} = 2700$$

$\Rightarrow \quad 10x + 12y = 280000 \quad \ldots(i)$

and $\quad 12x + 10y = 270000 \quad \ldots(ii)$

Above system of equation can be written in matrix form as

$AX = B$, where

$$A = \begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \ldots \quad (iii)$

Now, $|A| = \begin{vmatrix} 10 & 12 \\ 12 & 10 \end{vmatrix} = 100 - 144 = -44$

$$A^{-1} = \frac{1}{|A|}.\text{ adj } A = \frac{1}{|A|}.\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}^T = \frac{1}{-44}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}$$

Putting the value of X, A^{-1} and B in (iii), we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-44}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}\begin{bmatrix} 280000 \\ 270000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{44}\begin{bmatrix} 2800000 - 3240000 \\ -3360000 + 2700000 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{44}\begin{bmatrix} -440000 \\ -660000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$$

$\Rightarrow \quad x = 10000, \; y = 15000.$

Hence, invested amount in 1st and 2nd bonds are ₹ 10000 and ₹ 15000.

Long Answer Questions-I (OIQ)

[4 Mark]

Q.1. If A, B are square matrices of the same order, then prove that adj (AB) = (adj B) (adj A).

Ans.

We know that

$(AB)\ adj\ (AB) = |AB|I = adj\ (AB)(AB)$ …(i)

$\Rightarrow \quad (AB)\ (adj\ B\ .\ adj\ A) = A\ .\ B\ adj\ B\ .\ adj\ A = A(B\ adj\ B)\ adj\ A$

$= A(|B|I)\ adj\ A \qquad [\because B\ adj\ B = |B|I]$

$= |B|(A\ .\ adj\ A)$

$= |B||A|I \qquad [\because A\ adj\ A = |A|I]$

$= |A||B|I$

$= |AB|I$ …(ii)

From (i) and (ii), we get

$(AB)\ (adj\ AB) = AB\ (adj\ B\ .\ adj\ A)$

Pre-multiplying both sides by $(AB)^{-1}$, we get

$(AB)^{-1}\ ((AB)\ adj\ AB) = (AB)^{-1}\ ((AB)\ adj\ B\ .\ adj\ A)$

$\Rightarrow \quad adj\ AB = adj\ B\ .\ adj\ A$

Q.2. In a survey of 20 richest persons of three residential society *A, B, C* it is found that in society *A*, 5 believe in honesty, 10 in hard work and 5 in unfair means while in *B*, 5 believe in honesty, 8 in hard work and 7 in unfair means and in *C*, 6 believe in honesty, 8 in hard work and 6 in unfair means. If the per day income of 20 richest persons of society *A, B, C* are ₹ 32,500, ₹ 30,500, ₹31,000 respectively, then find the per day income of each type of people by matrix method.

Ans.

Let x, y, z be the per day income of person believing in honesty, hard work and unfair means respectively. The given situation can be written in matrix form as $AX = B$

Where, $A = \begin{bmatrix} 5 & 10 & 5 \\ 5 & 8 & 7 \\ 6 & 8 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 32500 \\ 30500 \\ 31000 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 5 & 10 & 5 \\ 5 & 8 & 7 \\ 6 & 8 & 6 \end{vmatrix} = 5(48 - 56) - 10(30 - 42) + 5(40 - 48) = 40 \neq 0$

Hence, A^{-1} exists and system have unique solution.

Also,

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 8 & 7 \\ 8 & 6 \end{vmatrix} = (48-56) = -8, \quad C_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 7 \\ 6 & 6 \end{vmatrix} = -(30-42) = 12$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 8 \\ 6 & 8 \end{vmatrix} = (40-48) = -8; \quad C_{21} = (-1)^{2+1}\begin{vmatrix} 10 & 5 \\ 8 & 6 \end{vmatrix} = -(60-40) = -20$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 5 \\ 6 & 6 \end{vmatrix} = (30-30) = 0; \quad C_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 10 \\ 6 & 8 \end{vmatrix} = -(40-60) = 20$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 10 & 5 \\ 8 & 7 \end{vmatrix} = (70-40) = 30; \quad C_{32} = (-1)^{3+2}\begin{vmatrix} 5 & 5 \\ 5 & 7 \end{vmatrix} = -(35-25) = -10$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 5 & 10 \\ 5 & 8 \end{vmatrix} = (40-50) = -10$$

$$\text{Adj } (A) = \begin{bmatrix} -8 & 12 & -8 \\ -20 & 0 & 20 \\ 30 & -10 & -10 \end{bmatrix}^T = \begin{bmatrix} -8 & -20 & 30 \\ 12 & 0 & -10 \\ -8 & 20 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } (A)}{|A|} = \begin{bmatrix} \frac{-8}{40} & \frac{-20}{40} & \frac{30}{40} \\ \frac{12}{40} & 0 & \frac{-10}{40} \\ \frac{-8}{40} & \frac{20}{40} & \frac{-10}{40} \end{bmatrix} = \begin{bmatrix} \frac{-1}{5} & \frac{-1}{2} & \frac{3}{4} \\ \frac{3}{10} & 0 & \frac{-1}{4} \\ \frac{-1}{5} & \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$$

Putting the value of X, A^{-1}, B in $X = A^{-1}B$, we get

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-1}{5} & \frac{-1}{2} & \frac{3}{4} \\ \frac{3}{10} & 0 & \frac{-1}{4} \\ \frac{-1}{5} & \frac{1}{2} & \frac{-1}{4} \end{bmatrix} \cdot \begin{bmatrix} 32500 \\ 30500 \\ 31000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1500 \\ 2000 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 1500, \; y = 2000, \; z = 1000$

Hence, per day income of person who believes in honesty = ₹ 1,500

Per day income of person who believes in hard work = ₹ 2,000

Per day income of person who believes in unfair means = ₹ 1,000

Q.3. Two Trusts *A* and *B* receive ₹ 70000 and ₹ 55000 respectively from central government to award prize to persons of a district in three fields agriculture, education and social service. Trust *A* awarded 10, 5 and 15 persons in the field of agriculture, education and social service respectively while trust *B* awarded 15, 10 and 5 persons respectively. If all three prizes together amount to ₹ 6000, then find the amount of each prize by matrix method.

Ans.

Let x, y, z be amount of prize to be awarded in the field of agriculture, education and social service respectively. The given situation can be written in matrix form as $AX = B$

Where $A = \begin{bmatrix} 10 & 5 & 15 \\ 15 & 10 & 5 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 70000 \\ 55000 \\ 6000 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 10 & 5 & 15 \\ 15 & 10 & 5 \\ 1 & 1 & 1 \end{vmatrix}$

$= 10\,(10 - 5) - 5\,(15 - 5) + (15 - 10) = 75 \neq 0$

Hence, A^{-1} exists and system have unique solution.

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 10 & 5 \\ 1 & 1 \end{vmatrix} = (10 - 5) = 5; \qquad C_{12} = (-1)^{1+2} \begin{vmatrix} 15 & 5 \\ 1 & 1 \end{vmatrix} = -(15 - 5) = -10$$

Also,

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 15 & 10 \\ 1 & 1 \end{vmatrix} = (15 - 10) = 5; \qquad C_{21} = (-1)^{2+1}\begin{vmatrix} 5 & 15 \\ 1 & 1 \end{vmatrix} = -(5 - 15) = 10$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 10 & 15 \\ 1 & 1 \end{vmatrix} = (10 - 15) = -5; \qquad C_{23} = (-1)^{2+3}\begin{vmatrix} 10 & 5 \\ 1 & 1 \end{vmatrix} = -(10 - 5) = -5$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 5 & 15 \\ 10 & 5 \end{vmatrix} = (25 - 150) = -125; \quad C_{32} = (-1)^{3+2}\begin{vmatrix} 10 & 15 \\ 15 & 5 \end{vmatrix} = -(50 - 225) = 175$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 10 & 5 \\ 15 & 10 \end{vmatrix} = (100 - 75) = 25$$

$$\text{Adj } |A| = \begin{bmatrix} 5 & -10 & 5 \\ 10 & -5 & -5 \\ -125 & 175 & 25 \end{bmatrix}^T = \begin{bmatrix} 5 & 10 & -125 \\ -10 & -5 & 175 \\ 5 & -5 & 25 \end{bmatrix}$$

$$\text{Adj } |A| = \begin{bmatrix} 5 & -10 & 5 \\ 10 & -5 & -5 \\ -125 & 175 & 25 \end{bmatrix}^T = \begin{bmatrix} 5 & 10 & -125 \\ -10 & -5 & 175 \\ 5 & -5 & 25 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } (A)}{|A|} = \begin{bmatrix} \frac{5}{75} & \frac{10}{75} & \frac{-125}{75} \\ \frac{-10}{75} & \frac{-5}{75} & \frac{175}{75} \\ \frac{5}{75} & \frac{-5}{75} & \frac{25}{75} \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 1 & 2 & -25 \\ -2 & -1 & 35 \\ 1 & -1 & 5 \end{bmatrix}$$

Putting the value of X, A^{-1}, B in $X = A^{-1} B$, we get

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 1 & 2 & -25 \\ -2 & -1 & 35 \\ 1 & -1 & 5 \end{bmatrix} . \begin{bmatrix} 70000 \\ 55000 \\ 6000 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 70000 + 110000 - 150000 \\ 740000 - 55000 + 210000 \\ 70000 + 55000 + 30000 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 30000 \\ 15000 \\ 45000 \end{bmatrix} \qquad \Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

$\Rightarrow$ $x = 2000$, $y = 1000$, $z = 3000$.

Hence, prize in the field of agriculture = ₹ 2000

prize in the field of education = ₹ 1000

prize in the field of Social service = ₹ 3000