Chapter—7

DIRECT AND INVERSE VARIATION

Introduction

Sometimes we hear statements like in a year of heavy rainfall the water level in ponds and wells increases. Consumption of water increases as the population grows. As the number of ponds decreases, the capacity to store water decreases. With the fall in electricity production the quantity of goods produced in factories decrease. As the rainfall has been more water level in the pond has increased.

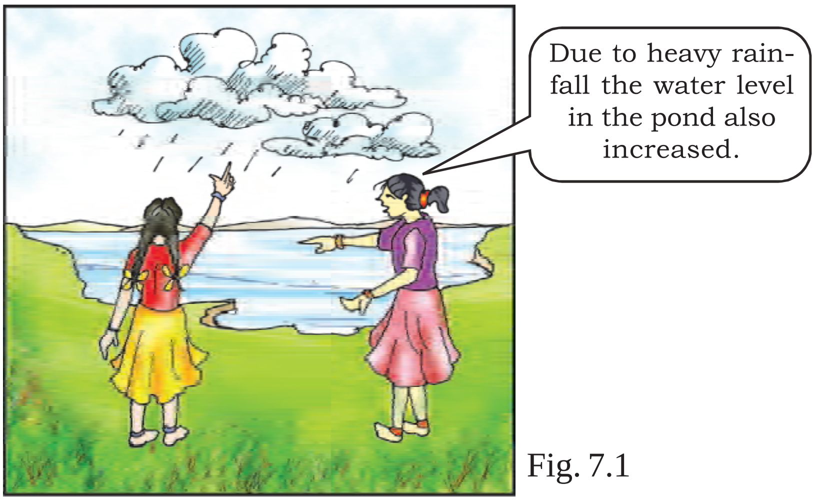

Fig. 7.1

If we consider the above statements we notice that the two quantities depend on one another. Change in the value of one also changes the value of the other. Thus for two related quantities, the change in the value of the second when the value of the first quantity changes is called variation. When on the increase or decrease in one value, the value of other also increases or decreases, then such a variation is called direct variation.

Think and write such examples of changes from your surrounding.

Raju wrote that if farm land is larger then the quantity of crop produced would be more i.e. if 1 acre land produced 24 bags rice then 3 acres of land would produce 72 bags rice.

Sudha wrote a relation between the quantity and the cost of a crop. She said if price of the 1 kg rice is Rs. 9 then the price of 2 kg rice will Rs. 18 and price of 5 kg rice will Rs. 45".

After reading these examples Mary said, "But this does not happen every time. It is not necessary that on increasing the value of one, the value of other increases in the same ratio. It also does not happen that on a decrease in the value of one, leads to a decrease in the value of the other in the same ratio. Some time we may have situations in which the increase in the value of one quantity leads to a decrease in the value of the other or on the decrease in the value of one results in increase of the other". For example – unemployment decreases with increase in industries. The water level in ponds decreases with increase in temperature. Hiring more

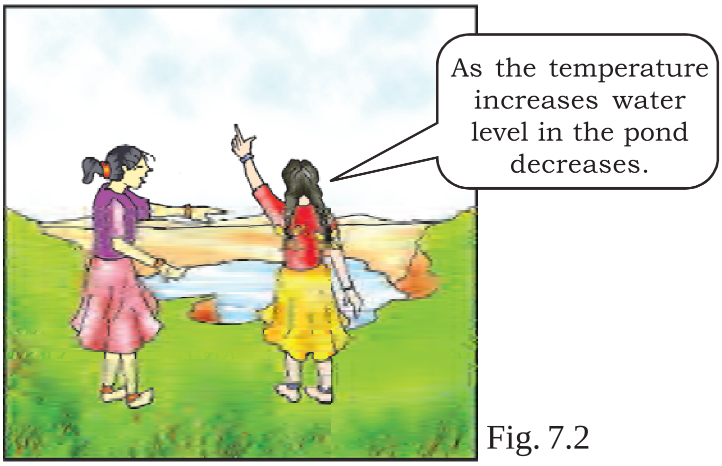

Fig. 7.2

labourers reduces the amount of time required for completion of the task. Such relations are not direct variations. What will these relations be called?

As the temperature increases the water level in the pond keeps falling.

Raju was listening carefully to Mary, he said, "In direct variation, if the value of one increases, the value of the other increases or if the value of one decreases the value of the other also decreases. But in a situation where when one quantity increase the other decrease or when on decreasing one quantity the other increases, we have inverse variation. This is the opposite of direct variation.

Raju was right in his thinking. The opposite of direct variation is called inverse variation. Let us first understand direct variation through some more examples:-

Direct Variation

Example 1: The price of 5 pens is Rs. 20. What will be the price of 10, 15, 20, 25 and 30 pens of the same type?

Solution: If the numbers of pens is denoted by x and their corresponding price by y then we can solve such questions by unitary method and make the following table.

No. of pens x	5	10	15	20	25	30
Price of pens y(Rs.)	20	40	60	80	100	120
Ratio $\frac{x}{y}$	$\frac{5}{20}$	$\frac{10}{40}$	$\frac{15}{60}$	$\frac{20}{80}$	$\frac{25}{100}$	$\frac{30}{120}$
Ratio $\frac{x}{y}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

From the above table it is clear that as the number of pens increases, their corresponding price also increases. In each case the ratio between number of pens and their price remains the same ($\frac{1}{4}$). An increase of this kind where the price increases in direct proportion to the number of pens is called **direct variation**.

Activity 1

In the following tables, number of balls along with their prices are given. On the basis of this fill the blank boxes in the table and answer the questions given below the table.

No. of balls x	10	6	4	3	2	1
Price (Rs.) y	30			9		3
Ratio $\frac{x}{y}$	$\frac{10}{30}$			$\frac{3}{9}$		$\frac{1}{3}$
Ratio $\frac{x}{y}$	$\frac{1}{3}$			$\frac{1}{3}$		$\frac{1}{3}$

(1) What is the price of 6 balls?

(2) What is the price of 4 balls?

(3) What is the price of 2 balls?

(4) What is the nature of variation between the number of balls and their price?

The decrease in the number of balls is resulting reducing the corresponding price by a constant ratio i.e. number of balls is changing together with the corresponding price and their ratio is the same $\frac{1}{3}$. Therefore the relation between them is called direct variation.

Activity 2

The value of y for each x is given in the following table. Which of the following values have direct variation? Identify and write.

x	4	9	12	15	20	7	13
y	28	63	72	105	100	49	91
$\frac{x}{y}$	----	----	----	----	----	----	----

Write down five examples of direct variation related to your daily life. If two variables have direct variation the ratio between them is always equal. The ratio is a constant or non variable quantity.

Or if x and y are in direct variation then $\frac{x}{y} = k$ (constant)

If the ratio between x_1, y_1 is same as the ratio between x_2, y_2 then $\frac{x_1}{y_1} = \frac{x_2}{y_2} = k$

Activity 3

If x and y are in direct variation then fill the blanks in the following table:

S.No.	x	y	$\frac{x}{y}$
(1)	3	18	---
(2)	25	---	$\frac{1}{6}$
(3)	9	---	$\frac{1}{6}$

S.No.	x	y	$\frac{x}{y}$
(4)	---	24	$\frac{1}{6}$
(5)	7	42	---
(6)	---	66	$\frac{1}{6}$

Example 2: The price of 3 kg of wheat is Rs. 36 .Find the price of 18 kg of wheat.

Solution: Since on increasing the amount of wheat its corresponding price will also increase. This relation is of a direct variation. Let the price of 18 kg of wheat be Rs. x. It can be written in the form of the following table

Quantity wheat (kg)	3	18
Price (Rs)	36	x

Here ratio between 18 and x would be the same as between 3 and 36.

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{3}{36} = \frac{18}{x}$$

Or $3 \times x = 18 \times 36$ (cross multiplying)

Or $x = \frac{18 \times 36}{3}$

Or $\boxed{x = \text{Rs. } 216}$

Thus the price of 18 kg of wheat is Rs. 216

Example 3: The distance covered by a train in 2 hours is 120 km. Find the distance that the train will cover in 5 hours with the same speed?

Solution: Since the distance covered will increase with the increase in time. Therefore here the relation is of direct variation. Let the distance covered in 5 hours be x km. It can be written in the form of the following table:

Time (in hours)	2	5
Distance covered (in km)	120	x

Here the ratio between 5 and x is equal to the ratio between 2 and 120.

Or $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2}$

$\therefore \quad \dfrac{2}{120} = \dfrac{5}{x}$

Or $2 \times x = 5 \times 120$ (cross multiplying)

Or $x = \dfrac{5 \times 120}{2}$

$\therefore\ x = 300$ km

Therefore, the distance covered by train in 5 hours is 300 km.

Example 4: A man gets Rs. 32 by working for 4 hours, what will he get by working for 7 hours?

Solution: Since when you work for more time the amount of labor is more. Here the relation is of direct variation. Let the man gets Rs x for working 7 hours.

Then table will be –

Time (in hours)	4	7
Wages (Rs.)	32	x

Here the ratio between 7 and x is the same as that between to 4 and 32.

Or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\therefore$ $\frac{4}{32} = \frac{7}{x}$

Or $4 \times x = 7 \times 32$ (cross multiplying)

Or $x = \frac{7 \times 32}{4}$

$x = 56$

Or the man will get Rs. 56 for working 7 hours.

Example 5: If the weight of 6 letters is 45 grams than how many letters would have a weight of $1\frac{1}{2}$ kg?

Solution: Since, on increasing the number of letters their weight will also increase in the same ratio. Therefore the relation is of direct variation. Let the weight of x letters be 1½ kg or 1500 gm.

(Here weight is changed into same units (grams))

Table is as follows:

No. of letters	6	x
Weight (in gm)	45	1500

Here ratio between x and 1500 is same as 6 and 45.

Or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\therefore$ $\frac{6}{45} = \frac{x}{1500}$

Or $6 \times 1500 = 45 \times x$ (cross multiplying)

Or $x = \frac{6 \times 1500}{45}$ or $\frac{6 \times 1500}{45} = x$

$\therefore$ $\boxed{x = 200 \text{ letters}}$

Or the weight of 200 letters is $1\frac{1}{2}$ kg.

Exercise 7.1

Q1. In the following tables are x and y are in direct variation. Find the value of the constant ratio also.

Table I.

x	7	9	13	21	25	30	41
y	21	27	39	63	75	90	123

Table II.

x	2.5	7.5	11	17.5	19
y	2.5	7.5	11	17.5	19

Table III.

x	5	6	7	8	9	11
y	25	24	35	40	50	66

Table IV.

x	1	2	3	4	5
y	2	1	2/3	½	2/5

Q2. Fill in the blanks in the following direct variation table:-

No. of workers	1	2	....	4	5	....
Wages (Rs.)	50	....	150	200	....	300

Q3. In following table x & y are in direct variation. Find the value of constant ratio k.

x	2	4	8	16	32
y	14	28	56	112	224
$k = \frac{x}{y}$	-------	-------	-------	-------	---------

Q4: A car runs 600 km in 3 hours. How far will it travel in 5 hours?

Q. 5: Which of the following quantities are in direct variation?

(i) Number of objects and their prices.

(ii) Number of Mathematics books of class 7th and their prices.

(iii) Area of a field and its price

(iv) Quantity of milk (in lt.) and its price.

(v) Number of labourers and the total number of days in which the work is completed.

(vi) Speed and time when the distance travelled remains the same.

Q6: Price of 15 tickets (of equal price) is Rs.18. How many tickets of the same price can be bought in Rs. 72?

Q 7: A car covers 432 km in 48 lt of petrol. What distance will it travel in 20 lt. of petro1?

Q8. The cost of 2 dozen oranges is Rs. 48. Find the cost of 108 oranges?

Q9: A machine prints 200 pages in 5 min. How much time will it take to print 2 x 10^3 pages?

Q10. A cyclist covers 12km in 3 hours. How much time will be taken by him to cover 20 km?

Q11. The wages of 25 labourers is Rs. 1250. What is the amount of wages for 40 labourers?

Q12. A labourers gets Rs. 806 for working 13 days. How many days would he have worked if he was paid Rs. 1798?

Q13. Namrata takes 1225 steps to cover a distance of 100m. How much distance will she cover in 2835 steps?

Q14. A dealer gets a commission of Rs. 73 for selling items worth Rs.1000/-. How much commission will he get on selling items of Rs. 100?

Q15. The thickness of 500 sheets of paper is 3.5 cm, find the thickness of 275 sheets.

Q16. A man can read 180 words in 1 min. How much time will it take him to read 768 words?

Q17. Sunita types 1080 words in 1 hour. Find her rate of typing per minute.

Q18. 25 labourers make a 7.5 km long road in one week. How many labourers will make a road of 10.2 km in 1 week?

Q19. The weight of 10 bags of cement is 4.5 quintal. What will be the weight of 35 such bags?

Q20. The speed of a car is 60 km per hour. The distance traveled and corresponding time taken are in direct variation (in the following table). Match the correct distance and time in the columns.

Distance (KM)		Time (in hours)
(i)	120	(a) 3
(ii)	180	(b) 2
(iii)	210	(c) 4
(iv)	240	(d) 3½

INVERSE VARIATION

In our daily life, we see sometimes that on increasing a quantity another quantity starts decreasing in a constant ratio or with a decrease in the first quantity the second quantity starts to increase in a constant ratio. Such proportional relations are called inverse ratio.

Fig. 7.3

I made this wall alone in 10 days but five of us would have made it in 2 days.

Let us see an example:-

The number of labourers and the days required by them to spread sand on the road is given below:

No. of labourers (x)	5	10	15	20	30
No. of days (y)	60	30	20	15	10

In above table the number of labourers (x) and number of days (y) are given. Can you find a relation between each x and each y? A relation that is constant for all values x and corresponding y.

Ashu considered the examples and thought that doubling the number of labourer the number of days became half and when number of labourers were tripled the numbers of days became $\frac{1}{3}$rd. In the same way if the number of labourers is made10 times then the number of days will become $\frac{1}{10}$. If we multiply the values of x and y, we will get a constant. In direct relation as $\frac{x}{y}$ or x/y is constant similarly here x × y or $\frac{x}{1/y}$ or $x : \frac{1}{y}$ is a constant. This is the inverse of direct ratio therefore it is called inverse ratio.

Do you agree with Ashu?

Here we find that the manner in which the number of labourers increases, the number of days decreases in the opposite ratio and the number of days increases in the opposite ratio to the manner in which number of labourers decrease. Such a ratio is called opposite ratio or inverse ratio. In the above example the number of labourers is in inverse ratio with the number of days taken i.e. the variation between both quantities is an inverse ratio.

Activity 4

A passenger train covers a distance in 4 hours with a speed of 12 km/hour.
Answer the following:
(i) If the speed is increased to 24 km/hr, how much time will it take to cover the same distance?
(ii) If the speed is increased to 36 km/hr, how much time will it take to cover the same distance?
Also fill up the following table.

Speed (km/hr) on increasing				
Time (in hours)				

Result: If the speed is increased, time taken is ....

Speed (km/hr)	48	32	16	6
Time (in hours)	1	-----	-----	-----

Result: If the speed is decreased, time taken is ....

Construct five example of inverse ratio from your daily life.

Let us discuss one more example:

A book can be finished in 15 days, if 16 pages are read every day. If 8 pages are read in a day, how many days will it require to finish the book? If 12, 15, 24 pages are read in a day, how many days will it require to finish the book in each case? If the number of pages read each day is denoted by x and the number of days taken to finish the book are denoted by y, the answer to the questions can be written in the following table:

No. of pages read each day (x)	16	8	12	15	24
No. of days (y)	15	30	20	16	10

$\frac{1}{y}$	$\frac{1}{15}$	$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{16}$	$\frac{1}{10}$

$x:\frac{1}{y}=$	$\frac{x}{1/y}$	$\frac{16}{1/15}$	$\frac{8}{1/30}$	$\frac{12}{1/20}$	$\frac{15}{1/16}$	$\frac{24}{1/10}$

x × y in standard form	16×15 =240	8×30 =240	12×20 =240	15×16 =240	24×10 =240

$$x:\frac{1}{y}=\frac{x}{\frac{1}{y}}=xy=240=\text{k (say)}$$

Here numbers of pages read per day are in inverse ratio to the number of days taken. The inverse relation between the numbers of pages read per day and numbers of days taken gives a constant value for the product each time. In other words, we can say that the product of the number of pages read per day x and the corresponding number of days taken y is a fixed quantity i.e. xy=k

In general for any values x_1 and x_2 of number of pages read per day and corresponding values y_1 and y_2 of the days taken we conclude. $x_1y_1=x_2y_2$

Conclusion: We find that if the relation between two variable quantities is such that on increasing one quantity the other starts decreasing or on decreasing one quantity the other starts increasing and the product of both quantities remains constant then the relation between them is called inverse variation. Mathematically, we can write that if x and y are in inverse variation then $\boxed{xy=k}$

If x has two values x_1, x_2 and the corresponding y has $y_1$and y_2 then $\mathbf{x_1y_1=x_2y_2}$

Activity 5

In which of the following tables x and y are in inverse variation-

(i)

x	6	2	3	18
y	3	9	6	1

(ii)

x	40	20	16	10	2.5
y	2	4	5	8	32

(iii)

x	10	5	2	4
y	3	6	15	8

(iv)

x	9	10	12	15
y	5	4.5	3.75	3

Activity 6

If x and y are in inverse variation, fill in the blanks in the following table.

(i)

x	9	18	20	---	30
y	4	---	---	1.5	----

(ii)

x	16	8	--	---
y	3	---	12	24

(iii)

x	20	50	25	----
y	---	4	----	5

Let us see some more examples of inverse variation:-

Example 6: 12 labours can build a wall in 10 days. How many days will 20 labours take to build the same wall?

Solution: As by increasing the number of labourers, the time taken to complete the work will decrease. Hence this is a case of inverse variation.

Let 20 labours make the wall in y days. The table will be made as follows:-

No. of labourers (x)	12	20
No. of days (y)	10	y

For inverse variation

$$x_1y_1 = x_2y_2$$

$$\therefore\ 12\times10 = 20\times y$$

$$\text{or}\quad \frac{12\times10}{20} = y$$

$$\text{or}\quad 6 = y \qquad \text{or} \qquad y = 6$$

$\therefore$ 20 labours will complete the wall in 6 days.

Example7. A hostel has 24 days food for 200 students.

If 100 more students join that hostel, then how many days will the food last?

Solution: The joining of 100 students in the hostel, means that the total number of students = 200 + 100 = 300.

Since, quantity of food is the same, the increase in the numbers of students means the food will be consumed faster therefore this is of an inverse variation.

Let the food be finished in y days.

The table is as follows:-

No. of students (x)	200	300
No. of days (y)	24	y

In inverse variation

$$x_1y_1 = x_2y_2$$

$$\therefore \quad 200 \times 24 = 300 \times y$$

$$\text{or} \quad \frac{200 \times 24}{300} = y$$

$$\text{or} \quad 16 = y \qquad \text{or} \qquad y = 16$$

$\therefore$ The food will finish in 16 days.

Example8. Shallu goes to school on a cycle with an average speed of 12 km/hr. She reaches the school in 20 min. If she wants to reach the school in 15 min, what should be her average speed?

Solution: Since to reach school in less time the speed has to increase, therefore this is a case of inverse variation. Let the average speed be x km/hr. The table is :-

Sped (km/h) (x)	12	x
Time (in min y)	20	15

In inverse variation

$$x_1y_1 = x_2y_2$$

$$\therefore \quad 12 \times 20 = x \times 15$$

$$\text{or} \quad \frac{12 \times 20}{15} = x$$

$$\text{or} \quad 16 = x$$

$$\therefore \quad x = 16 \text{ Km/hr}$$

Therefore the average speed of Shallu should be 16 km/hr to reach the school in 15 min.

Exercise 7.2

1. If x and y are in inverse ratio than fill in the table appropriately.

x	8	6	4	-----	36
y	9	12	-----	10	----

2. Fill in the blanks in variation table.

Speed (in km /hr)	4	8	----	-----	64
Time taken (in min)	-----	40	20	10	------

3. 10 labours finish a task in 2 days. How many days will 2 labours take to finish the same task?

4. 45 labours finish the work in 27 days. How many labourers will complete the same work in 15 days?

5. A bus takes 6 hours to cover a certain distance with a speed of 30 km/hr. What speed the bus should have to cover the same distance in 4 hours?

6. 40 horses eat 1 quintal of grain in 7 days. How many horses will eat the same quantity of grain in 28 days?

7. A hostel has 15 days of food for 300 students. If 200 students leave the hostel due to vacations then for how many days will the food last?

8. A military camp has 25 days of food for 700 soldiers. Due to the arrival of some more soldiers the food finishes in 20 days. Find the number of new soldiers that arrived in the camp.

9. A man completes a book in 15 days by reading 8 pages per day. If he reads 12 pages daily, then how many days will he take to finish the book?

10. A military camp has 21 days of ration for 105 soldiers. If 42 more soldiers arrive in the camp, then how many days will the ration last?

11. Which are the inverse variation in the following:-

    (i) Number of books bought and the cost of each book.

(ii) Distance traveled by the bus and the cost of petrol consumed.

(iii) Time taken by a cycle to cover a fixed distance with its speed.

(iv) The number of men employed to make a bridge and the time taken to make the bridge.

(v) Number of students and the weight of sweets distributed among them if 40 kg sweets are to be distributed.

(vi) Wages and hours of work

(vii) Number of objects and their total price.

12. On 26th January 100 gm sweets are distributed per student among 800 students. If the same amount of sweets is to be distributed among 1000 students, how many grams of sweets will each student get?

13. If a tap takes 1 hr in filling 640 lt. water, then a water tank is filled in 10hours. If the same tank was filled by another tap in 8 hours, how many liters will be filled in 1 hour by this tap?

We have learnt

1. When two variable quantities are related to each other such that on increasing (or decreasing) the value of one variable, the value of the other increases (or decreases) in the same ratio, then they are in direct variation.

2. When two variables quantities x and y are in direct variation then their ratio is a constant or $\boxed{\frac{x}{y} = k}$

3. If two quantities are in direct ratio, and one quantity has values x_1, x_2 and the corresponding values for the other quantity are y_1 and y_2 respectively. Then $\boxed{\frac{x_1}{y_1} = \frac{x_2}{y_2}}$

4. If two variable quantities are related to each other such that on increasing (or decreasing) the value of one variable, the value of the second variable quantity

decreases (or increases) in the same ratio, then the two quantities are in inverse ratio.

5. When two quantities are in inverse ratio then their product is a constant i.e. $xy = k$ where k is a constant.

6. If two quantities are in inverse ratio, and one quantity has values x_1, x_2 with the corresponding values for the second quantity y_1 & y_2 respectively then, $x_1 \times y_1 = x_2 \times y_2$