Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. Write the vector equation of the following line.

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$$

Ans.

Cartesian form of the line is given as

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{-2}$$

The standard form of line's equation

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

We get by comparing that the given line passes through the point (x_1, y_1, z_1) *i.e.*, $(5, -4, 6)$ and direction ratios are (a, b, c) *i.e.*, $(3, 7, -2)$.

Now, we can write vector equation of line as

$$\vec{a} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$$

Q.2. Find the direction cosines of the line passing through two points (–2, 4, –5) and (1, 2, 3).

Ans.

We know that direction cosines of the line passing through two points $P\ (x_1, y_1, z_1)$ and Q (x_2, y_2, z_2) are given by

$$\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}, \quad \text{where } PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Here P is $(-2, 4, -5)$ and Q is $(1, 2, 3)$.

So $PQ = \sqrt{(1-(-2))^2 + (2-4)^2 + (3-(-5))^2} = \sqrt{77}$

Thus, the direction cosines of the line joining two points are $\frac{1+2}{\sqrt{77}}, \frac{2-4}{\sqrt{77}}, \frac{3+5}{\sqrt{77}}$

or $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

Q.3. Find the value of l so that the lines $\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ **and** $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ **are perpendicular to each other.**

Ans.

The given lines can be expressed as

$$\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2} \text{ and } \frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-7}$$

The direction ratios of these lines are –3, 2l, 2 and 3l, 1, –7 respectively.

Since the lines are perpendicular, therefore

$$-3(3\lambda)+(2\lambda(1)+2(-7)=0 \Rightarrow \quad -9\lambda+2\lambda-14=0$$

$$\Rightarrow \ -7\lambda=14 \Rightarrow \quad \lambda=-2$$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. Show that the points *A* (2, 3, – 4), *B* (1, –2, 3) and *C* (3, 8, –11) are collinear.

Ans.

Direction ratios of the line joining *A* and *B* are (1 – 2), (–2 –3), (3 + 4) *i.e.*, –1, –5, 7

Direction ratios of the line joining *A* and *C* are (3 – 2), (8 – 3), (–11 + 4) *i.e.*, 1, 5, –7

It is clear that direction ratios of line *AB* and *AC* are proportional.

⇒ Parallel vectors of both lines are parallel to each other.

⇒ Both given lines are parallel.

⇒ But point A is common. So, points *ABC* are collinear.

Q.2. Find the equation of line in vector and cartesian form that passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ **and is in the direction** $\hat{i}+2\hat{j}-\hat{k}$

Ans.

P.V. of a point of line = $2\hat{i} - \hat{j} + 4\hat{k}.$

$\Rightarrow$the coordinate of that point = $(2, -1, 4)$

Therefore vector equation of the plane is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

Also its cartesian form is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Here, $x_1 = 2, y_1 = -1, z_1 = 4$ and $a = 1, b = 2, c = -1,$

Therefore required equation in cartesian form is

$$\frac{x - 2}{1} = \frac{y+1}{2} = \frac{z - 4}{-1}$$

Q.5. Find the distance of the point whose position vector is $(4\hat{i} + 3\hat{j} - \hat{k})$ from the plane $\vec{r}(\hat{i} - 2\hat{j} + 3\hat{k}) = 4.$

Ans.

Given P.V. of point = $4\hat{i} + 3\hat{j} - \hat{k}$

$\Rightarrow$ Coordinates of point = $(4, 3, -1)$

Equation of plane $\vec{r}(\hat{i} - 2\hat{j} + 3\hat{k}) = 4$

$\therefore$ Cartesian form of equation

$x - 2y + 3z - 4 = 0$

If d be the distance of point from the given plane then.

$$d = \left| \frac{4\times1+3\times(-2)+(-1)\times3-4}{\sqrt{(1)^2+(-2)^2+3^2}} \right|$$

$$\Rightarrow \quad d = \left| \frac{4-6-3-4}{\sqrt{14}} \right|$$

$$\Rightarrow \quad d = \frac{9}{\sqrt{14}} \text{ units}$$