*Eigen value and Eigen vectors *

$A_{n \times n}$

$|A - \lambda I| = 0$

↳ characteristics equation

→ roots ⇒ eigen values/ proper value/ latent value
characteristics value

Properties of eigen values:-

- Let A be the square matrix of order $n \times n$ and I be the unit matrix of order $n \times n$ then $|A - \lambda I| = 0$ is called characteristics eqⁿ and λ is parameter.

The roots of characteristics eqⁿ are called characteristics root / latent root/ proper root / eigen value

- The value $x = x_1, x_2, x_3$ are called as corresponding $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ eigen vectors

*Properties of Eigen vectors

1. The sum of eigen value of any matrix is equal to the sum of elements of it's principale diagonal (TRACE).

2. The product of eigen value of any matrix is equal to its determinant.

3. The eigen value of symmetric matrix $(A = A^T)$ is purely real.

④ The eigen value of skew symmetrix matrix ($A^T = -A$) are purely imaginary OR zero.

⑤ If the matrix is either upper triangular OR lower triangular then the principle diagonal elements are called eigen value

⑥ If λ is the eigen value of A then λ^2 is the eigen value of A^2 and so on...

Q:- Find the eigen values $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda = \frac{7 \pm \sqrt{49-24}}{2}$$

$$\lambda = \frac{7 \pm 5}{2}$$

$$\lambda = 1, 6$$

$$\lambda_1 + \lambda_2 = 5 + 2$$

$$\lambda_1 + \lambda_2 = 7$$

$$\lambda_1 \lambda_2 = \begin{vmatrix} 5 & 4 \\ 1 & 2 \end{vmatrix} = 10 - 4$$

$$\lambda_1 \lambda_2 = 6$$

$$\lambda_1 + \frac{6}{\lambda_1} = 7$$

$$\lambda_1^2 + 6 = 7\lambda_1$$

$$\lambda_1^2 - 7\lambda_1 + 6 = 0$$

$$\lambda_1 = 1, 6$$

$$\lambda_2 = 6, 1$$

Q:- Find characteristics root

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(a) 3 7 8
(b) 2 2 14
(c) 0 3 15
(d) 1 4 9

$$\lambda_1 + \lambda_2 + \lambda_3 = 18$$

$$= 0$$

$$\lambda_1 \lambda_2 \lambda_3 = 8(5) + 6(-10) + 2(10)$$

$$= 40 - 60 + 20 = 0$$

Q:- Find latent root

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(a) 3 3 6
(b) 2 2 8
(c) 1 4 7
(d) 2 3 9

$\lambda_1 + \lambda_2 + \lambda_3 = 12$

$6(9-1) + 2(-6+2) + 2(2-6) = \lambda_1 \lambda_2 \lambda_3$

$6(8) + 2(-4) + 2(-4) = \lambda_1 \lambda_2 \lambda_3$

$48 - 8 - 8 = 32 = \lambda_1 \lambda_2 \lambda_3$

Q:- Find eigen value

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) 1 2 3
(b) 0 2 4
(c) 1 1 4
(d) 2 2 2

$\lambda_1 + \lambda_2 + \lambda_3 = 1 + 2 + 3 = 6$

$\lambda_1 \lambda_2 \lambda_3 = 1(6-2) + 0 - 1(2-4) = 4 + 0 + 2 = 6$

Q:- The smallest & largest eigen value of following matrix are:-

$$\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$$

(a) 1.5, 2.5
(b) 0.5, 2.5
(c) 1, 3
(d) 1, 2

$\lambda_1 + \lambda_2 + \lambda_3 = 4$

$\lambda_1 \lambda_2 \lambda_3 = 3(-20+18) + 2(20-12) + 2(-12+8)$

$= 3(-2) + 2(8) + 2(-4) = -6 - 8 + 16 = 2$

Q:- Find eigen value and eigen vector of matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

Sol:- $\Rightarrow \lambda_1 + \lambda_2 = 7$

$\Rightarrow \lambda_1 \lambda_2 = 10 - 4 = 6$

$\lambda_1 \lambda_2 = 6$

$\lambda_1 = 6 \;, \; \lambda_2 = 1$

$\cancel{\begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix}}$ For eigen vectors, eigen values should be subtracted from principle diagonal

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\lambda = 1$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$4x_1 + 4x_2 = 0$

$x_1 + x_2 = 0$

$\Rightarrow x_1 = -x_2$

$\frac{x_1}{1} = \frac{x_2}{-1}$

$X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \rightarrow$ Eigen vector

$\lambda = 6$

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 + 4x_2 = 0$

$x_1 - 4x_2 = 0$

$\Rightarrow x_1 = 4x_2$

$\frac{x_1}{4} = \frac{x_2}{1}$

$X_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \rightarrow$ Eigen vector

Q:- Find eigen value and corresponding eigen vector

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

Eigen value = 1, −1, 7 (upper triangular matrix

$\Rightarrow \lambda = 1$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & -5 & 2 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$2x_2 + 3x_3 = 0$$
$$-5x_2 + 2x_3 = 0$$
$$6x_3 = 0$$

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{eigen vector}$$

L.I. (linearly independent)

$\Rightarrow \lambda = -4$

$$\begin{bmatrix} 5 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$5x_1 + 2x_2 + 3x_3 = 0$$
$$2x_3 = 0$$
$$11x_3 = 0$$
$$x_3 = 0$$

$$\frac{x_1}{2} = \frac{x_2}{-5}$$

$$\cancel{5x_1 + 2k = 0}$$
$$\cancel{x_1 = \frac{-2k}{5}}$$
$$\cancel{X_2 = \begin{bmatrix} -2k/5 \\ k \\ 0 \end{bmatrix} = \begin{bmatrix} -2/5 \\ 1 \\ 0 \end{bmatrix}}$$

$$x_2 = \begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix} \rightarrow \text{eigen vector}$$

$\Rightarrow \lambda = 7$

$$\begin{bmatrix} -6 & 2 & 3 \\ 0 & -11 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-6x_1 + 2x_2 + 3x_3 = 0$$
$$-11x_2 + 2x_3 = 0$$
$$0x_3 = 0$$

$$\frac{x_2}{2} = \frac{0x_3}{11}$$

$$-6x_1 + 4 + 33 = 0$$
$$-6x_1 = 37$$
$$x_1 = 37/6$$

$$X_3 = \begin{bmatrix} 37/6 \\ 2 \\ 11 \end{bmatrix} \rightarrow \text{eigen vector}$$

Q:- For the matrix, $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen value is -2 . What is the eigen vector

$$\begin{bmatrix} 5 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$5x_1 - 2x_2 + 2x_3 = 0$

$x_3 = 0$

$3x_3 = 0$

$\frac{x_1}{2} = \frac{x_2}{5}$

$X_1 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ ← eigen vector

Q:- Find the eigen vector $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$\lambda = 1$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_2 = 0$

$x_2 + 2x_3 = 0$

$2x_3 = 0$

$X_1 = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 2$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 + x_2 = 0$ $x_1 = x_2$

$2x_3 = 0$

$x_3 = 0$

$X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-2x_1 + x_2 = 0$

$-x_2 + 2x_3 = 0$

$\frac{x_2}{2} = \frac{x_3}{1}$

$X_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$\boxed{(b) \; [1 \; 2 \; 1]^T}$

Q:- A real $n \times n$ matrix $A = [a_{ij}]_{n \times n}$ defines as follows:-
IISC

$$a_{ij} = \begin{cases} i, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

. Sum of the eigen value is.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & n-1 & 0 \\ 0 & 0 & 0 & 0 & n \end{bmatrix} = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

* Cayley Hamilton Theorem

- Every square matrix satisfies it's own characteristics equation. (for finding inverse without adjoint)

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$A^2 - 7A + 6I = 0$$

$$6I = 7A - A^2$$

$$(6I)A^{-1} = (7A - A^2)A^{-1}$$

$$6A^{-1} = 7I - A$$

$$\boxed{A^{-1} = \frac{1}{6}(7I - A)}$$

$$|A - \lambda I| X = 0$$

$$AX - \lambda X = 0$$

$$\boxed{AX = \lambda X}$$

eigen vector.   eigen value

Q:- If M be (3×3) matrix with characteristics eqⁿ $x^3+1=0$ find M^{-1}.

$\Rightarrow$ $|A-\lambda I| = x^3+1$

$\boxed{AX = \lambda X}$

$\cancel{|A-\lambda I|X = 0}$

$\cancel{(x^3+1)X = 0}$

$\cancel{x^3X + X = 0}$

$|A-\lambda I| = 0$

$A^3+1=0$

$1 = -A^3$

$A^{-1} = -A^2$

$m^{-1} = -m^2$

Q:- If P be a (3×3) matrix with characteristics eqⁿ $\lambda^3+\lambda^2+2\lambda+1=0$. Find P^{-1}.

$\Rightarrow$ $|A-\lambda I| = 0$

$A^3+A^2+2A+1=0$

$1 = -(A^3+A^2+2A)$

$A^{-1} = -(A^2+A+2)$

$P^{-1} = -(P^2+P+2)$

Q:- If A is (3×4) real matrix AX = B is an inconsistent system of eqⁿ. The higest possible rank of A is ?

$\Rightarrow$ $\rho(A) \neq \rho(AB)$ [inconsistent]

3×4 — hig. Possible 3

3×5 — hig. possible RANK = 3

But if 3 is there then inconsistent system will not satisfy.

$\therefore$ $\boxed{\text{RANK} = 2}$

Q:- which of following is an eigen vector

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

✓ (a) $\begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$

$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 13$ ×

$\lambda = 5, 5, 2, 1$ (lower triangular matrix)

$\lambda_1 \lambda_2 \lambda_3 \lambda_4 = 5\,[10] = 50$ ×

$$\boxed{A \cdot X = \lambda X}$$

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \\ 0 \\ 0 \end{bmatrix} = \underbrace{5}_{\text{eigenvalue}} \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Q:- Find normalised eigen vector of $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$

$\lambda_1 + \lambda_2 = 8$

$\lambda_1 \lambda_2 = 12$

6 2

$\lambda_1 = 6 \qquad \lambda_2 = 2$

$$\left[\text{normal vector} = \frac{\text{vector}}{|\text{mod}|} \right]$$

$$\begin{bmatrix} -1 & 3 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 + 3x_2 = 0 \qquad \frac{x_1}{3} = \frac{x_2}{1}$

$\cancel{3x_1 - 3x_2 = 0}$

$\cancel{x_1 = x_2} \qquad \begin{bmatrix} 3 \\ 1 \end{bmatrix} = X_1$

$\lambda_2 = 2$

$$\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$3x_1 + 3x_2 = 0$

$x_1 + x_2 = 0$

$\frac{x_1}{1} = \frac{x_2}{-1}$

$X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

normalised $X_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

Q:- If $-1, 1, 0$ are eigen values of matrix A then determinant of $|A^{100}+3I| = ?$

$\Rightarrow |A - \lambda I| = 0$

If λ is eigen value of A

then λ^n is " " of A^n

$(-1)^{100} = 1, 1, 0$

$\lambda = 1$

$|(1)^{100} + 3| = 4$

$\lambda = -1$

$(-1)^{100} = 1$

$|(-1)^{100} + 3| = 4$

$\lambda = 0$

$|(-0)^{100} + 3|$

$= 3$

$|A^{100} + 3I| = 4 \times 4 \times 3 = 48$

Q:- The value of a for which the set of following equation $y+2z=0$, $2x+y+z=0$, $ax+2y=0$ have non-trivial solution. Find value of a.

NOTE: For non-trivial solution $|A| = 0$
determinant of co-efficient matrix is zero.
Only applicable for homogeneous ~~soln~~ eq^n.

$$\begin{vmatrix} 0 & 1 & 2 \\ 2 & 1 & 1 \\ a & 2 & 0 \end{vmatrix} = 0 - 1(-a) + 2(4-a) = 0$$

$$a + 8 - 2a = 0$$

$$\boxed{a = 8}$$

Q:- The rank of 9×9 diagonal matrix is

$$\begin{bmatrix} 4 & & & & & & & & \\ & 0 & & & & & & & \\ & & 0 & & & & & & \\ & & & 4 & & & & & \\ & & & & 0 & & & & \\ & & & & & 3 & & & \\ & & & & & & 0 & & \\ & & & & & & & 0 & \\ & & & & & & & & 3 \end{bmatrix}_{9 \times 9}$$

NOTE: The RANK of diagonal matrix is equal to no. of non-zero element in the diagonal

$\boxed{4}$

NOTE:-

If sum of each row or column of matrix A is equal to s, then s is the eigen value of matrix A.

Q:- If A, B, C, D be $n \times n$ matrix each be non-zero determinant. If product of ABCD is I Then find B^{-1}.

Soln:- $ABCD = I$

$$A^{-1}(ABCD) = A^{-1}$$

$$IBCD = A^{-1}$$

$$(IBCD)D^{-1} = A^{-1} \cdot D^{-1}$$

$$BC = A^{-1}D^{-1}$$

$$(BC)C^{-1} = A^{-1}D^{-1}C^{-1}$$

$$BI = A^{-1}D^{-1}C^{-1}$$

$$B = A^{-1}D^{-1}C^{-1} \qquad [(AB)^{-1} = B^{-1}A^{-1}]$$

$$(B)^{-1} = (A^{-1}D^{-1}C^{-1})^{-1} = CDA$$

A B CD
$B^{-1} = CDA$
A B C D E
$C^{-1} = DEAB$

Q:- Consider 5X5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}_{5\times5}$$

. It is given that A has only one eigen value is

(a) −2.5

(b) 0

(c) 25

✓(d) 15

$S = 15$ (NOTE).

sum of all rows are 15 ⇒ Eigen value is 15.

Q:- Consider the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ whose eigen values are 1, -1 and 3, then trace of $A^3 - 3A^2$ is ___.

$\Rightarrow$ $1 - 3 = -2$

$-1 - 3 = -4$ TRACE $= -2 - 4 = \underline{-6}$

$27 - 27 = 0$

Q:- Consider a LTI (linear time invariant) system $\dot{x} = Ax$ with initial conditions $x(0)$ at $t = 0$. Suppose a and b are eigen vector of 2×2 matrix A corresponding to distinct eigen values. λ_1 and λ_2 respectively, then the response $x(t)$ of system due to initial conditions $x(0) = \alpha$ is

(a) $\alpha e^{\lambda_1 t}$

(b) $e^{\lambda_2 t \alpha} \beta$

(c) $e^{\lambda_2 t \alpha} \alpha$

(d) $e^{\lambda_1 t \alpha} + e^{\lambda_2 t \beta}$

Sol:- $\dot{x} = Ax$

Q:- The max^m value of a such that matrix $\begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$ has three lineally independent eigen ~~value~~ vector is

(a) $2/3\sqrt{3}$ (b) $1/3\sqrt{3}$ (c) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$ (d) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

$-3-1-2 = \lambda_1+\lambda_2+\lambda_3$

$\lambda_1+\lambda_2+\lambda_3 = -6$

$\lambda_1\lambda_2\lambda_3 = -3(2) - 2(a)$

$\lambda_1\lambda_2\lambda_3 = -6-2a$

$$\begin{vmatrix} -3-\lambda & 0 & 2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda \end{vmatrix} = (-3-\lambda)\left[(-1-\lambda)(-2-\lambda)\right] + 2a$$

$= (-3-\lambda)\left[2+\lambda+2\lambda+\lambda^2\right] + 2a$

~~$= (-3-\lambda)[\lambda^2+2\lambda+2a+2]$~~

~~$= (-3-\lambda)[\lambda^2+\lambda(2+a)+2]$~~

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 4 & 0 \\ 0 & a & 1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$-2x_3 = 0$

$x_1 + 4x_2 = 0$

$ax_2 + x_3 = 0$

$\frac{x_1}{4} = \frac{x_2}{-1}$

$\begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$

$-3-\lambda = 0$ $\boxed{\lambda = -3}$

$\lambda^2 + \lambda(2+a) + 2 = 0$

$$\frac{-(2+a) \pm \sqrt{(4+4a+a^2-8}}{2}$$

$$\frac{-(2+a) \pm \sqrt{a^2+4a-4}}{2}$$

$$\frac{-(2+a) \pm a-2}{2}$$

$\frac{-2a-4}{2}$

~~2a-4~~

$-2, -a-2$

$$\begin{bmatrix} -3+(a+2) & 0 & -2 \\ 1 & 4+a+2 & 0 \\ 0 & a & -2+a+2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$[-3+(a+2)]x_1 - 2x_3 = 0$$

$$x_1 + (6+a)x_2 = 0$$

$$ax_2 + (a)x_3 = 0$$

$$\frac{x_1}{2} = \frac{x_3}{a-1} \quad \Bigg| \quad \frac{x_1}{6+a} = \frac{x_2}{-1}$$

$$\begin{bmatrix} 2 \\ -2/6+a \\ a-1 \end{bmatrix} \qquad 2+(6+a)x_2 = 0$$

$$x_2 = \frac{-2}{6+a}$$

$$(-3-\lambda)(\lambda^2+2\lambda+2a+2) = 0$$

$$-3\lambda^2 - 6\lambda - 6a - 6 - \lambda^3 - 2\lambda^2 - 2\lambda a - 2\lambda = 0$$

$$-\lambda^3 - 5\lambda^2 - 8\lambda - 2\lambda a - 6a - 6 = 0$$

$$-\lambda^3 - 5\lambda^2 - 2\lambda(4+a) - 6a - 6 = 0$$

$$-3\lambda^2 - 10\lambda - 2(4+a) = 0$$

Q:- How many trailing zeros ends with 100!

$(100!) = \cancel{1}\ 100 \times 99 \times 98 \times \dots \times 50 \times \dots \times 1$

$= 2(10+10+10+10+10)!$

$= 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times \dots \times 100$

surplus scarcity

5×5 5×10 5×15 5×20

$\frac{100}{5} = 20$ but $\frac{20}{5} = 4\ [25, 50, 75, 100]$

$\left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] = 20 + 4 = 24$

$20 + 4 = 24$

$(-3-\lambda)\{(-1-\lambda)(-2-\lambda)\} - 2a = 0$

$2a = -(\lambda+3)(\lambda+1)(\lambda+2)$ —— (1)

$2a = -\{\lambda^2 + 4\lambda + 3\}(\lambda+2)$

$2a = -\{\lambda^3 + 2\lambda^2 + 4\lambda^2 + 8\lambda + 3\lambda + 6\}$

$2a = -\{\lambda^3 + 6\lambda^2 + 11\lambda + 6\}$

$\frac{da}{d\lambda} = 0$

$-\{3\lambda^2 + 12\lambda + 11\} = 0$

$\Delta = 12^2 - 4(3)(11) = 144 - 132 = 12$

$\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a} = \frac{-12 \pm \sqrt{12}}{2(3)} = \frac{-12 \pm 2\sqrt{3}}{6}$

$= -2 \pm \frac{1}{\sqrt{5}}$

$\alpha = -2 + \frac{1}{\sqrt{3}}$, $\beta = -2 - 1/\sqrt{3}$

In eq^n (1)

Substitute $\alpha = -2 + 1/\sqrt{3}$

$$2a = -(\lambda+3)(\lambda+1)(\lambda+2)$$

$$2a = -\left(-2+1/\sqrt{3}+3\right)\left(-2+1/\sqrt{3}+1\right)\left(-2+1/\sqrt{3}+2\right)$$

$$2a = -\left(1/\sqrt{3}+1\right)\left(1/\sqrt{3}-1\right)\left(1/\sqrt{3}\right)$$

$$2a = -\left(\left(1/\sqrt{3}\right)^2-1\right)\left(1/\sqrt{3}\right)$$

$$2a = -\left(1/3-1\right)\left(1/\sqrt{3}\right) = -\left(\frac{-2}{3}\right)\cdot\frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}}$$

$$\boxed{a = \frac{1}{3\sqrt{3}}}$$

Q:- The no. of linearly independent eigen vector of matrix

$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is ,

$\lambda = 2, 2, 3$

$\lambda_1 = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_3 = 0$

$x_2 = 0$ $X_1 = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} = X_2$

$\lambda_2 = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$1 + 1 = 2$

$\lambda = 3$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 + x_2 = 0$

$-x_2 = 0$

$X_3 = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}$