CBSE Board
Class XII Mathematics
Board Paper 2014
Set – 3

Time: 3 hrs **Total Marks: 100**

Note:

- Please check that this question paper contains 5 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A. are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

2. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of x + y.

3. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}, xy < 1$, then write the value of x + y + xy.

4. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.

5. If $f(x) = \int_0^x t \sin t \, dt$, write the value of $f'(x)$.

6. Find the value of 'p' for which the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}-2p\hat{j}+3\hat{k}$ are parallel.

7. If R = {(x, y): x + 2y = 8} is a relation on N, write the range of R.

8. If the cartesian equations of a line are $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$, write the vector equation for the line.

9. If $\int_0^a \frac{1}{4+x^2}dx=\frac{\pi}{8}$ find the value of a.

10. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5$ and find the value of $|\vec{b}|$.

SECTION – B

11. Solve the differential equation $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$.

12. Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.

OR

The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

13. Evaluate:

$$\int_0^{\pi}\frac{4x \sin x}{1+\cos^2 x}dx$$

OR

Evaluate:

$$\int\frac{x+2}{\sqrt{x^2+5x+6}}dx$$

14. Find the value(s) of x for which $y = [x(x-2)]^2$ is an increasing function.

OR

Find the equations of the tangent and normal to the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $\left(\sqrt{2}\,a,b\right)$.

15. If the function $f: R \to R$ be given by $f(x) = x^2 + 2$ and $g: R \to R$ be given by $g(x)=\frac{x}{x-1}, x \neq 1$, find fog and gof and hence find fog (2) and gof (–3).

16. Prove that

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \frac{-1}{\sqrt{2}}\le x\le 1$$

OR

If $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$, find the value of x.

17. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.

18. If $y = Pe^{ax} + Q\,e^{hx}$, show that

$$\frac{d^2y}{dx^2}-(a+b)\frac{dy}{dx}+aby=0$$

19. Using properties of determinants, prove that:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$$

20. If $x = \cos t\,(3 - 2\cos^2 t)$ and $y = \sin t\,(3 - 2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

21. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that $y = 0$ when $x = 0$.

22. Find the value of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equations of a line passing through a point (3, 2, – 4) and parallel to line l_1.

SECTION – C

23. Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x- y + z = 0. Also find the distance of the plane obtained above, from the origin.

OR

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r} \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$.

24. Using integration, find the area of the region bounded by the triangle whose vertices are (-1, 2), (1, 5) and (3, 4).

25. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of 80 on each piece of type A and 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

26. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of The three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

27. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award x each, y each and z each for the three respective values to 3, 2 and 1 students respectively with a total award money of 1,600. School B wants to spend 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount for one prize on each value is 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

28. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum, when the angle between them is 60°.

29. Evaluate:

$$\int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx$$

CBSE Board
Class XII Mathematics
Board Paper 2014 Solution
Delhi Set – 3

SECTION – A

1. Given that $A^2 = A$.

We need to find the value of $7A-(I+A)^3$, where I is the identity matrix.

Thus,

$$7A-(I+A)^3 = 7A-(I^3+3I^2A+3IA^2+A^3)$$

$$\Rightarrow 7A-(I+A)^3 = 7A-(I^3+3A+3A^2+A^2\times A) \quad [I^3=I, I^2A=A, IA^2=A^2]$$

$$\Rightarrow 7A-(I+A)^3 = 7A-(I+3A+3A+A) \quad [\because A^2=A]$$

$$\Rightarrow 7A-(I+A)^3 = 7A-I-3A-3A-A$$

$$\Rightarrow 7A-(I+A)^3 = 7A-I-7A$$

$$\Rightarrow 7A-(I+A)^3 = -I$$

2. Given that $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

We need to find the value of x + y.

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Two matrices A and B are equal to each other, if they have the same dimensions and the same elements $a_{ij} = b_{ij}$, for i = 1,2,...,n and j = 1,2,...,m.

$x-y=-1$...(1)

$2x-y=0$...(2)

Equation $(2)-(1)$ is x = 1

Substituting the value of x = 1 in equation (1), we have

$1-y=-1$

$\Rightarrow y=2$

Therefore, x + y = 1 + 2 = 3

3. Given that $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$ and xy<1.

We need to find the value of x+y+xy.

$$\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right)=\frac{\pi}{4}\ [\because xy<1]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]=\tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{x+y}{1-xy}=1$$

$$\Rightarrow x+y=1-xy$$

$$\Rightarrow x+y+xy=1$$

4. Given that $\begin{vmatrix}3x & 7\\ -2 & 4\end{vmatrix}=\begin{vmatrix}8 & 7\\ 6 & 4\end{vmatrix}$.

We need to find the value of x

$$\begin{vmatrix}3x & 7\\ -2 & 4\end{vmatrix}=\begin{vmatrix}8 & 7\\ 6 & 4\end{vmatrix}$$

$$\Rightarrow 12x-(-14)=32-42$$

$$\Rightarrow 12x+14=-10$$

$$\Rightarrow 12x=-10-14$$

$$\Rightarrow 12x=-24$$

$$\Rightarrow x=-2$$

5. Since differentiation operation is the inverse operation of integration, we have

$f'(x) = x\sin x$

Let $f(x) = \int_0^x t\sin t dt$

Let us do this by integration by parts.

Therefore assume u = t; du = dt

$\int \sin t dt = \int dv$

$-\cos t = v$

Therefore,

$f(x) = \left[t(-\cos t)\right]_0^x - \int_0^x (-\cos t) dt$

$f(x) = -x\cos x + \sin x + C$

Differentiating the above function with respect to x,

$f'(x) = -\left[x(-\sin x) + \cos x\right] + \cos x = x\sin x$

6. Since the vectors are parallel, we have

$\vec{a} = \lambda\vec{b}$

$\Rightarrow 3\hat{i} + 2\hat{j} + 9k = \lambda\left(\hat{i} - 2p\hat{j} + 3k\right)$

$\Rightarrow 3\hat{i} + 2\hat{j} + 9k = \lambda\hat{i} - 2\lambda p\hat{j} + 3\lambda k$

Comparing the respective coefficients, we have

$\Rightarrow \lambda = 3;$

$-2\lambda p = 2$

$\Rightarrow -2 \times 3 \times p = 2$

$\Rightarrow p = \frac{-1}{3}$

7. The set of natural numbers, N = $\{1, 2, 3, 4, 5, 6....\}$

The relation is given as

R = $\{(x, y) : x + 2y = 8\}$

Thus, R = $\{(6, 1), (4, 2), (2, 3)\}$

Domain = $\{6, 4, 2\}$

Range = $\{1, 2, 3\}$

8. Given that the cartesian equation of the line as

$$\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$$

That is,

$$\frac{-(x-3)}{5}=\frac{y-(-4)}{7}=\frac{2(z-3)}{4}$$

$$\Rightarrow \frac{x-3}{-5}=\frac{y-(-4)}{7}=\frac{z-3}{2}=\lambda$$

Any point on the line is of the form:

$-5\lambda+3, 7\lambda-4, 2\lambda+3$

Thus, the vector equation is of the form:

$\vec{r}=\vec{a}+\lambda\vec{b}$, where $\vec{a}$ is the position vector of any point on the line and $\vec{b}$ is the vector parallel to the line.

Therefore, the vector equation is

$$\vec{r}=(-5\lambda+3)\hat{i}+(7\lambda-4)\hat{j}+(2\lambda+3)\widehat{k}$$

$$\Rightarrow \vec{r}=-5\lambda\hat{i}+7\lambda\hat{j}+2\lambda\widehat{k}+3\hat{i}-4\hat{j}+3\widehat{k}$$

$$\Rightarrow \vec{r}=3\hat{i}-4\hat{j}+3\widehat{k}+\lambda\left(-5\hat{i}+7\hat{j}+2\widehat{k}\right)$$

9. Given that $\int_0^a \frac{dx}{4+x^2}=\frac{\pi}{8}$

We need to find the value of a.

Let I= $\int_0^a \frac{dx}{4+x^2}=\frac{\pi}{8}$

Thus, I= $\frac{1}{2}\left(\tan^{-1}\frac{x}{2}\right)_0^a=\frac{\pi}{8}$

$$\Rightarrow \frac{1}{2}\tan^{-1}\frac{a}{2}=\frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}\frac{a}{2}=2\times\frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}\frac{a}{2}=\frac{\pi}{4}$$

$$\Rightarrow \frac{a}{2}=1$$

$$\Rightarrow a=2$$

10. Given that $\vec{a}$ and $\vec{b}$ are two perpendicular vectors.

Thus, $\vec{a} \cdot \vec{b} = 0$

Also given that, $\left|\vec{a} + \vec{b}\right| = 13$ and $\left|\vec{a}\right|$=5.

We need to find the value of $\vec{b}$.

Consider $\left|\vec{a} + \vec{b}\right|^2$:

$$\left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{a} \cdot \vec{b}\right| + \left|\vec{b}\right|^2$$

$$13^2 = 5^2 + 2 \times 0 + \left|\vec{b}\right|^2$$

$$169 = 25 + \left|\vec{b}\right|^2$$

$$\left|\vec{b}\right|^2 = 169 - 25$$

$$\left|\vec{b}\right|^2 = 144$$

$$\vec{b} = 12$$

SECTION – B

11. Given differential equation is:

$$\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{\left(1+x^2\right)}=\frac{e^{\tan^{-1}x}}{\left(1+x^2\right)}$$

This a linear differential equation of the form

$$\frac{dy}{dx}+Py=Q$$

where $P=\dfrac{1}{\left(1+x^2\right)}$ and $Q=\dfrac{e^{\tan^{-1}x}}{\left(1+x^2\right)}$

Therefore,

$$\text{I.F.}=e^{\int P dx}=e^{\tan^{-1}x}$$

Thus the solution is,

$$y\left(I.F\right)=\int Q\left(I.F\right)dx$$

$$\Rightarrow y\left(e^{\tan^{-1}x}\right)=\int\frac{e^{\tan^{-1}x}}{\left(1+x^2\right)}\times e^{\tan^{-1}x}dx$$

Substitute $e^{\tan^{-1}x}=t$;

$$e^{\tan^{-1}x}\times\frac{1}{\left(1+x^2\right)}dx=dt$$

Thus,

$$y\left(e^{\tan^{-1}x}\right)=\int t\,dt$$

$$\Rightarrow y\left(e^{\tan^{-1}x}\right)=\frac{t^2}{2}+C$$

$$\Rightarrow y\left(e^{\tan^{-1}x}\right)=\frac{\left(e^{\tan^{-1}x}\right)^2}{2}+C$$

12. Given position vectors of four points A,B,C and D are:

$\overrightarrow{\text{OA}} = 4\hat{i} + 5\hat{j} + k$

$\overrightarrow{\text{OB}} = -\hat{j} - k$

$\overrightarrow{\text{OC}} = 3\hat{i} + 9\hat{j} + 4k$

$\overrightarrow{\text{OD}} = 4\left(-\hat{i} + \hat{j} + k\right)$

These points are coplanar, if the vectors, $\overrightarrow{\text{AB}}, \overrightarrow{\text{AC}}$ and $\overrightarrow{\text{AD}}$ are coplanar.

$\overrightarrow{\text{AB}} = \overrightarrow{OB} - \overrightarrow{OA}$

$= -\hat{j} - k - \left(4\hat{i} + 5\hat{j} + k\right) = -4\hat{i} - 6\hat{j} - 2k$

$\overrightarrow{\text{AC}} = \overrightarrow{OC} - \overrightarrow{OA}$

$= 3\hat{i} + 9\hat{j} + 4k - \left(4\hat{i} + 5\hat{j} + k\right) = -\hat{i} + 4\hat{j} + 3k$

$\overrightarrow{\text{AD}} = \overrightarrow{OD} - \overrightarrow{OA}$

$= 4\left(-\hat{i} + \hat{j} + k\right) - \left(4\hat{i} + 5\hat{j} + k\right) = -8\hat{i} - \hat{j} + 3k$

These vectors are coplanar if and only if, they can be expressed as a linear combination of other two.

So let

$\overrightarrow{\text{AB}} = x\overrightarrow{\text{AC}} + \text{y}\overrightarrow{\text{AD}}$

$\Rightarrow -4\hat{i} - 6\hat{j} - 2k = x\left(-\hat{i} + 4\hat{j} + \widehat{3k}\right) + y\left(-8\hat{i} - \hat{j} + \widehat{3k}\right)$

$\Rightarrow -4\hat{i} - 6\hat{j} - 2\widehat{k =} \left(-x - 8y\right)\hat{i} + \left(4x - y\right)\hat{j} + \left(3x + 3y\right)\hat{k}$

Comparing the coefficients, we have,

$-x - 8y = -4; 4x - y = -6; 3x + 3y = -2$

Thus, solving the first two equations, we get

x=$\frac{-4}{3}$ and y=$\frac{2}{3}$

These values of x and y satisfy the equation $3x + 3y = -2$.

Hence the vectors are coplanar.

OR

Given that

$\vec{b} = 2\hat{i} + 4\hat{j} - 5k$

$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3k$

Now consider the sum of the vectors $\vec{b} + \vec{c}$:

$\vec{b} + \vec{c} = \left(2\hat{i} + 4\hat{j} - 5k\right) + \left(\lambda\hat{i} + 2\hat{j} + 3k\right)$

$\Rightarrow \vec{b} + \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2k$

Let $\hat{n}$ be the unit vector along the sum of vectors $\vec{b} + \vec{c}$:

$$\hat{n} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

The scalar product of $\vec{a}$ and n is 1. Thus,

$$\vec{a} \cdot \hat{n} = \left(\hat{i} + \hat{j} + k\right) \cdot \left(\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}} \right)$$

$$\Rightarrow 1 = \frac{1(2+\lambda) + 1 \cdot 6 - 1 \cdot 2}{\sqrt{(2+\lambda)^2 + 6^2 + 2^2}}$$

$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = 2 + \lambda + 6 - 2$

$\Rightarrow \sqrt{(2+\lambda)^2 + 6^2 + 2^2} = \lambda + 6$

$\Rightarrow (2+\lambda)^2 + 40 = (\lambda + 6)^2$

$\Rightarrow \lambda^2 + 4\lambda + 4 + 40 = \lambda^2 + 12\lambda + 36$

$\Rightarrow 4\lambda + 44 = 12\lambda + 36$

$\Rightarrow 8\lambda = 8$

$\Rightarrow \lambda = 1$

Thus, n is :

$$n = \frac{(2+1)\hat{i} + 6\hat{j} - 2k}{\sqrt{(2+1)^2 + 6^2 + 2^2}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{\sqrt{3^2 + 6^2 + 2^2}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{\sqrt{49}}$$

$$\Rightarrow n = \frac{3\hat{i} + 6\hat{j} - 2k}{7}$$

$$\Rightarrow n = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}k$$

13. We need to evaluate the integral

$$I=\int_0^{\pi}\frac{4x\sin x}{1+\cos^2 x}dx....(1)$$

Using the property $\int f(a-x)dx=\int f(x)dx$, we have

$$I=\int_0^{\pi}\frac{4(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{4\pi\sin x}{1+\cos^2 x}dx-\int_0^{\pi}\frac{4x\sin x}{1+\cos^2 x}dx....(2)$$

Adding equations (1) and (2), we have,

$$\Rightarrow 2I=\int_0^{\pi}\frac{4x\sin x}{1+\cos^2 x}dx+\int_0^{\pi}\frac{4\pi\sin x}{1+\cos^2 x}dx-\int_0^{\pi}\frac{4x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow 2I=\int_0^{\pi}\frac{4\pi\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow 2I=4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

Substitute t = cosx; dt $=-\sin x dx$

when x = 0,t = 1

when x = π, t $=-1$

$$\Rightarrow 2I = 4\pi\int_1^{-1}\frac{(-1)dt}{1+t^2}$$

$$\Rightarrow I = 2\pi\int_{-1}^{1}\frac{dt}{1+t^2}$$

$$\Rightarrow I=2\times 2\pi\int_0^{1}\frac{dt}{1+t^2}$$

$$\Rightarrow I=2\times 2\pi\left(\tan^{-1}t\right)_0^1$$

$$\Rightarrow I=4\pi\tan^{-1}(1)$$

$$\Rightarrow I=4\pi\times\frac{\pi}{4}=\pi^2$$

OR

We need to evaluate the integral

$$\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

$$Let\ I=\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

Consider the integrand as follows:

$$\frac{x+2}{\sqrt{x^2+5x+6}} = \frac{A\frac{d}{dx}\left(x^2+5x+6\right)+B}{\sqrt{x^2+5x+6}}$$

$$\Rightarrow x+2 = A\left(2x+5\right)+B$$

$$\Rightarrow x+2 = \left(2A\right)x+5A+B$$

Comparing the coefficients, we have

$2A=1;5A+B=2$

Solving the above equations, we have

$A=\frac{1}{2}$ and $B=-\frac{1}{2}$

Thus,

$$I=\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

$$=\int \frac{\frac{2x+5}{2}-\frac{1}{2}}{\sqrt{x^2+5x+6}}dx$$

$$=\frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}dx-\frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+6}}dx$$

$$I=\frac{1}{2}I_1-\frac{1}{2}I_2,$$

where $I_1 = \int \frac{2x+5}{\sqrt{x^2+5x+6}} dx$

and $I_2 = \int \frac{1}{\sqrt{x^2+5x+6}} dx$

Now consider I_1 :

$$I_1 = \int \frac{2x+5}{\sqrt{x^2+5x+6}} dx$$

Substitute

$x^2+5x+6 = t; (2x+5)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t}$$

$$= 2\sqrt{x^2+5x+6}$$

Now consider I_2 :

$$I_2 = \int \frac{1}{\sqrt{x^2+5x+6}} dx$$

$$= \int \frac{1}{\sqrt{x^2+5x+\left(\frac{5}{2}\right)^2+6-\left(\frac{5}{2}\right)^2}} dx$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2+6-\frac{25}{4}}} dx$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2-\frac{1}{4}}} dx$$

$$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}} dx$$

$$I_2 = \log\left|x+\frac{5}{2}-\sqrt{x^2+5x+6}\right| + C$$

Thus, $I = \frac{1}{2}I_1 - \frac{1}{2}I_2$

$$I = \sqrt{x^2+5x+6} - \frac{1}{2}\log\left|x+\frac{5}{2}-\sqrt{x^2+5x+6}\right| + C$$

14. Given function is

$$f(x)=\left[x(x-2)\right]^2$$

$$\Rightarrow f'(x)=x^2\times 2(x-2)+(x-2)^2\times 2x$$

$$\Rightarrow f'(x)=2x(x-2)\left[x+(x-2)\right]$$

$$\Rightarrow f'(x)=2x(x-2)[2x-2]$$

$$\Rightarrow f'(x)=2x(x-2)\left[2(x-1)\right]$$

$$\Rightarrow f'(x)=4x(x-1)(x-2)$$

Since $f'(x)$ is an increasing function, $f'(x)>0$.

$$\Rightarrow f'(x)=4x(x-1)(x-2)>0$$

$$\Rightarrow x(x-1)(x-2)>0$$

$$\Rightarrow\ 0<x<1 \text{ or } x>2$$

$$\Rightarrow x\in(0,1)\cup(2,\infty)$$

OR

Let $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2}=\frac{x^2}{a^2}-1$$

$$\Rightarrow y^2=\frac{b^2}{a^2}x^2-b^2$$

Differentiating the above function w.r.t. x, we get,

$$2y\frac{dy}{dx}=\frac{b^2}{a^2}2x$$

$$\Rightarrow \frac{dy}{dx}=\frac{b^2}{a^2}\frac{x}{y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{\left(\sqrt{2}a,b\right)}=\frac{b^2}{a^2}\frac{\sqrt{2}a}{b}=\frac{\sqrt{2}b}{a}$$

Slope of the tangent is m=$\frac{\sqrt{2}b}{a}$

Equation of the tangent is

$$\left(y-y_1\right)=m\left(x-x_1\right)$$

$$\Rightarrow \left(y-b\right)=\frac{\sqrt{2}b}{a}\left(x-\sqrt{2}a\right)$$

$$\Rightarrow a\left(y-b\right)=\sqrt{2}b\left(x-\sqrt{2}a\right)$$

$$\Rightarrow \sqrt{2}bx-ay+ab-2ab=0$$

$$\Rightarrow \sqrt{2}bx-ay-ab=0$$

Slope of the normal is $-\frac{1}{\frac{\sqrt{2}b}{a}}$

Equation of the normal is

$$\left(y-y_1\right)=m\left(x-x_1\right)$$

$$\Rightarrow \left(y-b\right)=\frac{-a}{\sqrt{2}b}\left(x-\sqrt{2}a\right)$$

$$\Rightarrow \sqrt{2}b\left(y-b\right)=-a\left(x-\sqrt{2}a\right)$$

$$\Rightarrow ax+\sqrt{2}by-\sqrt{2}b^2+\sqrt{2}a^2=0$$

$$\Rightarrow ax+\sqrt{2}by+\sqrt{2}\left(a^2-b^2\right)=0$$

15. Given that $f(x)=x^2+2$ and $g(x)=\frac{x}{x-1}$

Let us find $f \circ g$:

$f \circ g = f(g(x))$

$\Rightarrow f \circ g = (g(x))^2 + 2$

$\Rightarrow f \circ g = \left(\frac{x}{x-1}\right)^2 + 2$

$\Rightarrow f \circ g = \frac{x^2+2(x-1)^2}{(x-1)^2}$

$\Rightarrow f \circ g = \frac{x^2+2(x^2-2x+1)}{x^2-2x+1}$

$\Rightarrow f \circ g = \frac{3x^2-4x+2}{x^2-2x+1}$

Therefore, $(f \circ g)(2) = \frac{3\times 2^2 - 4\times 2 + 2}{2^2 - 2\times 2 + 1}$

$\Rightarrow (f \circ g)(2) = \frac{12-8+2}{4-4+1} = 6$

Now let us find $g \circ f$:

$g \circ f = g(f(x))$

$\Rightarrow g \circ f = \frac{f(x)}{f(x)-1}$

$\Rightarrow g \circ f = \frac{x^2+2}{x^2+2-1}$

$\Rightarrow g \circ f = \frac{x^2+2}{x^2+1}$

Therefore, $(g \circ f)(-3) = \frac{(-3)^2+2}{(-3)^2+1} = \frac{9+2}{9+1} = \frac{11}{10}$

16. We need to prove that

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,-\frac{1}{\sqrt{2}}\leq x\leq 1$$

Consider x=cos2t;

$$\text{L.H.S}=\tan^{-1}\left[\frac{\sqrt{1+\cos 2t}-\sqrt{1-\cos 2t}}{\sqrt{1+\cos 2t}+\sqrt{1-\cos 2t}}\right]$$

$$=\tan^{-1}\left[\frac{\sqrt{2}\cos t-\sqrt{2}\sin t}{\sqrt{2}\cos t+\sqrt{2}\sin t}\right]$$

$$=\tan^{-1}\left[\frac{1-\tan t}{1+\tan t}\right]$$

$$=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan t}{1+\tan\frac{\pi}{4}\times\tan t}\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-t\right)\right]$$

$$=\frac{\pi}{4}-t$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

$$=\text{R.H.S}$$

OR

Given that $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$

We need to find the value of x.

$$\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\left(\frac{x-2}{x-4}\right)\left(\frac{x+2}{x+4}\right)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\left(\frac{x-2}{x-4}\right)\left(\frac{x+2}{x+4}\right)}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)-(x-2)(x+2)}=1$$

$$\Rightarrow \frac{\left(x^2+2x-8\right)+\left(x^2-2x-8\right)}{\left(x^2-16\right)-\left(x^2-4\right)}=1$$

$$\Rightarrow \frac{2x^2-16}{-12}=1$$

$$\Rightarrow 2x^2-16=-12$$

$$\Rightarrow 2x^2=4$$

$$\Rightarrow x^2=2$$

$$\Rightarrow x=\pm\sqrt{2}$$

17. An experiment succeeds thrice as often as it fails.
Therefore, there are 3 successes and 1 failure.

Thus the probability of success $=\frac{3}{4}$

And the probability of failure $=\frac{1}{4}$

We need to find the probability of atleast 3 successes in the next five trials.

Required Probability=$P(X=3)+P(X=4)+P(X=5)$

$$={}^5C_3p^3q^2+{}^5C_4p^4q^1+{}^5C_5p^5q^0$$

$$={}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2+{}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1+{}^5C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$$

$$=10\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2+5\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1+\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$$

$$=\frac{918}{1024}$$

$$=\frac{459}{512}$$

18. Given that

$y = Pe^{ax} + Qe^{bx}$

Differentiating the above function w.r.t. x,

$$\frac{dy}{dx} = Pae^{ax} + Qbe^{bx}$$

Differentiating once again, we have,

$$\frac{d^2y}{dx^2} = Pa^2e^{ax} + Qb^2e^{bx}$$

Let us now find $(a+b)\frac{dy}{dx}$:

$$(a+b)\frac{dy}{dx} = (a+b)\left(Pae^{ax} + Qbe^{bx}\right)$$

$$\Rightarrow (a+b)\frac{dy}{dx} = Pa^2e^{ax} + Qabe^{bx} + Pabe^{bx} + Qb^2e^{bx}$$

$$\Rightarrow (a+b)\frac{dy}{dx} = Pa^2e^{ax} + (P+Q)abe^{bx} + Qb^2e^{bx}$$

Also we have,

$$aby = ab\left(Pe^{ax} + Qe^{bx}\right)$$

Thus, $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$

$$= Pa^2e^{ax} + Qb^2e^{bx} - Pa^2e^{ax} - (P+Q)abe^{bx} - Qb^2e^{bx} + abPe^{ax} + abQe^{bx}$$

$$= 0$$

19. Consider the detrminant

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

Taking abc common outside, we have

$$\Delta = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

Apply the transformation, $C_1 \to C_1 + C_2 + C_3$,

$$\Delta = abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

$$\Rightarrow \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

Apply the transformations, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_1, we have

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\times 1 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc + ab + bc + ca$$

20. $x = \cos t\left(3 - 2\cos^2 t\right)$

and

$y = \sin t\left(3 - 2\sin^2 t\right)$

We need to find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Let us find $\frac{dx}{dt}$:

$$x = \cos t\left(3 - 2\cos^2 t\right)$$

$$\frac{dx}{dt} = \cos t\left(4\cos t\sin t\right) + \left(3 - 2\cos^2 t\right)\left(-\sin t\right)$$

$$\Rightarrow \frac{dx}{dt} = -3\sin t + 4\cos^2 t\sin t + 2\cos^2 t\sin t$$

Let us find $\frac{dy}{dt}$:

$$y = \sin t\left(3 - 2\sin^2 t\right)$$

$$\frac{dy}{dt} = \sin t\left(-4\sin t\cos t\right) + \left(3 - 2\sin^2 t\right)\left(\cos t\right)$$

$$\Rightarrow \frac{dy}{dt} = 3\cos t - 4\sin^2 t\cos t - 2\sin^2 t\cos t$$

Thus, $\frac{dy}{dx} = \frac{3\cos t - 4\sin^2 t\cos t - 2\sin^2 t\cos t}{-3\sin t + 4\cos^2 t\sin t + 2\cos^2 t\sin t}$

$$\Rightarrow \frac{dy}{dx} = \frac{3\cos t - 6\sin^2 t\cos t}{-3\sin t + 6\cos^2 t\sin t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3\cos t\left(1 - 2\sin^2 t\right)}{-3\sin t\left(1 - 2\cos^2 t\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3\cos t\left(1 - 2\sin^2 t\right)}{3\sin t\left(2\cos^2 t - 1\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos t}{\sin t} \left[\because 2\cos^2 t - 1 = 1 - 2\sin^2 t\right]$$

$$\Rightarrow \frac{dy}{dx} = \cot t$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$$

21. Consider the differential equation,

$$\log\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = 3x + 4y$$

Taking exponent on both the sides, we have

$$\mathrm{e}^{\log\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)} = e^{3x+4y}$$

$$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = e^{3x+4y}$$

$$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = e^{3x} \cdot e^{4y}$$

$$\Rightarrow \frac{\mathrm{dy}}{e^{4y}} = e^{3x} dx$$

Integration in both the sides, we have

$$\int \frac{\mathrm{dy}}{e^{4y}} = \int e^{3x} dx$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

We need to find the particular solution.

We have, y=0, when x=0

$$\frac{1}{-4} = \frac{1}{3} + C$$

$$\Rightarrow C = -\frac{1}{4} - \frac{1}{3}$$

$$\Rightarrow C = \frac{-3-4}{12} = -\frac{7}{12}$$

Thus, the solution is $\frac{e^{3x}}{3} + \frac{e^{-4y}}{4} = \frac{7}{12}$

22. The equation of line L_1 :

$$\frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$$

$$\Rightarrow \frac{x-1}{-3}=\frac{y-2}{\frac{p}{7}}=\frac{z-3}{2} \text{....(1)}$$

The equation of line L_2 :

$$\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$$

$$\Rightarrow \frac{x-1}{\frac{-3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5} \text{....(2)}$$

Since line L_1 and L_2 are perpendicular to each other, we have

$$-3\times\left(\frac{-3p}{7}\right)+\frac{p}{7}\times 1+2\times(-5)=0$$

$$\Rightarrow \frac{9p}{7}+\frac{p}{7}=10$$

$$\Rightarrow 10p=70$$

$$\Rightarrow p=7$$

Thus equations of lines L_1 and L_2 are:

$$\frac{x-1}{-3}=\frac{y-2}{1}=\frac{z-3}{2}$$

$$\frac{x-1}{-3}=\frac{y-5}{1}=\frac{z-6}{-5}$$

Thus the equation of the line passing through the point $(3, 2, -4)$ and parallel to the line L_1 is:

$$\frac{x-3}{-3}=\frac{y-2}{1}=\frac{z+4}{2}$$

SECTION – C

23. Equation of the plane passing through the intersection of the planes x+y+z=1 and 2x+3y+4z=5 is :

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$$

$$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-(1+5\lambda)=0$$

This plane has to be perpendicular to the plane x-y+z=0.

Thus,

$$(1+2\lambda)1+(1+3\lambda)(-1)+(1+4\lambda)1=0$$

$$1+2\lambda-1-3\lambda+1+4\lambda=0$$

$$1+3\lambda=0$$

$$\lambda=-\frac{1}{3}$$

Thus, the equation of the plane is :

$$\left(1+2\left(-\frac{1}{3}\right)\right)x+\left(1+3\left(-\frac{1}{3}\right)\right)y+\left(1+4\left(-\frac{1}{3}\right)\right)z-\left(1+5\left(-\frac{1}{3}\right)\right)=0$$

$$\left(1-\frac{2}{3}\right)x+(1-1)y+\left(1-\frac{4}{3}\right)z-\left(1-\frac{5}{3}\right)=0$$

$$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

$$x-z=-2$$

Thus, the distance of this plane form the origin is :

$$\left|\frac{-(-2)}{\sqrt{1^2+0^2+1^2}}\right|=\left|\frac{2}{\sqrt{2}}\right|=\sqrt{2}$$

OR

Any point in the line is

$2+3\lambda, -4+4\lambda, 2+2\lambda$

The vector equation of the plane is given as

$\vec{r}\cdot\left(\hat{i}-2\hat{j}+k\right)=0$

Thus the cartesian equation of the plane is $x-2y+z=0$

Since the point lies in the plane

$(2+3\lambda)1+(-4+4\lambda)(-2)+(2+2\lambda)1=0$

$\Rightarrow 2+8+2+3\lambda-8\lambda+2\lambda=0$

$\Rightarrow 12-3\lambda=0$

$\Rightarrow 12=3\lambda$

$\Rightarrow \lambda=4$

Thus, the point of intersection of the line and the plane is: $2+3\times4, -4+4\times4, 2+2\times4$

$\Rightarrow 14,12,10$

Distance between $(2, 12, 5)$ and $(14, 12, 10)$ is:

$d=\sqrt{(14-2)^2+(12-12)^2+(10-5)^2}$

$\Rightarrow d=\sqrt{144+25}$

$\Rightarrow d=\sqrt{169}$

$\Rightarrow d=13\ units$

24. Consider the vertices, $A(-1, 2)$, $B(1, 5)$ and $C(3, 4)$.

Let us find the equation of the sides of ΔABC.

Thus, the equation of AB is:

$\frac{y-2}{5-2}=\frac{x+1}{1+1}$

$\Rightarrow 3x-2y+7=0$

Similarly, the equation of BC is:

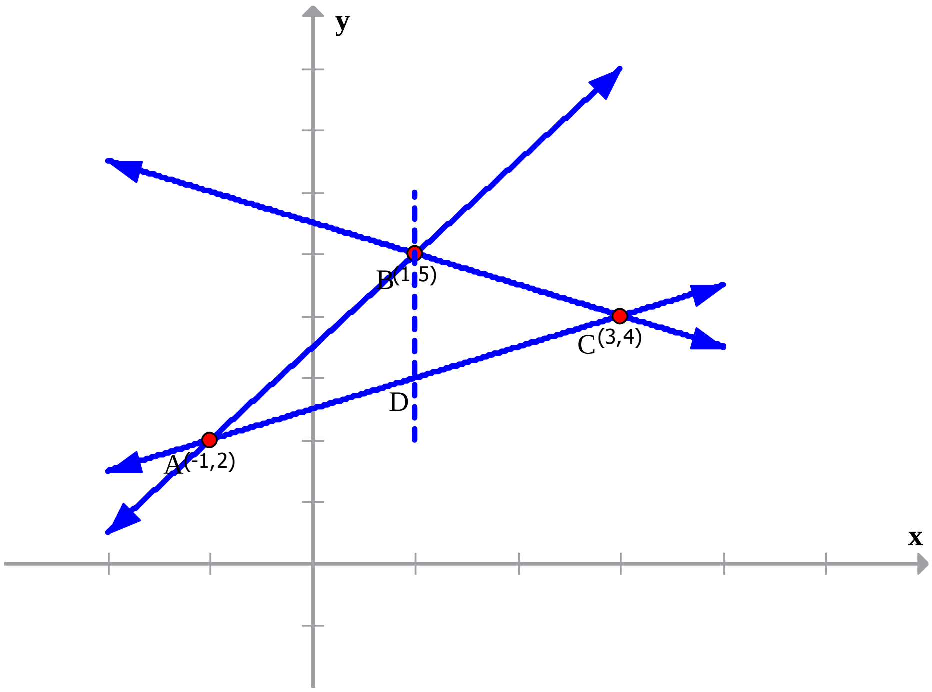
$\frac{y-5}{4-5}=\frac{x-1}{3-1}$

$\Rightarrow x+2y-11=0$

Also, the equation of CA is:

$\frac{y-4}{2-4}=\frac{x-3}{-1-3}$

$\Rightarrow x-2y+5=0$



Now the area of ΔABC=Area of ΔADB + Area of ΔBDC

$\therefore$ Area of ΔADB=$\int_{-1}^{1}\left[\frac{3x+7}{2}-\frac{x+5}{2}\right]dx$

Similarly, Area of ΔBDC=$\int_{1}^{3}\left[\frac{11-x}{2}-\frac{x+5}{2}\right]dx$

Thus, Area of ΔADB+Area of ΔBDC

$$=\int_{-1}^{1}\left[\frac{3x+7}{2}-\frac{x+5}{2}\right]dx+\int_{1}^{3}\left[\frac{11-x}{2}-\frac{x+5}{2}\right]dx$$

$$=\int_{-1}^{1}\left[\frac{2x+2}{2}\right]dx+\int_{1}^{3}\left[\frac{6-2x}{2}\right]dx$$

$$=\int_{-1}^{1}[x+1]dx+\int_{1}^{3}[3-x]dx$$

$$=\left[\frac{x^2}{2}+x\right]_{-1}^{1}+\left[3x-\frac{x^2}{2}\right]_{1}^{3}$$

$$=2+9-\frac{9}{2}-3+\frac{1}{2}$$

$$=2+\frac{9}{2}-\frac{5}{2}$$

$=4$ square units

25. Let x be the number of pieces manufactured of type A
and y be the number of pieces manufactured of type B.
Let us summarise the data given in the problem as follows:

Product	Time for Fabricating (in hours)	Time for Finishing (in hours)	Maximum labour hours available
Type A	9	1	180
Type B	12	3	30
Maximum Profit(in Rupees)	80	120	

Thus, the mathematical form of above L.P.P. is

Maximise $Z = 80x + 120y$

subject to

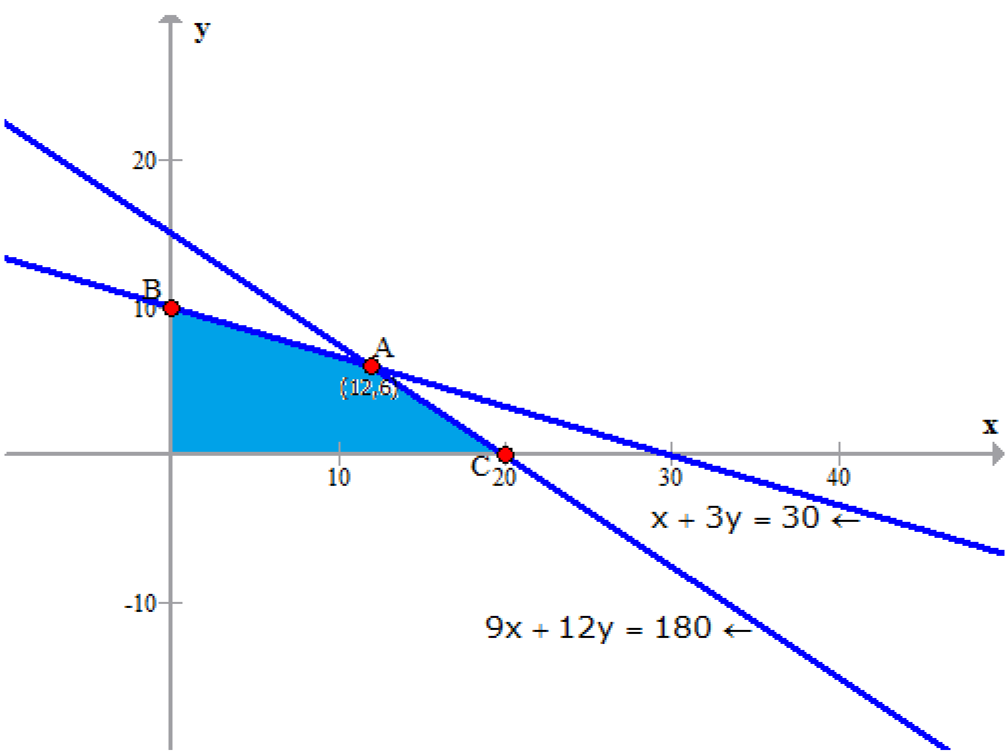
$9x + 12y \leq 180$

$x + 3y \leq 30$

Also, we have $x \geq 0, y \geq 0$

Let us now find the feasible region, which is the set of all points whose coordinates satisfy all constraints.

Consider the following figure.



Thus, the feasible region consists of the points A, B and C.
The values of the objective function at the corner points are given below in the following table:

Points	Value of Z
$A(12, 6)$	$Z = 80\times12+120\times6 = \text{Rs. } 1680$
$B(0, 10)$	$Z = 80\times0+120\times10 = \text{Rs. } 1200$
$C(20, 0)$	$Z = 80\times20+120\times0 = \text{Rs. } 1600$

Clearly, Z is maximum at $x = 12$ and $y = 6$ and the maximum profit is Rs. 1680.

26. Let E_1, E_2, E_3 and A be the events defined as follows:

$E_1 = \text{Choosing 2 headed coin}$

$E_2 = \text{Choosing coin with 75\% chance of getting heads}$

$E_3 = \text{Choosing coin with 40\% chance of getting heads}$

A= Getting heads

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, $P(A/E_1) = 1, P(A/E_2) = \frac{75}{100} = \frac{3}{4}, P(A/E_3) = \frac{40}{100} = \frac{2}{5}$

Required probability

$= P(E_1/A)$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{3}\times\frac{2}{5}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{4} + \frac{2}{15}}$$

$$= \frac{\frac{1}{3}}{\frac{43}{60}} = \frac{20}{43}$$

OR

If 1 is the smallest number,

the other numbers are:2,3,4,5,6

If 2 is the smallest number,

the other numbers are:3,4,5,6

If 3 is the smallest number,

the other numbers are:4,5,6

If 4 is the smallest number,

the other numbers are:5,6

If 5 is the smallest number,

the other number is:6

$$\text{Thus, the sample space is } S=\begin{Bmatrix} 12,13,14,15,16 \\ 23,24,25,26 \\ 34,35,36 \\ 45,46 \\ 56 \end{Bmatrix}$$

Thus, there are 15 set of numbers in the sample space.

Let X be

$$\begin{array}{llllll} X & :2 & 3 & 4 & 5 & 6 \\ P(X) & :\frac{1}{15} & \frac{2}{15} & \frac{3}{15} & \frac{4}{15} & \frac{5}{15} \end{array}$$

We know that,

$$E(X)=X_iP(X_i)$$

$$=2\times\frac{1}{15}+3\times\frac{2}{15}+4\times\frac{3}{15}+5\times\frac{4}{15}+6\times\frac{5}{15}$$

$$=\frac{2+6+12+20+30}{15}$$

$$=\frac{70}{15}$$

$$\approx 4.66$$

27. From the given data, we write the following equations:

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 1600$$

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} = 2300$$

$$\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 900$$

From above system, we get:

$3x+2y+z=1600$

$4x+y+3z=2300$

$x+y+z=900$

Thus we get:

$$\begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

This is of the form

$$AX=B, \text{ where } A=\begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}; X=\begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } B=\begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3(1-3)-2(4-3)+1(4-1) = -6-2+3 = -5 \neq 0$$

We need to find A^{-1}:

$C_{11}=-2; C_{12}=-1; C_{13}=3$

$C_{21}=-1; C_{22}=2; C_{23}=-1$

$C_{31}=5; C_{32}=-5; C_{33}=-5$

$$\text{Therefore, adj } A=\begin{pmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{pmatrix}^{T} = \begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}$$

$$\text{Thus, } A^{-1} = \frac{\text{adj}A}{|A|} = -\frac{1}{5}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}$$

$\text{Therefore}, X = A^{-1}B$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2\times1600-1\times2300+5\times900 \\ -1\times1600+2\times2300-5\times900 \\ 3\times1600-1\times2300-5\times900 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -1000 \\ -1500 \\ -2000 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 200 \\ 300 \\ 400 \end{pmatrix}$$

Awards can be given for discipline.

28. Let ΔABC be the right angled triangle with base b and hypotenuse h.

Given that b + h = k

Let A be the area of the right triangle.

$$A=\frac{1}{2}\times b\times\sqrt{h^2-b^2}$$

$$\Rightarrow A^2=\frac{1}{4}b^2\left(h^2-b^2\right)$$

$$\Rightarrow A^2=\frac{b^2}{4}\left(\left(k-b\right)^2-b^2\right) \qquad \left[\because h=k-b\right]$$

$$\Rightarrow A^2=\frac{b^2}{4}\left(k^2+b^2-2kb-b^2\right)$$

$$\Rightarrow A^2=\frac{b^2}{4}\left(k^2-2kb\right)$$

$$\Rightarrow A^2=\frac{b^2k^2-2kb^3}{4}$$

Differentiating the above function w.r.t. x, we have

$$2A\frac{dA}{db}=\frac{2bk^2-6kb^2}{4}....(1)$$

$$\Rightarrow \frac{dA}{db} = \frac{bk^2 - 3kb^2}{2A}$$

For the area to be maximum, we have

$$\frac{dA}{db} = 0$$

$$\Rightarrow bk^2 - 3kb^2 = 0$$

$$\Rightarrow bk = 3b^2$$

$$\Rightarrow b = \frac{k}{3}$$

Again differentiating the function in equation (1), with respect to b, we have

$$2\left(\frac{dA}{db}\right)^2 + 2A\frac{d^2A}{db^2} = \frac{2k^2 - 12kb}{4} \text{....(2)}$$

Now substituting $\frac{dA}{db} = 0$ and $b = \frac{k}{3}$ in equation (2), we have

$$2A\frac{d^2A}{db^2} = \frac{2k^2 - 12k\left(\frac{k}{3}\right)}{4}$$

$$\Rightarrow 2A\frac{d^2A}{db^2} = \frac{6k^2 - 12k^2}{12}$$

$$\Rightarrow 2A\frac{d^2A}{db^2} = -\frac{k^2}{2}$$

$$\Rightarrow \frac{d^2A}{db^2} = -\frac{k^2}{4A} < 0$$

Thus area is maximum at b=$\frac{k}{3}$.

Now, $h = k - \frac{k}{3} = \frac{2k}{3}$

Let θ be the angle between the base of the triangle and the hypotenuse of the right triangle.

Thus, $\cos\theta = \frac{b}{h} = \frac{\frac{k}{3}}{\frac{2k}{3}} = \frac{1}{2}$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

29. We need to evaluate $\int \frac{dx}{\sin^4 x+\sin^2 x\cos^2 x+\cos^4 x}$

Let $I=\int \frac{dx}{\sin^4 x+\sin^2 x\cos^2 x+\cos^4 x}$

Multiply the numerator and the denominator by $\sec^4 x$, we have

$$I=\int \frac{\sec^4 x dx}{\tan^4 x+\tan^2 x+1}$$

$$I=\int \frac{\sec^2 x\times \sec^2 x dx}{\tan^4 x+\tan^2 x+1}$$

We know that $\sec^2 x=1+\tan^2 x$

Thus,

$$I=\int \frac{\left(1+\tan^2 x\right)\sec^2 x dx}{\tan^4 x+\tan^2 x+1}$$

Now substitute $t=\tan x; dt=\sec^2 x dx$

Therefore,

$$I=\int \frac{\left(1+t^2\right)dt}{1+t^2+t^4}$$

Let us rewrite the integrand as

$$\frac{\left(1+t^2\right)}{1+t^2+t^4}=\frac{\left(1+t^2\right)}{\left(t^2-t+1\right)\left(t^2+t+1\right)}$$

Using partial fractions, we have

$$\frac{\left(1+t^2\right)}{1+t^2+t^4}=\frac{At+B}{t^2-t+1}+\frac{Ct+D}{t^2+t+1}$$

$$\Rightarrow \frac{\left(1+t^2\right)}{1+t^2+t^4}=\frac{\left(At+B\right)\left(t^2+t+1\right)+\left(Ct+D\right)\left(t^2-t+1\right)}{\left(t^2-t+1\right)\left(t^2+t+1\right)}$$

$$\Rightarrow \frac{\left(1+t^2\right)}{1+t^2+t^4}$$

$$=\frac{At^3+At^2+At+Bt^2+Bt+B+Ct^3-Ct^2+Ct+Dt^2-Dt+D}{\left(t^2-t+1\right)\left(t^2+t+1\right)}$$

$$\Rightarrow \frac{(1+t^2)}{1+t^2+t^4}$$
$$= \frac{t^3(A+C)+t^2(A+B-C+D)+t(A+B+C-D)+(B+D)}{(t^2-t+1)(t^2+t+1)}$$

So we have,

$A+C=0; A+B-C+D=1; A+B+C-D=0; B+D=1$

Solving the above equations, we have

$A=C=0$ and $B=D=\frac{1}{2}$

$$I = \int \frac{(1+t^2)dt}{1+t^2+t^4}$$
$$= \int \left[\frac{1}{2(t^2-t+1)} + \frac{1}{2(t^2+t+1)} \right] dt$$
$$= \int \frac{dt}{2(t^2-t+1)} + \int \frac{dt}{2(t^2+t+1)}$$
$$= \frac{1}{2}\int \frac{dt}{t^2-t+1} + \frac{1}{2}\int \frac{dt}{t^2+t+1}$$
$$= I_1 + I_2$$

where, $I_1 = \frac{1}{2}\int \frac{dt}{t^2-t+1}$ and $I_2 = \frac{1}{2}\int \frac{dt}{t^2+t+1}$

Consider I_1:

$$I_1 = \frac{1}{2}\int \frac{dt}{t^2-t+1}$$
$$= \frac{1}{2}\int \frac{dt}{t^2-t+\frac{1}{4}+1-\frac{1}{4}}$$

$$= \frac{1}{2}\int \frac{dt}{\left(t-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{\frac{3}{4}}} \tan^{-1}\left(\frac{t-\frac{1}{2}}{\sqrt{\frac{3}{4}}} \right)$$
$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2t-1}{\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{2\tan x-1}{\sqrt{3}}$$

Similarly,

Consider I_2 :

$$I_2 = \frac{1}{2}\int \frac{dt}{t^2+t+1}$$

$$= \frac{1}{2}\int \frac{dt}{t^2+t+\frac{1}{4}+1-\frac{1}{4}}$$

$$= \frac{1}{2}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \frac{1}{2}\times\frac{1}{\sqrt{\frac{3}{4}}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\sqrt{\frac{3}{4}}}\right)$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{2t+1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x+1}{\sqrt{3}}$$

Thus, $I = I_1 + I_2$

$$\Rightarrow I = \frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x+1}{\sqrt{3}}$$

$$I = \frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\tan^{-1}\frac{2\tan x+1}{\sqrt{3}}\right]+C$$