

# Some Mechanical Properties of Matter

## Exercise Solutions

**Question 1:** A load of 10 kg is suspended by a metal wire 3 m long and having a cross-sectional area 4 mm<sup>2</sup>. Find (a) the stress (b) the strain and (c) the elongation. Young modulus of the metal is  $2.0 \times 10^{11} \text{ N m}^{-2}$ .

**Solution:**

(a) Stress = Force/Area =  $mg/A$

$$= (10 \times 10) / (4 \times 10^{-6})$$

$$= 2.5 \times 10^{-7} \text{ N/m}^2$$

(b) Strain = Stress/ $Y$

$$= [2.5 \times 10^{-7}] / [2 \times 10^{11}]$$

$$= 1.25 \times 10^{-4} \text{ m}$$

(c) Let elongation be  $l$

$$l = L \times \text{strain} = 3.75 \times 10^{-4} \text{ m}$$

**Question 2:** A vertical metal cylinder of radius 2 cm and length 2 m is fixed at the lower end and a load of 100 kg is put on it. Find (a) the stress (b) the strain and (c) the compression of the cylinder. Young modulus of the metal =  $2 \times 10^{11} \text{ N m}^{-2}$ .

**Solution:**

Area of cross-section of the cylinder =  $A = \pi r^2$

$$= \pi \times 0.02^2$$

$$= 4\pi \times 10^{-4} \text{ m}^2$$

(a) stress = Force/Area

$$= [10 \times 10] / [4\pi \times 10^{-4}]$$

$$= 7.96 \times 10^5 \text{ N/m}^2$$

$$(b) \text{ Strain} = \text{Stress}/Y$$

$$= [7.96 \times 10^5]/[2 \times 10^{11}]$$

$$= 3.98 \times 10^{-6}$$

$$(c) \text{ Let compression be } l$$

$$l = 2 \times (3.98 \times 10^{-6}) = 8 \times 10^{-6} \text{ m (approx)}$$

**Question 3:** The elastic limit of steel is  $8 \times 10^8 \text{ N m}^{-2}$  and its Young modulus  $2 \times 10^{11} \text{ Nm}^{-2}$ . Find the maximum elongation of a half-meter steel wire that can be given without exceeding the elastic limit.

**Solution:**

The elastic limit of steel is  $8 \times 10^8 \text{ N m}^{-2}$  and its Young modulus  $2 \times 10^{11} \text{ N m}^{-2}$ .

$$\text{Strain} = \text{Stress}/Y \text{ and Stress} = l/L$$

$$\Rightarrow \text{Stress}/Y = l/L$$

$$\Rightarrow l = (\text{stress } L)/Y$$

$$= [8 \times 10^8 \times 0.5]/[2 \times 10^{11}]$$

$$= 2 \times 10^{-3} \text{ m}$$

**Question 4:** A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of (a) the stresses developed in the two wires and (b) the strains developed.  $Y$  of steel =  $2 \times 10^{11} \text{ N m}^{-2}$ .  $Y$  of copper =  $1.3 \times 10^{11} \text{ N m}^{-2}$ .

**Solution:**

(a) Let the stress in steel wire be  $\sigma_s$  and in copper  $\sigma_c$

Now,  $\sigma_s = F/A$  and  $\sigma_c = F/A$

$$\Rightarrow \sigma_s/\sigma_c = 1$$

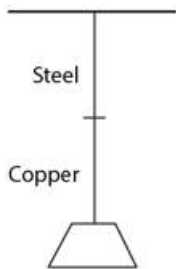
(b) Strain,  $\epsilon = \sigma/Y$

Strain in the steel wire =  $\epsilon_s = [F/A]/[2 \times 10^{11}]$

and strain in the copper wire =  $\epsilon_c = [F/A]/[1.3 \times 10^{11}]$

Hence,  $\epsilon_c/\epsilon_s = 20/13$

**Question 5:** In figure (below) the upper wire is made of steel and the lower of copper. The wires have equal cross section. Find the ratio of the longitudinal strains developed in the two wires.



**Solution:**

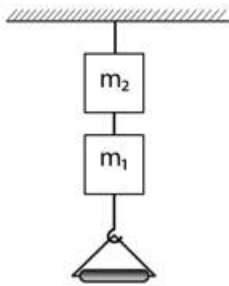
$$[\text{Strain in the steel wire}]/[\text{strain in the copper wire}] = [2 \times 10^{11}]/[1.3 \times 10^{11}]$$

$$= 20/13$$

$$= 1.54 \text{ (approx)}$$

**Question 6:** The two wires shown in figure (below) are made of the same material which has a breaking stress of  $8 \times 10^8 \text{ N m}^{-2}$ . The area of cross section of the upper wire is  $0.006 \text{ cm}^2$  and that of the lower wire is  $0.003 \text{ cm}^2$ . The mass  $m_1 = 10 \text{ kg}$ ,  $m_2 = 20 \text{ kg}$  and

the hanger is light. (a) Find the maximum load that can be put on the hanger without breaking a wire. Which wire will break first if the load is increased? (b) Repeat the above part if  $m_1 = 10 \text{ kg}$  and  $m_2 = 36 \text{ kg}$ .



**Solution:**

(a) Stress on the lower wire =  $\sigma = [(\text{total weight of lower wire})g]/[\text{Area of cross section of upper wire}]$

$$= [m+m_1]g/A_1$$

Let say,  $A_1 = \text{Area of cross section of upper wire}$

$m = \text{weight of the load and } m_1 = \text{mass of the block} = 10 \text{ kg}$

For maximum load, we take in the maximum stress that the wire can withstand.

$$\Rightarrow 8 \times 10^8 = [(m+10) \times 10]/[3 \times 10^{-7}]$$

$$\Rightarrow m = 14 \text{ kg}$$

The upper wire has a total weight of  $m+m_1+m_2$

Hence, stress on the lower wire =  $[(m+m_1+m_2)g]/A_2$

For maximum load,

$$\Rightarrow 8 \times 10^8 = [(m+10+20) \times 10]/[3 \times 10^{-7}]$$

Given,  $m_2 = 20 \text{ kg}$

$$\Rightarrow m = 18 \text{ kg}$$

(b) For  $m_2 = 36 \text{ kg}$ .

$$\Rightarrow 8 \times 10^8 = [(m+10+36) \times 10] / [6 \times 10^{-7}]$$

$$\Rightarrow m = 2 \text{ kg}$$

The maximum load that can be put is 2 kg, Upper wire will break first load is increased.

**Question 7:** Two persons pull a rope towards themselves. Each person exerts a force of 100 N on the rope. Find the Young modulus of the material of the rope if it extends in length by 1 cm. Original length of the rope = 2 m and the area of cross section =  $2 \text{ cm}^2$ .

**Solution:**

We know, stress =  $F/A$  and strain =  $l/L$

$$\text{Young's modulus} = Y = FL/AI$$

$$= [100 \times 2] / [2 \times 10^{-4} \times 0.01]$$

$$= 1 \times 10^8 \text{ N/m}^2$$

**Question 8:** A steel rod of cross-sectional area  $4 \text{ cm}^2$  and length 2 m in shrinks by 0.1 cm as the temperature decreases in night. If the rod is clamped at both ends during the day hours, find the tension developed in it during night hours. Young modulus of steel =  $1.9 \times 10^{11} \text{ N m}^{-2}$ .

**Solution:**

We know, stress =  $T/A$  and strain =  $l/L$

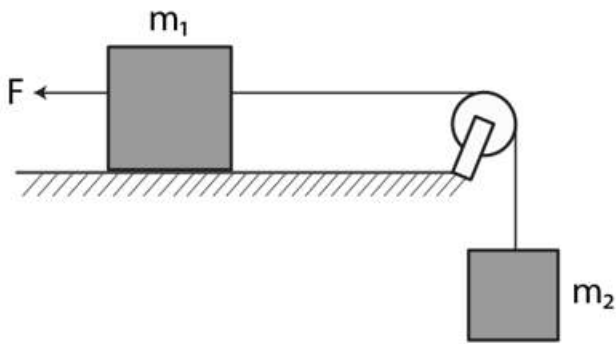
$$\text{Young's modulus} = Y = TL/AI$$

$$\Rightarrow T = YAI/L$$

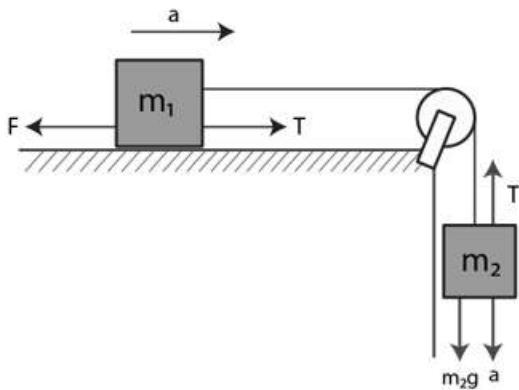
$$= [1.9 \times 10^{11} \times 4 \times 10^{-4} \times 0.001] / [2]$$

$$\Rightarrow T = 3.8 \times 10^4 \text{ N}$$

**Question 9:** Consider the situation shown in figure (below). The force  $F$  is equal to the  $m_2 (g/2)$ , If the area of cross section of the string is  $A$  and its Young Modulus  $Y$ , find the strain developed in it. The string is light and there is no friction anywhere.



**Solution:**



$$\text{Force} = F = m_2 (g/2) \dots\dots(\text{given})$$

From free body diagram:

$$T - F = m_1 a \quad [\text{for mass } m_1]$$

$$\Rightarrow T - m_2 g/2 = m_1 a \dots(1)$$

$$m_2 g - T = m_2 a \dots(2) \quad [\text{for mass } m_2]$$

Using (1) and (2)

$$\Rightarrow T - \frac{m_2 g}{2} = m_1 \left( g - \frac{T}{m_2} \right)$$

$$\Rightarrow T = \frac{m_2 g (2m_1 + m_2)}{2(m_1 + m_2)}$$

We know, stress =  $T/A$

and Young's modulus =  $Y = \text{stress/strain}$

Therefore, strain =  $T/AY$

$$= [m_2 g (2m_1 + m_2)] / [2AY(m_1 + m_2)]$$

**Question 10:** A sphere of mass 20 kg is suspended by a metal wire of unstretched length 4 m and diameter 1 mm. When in equilibrium, there is a clear gap of 2 mm between the sphere and the floor. The sphere is gently pushed aside so that the wire makes an angle  $\theta$  with the vertical and is released. Find the maximum value of  $\theta$  so that the sphere does not rub the floor. Young modulus of the metal of the wire is  $2.0 \times 10^{11} \text{ N m}^{-2}$ . Make appropriate approximations.

**Solution:**

At the point of release, the tension =  $T = mg + mv^2/r$

Force which brings elongation is centrifugal force =  $F = mv^2/r \dots (1)$

From work energy theorem,

$$(1/2)mv^2 = mgr(1 - \cos\theta)$$

$$\text{or } v^2 = 2gr(1 - \cos\theta)$$

$$(1) \Rightarrow F = (m/r) 2gr(1 - \cos\theta) = 2mg(1 - \cos\theta)$$

Again, Young's modulus =  $Y = FL/AI$

$$\Rightarrow Y = [2mg(1 - \cos\theta)L]/AI$$

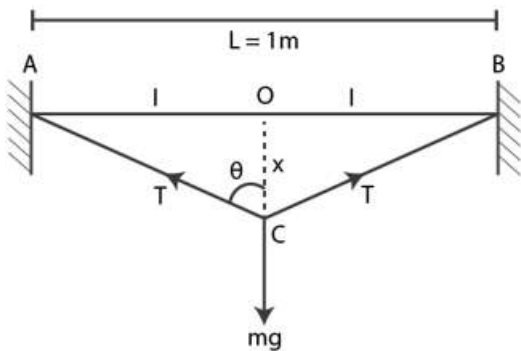
$$\cos\theta = 1 - \frac{2 \times 10^{11} \times \pi \times 0.25 \times 10^{-6} \times 0.002}{2 \times 20 \times 10 \times 4} = 0.803$$

$$\Rightarrow \theta = \cos^{-1}(0.803) \text{ or } 36.4^\circ \text{ (approx)}$$

**Question 11:** A steel wire of original length 1 m and cross-sectional area  $4.00 \text{ mm}^2$  is clamped at the two ends so that it lies horizontally and without tension. If a load of 2.16 kg is suspended from the middle point of the wire, what would be its vertical depression?

Y of the steel =  $2.0 \times 10^{11} \text{ N m}^{-2}$ . Take  $g = 10 \text{ ms}^{-2}$ .

**Solution:**



From the figure,

$$\cos \theta = \frac{x}{\sqrt{x^2 + l^2}}$$

$$= \left(\frac{x}{l}\right) \left(1 - \frac{x^2}{2l^2} + \dots\right)$$

[Using binomial theorem]

$$\text{If } x \ll l, \frac{x^2}{l^2} = 0$$

$$\Rightarrow \cos \theta = \left(\frac{x}{l}\right)$$

Again, change in length,



$$\Delta L = (AC + CB) - AB = 2(l^2 + x^2)^{1/2} - 2l$$

$$\Delta L = 2l \left( 1 + \frac{x^2}{l^2} \right)^{1/2} - 2l$$

$$= \frac{x^2}{l}$$

Now, Using Young's modulus,

$$T = (YA \Delta l)/L$$

$$\Rightarrow T = 8 \times 10^5 \times (x^2/l)$$

Forces are balance,  $2T \cos \theta = mg$

$$\Rightarrow 2 \times 8 \times 10^5 \times (x^2/l) \times (x/l) = 2.16 \times 10$$

$$\Rightarrow x = 1.5 \text{ cm}$$

**Question 12:** A topper wire of cross-sectional area 0.01 cm is under tension of 20 N. Find the decrease in the cross-sectional area Young modulus of copper =  $1.1 \times 10^{11} \text{ N m}^{-2}$  and Poisson ratio = 0.32.  
[Hint:  $\Delta A/A = 2(\Delta r/r)$ ]

**Solution:**

The longitudinal strain =  $T/A Y$

Where T = tension, A = cross-sectional area and Y = Young's modulus

$$\Rightarrow \text{longitudinal strain} = 20/[0.01 \times 10^{-4} \times 1.1 \times 10^{11}] = 1.82 \times 10^{-4}$$

The area of circular section be of the diameter D,  $A = \pi D^2/4$

$$\Rightarrow dA = 2\pi D dD/4$$

$$\Rightarrow dA = 2 \times 0.01 \times 5.82 \times 10^{-5}$$

$$= 1.164 \times 10^{-6} \text{ cm}^2$$

**Question 13:** Find the increase in pressure required to decrease the volume of a water sample by 0.01%. Bulk modulus of water =  $2.1 \times 10^9 \text{ N m}^{-2}$ .

**Solution:**

Let  $v$  be initial volume and  $v'$  be the final volume.

$$\text{Change in volume} = \Delta V = -(v - v')$$

$$\text{and pressure, } P = -B(\Delta v/v)$$

$$= B(1 - v'/v)$$

$$\text{Therefore, } dP = B (dv'/v)$$

$$\text{We are given that, } dv'/v' = (0.01/100)$$

$$\Rightarrow dP = B \times dv'/v' \times v'/v$$

$$[\text{Assuming, } v'/v = 1]$$

$$\Rightarrow dP = B (dv'/v')$$

$$= 2.1 \times 10^9 \times (0.01/100)$$

$$= 2.1 \times 10^5 \text{ N/m}^2$$

**Question 14:** Estimate the change in the density of water in ocean at a depth of 400 m below the surface. The density of water at the surface  $1030 \text{ kg m}^{-3}$  and the bulk modulus of water =  $2 \times 10^9 \text{ N m}^{-2}$ .

**Solution:**

The bulk strain at depth  $h$ :

$$\Delta v/v = P/B = \rho gh/B$$

Now, the density at depth is  $\rho' = \rho (v/v')$

$V'$  is compressed volume at depth  $h$  and  $v' = v - \Delta v$

$$\Rightarrow \rho' = \rho \times [v/(v-\Delta v)]$$

$$= \rho B/(B-\rho gh)$$

$$= [1030 \times 2 \times 10^9] / [2 \times 10^9 - (1030 \times 10 \times 400)]$$

$$= 1032 \text{ kg/m}^3$$

Therefore, change in density =  $1032 - 1030 = 2 \text{ kg/m}^3$

**Question 15:** A steel plate of face area  $4 \text{ cm}^2$  and thickness  $0.5 \text{ cm}$  is fixed rigidly at the lower surface. A tangential force of  $10 \text{ N}$  is applied on the upper surface. Find the lateral displacement of the upper surface with respect to the lower surface. Rigidity modulus of steel =  $8.4 \times 10^{10} \text{ N m}^{-2}$ .

**Solution:**

$$\text{Shearing strain} = \Delta x/d = \Delta x/0.0005 = 200 \Delta x$$

$$\text{Shearing Stress} = F/A = 10/(4 \times 10^{-4}) = 25000 \text{ N/m}^2$$

$$\text{Rigidity modulus} = (\text{shearing stress})/(\text{shearing strain})$$

$$\Rightarrow 8.4 \times 10^{10} = 25000/200\Delta x$$

$$\Rightarrow \Delta x = 1.5 \times 10^{-9} \text{ m}$$

**Question 16:** A  $5.0 \text{ cm}$  long straight piece of thread is kept on the surface of water. Find the force with which the surface on one side of the thread pulls it. Surface tension of water =  $0.076 \text{ N m}^{-1}$ .

**Solution:**

Length of thread =  $L = 5 \text{ cm}$  or  $0.05 \text{ m}$

Surface tension of water =  $S = 0.076 \text{ N/m}$

Therefore, force due to surface tension:  $F = SL = 0.076 \times 0.05$

$$= 3.8 \times 10^{-3} \text{ N}$$

**Question 17:** Find the excess pressure inside (a) a drop of mercury of radius 2 mm (b) a soap bubble of radius 4 mm and (c) an air bubble of radius 4 mm formed inside a tank of water. Surface tension of mercury, soap solution and water are  $0.465 \text{ N m}^{-1}$ ,  $0.03 \text{ N m}^{-1}$  and  $0.076 \text{ N m}^{-1}$  respectively.

**Solution:**

(a) excess pressure inside a drop of mercury of radius 2 mm:

$$P = 2S/r = [2 \times 0.465]/0.002 = 465 \text{ N/m}^2$$

(b) excess pressure inside a soap bubble of radius 4 mm

$$P = 4S/r = [4 \times 0.03]/0.004 = 30 \text{ N/m}^2$$

(c) excess pressure inside an air bubble of radius 4 mm formed inside a tank of water.

$$P = 2S/r = [2 \times 0.076]/0.004 = 38 \text{ N/m}^2$$

**Question 18:** Consider a small surface area of  $1 \text{ mm}^2$  at the top of a mercury drop of radius 4.0 mm. Find the force exerted on this area (a) by the air above it (b) by the mercury below it and (c) by the mercury surface in contact with it. Atmospheric pressure =  $1.0 \times 10^5 \text{ Pa}$  and surface tension of mercury =  $0.465 \text{ N m}^{-1}$ . Neglect the effect of gravity. Assume all numbers to be exact.

**Solution:**

(a) Force exerted by air above:

$$F = PA = 10^5 \times 10^{-6} = 0.1 \text{ N}$$

(b) Force exerted by the mercury below the surface area:

Let  $P'$  be the required pressure,

$$\Rightarrow P' = P_o + 2T/r$$

$$\text{Now, } F = P'A = (P_o + 2T/r)r$$

$$= (0.1 + (2 \times 0.465)/(4 \times 10^{-3})) \times 10^{-6}$$

$$= 0.10023 \text{ N}$$

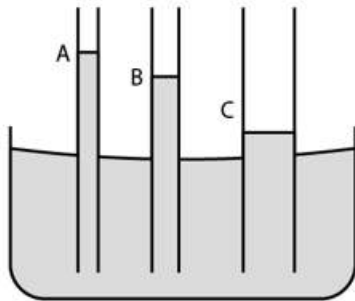
(c) Force exerted by the mercury surface in contact with it:

$$F = PA = (2T/r) A$$

$$= (2 \times 0.465)/(4 \times 10^{-3})$$

$$= 0.00023 \text{ N}$$

**Question 19:** The capillaries shown in figure (below) have inner radii 0.5 mm, 1.0 mm and 1.5 mm respectively. The liquid in the beaker is water. Find the heights of water level in the capillaries. The surface tension of water is  $7.5 \times 10^{-2} \text{ N m}^{-1}$ .



**Solution:**

Surface tension of water =  $T = 7.5 \times 10^{-2} \text{ N/m}$

Considering  $\cos \theta = 1$

Radius of capillary A =  $r_A = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

Height of water level in capillary A:

$$h_A = [2T \cos \theta] / [r_A \rho g]$$

$$= [2 \times 7.5 \times 10^{-2}] / [0.5 \times 10^{-3} \times 1000 \times 10]$$

$$= 3 \times 10^{-2} \text{ m or } 3 \text{ cm}$$

Radius of capillary B =  $r_B = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Height of water level in capillary B:

$$h_B = [2T \cos \theta] / [r_B \rho g]$$

$$= [2 \times 7.5 \times 10^{-2}] / [1 \times 10^{-3} \times 1000 \times 10]$$

$$= 15 \times 10^{-3} \text{ m or } 1.5 \text{ cm}$$

Radius of capillary C =  $r_C = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Height of water level in capillary C:

$$h_C = [2T \cos \theta] / [r_C \rho g]$$

$$= [2 \times 7.5 \times 10^{-2}] / [1.5 \times 10^{-3} \times 1000 \times 10]$$

$$= 15 / 1.5 \times 10^{-3} \text{ m or } 1 \text{ cm}$$

**Question 20:** The lower end of a capillary tube is immersed in mercury. The level of mercury in the tube is found to be 2 cm below the outer level. If the same tube is immersed in water, up to what height will the water rise in the capillary?

**Solution:**

We know, The rise or depression in capillary =  $h = [2T \cos\theta]/[r\rho g]$

Let the angle of mercury,  $\theta = 0^\circ$

depression in mercury = -2 cm = -0.02 cm (Given)

For mercury =  $h_{hg} = -0.02 = [2 T_{hg}]/[r\rho_{hg} g] \dots(1)$

Let  $h_w$  be the rise in water.

$h_w = [2T_w]/[r_w w g] \dots(2)$

Dividing (2) by (1), we get

$h_w/h_{hg} = (0.075/0.465) \times 13.6$

= 2.19

Now, height of the water level =  $h_w = 2 \times 2.19 = 4.38 \text{ cm}$

**Question 21:** A barometer is constructed with its tube having radius 1.0 mm Assume that the surface of mercury in the tube is spherical in shape. if the atmospheric pressure is equal to 76 cm of mercury, what will be the height raised in the barometer tube? The contact angle of mercury with glass =  $135^\circ$  and surface tension of mercury =  $0.465 \text{ N m}^{-1}$ . Density of mercury  $13600 \text{ kg m}^{-3}$ .

**Solution:**

Rise of mercury column =  $h = [2 T \cos\theta]/[r\rho g]$

Substituting given values,

$h = [2 \times 0.466 \times \cos 130^\circ]/[0.001 \times 13600 \times 10]$

$$= -0.005 \text{ m or } -0.5 \text{ cm}$$

[The negative sign is used as mercury goes down in a capillary tube.]

$$\text{Height of mercury in the tube} = h + H = -0.5 + 76 = 75.5 \text{ cm}$$

**Question 22:** A capillary tube of radius 0.50 mm is dipped vertically in a pot of water. Find the difference between the pressure of the water in the tube 5.0 cm below the surface and the atmospheric pressure. Surface tension of water =  $0.075 \text{ N m}^{-2}$ .

**Solution:**

$$\text{Pressure} = p = \frac{2T}{r}$$

Where  $T$  = surface tension =  $0.075 \text{ N/m}$  (given)

and  $r$  = Radius of the capillary tube =  $0.5 \text{ mm}$  or  $0.0005 \text{ m}$  (given)

$$\Rightarrow p = 300 \text{ N/m}^2$$

$$\text{Pressure at a depth of } 5 \text{ cm} = p' = \rho gh = 1000 \times 9 \times 0.05 = 490 \text{ N/m}^2$$

Let  $P$  be the pressure of water just below surface, then

$$\text{Difference in pressure : } \Delta P = (P + p') - (P + p)$$

$$= (P + 490) - (P + 300)$$

$$= 190 \text{ N/m}^2$$

**Question 23:** Find the surface energy of water kept in a cylindrical vessel of radius 6.0 cm. Surface tension of water  $0.075 \text{ J m}^{-2}$ .

**Solution:**

$$\text{Area of surface} = A = \pi r^2$$

where  $r = 6 \text{ cm} = 0.06 \text{ m}$  (given)

$$\Rightarrow A = 0.0113 \text{ m}^2$$

$$\text{Now, surface energy} = E = SA = 0.075 \times 0.0113 = 8.4 \times 10^{-4} \text{ J}$$



**Question 24:** A drop of mercury of radius 2 mm is split into 8 identical droplets. Find the increase in surface energy. Surface tension of mercury  $0.465 \text{ J m}^{-2}$ .

**Solution:**

$$\text{Volume of drop} = v = \frac{4}{3}\pi r^3$$

$$\text{Volume of each drop after division} = v' = v/8 = \frac{1}{6}\pi r^3$$

Now the new radius of each drop be  $r'$ :

$$\frac{4}{3}\pi r'^3 = \frac{1}{6}\pi r^3$$

$$\text{or } r' = r/2$$

$$\text{The total area of all droplets} = A' = 8 \times 4\pi r'^2 = 8\pi r^2$$

$$\text{Therefore, increase in area} = 8\pi r^2 - 4\pi r^2 = 4\pi r^2$$

$$\text{Therefore, increase in energy} = E = S \times (\text{increase in area}) = 2.34 \times 10^{-5} \text{ J}$$

$$[S = 0.465 \text{ J m}^{-2} \text{ and } r = 0.002 \text{ cm}]$$

**Question 25:** A capillary tube of radius 1 mm is kept vertical with the lower end in water. (a) Find the height of water raised in the capillary. (b) If the length of the capillary tube is half the answer of part (a), find the angle  $\theta$  made by the water surface in the capillary with the wall.

**Solution:**

Let  $h$  be the height of water raised in the capillary.

$$h = [2T \cos\theta]/r\rho g \dots(1)$$

$$\text{Here } T = \text{Surface tension of water} = 0.076 \text{ N/m}, \theta = 0^\circ, r = 0.001, \rho = 1000 \text{ and } g = 10]$$

$$\Rightarrow h = 15 \text{ cm}$$

(b) length of capillary,  $h' = h/2 \dots(A)$ , then

$$h' = [2T \cos\theta]/r\rho g \dots(2)$$

Dividing (2) by (1)

$$h'/h = \cos \theta$$

using (A)

$$\text{or } \theta = 60 \text{ degrees}$$

**Question 26:** The lower end of a capillary tube of radius 1 mm is dipped vertically into mercury. (a) Find the depression of mercury column in the capillary. (b) If the length dipped inside is half the answer of part (a), find the angle made by the mercury surface at the end of the capillary with the vertical. Surface tension of mercury =  $0.465 \text{ N m}^{-1}$  and the contact angle of mercury with glass =  $135^\circ$ .

**Solution:**

(a) depression of mercury column in the capillary

$$h = [2T \cos\theta]/r\rho g$$

$$= [2 \times 0.465 \times \cos 135^\circ] / [0.001 \times 13600 \times 9.8]$$

$$= 4.9 \text{ cm}$$

(b) Let  $\phi$  be the angle made by mercury.

$$\text{new height is } h' = h/2$$

$$\text{Now, } h' = [2T \cos\phi]/r\rho g$$

$$\text{From above equations, } h'/h = \cos\phi/\cos(135^\circ)$$

$$\Rightarrow \cos\phi = -0.3535$$

$$\text{or } \phi = 111 \text{ degrees (approx)}$$

**Question 27:** Two large glass plates are placed vertically and parallel to each other inside a tank of water with separation between the plates equal to 1 mm. Find the rise of water in the space between the plates. Surface tension of water = 0.075 N m<sup>-1</sup>.

**Solution:**

The weight of water in unit volume =  $W = \rho h dg \times 1$

and given force =  $F = 2Tl = 2 \times 0.0075 \times 1 = 0.150 \text{ N}$

Now,  $W = \rho h dg = 1000h \times 0.001 \times 10 = 10h \text{ N}$

or  $10h = 0.150$

or  $h = 0.15/10 = 1.5 \text{ cm}$

**Question 28:** Consider an ice cube of edge 1.0 cm kept in a gravity-free hail. Find the surface area of the water when the ice melts. Neglect the difference in densities of ice and water.

**Solution:**

Volume of ice cube of edge =  $V = 1 \text{ cm}^3 = 10^{-6} \text{ m}^3$

keeping in mind that, volumes do not change when state changes.

As gravity has no effect here, the water after melting will form a sphere,

$$\left(\frac{4}{3}\right)\pi r^3 = 10^{-6}$$

$$\text{or } r = \left[\frac{10^{-6} \times 3}{4\pi}\right]^{1/3} \text{ m}$$

Now, The surface area of the sphere

$$A = 4\pi r^2$$

Using value of  $r$ , and solving we get

$$A = (36\pi)^{1/3} \text{ cm}^2$$

**Question 29:** A wire forming a loop is dipped into soap solution and taken out so that a film of soap solution is formed. A loop of 6.28 cm long thread is gently put on the film and the film is pricked with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution =  $0.030 \text{ N m}^{-1}$ .

**Solution:**

The loop will take circular shape after pricking. Radius can be calculated by using relation,

$$l = 2\pi R$$

$$\Rightarrow R = (6.28)/(2 \times 3.14)$$

$$= 1 \text{ cm or } 10^{-2} \text{ m}$$

$2T \sin(d\theta)$  force inward direction is balanced by surface tension force in outward direction.

Therefore,  $2T \sin(d\theta) = \text{length of arc} \times \text{surface tension}$

for small angle,  $\sin(d\theta) = d\theta$

$$2T d\theta = 2R d\theta \times S$$

Where  $S$  is the surface tension.

$$\text{Now, } T = SR$$

$$= 0.030 \times 10^{-2}$$

$$= 3 \times 10^{-4} \text{ N}$$

**Question 30:** A metal sphere of radius 1 mm and mass 50 mg falls vertically in glycerin. Find (a) the viscous force exerted by the glycerin on the sphere when the speed of the sphere is  $1 \text{ cm s}^{-1}$ , (b) the hydrostatic force exerted by the glycerin on the sphere and (c)

the terminal velocity with which the sphere will move down without acceleration.  
Density of glycerin =  $1260 \text{ kg m}^{-3}$  and its co-efficient of viscosity at room temperature = 8.0 poise.

**Solution:**

(a) viscous force =  $F = 6\pi\eta r v$

$$= 6 \times 0.8 \times 3.14 \times 0.001 \times 0.01$$

$$= 1.5 \times 10^{-4} \text{ N}$$

(b) The hydrostatic force exerted by the glycerin on the sphere:

$$\text{Force} = F_B = \left(\frac{4}{3}\right) \pi r^3 \times \rho g$$

$$= \left(\frac{4}{3}\right) \times 3.14 \times 0.001^3 \times 1260 \times 9.8$$

$$= 5.2 \times 10^{-5} \text{ N (approx)}$$

(c) Terminal velocity with which the sphere will move down without acceleration

$$\text{viscous force} = F = 6\pi\eta r v = 6\pi \times 0.8 \times 0.001 \times v = 0.0048 \pi v$$

The upwards force is due to the buoyant force in glycerin.

$$F_B = 5.2 \times 10^{-5} \text{ N}$$

$$\text{and the weight downwards, } F_m = 50 \times 10^{-6} \times 9.8$$

$$= 4.9 \times 10^{-4} \text{ N}$$

As per situation, there should be no acceleration downwards:

$$F + F_B = F_m$$

$$\Rightarrow 0.0048\pi v + 5.2 \times 10^{-5} = 4.9 \times 10^{-4}$$

$$\Rightarrow v = 0.029 \text{ m/s}$$

$$\Rightarrow v = 2.9 \text{ cm/s}$$

**Question 31:** Estimate the speed of vertically falling raindrops from the following data. Radius of the drops 0.02 cm, viscosity of air =  $1.8 \times 10^{-4}$  poise,  $g = 9.9 \times 10 \text{ m s}^{-2}$  and density of water =  $1000 \text{ kg m}^{-3}$ .

**Solution:**

Viscous force is balanced by the weight:

$$6\pi\eta r v = \rho v g$$

$$\Rightarrow v = [1000 \times (4/3) \pi r^3 \times 9.9] / [6\pi\eta r]$$

$$\Rightarrow v = [2000 \times 0.002^2 \times 1.1] / [1.8 \times 10^{-5}] = 4.9 \text{ m/s}$$

**Question 32:** Water flows at a speed of  $6 \text{ cm s}^{-1}$  through a tube of radius 1 cm. Coefficient of viscosity of water at room temperature is 0.01 poise. Calculate the Reynolds number. Is it a steady flow?

**Solution:**

$$\text{Reynolds number} = N = \rho v (2r) / \eta$$

$$= [1000 \times 0.06 \times 2 \times 0.01] / [0.001]$$

$$N = 1200 \text{ (approx)}$$