Class-XII
Session - 2022-23
Subject - Mathematics (041)
Sample Question Paper - 28
With Solution

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Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Relations and Functions	8		Q.19			Q.32		6
2	Inverse Trigonometry Functions		Q.2,4						2
3	Matrices	10	Q.1,3	Q.20					3
4	Determinants				Q.21		Q.33		7
5	Continuity and Differentiability	35	Q.7,5		Q.22				4
6	Applications of Derivatives		Q.10				Q.34	Q.36	10
7	Integrals		Q.13,11		Q.23	Q.26,27			10
8	Applications of Integrals		Q.9,8			Q.28			5
9	Differential Equations		Q.16,14		Q.25,24				6
10	Vector Algebra	14				Q.29		Q.37	7
11	Three Dimensional Geometry		Q.17,12				Q.35		7
12	Linear Programming	5	Q.18,15			Q.30			5
13	Probability	8	Q.6			Q.31		Q.38	8
	Total Marks (Total Questions)		18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then AB is equal to

(a) B (b) A (c) O (d) 1

2. If $6 \sin^{-1}(x^2 - 6x + 8.5) = \pi$, then x is equal to

(a) 1 (b) 2 (c) 3 (d) 8

3. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$.

4. If $\tan^{-1}k - \tan^{-1}\sqrt{3} = \tan^{-1}\frac{1}{\sqrt{3}}$, then k =

(a) 1 (b) 2 (c) ∞ (d) 5

5. Let $f(x) = \begin{cases} 3x-4, & 0 \le x \le 2 \\ 2x+\ell, & 2 < x \le 9 \end{cases}$. If f is continuous at x = 2, then what is the value of ℓ ?

(a) 0 (b) 2 (c) -2 (d) -1

6. If $P(B) = \frac{3}{5}$, $P(A|B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A \cup B)' + P(A' \cup B) =$

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2}$ (d) 1

7. If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then the value of $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

(a) $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\frac{\pi}{6}}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\frac{\pi}{\sqrt{6}}$

8. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$ (c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) $(2, 4)$

9. The area bounded by the curve $y = \sin^{-1}x$ and the line $x = 0$, $|y| = \frac{\pi}{2}$ is

(a) 1 (b) 2 (c) π (d) 2π

10. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing:

(a) $(-\infty, 2) \cup (2, \infty)$ (b) $(2, \infty)$ (c) $(-\infty, 2)$ (d) $(-\infty, 2] \cup (2, \infty)$

11. Evaluate : $\int (x^2 + x)^2 dx$

(a) $\frac{x^5}{5} + \frac{x^3}{3} + \frac{x^4}{2}$ (b) $\frac{x^5}{5} + \frac{x^3}{3} + \frac{x^4}{2} + c$ (c) $5x^5 + 3x^5 + 8x^4$ (d) $5x^5 + 3x^3 + 8x^4 + c$

12. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are

(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$ (c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) $6, -3, 2$

13. If $\int \frac{3x+4}{x^3 - 2x - 4} dx = \log|x-2| + k \log f(x) + c$, then

(a) $f(x) = |x^2 + 2x + 2|$ (b) $f(x) = x^2 + 2x + 2$ (c) $k = -\frac{1}{2}$ (d) All of these

14. The family of curves $y = e^{a \sin x}$, where a is an arbitrary constant, is represented by the differential equation

(a) $\log y = \tan x \frac{dy}{dx}$ (b) $y \log y = \tan x \frac{dy}{dx}$ (c) $y \log y = \sin x \frac{dy}{dx}$ (d) $\log y = \cos x \frac{dy}{dx}$

15. L.P.P is a process of finding

(a) Maximum value of objective function (b) Minimum value of objective function
(c) Optimum value of objective function (d) None of these

16. The solution of $\frac{dv}{dt} + \frac{k}{m}v = -g$ is

(a) $v = ce^{-\frac{k}{m}t} - \frac{mg}{k}$ (b) $v = c - \frac{mg}{k} e^{-\frac{k}{m}t}$ (c) $ve^{-\frac{k}{m}t} = c - \frac{mg}{k}$ (d) $ve^{\frac{k}{m}t} = c - \frac{mg}{k}$

17. The distance between the lines given by

$\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is

(a) $\sqrt{\frac{59}{14}}$ (b) $\sqrt{\frac{59}{7}}$ (c) $\sqrt{\frac{118}{7}}$ (d) $\frac{\sqrt{59}}{7}$

18. The solution set of constraints $x + 2y \geq 11$, $3x + 4y \leq 30$, $2x + 5y \leq 30$ and $x \geq 0$, $y \geq 0$, includes the point

(a) (2, 3) (b) (3, 2) (c) (3, 4) (d) (4, 3)

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. Let for real numbers x and y we define the relation R such that $xRy \Leftrightarrow x^2 + y^2 = 1$

Assertion : The relation R is an equivalence relation.

Reason : A relation R is an equivalence relation if it is reflexive, transitive and symmetric.

20. **Assertion :** The possible dimensions of a matrix containing 32 elements is 6.

Reason : The no. of ways of expressing 32 as a product of two positive integers is 6.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If $A^{-1}=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$, Find $(AB)^{-1}$,

OR

Two schools A and B decided to award prizes to their students for the three values Truth, Patriotism and Non-violence. School A decided to award a total of ₹ 11000 for the three values to 5, 4 and 3 students respectively, while school B decided to award ₹ 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to ₹ 2700, then

(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.

(ii) Is it possible to solve the system of equations so obtained using matrices ?

22. Find $\frac{dy}{dx}$, if $ax^2+2hxy+by^2+2gx+2fy+c=0$.

OR

Find $\frac{dy}{dx}$, if $\sin(x+y)+\sin(xy)=\sin^2 x$.

23. Evaluate: $\int \frac{e^{6\log x}-e^{5\log x}}{e^{4\log x}-e^{3\log x}}dx$.

24. Find the order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$.

25. Solve: $(x+y)^2\frac{dy}{dx}=a^2$

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Evaluate $\int_{-1}^{2}\left|x^3-x\right|dx$.

27. Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

28. Find the area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$.

29. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis.

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that each one is perpendicular to the vector obtained by sum of the other two and

$\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=4$, and $\left|\vec{c}\right|=5$, then prove that $\left|\vec{a}+\vec{b}+\vec{c}\right|=5\sqrt{2}$.

30. Find number of point at which the maximum value of $z = 2x + 5y$ subject to the constraints $2x + 5y \le 10$, $x+2y\ge 1, x-y\le 4, x \ge y\ge 0$.

OR

Maximize $Z = 3x + 5y$, subject to $x + 4y \le 24$, $3x + y \le 21$, $x + y, \le 9$, $x\ge 0$, $y\ge 0$. Then find maximum value.

31. A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?

OR

A die is thrown three times. Events A and B are defined as below:

A: 5 on the first and 6 on the second throw.

B: 3 or 4 on the third throw.

Find the probability of B. given that A has already occurred.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

OR

Show that the function $f: R \to R$ defined by $f(x) = \frac{1}{x}$ is one-one onto, where R is the set of all non-zero real numbers. Is the result true, if the domain R is replaced by N with co-domain being same as R?

33. Using matrices, solve the following system of equations

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

34. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

35. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

OR

Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

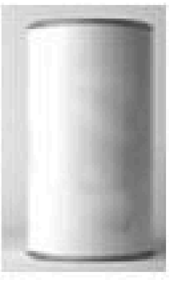
$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case - Study 1:** Read the following passage and answer the questions given below.

A manufacturer designs a cylindrical tin can for milk company to store milk. The tin can is made to hold 3 litres of milk.



(i) If *r* cm be the radius and *h* cm be the height of the cylindrical tin can, then find the surface area in term of r.

(ii) Find the radius that will minimize the cost of the material to manufacture the tin can.

(iii) Find the height that will minimize the cost of the material to manufacture the tin can.

OR

If the cost of material used to manufacture the tin can is ₹ $100/m^2$ then find minimum cost.

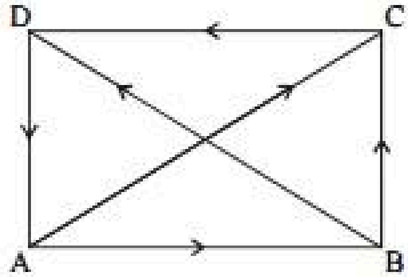
37. Case - Study 2: Read the following passage and answer the questions given below.

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

(i) If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then $\vec{q} + \vec{r} =$

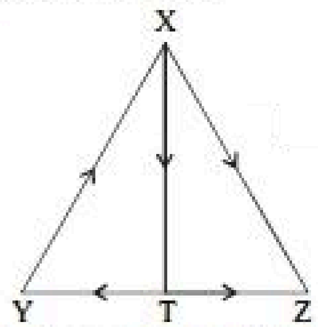
(ii) If *ABCD* is a parallelogram and *AC* and *BD* are its diagonals, then $\overline{AC} + \overline{BD} =$

(iii) If ABCD is a quadrilateral whose diagonals are $\overline{AC}$ and $\overline{BD}$, then $\overline{BA} + \overline{CD} =$



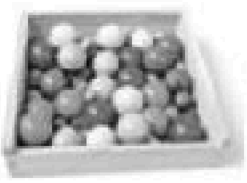
OR

If T is the mid point of side YZ of ΔXYZ, then $\overline{XY} + \overline{XZ} =$



38. Case - Study 3: Read the following passage and answer the questions given below.

In a play zone, students is playing game. A bag contain 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. If a student draws two balls one after the other without replacement.



(i) What is the probability that the one ball is blue and one ball is green?

(ii) What is the probability that both the balls are red?

Solutions

SAMPLE PAPER-8

1. (c) $AB=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}$

$$AB=\begin{bmatrix}abc-abc & b^2c-b^2c & bc^2-bc^2\\ -a^2c+a^2c & -abc+abc & -ac+ac\\ a^2b-a^2b & ab^2-ab^2 & abc-abc\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

2. (b) We have. $6\sin^{-1}(x^2-6x+8.5)=\pi$

$\Rightarrow \sin^{-1}(x^2-6x+8.5)=\frac{\pi}{6}$

$\Rightarrow x^2-6x+8.5=\sin\frac{\pi}{6}=\frac{1}{2}$

$\Rightarrow x^2-6x+8.5-0.5=0 \Rightarrow x^2-6x+8=0$

$\Rightarrow (x-4)(x-2)=0 \Rightarrow x=4$ or $x=2$

3. (b) $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, $A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$$A+A'=\begin{bmatrix}\cos\alpha+\cos\alpha & -\sin\alpha+\sin\alpha\\ \sin\alpha-\sin\alpha & \cos\alpha+\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=I=\begin{bmatrix}1&0\\0&1\end{bmatrix}\text{(given)}$$

$\Rightarrow 2\cos\alpha=1, \Rightarrow \cos\alpha=\frac{1}{2} \therefore \alpha=\frac{\pi}{3}$.

4. (c) Given that, $\tan^{-1}k-\tan^{-1}\sqrt{3}=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\tan^{-1}k=\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}$

$\therefore k=\infty$

5. (c) Given function is : $f(x)=\begin{cases}3x-4, & 0\le x\le 2\\ 2x+\ell, & 2<x\le 9\end{cases}$

and also given that $f(x)$ is continuous at $x=2$

For a function to be continuous at a point

LHL = RHL = Value of a function at that point.

$f(2)=2 \Rightarrow \text{RHL}: \lim_{x\to 2}(2x+\ell)=3(2)-4$

$\Rightarrow \lim_{h\to 0}\{2(2+h)+\ell\}=6-4 \Rightarrow 4+\ell=2 \Rightarrow \ell=-2$

6. (d) $P(B)=\frac{3}{5}$, $P(A|B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$

$P(A\cap B)=P(A|B)P(B)=\frac{1}{2}\cdot\frac{3}{5}=\frac{3}{10}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A)=\frac{4}{5}-\frac{3}{10}=\frac{1}{2}$. $P(A')=1-P(A)=\frac{1}{2}$

We know, $P(A\cap B)+P(A'\cap B)=P(B)$

[as $A\cap B$ and $A'\cap B$ are mutually exclusive events]

$\Rightarrow \frac{3}{10}+P(A'\cap B)=\frac{3}{5}\Rightarrow P(A'\cap B)=\frac{3}{5}-\frac{3}{10}=\frac{3}{10}$

Now, $P(A'\cup B)=P(A')+P(B)-P(A'\cap B)$

$=\frac{1}{2}+\frac{3}{5}-\frac{3}{10}=\frac{5+6-3}{10}=\frac{4}{5}$

$P\left((A\cup B)'\right)=1-P(A\cup B)=1-\frac{4}{5}=\frac{1}{5}$

$\therefore P\left((A\cup B)'\right)+P(A'\cup B)=\frac{1}{5}+\frac{4}{5}=1$

7. (b) We have, $f(x)=\sqrt{1+\cos^2(x^2)}$ ...(i)

On differentiating (i) w.r.t.x, we get

$f'(x)=\frac{-2\sin x^2\cos x^2}{\sqrt{1+\cos^2 x^2}}(x)$

$\Rightarrow f'(x)=\frac{-\sin 2x^2}{\sqrt{1+\cos^2 x^2}}(x)$ ...(ii)

Put, $x=\frac{\sqrt{\pi}}{2}$ in (ii), we get

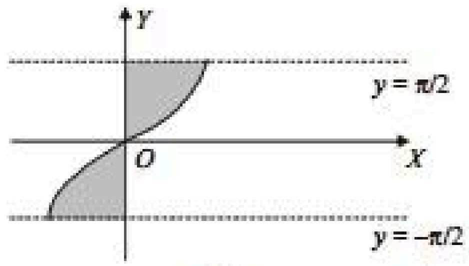
$f'\left(\frac{\sqrt{\pi}}{2}\right)=-\frac{\sqrt{\pi}}{2}\cdot\frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1+\frac{1}{2}}}=-\frac{\sqrt{\pi}}{2}\cdot\frac{\sin\frac{\pi}{2}}{\sqrt{\frac{3}{2}}}=-\sqrt{\frac{\pi}{6}}$

8. (a) $y^2=18x\Rightarrow 2y\frac{dy}{dx}=18\Rightarrow\frac{dy}{dx}=\frac{9}{y}$

Given $\frac{dy}{dx}=2\Rightarrow\frac{9}{y}=2\Rightarrow y=\frac{9}{2}$

Putting in $y^2=18x\Rightarrow x=\frac{9}{8}$ $\therefore$ Req. point is $\left(\frac{9}{8},\frac{9}{2}\right)$

9. (b) The required area is shown by shaded portion in the figure.



The req. area is $A=\int_{-\pi/2}^{\pi/2}|\sin y|\,dy=2\int_{0}^{\pi/2}\sin y\,dy=2$

10. **(b)** $f(x)=x^2-4x+6$

$f'(x)=2x-4$

Let $f'(x)=0\Rightarrow x=2$

$-\quad +$

$-\infty \quad 2 \quad \infty$

$\Rightarrow$ $f(x)$ is strictly increasing in $(2,\infty)$

11. **(a)** $\int(x^4+x^2+2x^3)dn=\frac{x^5}{5}+\frac{x^3}{3}+\frac{x^4}{2}+c.$

12. **(b)** Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6, – 3, 2 then

$x_2-x_1=6;\ \ y_2-y_1=-3;\ \ z_2-z_1=2$

So that direction ratios of $\overline{PQ}$ are 6, – 3, 2

$\therefore$ Direction cosines of $\overline{PQ}$ are

$$\frac{6}{\sqrt{6^2+(-3)^2+2^2}},\ \frac{-3}{\sqrt{6^2+(-3)^2+2^2}},$$

$$\frac{2}{\sqrt{6^2+(-3)^2+2^2}}=\frac{6}{7},\ \frac{-3}{7},\ \frac{2}{7}$$

13. **(d)** $\frac{3x+4}{x^3-2x-4}=\frac{3x+4}{(x-2)(x^2+2x+2)}$

$$=\frac{A}{x-2}+\frac{Bx+C}{x^2+2x+2}$$

$\Rightarrow$ $3x+4=A(x^2+2x+2)+(Bx+C)(x-2)$

$\therefore$ $A+B=0$

$2A-2B+C=3$

$2A-2C=4$

$\Rightarrow$ $A=1, B=C=-1$

$\therefore \int\frac{3x+4}{x^3-2x-4}dx=\int\frac{dx}{x-2}-\frac{1}{2}\int\frac{2x+2}{x^2+2x+2}dx$

$=\log_e|x-2|-\frac{1}{2}\log|x^2+2x+2|+C$

$\Rightarrow k=-\frac{1}{2}$ and $f(x)=|x^2+2x+2|$

14. **(b)** $y=e^{a\sin x}\Rightarrow\log y=a\sin x\log e$

$\Rightarrow\frac{1}{y}\frac{dy}{dx}=a\cos x\Rightarrow\frac{dy}{dx}=y\left(\frac{\log y}{\sin x}\right)\cos x$

$\Rightarrow\frac{dy}{dx}=\frac{y\log y}{\tan x}\Rightarrow\tan x\frac{dy}{dx}=y\log y$

15. **(c)**

16. **(a)** $\frac{dv}{dt}+\frac{k}{m}v=-g\Rightarrow\frac{dv}{dt}=-\frac{k}{m}\left(v+\frac{mg}{k}\right)$

$\Rightarrow\ \frac{dv}{v+mg/k}=-\frac{k}{m}dt\Rightarrow\log\left(v+\frac{mg}{k}\right)=-\frac{k}{m}t+\log C$

$\Rightarrow\ v+\frac{mg}{k}=Ce^{-kt/m}\Rightarrow v=Ce^{-kt/m}-\frac{mg}{k}$

17. **(b)** The given lines are parallel and

$\vec{a}_1=\hat{i}+\hat{j},\ \vec{a}_2=2\hat{i}-3\hat{k}$

$\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$

Now, $\vec{a}_2-\vec{a}_1=(2\hat{i}-3\hat{k})-(\hat{i}+\hat{j})=\hat{i}-\hat{j}-3\hat{k}$

$$\vec{b}\times(\vec{a}_2-\vec{a}_1)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 1 & -1 & -3\end{vmatrix}$$

$=\hat{i}(6+3)-j(-3-3)+\hat{k}(-1+2)$

$=9\hat{i}+6j+\hat{k}$

$|\vec{b}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$

Distance, $d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|=\left|\frac{9\hat{i}+6\hat{j}+\hat{k}}{\sqrt{14}}\right|$

$=\frac{1}{\sqrt{14}}\sqrt{(9)^2+(6)^2+(1)^2}=\sqrt{\frac{59}{7}}$

18. **(c)** Obviously, solution set of constraints included the point (3, 4).

19. **(d)** Clearly the relation is not reflexive, neither transitive it is only symmetric.

20. **(c)** $32=2^5$

Number of ways of expressing 32 as product of two positive integers $=\frac{5+1}{2}=3$.

Possible dimensions of a matrix are

$\{1\times32, 32\times1, 2\times16, 16\times2, 4\times8, 8\times4\}=6$

$\Rightarrow$ Assertion is true and Reason is false

21. $|B|=\begin{vmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{vmatrix}=1\neq0$

$\therefore B^{-1}=$ exists,

$$B^{-1} = \frac{1}{|B|}(\text{adj } B) = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now $(AB)^{-1} = B^{-1}A^{-1}$ [1 Mark]

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$ [1 Mark]

OR

(i) Let the award prize for Truth be ₹ x, Patriotism be ₹ y and Non-violence be ₹ z.

$5x + 4y + 3z = 11000$

$4x + 3y + 5z = 10700$

$x + y + z = 2700$ [½ Mark]

The system of equations can be written as AX = B.

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11000 \\ 10700 \\ 2700 \end{bmatrix}$$

[½ Mark]

(ii) $\therefore |A| = 5(3-5) - 4(4-5) + 3(4-3)$

$= -10 + 4 + 3 = -3 \neq 0$ [½ Mark]

$\therefore A^{-1}$ exists. So, equations have a unique solution.

[½ Mark]

22. Consider $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Differentiating w.r.t. x, both sides, we get

$$2ax + 2h\left(x\frac{dy}{dx} + y\right) + 2by\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} + 0 = 0$$

[1 Mark]

$$\Rightarrow ax + hx\frac{dy}{dx} + hy + by\frac{dy}{dx} + g + f\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(hx + by + f) = -(ax + hy + g)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(ax + hy + g)}{hx + by + f}$$ [1 Mark]

OR

Consider $\sin(x+y) + \sin(xy) = \sin^2 x$.

Differentiating, w.r.t. x, we get

$$\cos(x+y).\left\{1 + \frac{dy}{dx}\right\} + \cos(xy).\left\{x\frac{dy}{dx} + y\right\}$$

$= 2\sin x \cos x$ [1 Mark]

$\Rightarrow \cos(x+y) + \cos(x+y)$

$$\frac{dy}{dx} + x\cos(xy).\frac{dy}{dx} + y\cos(xy) = \sin 2x$$

$$\Rightarrow \frac{dy}{dx}\{\cos(x+y) + x\cos(xy)\}$$

$= \sin 2x - \cos(x+y) - y\cos(xy)$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin 2x - \cos(x+y) - y\cos(xy)}{\cos(x+y) + x\cos(xy)}.$$ [1 Mark]

23. Let $I = \int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}dx$

$$= \int \frac{e^{\log x^6} - e^{\log x^5}}{e^{\log x^4} - e^{\log x^3}}dx = \int \frac{x^6 - x^5}{x^4 - x^3}dx$$ [1 Mark]

$$= \int \frac{x^2(x^4 - x^3)}{x^4 - x^3}dx = \int x^2 dx = \frac{x^3}{3} + C$$ [1 Mark]

24. Clearly order of the differential equation is 2.

Again $\frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3}$

$$\Rightarrow \left(\frac{d^2y}{dx^2} + x^{1/4}\right)^3 = -\frac{dy}{dx}$$ [1 Mark]

which shows that degree of the differential equation is 3.

[1 Mark]

25. Let $x + y = v$. Then,

$$1 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$$

Putting $x + y = v$ and $\frac{dy}{dx} = \frac{dv}{dx} - 1$ the given differential equation, we get

$$v^2\left(\frac{dv}{dx} - 1\right) = a^2 \quad \Rightarrow \quad v^2\frac{dv}{dx} = a^2 + v^2$$

$$\Rightarrow v^2 dv = \left(a^2 + v^2\right)dx$$ [1 Mark]

$$\Rightarrow \frac{v^2}{v^2 + a^2}dv = dx$$ [By separating the variables]

$$\Rightarrow \left(1 - \frac{a^2}{v^2 + a^2}\right)dv = dx$$

$$\Rightarrow \int 1.dv - a^2\int \frac{1}{v^2 + a^2}dv = \int dx + C$$

[On integration]

$$\Rightarrow v - a\tan^{-1}\left(\frac{v}{a}\right) = x + C$$

$$\Rightarrow (x+y) - a\tan^{-1}\left(\frac{x+y}{a}\right) = x + C$$ [1 Mark]

26. Let $I = \int_{-1}^{2}\left|x^3 - x\right|dx$

$f(x) = x^3 - x$

$\Rightarrow f(x) = x(x-1)(x+1)$

The signs 8 of f(x) for the different values are as follows:

$f(x) > 0$ for all $x \in (-1, 0) \cup (1, 2)$

$f(x) < 0$ for all $x \in (0, 1)$

Therefore,

$$\left|x^3-x\right|=\begin{cases}x^3-x, x\in(-1,0)\cup(1,2)\\-\left(x^3-x\right), x\in(0,1)\end{cases}$$ [1 Mark]

$$\therefore I=\int_{-1}^{2}\left|x^3-x\right|dx$$

$$=\int_{-1}^{0}\left|x^3-x\right|dx+\int_{0}^{1}\left|x^3-x\right|dx+\int_{1}^{2}\left|x^3-x\right|dx$$

$$=\int_{-1}^{0}\left(x^3-x\right)dx-\int_{0}^{1}\left(x^3-x\right)dx+\int_{1}^{2}\left(x^3-x\right)dx$$ [1 Mark]

$$=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}-\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{0}^{1}+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{1}^{2}$$

$$=-\left(\frac{1}{4}-\frac{1}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{16}{4}-\frac{4}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)$$

$$=\frac{3}{4}+(4-2)=\frac{11}{4}$$ [1 Mark]

27. Let $I=\int\frac{x^2+1}{(x-1)^2(x+3)}dx$

Let $\frac{x^2+1}{(x-1)^2(x+3)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3}$ ...(i)[½ Mark]

$$\Rightarrow\ \frac{x^2+1}{(x-1)^2(x+3)}$$

$$=\frac{A(x-1)(x+3)+B(x+3)+C(x-1)^2}{(x-1)^2+(x+3)}$$

$x^2+1=A(x-1)(x+3)+B(x+3)+C(x-1)^2$

$\Rightarrow\ x^2+1=A(x^2+2x-3)+B(x+3)+C(x^2+1-2x)$

$\Rightarrow\ x^2+1=(A+C)x^2+2A+B-2C)x+(-3A+3B+C)$

Comparing coefficients of x^2, x and constant on both sides, we get

$A+C=1$ ...(ii)

$2A+B-2C=0$ ...(iii)

$-3A+3B+C=1$ ....(iv)

Multiply Eq. (iii) by '3' and subtracting it from Eq. (iv), we get

$$\begin{array}{r}-3A+3B+C=1\\ \underline{6A\pm 3B\mp 6C=0}\\ -9A+7C=1\end{array}$$ ...(v)

Multiply Eq. (ii) by '7' and subtracting it from Eq. (v), we get

$$\begin{array}{r}-9A+7C=1\\ \underline{\_7A+\_7C=\_7}\\ -16A=-6\end{array}$$

$\therefore\ A=\frac{6}{16}=\frac{3}{8}$

Put $A=\frac{3}{8}$ in Eq. (ii), we get

$$\frac{3}{8}+C=1$$

$\Rightarrow\ C=1-\frac{3}{8}=\frac{5}{8}$

Put $A=\frac{3}{8}$ and $C=\frac{5}{4}=0$ in Eq. (iii), we get

$$\frac{3}{4}+B-\frac{5}{4}=0$$

$\Rightarrow\ B-\frac{2}{4}=0$

$\Rightarrow\ B=\frac{2}{4}=\frac{1}{2}$

$\therefore\ A=\frac{3}{8}, B=\frac{1}{2}$ and $C=\frac{5}{8}$ [1 Mark]

$\therefore$ Eq. (i) becomes

$$\frac{x^2+1}{(x-1)^2(x+3)}=\frac{3/8}{x-1}+\frac{1/2}{(x-1)^2}+\frac{5/8}{x+3}$$

Integrating both sides, we get

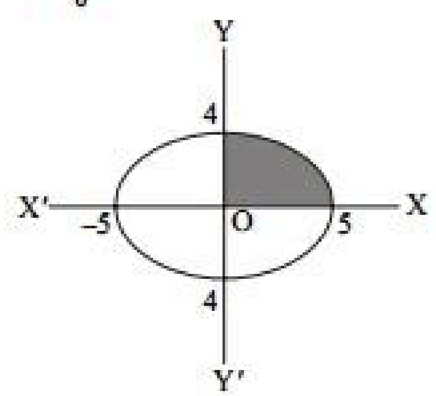
$$\int\frac{x^2+1}{(x-1)^2(x+3)}dx=\frac{3}{8}\int\frac{dx}{x-1}+\frac{1}{2}\int\frac{dx}{(x-1)^2}+\frac{5}{8}\int\frac{dx}{x+3}$$

$$=\frac{3}{8}\log|x-1|+\frac{1}{2}\left(\frac{-1}{x-1}\right)+\frac{5}{8}\log|x+3|+C$$

Hence, $I=\frac{3}{8}\log|x-1|-\frac{1}{2(x-1)}+\frac{5}{8}\log|x+3|+C$

[1½ Marks]

28. Area $=4\times\frac{4}{5}\int_{0}^{5}\sqrt{25-x^2}\ dx$



[2 Marks]

$= \frac{16}{5}\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\cdot \sin^{-1}\frac{x}{5}\right]_0^5$

$= \frac{16}{5}\left[0 + \frac{25}{2}\left(\frac{\pi}{2}\right) - 0 - 0\right]$

$= \frac{16}{5} \times \frac{25\pi}{4} = 20\pi$ sq. units [1 Mark]

29. We have $|\vec{a}| = 5\sqrt{2}$

$\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{2}, \gamma = \theta < \frac{\pi}{2}$

We know,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos\theta = 1$

$\frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \pm\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ or $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta = \frac{\pi}{4}, \theta = \frac{3\pi}{4}$

For $\theta = \gamma = \frac{\pi}{4}$ [1 Mark]

Now, $\vec{a} = |\vec{a}|\left\{\cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}\right\}$

$= 5\sqrt{2}\left\{\cos\frac{\pi}{4}\hat{i} + \cos\frac{\pi}{2}\hat{j} + \cos\frac{\pi}{4}\hat{k}\right\}$

$= 5\sqrt{2}\left\{\frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right\}$

$\vec{a} = 5\hat{i} + 5\hat{k}$ [1 Mark]

For $\theta = \frac{3\pi}{4}$

$\vec{a} = |\vec{a}|\left\{\cos\frac{\pi}{4}\hat{i} + \cos\frac{\pi}{2}\hat{j} + \cos\frac{3\pi}{4}\hat{k}\right\}$

$\vec{a} = 5\sqrt{2}\left\{\frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right\}$

$\vec{a} = 5\hat{i} - 5\hat{k}$

$\therefore$ The required vector is

$\vec{a} = 5\hat{i} + 5\hat{k}$ or $\vec{a} = 5\hat{i} - 5\hat{k}$ [1 Mark]

OR

Here, we have

$|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$

Now, $\vec{a}, \vec{b}, \vec{c}$ each one is perpendicular to the vector obtained by sum of the other two.

$\Rightarrow \vec{a}.\left(\vec{b} + \vec{c}\right) = 0$, $\vec{b}.\left(\vec{c} + \vec{a}\right) = 0$ and $\vec{c}.\left(\vec{a} + \vec{b}\right) = 0$

[1 Mark]

Consider $\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left(\vec{a} + \vec{b} + \vec{c}\right).\left(\vec{a} + \vec{b} + \vec{c}\right)$

$= \vec{a}.\vec{a} + \vec{a}.\left(\vec{b} + \vec{c}\right) + \vec{b}.\vec{b} + \vec{b}.\left(\vec{c} + \vec{a}\right)$

$+ \vec{c}.\vec{c} + \vec{c}.\left(\vec{a} + \vec{b}\right)$

[1 Mark]

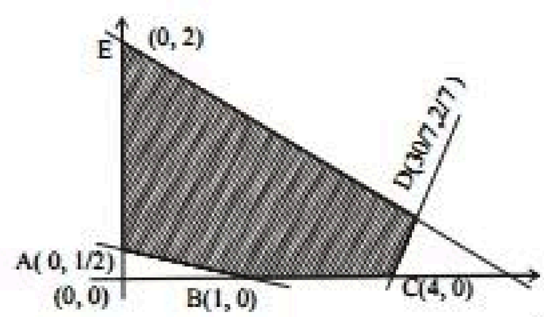
$= \left|\vec{a}\right|^2 + 0 + \left|\vec{b}\right|^2 + 0 + \left|\vec{c}\right|^2 + 0 = 9 + 16 + 25 = 50$

[1 Mark]

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{50} = 5\sqrt{2}$

Hence, $\left|\vec{a} + \vec{b} + \vec{c}\right| = 5\sqrt{2}$

30. We find that the feasible region is on the same side of the line $2x + 5y = 10$ as the origin, on the same side of the line $x - y = 4$ as the origin and on the opposite side of the line $x + 2y = 1$ from the origin. Moreover, the lines meet the coordinate axes at (5, 0), (0, 2), (1, 0), (0, 1/2) and (4, 0). The lines $x - y = 4$ and $2x + 5y = 10$ intersect at $\left(\frac{30}{7}, \frac{2}{7}\right)$.



[2 Marks]

The values of the objective function at the vertices of the pentagon are:

(i) $Z = 0 + \frac{5}{2} = \frac{5}{2}$

(ii) $Z = 2 + 0 = 2$

(iii) $Z = 8 + 0 = 8$

(iv) $Z = \frac{60}{7} + \frac{10}{7} = 10$

(v) $Z = 0 + 10 = 10$

The maximum value 10 occurs at the points $D\left(\frac{30}{7}, \frac{2}{7}\right)$ and E(0, 2). Since D and E are adjacent vertices, the objective function has the same maximum value 10 at all the points on the line DE. [1 Mark]

OR

Let $\ell_1 : x+4y=24$; $\ell_2 : 3x+y=21$;
$\ell_3 : x+y=9$; $\ell_4 : x=0$ and $\ell_5 : y=0$

On solving these equations we will get points as O(0, 0), A(7, 0), B(6, 3), C(4, 5), D(0, 6)

Now maximize $Z=3x+5y$

Z at O(0, 0) $=3(0)+5(0)=0$

Z at A(7, 0) $=3(7)+5(0)=21$

Z at B(6, 3) $=3(6)+5(3)=33$

Z at C(4, 5) $=3(4)+5(5)=37$

Z at D(0, 6) $=3(0)+5(6)=30$ [2 Marks]

Thus, Z is maximized at C(4, 5) and its maximum value is 37. [1 Mark]

31. Let S denote the success, i.e. getting a number greater than four and F denote the failure.i.e. getting a number less than four.

$\therefore P(S)=\frac{2}{6}=\frac{1}{3}, P(F)=1-\frac{1}{3}=\frac{2}{3}$ [½ Mark]

Now, B gets the second throw, if A fails in the first throw.

$\therefore$ P(B wins in the second throw) = P(FS) = P(F)P(S)

$=\frac{2}{3}\times\frac{1}{3}$ [½ Mark]

Similarly, P(B wins in the fourth throw) = P(FFFS)

$=P(F)P(F)P(F)P(S)=\left(\frac{2}{3}\right)^3\times\frac{1}{3}$ [½ Mark]

P(B wins in the sixth throw) = P(FFFFFS)

$=P(F)P(F)P(F)P(F)P(F)P(S)=\left(\frac{2}{3}\right)^5\times\frac{1}{3}$ and so on.

Hence. [½ Mark]

$P(B \text{ wins})=\frac{2}{3}\times\frac{1}{3}+\left(\frac{2}{3}\right)^3\times\frac{1}{3}+\left(\frac{2}{3}\right)^3\times\frac{1}{3}+...$

$=\frac{2}{3}\times\frac{1}{3}\times\left[1+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^4+...\right]$

$=\frac{2}{3}\times\frac{1}{3}\times\left(\frac{1}{1-\frac{2}{3}}\right)$ $[\because a+ar+ar^2+...=\frac{a}{1-r}]$

$=\frac{2}{3}$

Thus, the probability that B wins is $\frac{2}{3}$. [1 Mark]

OR

A is an event getting 5 on the first throw and 6 on the second throw

Then

A = {(5,6,1) (5,6,2) (5,6,3) (5,6,4) (5,6,5), (5,6,6)} [1 Mark]

Also B is an event of getting 3 or 4 on the third throw.

$\therefore A\cap B=\{(5.6,3),(5,6,4)\}$ [1 Mark]

Required probability, $P(B\mid A)=\frac{n(A\cap B)}{n(A)}=\frac{2}{6}=\frac{1}{3}$

Thus, the probability of B, given that A has already occurred is $\frac{1}{3}$. [1 Mark]

32. (i) In a set of triangles $R=\{(T_1, T_2): T_1$ is similar $T_2\}$

(a) Since A triangle T is similar to itself. Therefore $(T, T)\in R$ for all $T\in A$.
Since R is reflexive. [1 Mark]

(b) If triangle T_1 is similar to triangle T_2 then T_2 is similar triangle T_1
$\therefore$ R is symmetric. [1 Mark]

(c) Let T_1 is similar to triangle T_2 and T_2 to T_3 then triangle T_1 is similar to triangle T_3, $\therefore$ R is transitive.
Hence, R is an equivalence relation. [1 Mark]

(ii) Two triangles are similar if their sides are proportional now sides 3, 4, 5 of triangle T_1 are proportional to the sides 6, 8, 10 of triangle T_3.
$\therefore$ T_1 is related to T_3. [2 Marks]

OR

(a) We observe the following properties of f :

(i) $f(x)=\frac{1}{x}$, if $f(x_1)=f(x_2)$

$\Rightarrow\frac{1}{x_1}=\frac{1}{x_2}\Rightarrow x_1=x_2$

Each $x\in R$ has a unique image in codomain
$\Rightarrow$ f is one-one. [2 Marks]

(ii) For each y belonging codomain then

$y=\frac{1}{x}$ or $x=\frac{1}{y}$ there is a unique pre-image of y.

$\Rightarrow$ f is onto. [2 Marks]

(b) When domain R is replaced by N. codomain remaining the same, then $f: N\rightarrow R$ If $f(x_1)=f(x_2)$

$\Rightarrow\frac{1}{n_1}=\frac{1}{n_2}\Rightarrow n_1=n_2$ where $n_1, n_2\in N$

$\Rightarrow$ f is one-one.

But for every real number belonging to codomain may not have a pre-image in N.

e.g. $\frac{1}{2},\frac{3}{2}, N$ $\therefore$ f is not onto. [1 Mark]

33. Given equations are

$x-y+2z=7$

$3x+4y-5z=-5$

$2x-y+3z=12$

Let $A=\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}7\\ -5\\ 12\end{bmatrix}$

According to matrix, we have $X = A^{-1} B$ [1 Mark]

where, $A^{-1} = \frac{\text{adj}A}{|A|}$

and $\text{adj } A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}^T$

$\therefore \text{adj } A = \begin{bmatrix} 7 & 19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

Now, [2 Marks]

$|A| = 1\,(12 - 5) + 1\,(9 + 10) + 2\,(-3 - 8)$

$|A| = 1\,(7) + 1\,(19) + 2\,(-11) = 7 + 19 - 22 = 4$

$\therefore A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$ [1 Mark]

$\therefore X = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$

$= \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$\therefore x = 2, y = 1, z = 3$ [1 Mark]

34. Let the length and breadth of the rectangle inscribed in a circle of radius a be x and y respectively.

$\therefore \quad x^2 + y^2 = (2a)^2 \Rightarrow \quad x^2 + y^2 = 4a^2$ ...(i)

$\therefore$ Perimeter $= 2\,(x + y)$ [1 Mark]

$\Rightarrow P(x) = 2\left[x + \sqrt{4a^2 - x^2}\right]$

$\therefore P'(x) = 2\left[1 - \frac{x}{\sqrt{4a^2 - x^2}}\right]$ ...(ii)[1 Mark]

and $P''(x) = \frac{-8a^2}{(4a^2 - x^2)^{3/2}}$ ...(iii)

For P (x) to be minimum $P'(x) = 0$ and $P''(x) < 0$

$\therefore$ from (i), $P'(x) = 0 \Rightarrow 4a^2 - x^2 = x^2 \Rightarrow x = a\sqrt{2}$ [1 Mark]

from (iii) $P''(x) = \frac{-8a^2}{(2a^2)^{3/2}} \Rightarrow P(x)$ is maximum at $x = a\sqrt{2}$

from (i) $y = \sqrt{2}a = x$; Thus, $x = y$ [2 Marks]

Hence rectangle becomes square hence found.

35. The given equation are not in the standard form. The equation of the given lines

$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ ...(i)

and $\frac{x-1}{3p/-7} = \frac{y-5}{1} = \frac{z-6}{-5}$ ...(ii) [2 Marks]

The direction ratios of the given lines are -3, $\frac{2P}{7}$, 2 and $\frac{-3P}{7}$, 1, -5. [1 Mark]

The lines are perpendicualr to each other

$\therefore (-3)\left(\frac{-3P}{7}\right) + \left(\frac{2P}{7}\right)(1) + 2\,(-5) = 0 \Rightarrow P = \frac{70}{11}$ [2 Marks]

OR

The given equation

$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k})$ ...(i)

$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s\,(-\hat{i} + 2\hat{j} - 2\hat{k})$ ...(ii)

$\therefore \quad \vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \; \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$ and

$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}, \; \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$ [2 Marks]

Now, $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$ [1 Mark]

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ [1 Mark]

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{4 + 16 + 9} = \sqrt{29}$ and

$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (\hat{j} - 4\hat{k})\cdot(2\hat{i} - 4\hat{j} - 3\hat{k}) = -4 + 12 = 8$

$\therefore \quad d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \left|\frac{8}{\sqrt{29}}\right| = \frac{8}{\sqrt{29}}.$ [1 Mark]

36. (i) Volume : $\pi r^2 h = 3$ lit $= 3000$ cm^3

$\therefore h = \frac{3000}{\pi r^2}$

Surface area $s(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{6000}{r}$

[1 Mark]

(ii) $s'(r) = 4\pi r - \frac{6000}{r^2} = 0 \Rightarrow r = \left(\frac{1500}{\pi}\right)^{1/3}$

$s''(r)\left(r = \left(\frac{1500}{\pi}\right)^{1/3}\right) = 4\pi + \frac{12000 \times \pi}{1500} > 0$

$\therefore$ Surface area is minimum when $r = \left(\frac{1500}{\pi}\right)^{1/3}$.

[1 Mark]

(iii) $\because \; r = \left(\frac{1500}{\pi}\right)^{1/3} \Rightarrow \pi = \frac{1500}{r^3}$

Now $h = \frac{3000}{\pi r^2} = \frac{3000 \times r^3}{1500 r^2} = 2r = 2\left(\frac{1500}{\pi}\right)^{1/3}$

[2 Marks]

OR

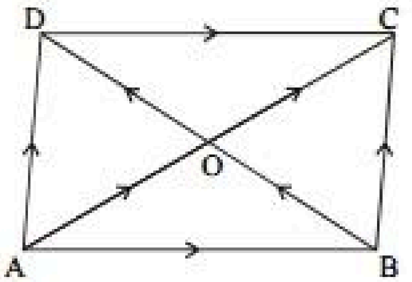
$$\text{Minimum surface area} = \frac{2\pi r^3 + 6000}{r} = 1153.84\ \text{cm}^3$$

$$\text{Minimum cost} = 1153.84 \times \frac{1}{100} = ₹11.538.$$ [2 Marks]

37. (i) $\vec{p} + \vec{q} + \vec{r} = 0 \Rightarrow \vec{q} + \vec{r} = -\vec{p}$ [1 Mark]

(ii) $\overline{AC} + \overline{BD} = \overline{AB} + \overline{BC} + \overline{BC} - \overline{DC}$

$= \overline{AB} + 2\overline{BC} - \overline{AB} = 2\overline{BC}.$ [1 Mark]



(iii) In ΔABC

$\overline{AB} + \overline{BC} = \overline{AC}$

$\overline{BC} = \overline{AC} + \overline{BA}$ ...(i)

In ΔBCD

$\overline{BD} = \overline{BC} + \overline{CD}$ ...(ii)

Adding (i) and (ii)

$\overline{BC} + \overline{BD} = \overline{AC} + \overline{BA} + \overline{BC} + \overline{CD}$

$\overline{BD} - \overline{AC} = \overline{BA} + \overline{CD}$

$\Rightarrow \overline{BA} + \overline{CD} = \overline{BD} + \overline{CA}.$ [2 Marks]

OR

$\overline{YT} = \overline{TZ}$

$\overline{XY} + \overline{XZ} = \left(\overline{XT} + \overline{TY}\right) + \left(\overline{XT} + \overline{TZ}\right)$

$= 2\overline{XT} + \overline{TY} + \overline{TZ} = 2\overline{XT}.$ [2 Marks]

38. Let B, R, Y, G denotes the blue, red yellow, green balls
Total = 35 balls
$\therefore$ n(B) = 12, n(R) = 8, n(Y) = 10 and n(G) = 5

(i) $P(B \cap G)$ or $P(G \cap B)$

$= P(B).P(G/B) + P(G).P(B/G)$

$= \frac{12}{35} \times \frac{5}{34} + \frac{5}{35} \times \frac{12}{34} = \frac{12}{119}$ [2 Marks]

(ii) $P(R \cap R) = P(R).P(R/R)$

$= \frac{8}{35} \times \frac{7}{34} = \frac{4}{85}$ [2 Marks]