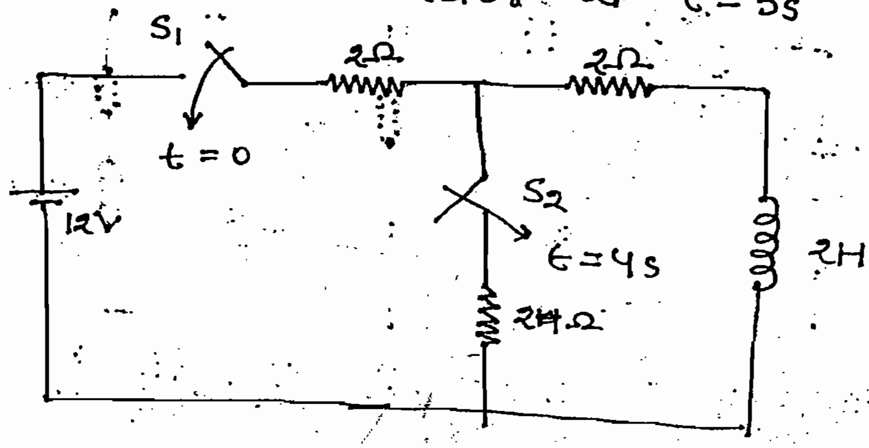
Lecture - 10

Ques:- Find current in the inductor at $t = 5s$



Sol^n:- S_1

$$i(t) = \frac{V}{R}(1 - e^{-Rt/L})$$

$$i(t) = \frac{12}{4}(1 - e^{-2t}) \Rightarrow i(t) = 3(1 - e^{-2t})$$

$t > 4s$, S_1 and $S_2 \rightarrow$ operated

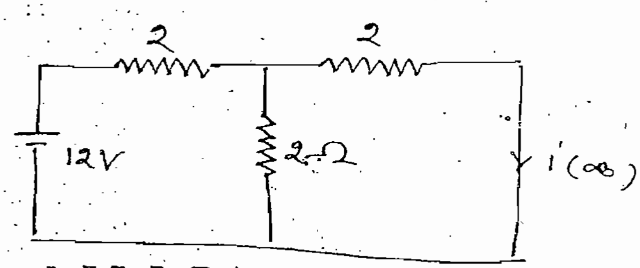
$$i(t) = [i(0^+) - i(\infty)]e^{-Rt/L} + i(\infty)$$

$$\Rightarrow i(t) = [i(4) - i(\infty)]e^{-\frac{R'}{L}(t-4)} + i(\infty) \quad -(1)$$

$t = 4$

$$i(4) = 3[1 - e^{-2(4)}] \simeq 2.99A$$

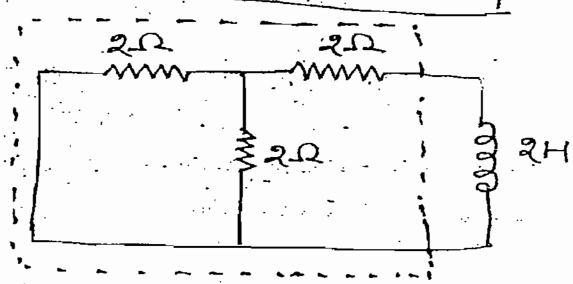
$t = \infty$



$$i(\infty) = 2A$$

Time constant for $T > 4s$

$$R' = 3 \quad -(II)$$

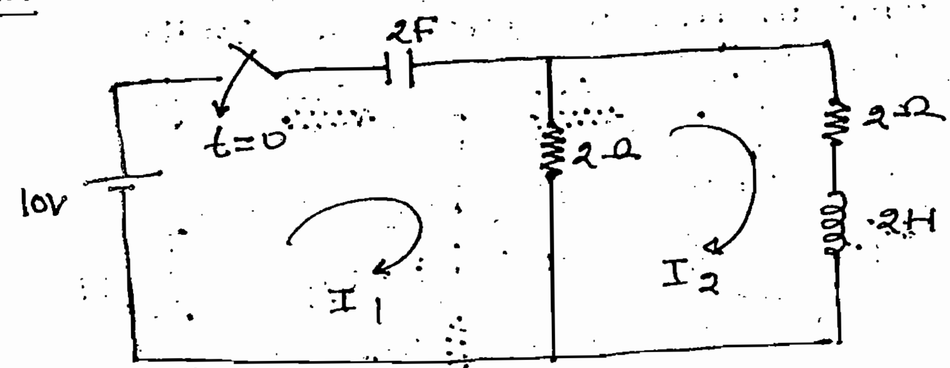


From - (I) & (II)

$$i(t) = [2.99 - 2]e^{-3/2(t-4)} + 2$$

$t = 5$ sec.

Ques:-



Find $i_1(0^+)$, $i_2(0^+)$, $V_c(0^+)$, $\frac{di_1(0^+)}{dt}$, $\frac{di_2(0^+)}{dt}$, $\frac{d^2i_1(0^+)}{dt^2}$, $\frac{d^2i_2(0^+)}{dt^2}$

Soln:- At $t=0^-$

$i_1(0^-)=0 \quad i_2(0^-)=0 \quad V_c(0^-)=0$

At $t=0^+$

$i_2(0^+)=0 \quad V_c(0^+)=0 \quad i_1(0^+)=\frac{10}{2}=5A$

$$(1) — \quad 4I_2 + 2\frac{di_2}{dt} - 2I_1 = 0 \longrightarrow t=0^+$$

$$4(0) + 2\frac{di_2}{dt} - 2(5) = 0$$

$$\Rightarrow \frac{di_2}{dt} = 5\ A/s$$

$$-10 + \frac{1}{2}\int i_1\,dt + 2(I_1 - I_2) = 0$$

Diff. w.r.t t

$$0 + \frac{i_1}{2} + 2\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0 \quad —(II)$$

$$\Rightarrow \frac{5}{2} + 2\left[\frac{di_1}{dt} - 5\right] = 0$$

$$\Rightarrow \frac{5}{2} - 10 + \frac{2di_1}{dt} = 0$$

$$\Rightarrow \frac{di_1}{dt} = \frac{15}{4} = 3.75\ A/s$$

Diff eq-(i) w.r.t 't'

$$4\frac{di_2}{dt} + 2\frac{d^2 i_2}{dt^2} - 2\frac{di_1}{dt} = 0$$

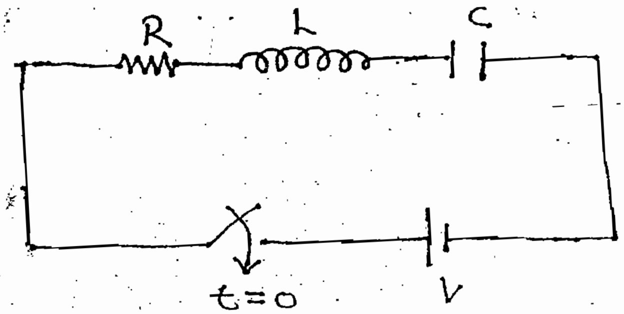
$$\Rightarrow \frac{d^2 i_2}{dt^2} = -6.25\ A/s^2$$

Diff eq-(ii), w.r.t 't'

$$\frac{1}{2}\frac{di_1}{dt} + 2\left[\frac{d^2 i_1}{dt^2} - \frac{d^2 i_2}{dt^2}\right] = 0$$

$$\Rightarrow \frac{d^2 i_1}{dt^2} = -7.18\ A/s^2$$

RLC Series Circuit with DC Excitation :-



By KVL

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Diff w.r.t 't'

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C}$$

Divide both sides by L we get

$$\boxed{\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0}$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \qquad \left(\because \frac{d}{dt} = D\right)$$

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$\boxed{D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$

Case-(I):-

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

overdamping

$$\alpha = \frac{-R}{2L} \qquad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$$\boxed{i(t) = C_1\, e^{(\alpha-\beta)t} + C_2\, e^{(\alpha+\beta)t}}$$

Case-(II):-

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \longrightarrow \text{Critical damping}$$

$$\boxed{i(t) = (C_1 + C_2 t)\, e^{\alpha t}}$$

Case-(III):-

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC} \longrightarrow \text{Underdamping}$$

$$\alpha = \frac{R}{2L}$$

$$\beta = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\boxed{i(t) = (C_1 \cos\beta t + C_2 \sin\beta t)\, e^{\alpha t}}$$

Damping coefficient $= \dfrac{R}{2L}$

Time Constant $= \dfrac{1}{\text{Damping Coefficient}} = \dfrac{2L}{R}$

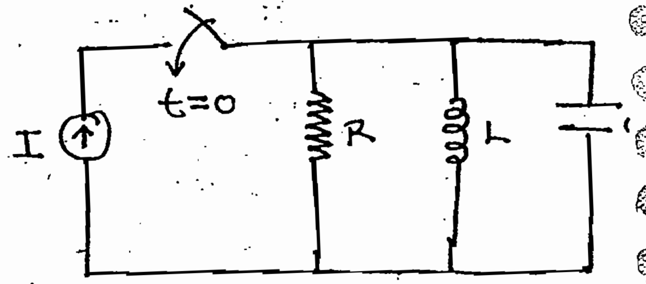
Case-(IV):-

$$R = 0 \longrightarrow \text{Underdamping}$$

$$\alpha = 0, \quad \beta = \frac{1}{\sqrt{LC}}$$

$$\boxed{i(t) = C_1 \cos\beta t + C_2 \sin\beta t}$$

RLC Parallel Circuit with DC Excitation:-



$$I = \frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int V dt$$

Diff. w.r.t t

$$0 = \frac{1}{R}\cdot\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V}{L}$$

Divide the whole equation by C, we get

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V}{LC} = 0$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = 0. \qquad \left(\because D = \frac{d}{dt}\right)$$

$$D_1, D_2 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_1, D_2 = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Case-(I):-

$$\left(\frac{1}{2RC}\right) > \frac{1}{LC} \longrightarrow \text{over damping}$$

$$\alpha = -\frac{1}{2RC}$$

$$\beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\boxed{V(t) = C_1 e^{(\alpha-\beta)t} + C_2 e^{(\alpha+\beta)t}}$$

Case-(II):-

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} \longrightarrow \text{Critical damping}$$

$$\boxed{v(t) = (C_1 + C_2 t)e^{\alpha t}}$$

Case-(III) :-

$$\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC} \longrightarrow \text{Under damping}$$

$$\alpha = -\frac{1}{2RC} \qquad \beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$\boxed{V(t) = (C_1 \cos\beta t + C_2 \sin\beta t)\, e^{\alpha t}}$$

$$\text{Damping coefficient} = \frac{1}{2RC}$$

$$\text{Time constant} = \frac{1}{\text{Damping coefficient}} = 2RC$$

Case-(IV) :-

$$G = 0 \qquad \frac{1}{R} = 0 \Rightarrow R = \infty$$

$\longrightarrow$ Underdamping

$$\alpha = 0, \quad \beta = \frac{1}{\sqrt{LC}}$$

$$\boxed{V(t) = C_1 \cos\beta t + C_2 \sin\beta t}$$

Note:-

1. Steady state current response of RLC series circuit with DC excitation = 0

2. At $t = \infty$

   $L \rightarrow$ Uncharged ($I = 0$)

   $C \rightarrow$ Charged ($V_c = V$)

3. $$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}} \longrightarrow \text{Series}$$

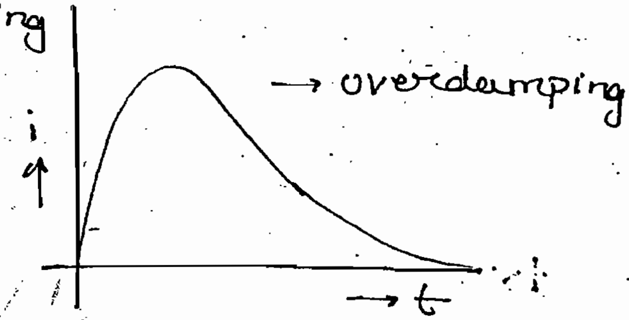
   $\xi > 1 \rightarrow$ Overdamping $\qquad \xi < 1 \rightarrow$ Underdamping

   $\xi = 1 \rightarrow$ Critical damping $\qquad \xi = 0 \rightarrow$ Undamping

$$\boxed{\delta = \frac{1}{2R}\sqrt{\frac{L}{C}}} \longrightarrow \text{Parallel}$$

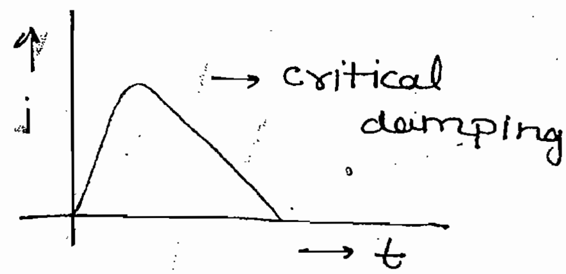
4. For over and critical damping system no oscillations are present

$$\frac{1}{2R}\sqrt{\frac{L}{C}} \geq 1 \longrightarrow \text{Parallel}$$



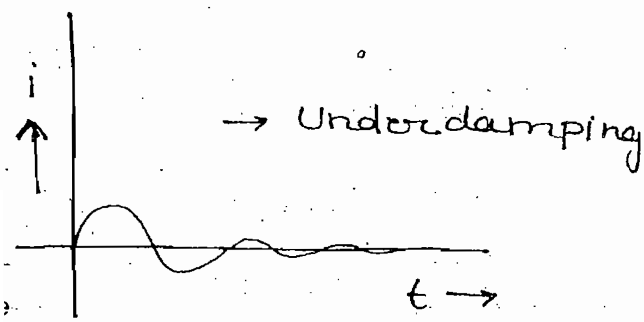
$$\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1 \longrightarrow \text{Series}$$

$$\delta \geq 1.$$



→ In the underdamping system more than 1 oscillation are present

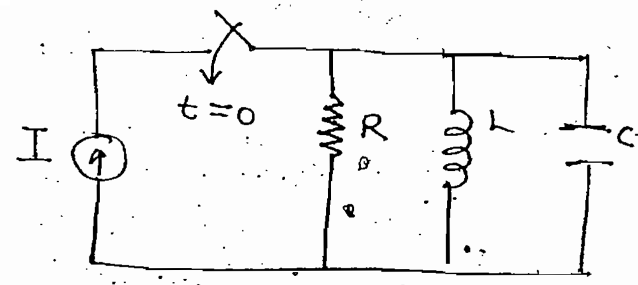
$$\delta < 1$$



5. At $t = \infty$

$L \rightarrow$ Charged $(I_L = I)$

$C \rightarrow$ Uncharged $(V_C = 0)$

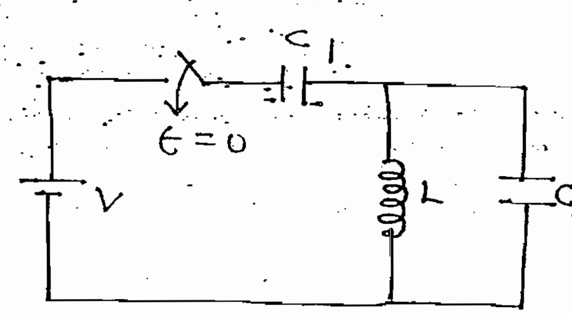


6. $t = \infty$

$C_1 \rightarrow$ charged $(V_{C_1} = V)$

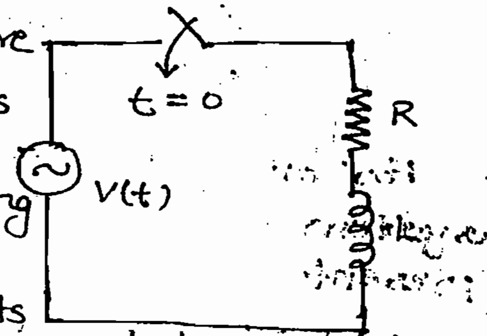
$C_2 \rightarrow$ Uncharged $(V_{C_2} = 0)$

$L \rightarrow$ Uncharged $(I_L = 0)$



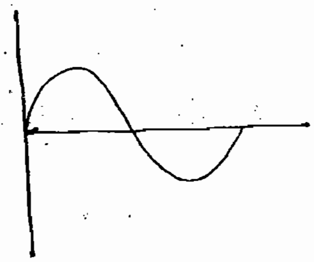
A.C Transients :-

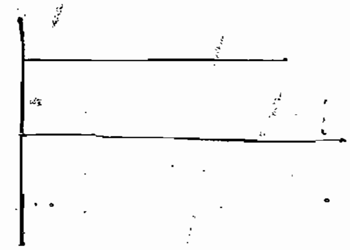
→ DC transients intensity is more than intensity of AC transients



→ Based on selection of operating frequency and switching operation and circuit elements it is possible to ~~op~~ obtain transient free response. But in DC circuit it is not possible to obtain transient free response

$V(t) = V_m \sin(\omega t + \theta)$





By KVL

$$V = iR + L\frac{di}{dt}$$

Divide both sides by L, we get

$$\boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}}$$

$$i(t) = \underbrace{C.F}_{\text{Transient Response}} + P.I$$

C.F :-

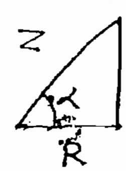
$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\Rightarrow i(t) = A e^{-\frac{Rt}{L}}$$

P.I → Steady state response

$$i = \frac{V}{Z\angle\alpha} = \frac{V\angle-\alpha}{Z}$$

$$\boxed{i(t) = \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}$$



$X_L = \omega L$

$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$i(t) = C.F + P.I$$

$$i(t) = Ae^{-\frac{R}{L}t} + \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)$$

$t = 0^-$, $i = 0$

$t = 0^+$, $i = 0$

$$0 = A + \frac{V_m}{Z}\sin(\theta - \alpha)$$

$$\Rightarrow \boxed{A = -\frac{V_m}{Z}\sin(\theta - \alpha)}$$

$$\boxed{i(t) = \underbrace{-\frac{V_m}{Z}\sin(\theta - \alpha)e^{-\frac{R}{L}t}}_{T.R.} + \underbrace{\frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}_{S.R}}$$

Case-(I) :-

$$V(t) = V_m \sin(\omega t + \theta) \quad , \quad t = 0$$

$$\theta - \alpha = 0$$

$$\theta = \alpha \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

→ condition for transient free response

Case-(II) :-

$$V(t) = V_m \cos(\omega t + \theta) \quad , \quad t = 0$$

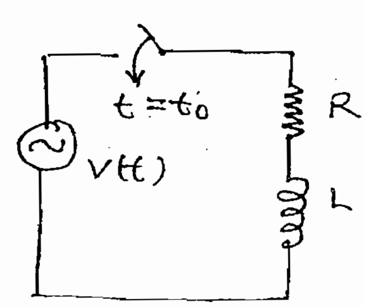
$$\boxed{\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}}$$

→ condition for transient free response

Case-(III) :-

$t = t_0^-$ $i = 0$

$t = t_0$ $i = 0$



$$0 = A + \frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha)$$

$$\Rightarrow \boxed{A = -\frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha)}$$

$$\boxed{i(t) = \underbrace{-\frac{V_m}{Z} \sin(\omega t_0 + \theta - \alpha)\, e^{-R/L (t - t_0)}}_{T.R} + \underbrace{\frac{V_m}{Z} \sin(\omega t + \theta - \alpha)}_{S.R}}$$

condition for transient free response for Case-(III) is

$$V(t) = V_m \sin(\omega t + \theta) \quad , \; t = t_0$$

$$\omega t_0 + \theta - \alpha = 0$$

$$\Rightarrow \omega t_0 = \alpha - \theta$$

$$\Rightarrow \boxed{\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta}$$

Case-(IV) :-

$$V(t) = V_m \cos(\omega t + \theta) \qquad t = t_0$$

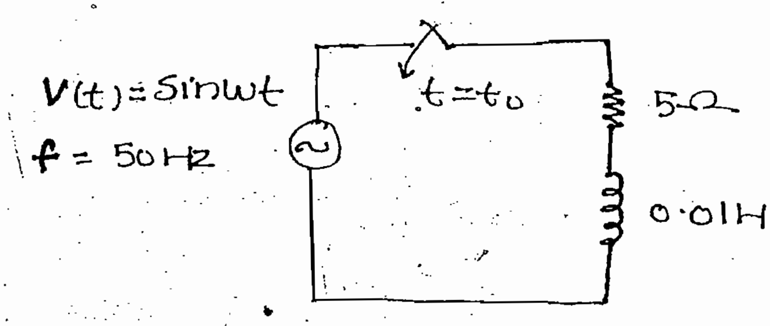
$$\boxed{\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}}$$

	RL		RC
(i) $V(t) = V_m \sin(\omega t + \theta)$, $t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$	$\theta = \tan^{-1}(\omega\tau)$	$\theta = \tan^{-1}(\omega R$
(ii) $V(t) = V_m \cos(\omega t + \theta)$, $t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}$	$\theta = \tan^{-1}(\omega\tau) + \frac{\pi}{2}$	$\theta = \tan^{-1}(\omega R$ $+ \frac{\pi}{2}$
(iii) $V(t) = V_m \sin(\omega t + \theta)$, $t = t_0$	$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$	$\omega t_0 = \tan^{-1}(\omega\tau) - \theta$	$\omega t_0 = \tan^{-1}(\omega R$ $- \theta$
(iv) $V(t) = V_m \cos(\omega t + \theta)$, $t = t_0$	$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$	$\omega t_0 = \tan^{-1}(\omega\tau) - \theta + \frac{\pi}{2}$	$\omega t_0 = \tan^{-1}(\omega R$ $- \theta + \frac{\pi}{2}$

→ The above condition for series circuit for transient free response and also for parallel circuit.

→ In the RLC circuit it is not possible to obtain transient free response since circuit is having two energy storage element (simultaneously both charging and discharging action are present)

Ques:- Find t_0 to obtain transient free response



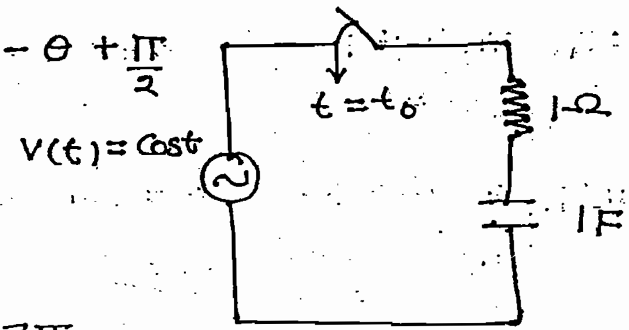
Soln:-

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$$

$$t_0 = \frac{\tan^{-1}\left(\frac{\omega L}{R}\right) \rightarrow \text{rad}}{\omega \rightarrow \text{rad/s}}$$

$$t_0 = \frac{32.1 \times \pi/180}{100\pi} = 1.78\,\text{ms}, \text{Ans}$$

Ques:- Find t_0 to obtain transient free response



Soln:- $\omega t_0 = \tan^{-1}(\omega RC) - \theta + \frac{\pi}{2}$

$$\Rightarrow t_0 = \frac{\tan^{-1}(1) + \pi/2}{1}$$

$$\Rightarrow t_0 = \frac{\frac{\pi}{4} + \frac{\pi}{2}}{1} = \frac{3\pi}{4} \quad (\pi = 22/7)$$

$$\Rightarrow t_0 = 7.35\,s \quad \text{Ans}$$

Laplace :-

$$A \rightarrow \frac{1}{s}$$

$$e^{at} \rightarrow \frac{1}{s-a}$$

$$\sin\omega t \rightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos\omega t \rightarrow \frac{s}{s^2+\omega^2}$$

$$e^{at}\sin\omega t \rightarrow \frac{\omega}{(s-a)^2+\omega^2}$$

$$e^{-at}\sin\omega t \rightarrow \frac{\omega}{(s-a)^2+\omega^2}$$

$$e^{-at}\cos\omega t \rightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

$$\frac{df}{dt} \Rightarrow sF(s) - f(0)$$

$$u(t) \rightarrow \frac{1}{s}$$

$$\delta(t) \rightarrow 1$$

$$\boxed{f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{t\to 0} f(t)} \rightarrow \text{Initial value theorem}$$

Ques:- Find initial value of the following functions:-

$$F(S) = \frac{(S+1)(S+3)}{S(2S+2)(S+4)}$$

Soln:- $f(0^+) = \lim_{S\to\infty} SF(S)$

$$= \lim_{S\to\infty} \frac{S \cdot S^2\left(1+\frac{1}{S}\right)\left(3+\frac{3}{S}\right)}{S \cdot S^2\left(2+\frac{2}{S}\right)\left(1+\frac{4}{S}\right)} = \frac{1}{2}$$

Ques:- Find initial value of the following function

$$f(t) = 3 + \delta(t)$$

Soln:- $F(S) = \frac{3}{S} + 1$

Note:-

For the above function initial value theorem can't be applied since to apply the initial value theorem denominator power should be greater than Numerator power

Final Value Theorem:-

$$\boxed{f(\infty) = \lim_{S\to 0} SF(S) = \lim_{t\to\infty} f(t)}$$

Ques:- Find final value of the following function

(I) $F(S) = \frac{(S+1)(S+3)}{S(S+2)(S+4)}$

(II) $f(t) = 3 + e^{2t}$

Soln:- (I) $f(\infty) = \lim_{S\to 0} SF(S)$

$$= \lim_{S\to 0} \frac{S(S+1)(S+3)}{S(S+2)(S+4)} = \frac{3}{8}$$

(II) $f(t) = 3 + e^{2t}$

$$F(S) = \frac{3}{S} + \frac{1}{S-2}$$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$\Rightarrow \quad V_c(t) = \frac{1}{C}\int_{-\infty}^{0^-} i\,dt + \frac{1}{C}\int_{0^-}^{t} i\,dt$$

$$\Rightarrow \quad V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i\,dt$$

At $t=0^+$

$$V_c(0^+) = V_c(0^-) + \underbrace{\frac{1}{C}\int_{0^+}^{0^+} i\,dt}_{=0}$$

$$V_c(0^+) = V_c(0^-) \quad \text{—— (I)}$$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i\,dt$$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\underbrace{\int_{0^-}^{0^+} \delta(t)\,dt}_{\text{Area} = 1} \qquad i \to \delta(t)$$

$$\boxed{V_c(0^+) = \frac{1}{C}} \quad \text{—— (II)}$$

$$w_c(0^+) = \frac{1}{2} C V_c^2(0^+) = \frac{1}{2} C \left(\frac{1}{C}\right)^2$$

$$\boxed{w_c(0^+) = \frac{1}{2C}} \quad \text{—— (III)}$$

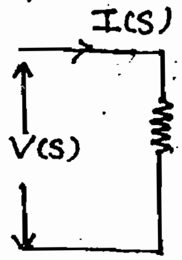
Note:—

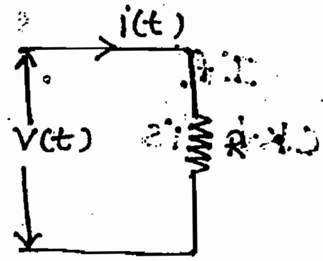
→ From eq-(I) it is concluded that capacitor does not allow instantaneous changes for given i/p.

→ From eq-(II) & (III) it is concluded that capacitor allows instantaneous changes for current impulse function.

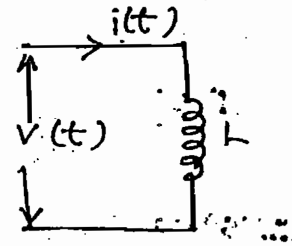
$V(t) = i(t) R$

$V(s) = I(s) R$





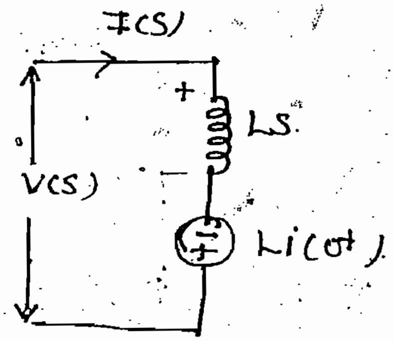
$V = L \dfrac{di}{dt}$ $\quad s = j\omega$



$V(s) = L\left[s I(s) - i(0^+)\right]$

$V(s) = L s I(s) - L i(0^+) \longrightarrow$ KVL

$\downarrow$

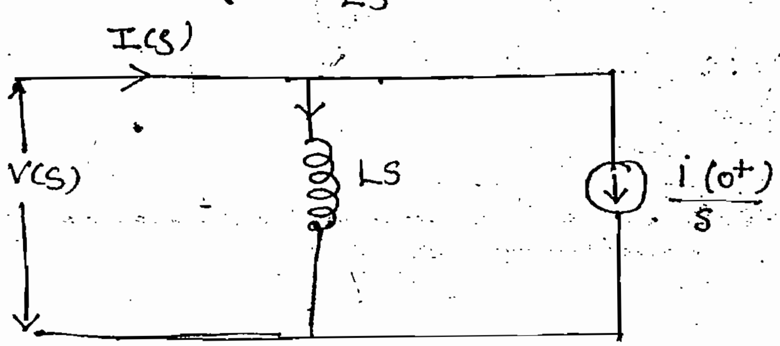


$Z(s) = \dfrac{V(s)}{I(s)}$

$\Rightarrow \boxed{Z(s) = Ls - \dfrac{L\, i(0^+)}{I(s)}}$

$L s I(s) = V(s) + L i(0^+)$

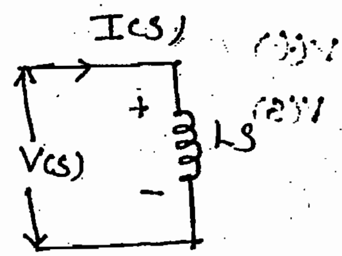
$I(s) = \dfrac{V(s)}{Ls} + \dfrac{L\, i(0^+)}{Ls} \longrightarrow$ KCL



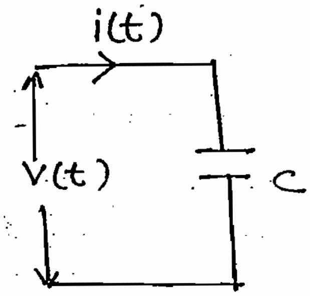
$Y(s) = \dfrac{I(s)}{V(s)}$

$\boxed{Y(s) = \dfrac{1}{Ls} + \dfrac{i(0^+)}{s V(s)}}$

If $i(0^+) = 0$ then eq. ckt is $\longrightarrow$



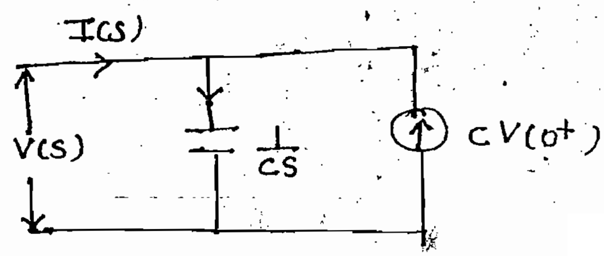
$i = C\frac{dV}{dt}$



$I(s) = C\left[sV(s) - V(0^+)\right]$

$\Rightarrow \quad I(s) = \frac{V(s)}{1/Cs} - CV(0^+) \longrightarrow$ KCL

$s = j\omega$

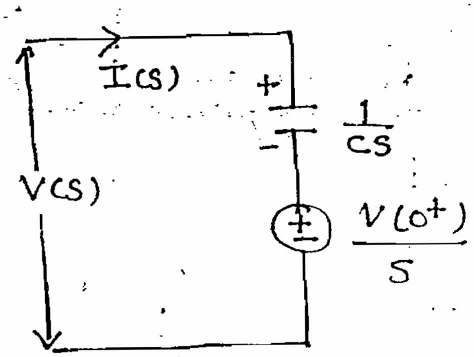


$Y(s) = \frac{I(s)}{V(s)}$

$$\boxed{Y(s) = Cs - \frac{CV(0^+)}{V(s)}}$$

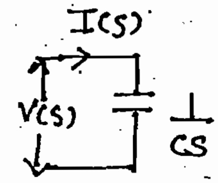
$\frac{V(s)}{1/Cs} = I(s) + CV(0^+)$

$\Rightarrow \quad V(s) = \frac{1}{Cs} I(s) + \frac{CV(0^+)}{Cs} \longrightarrow$ KVL

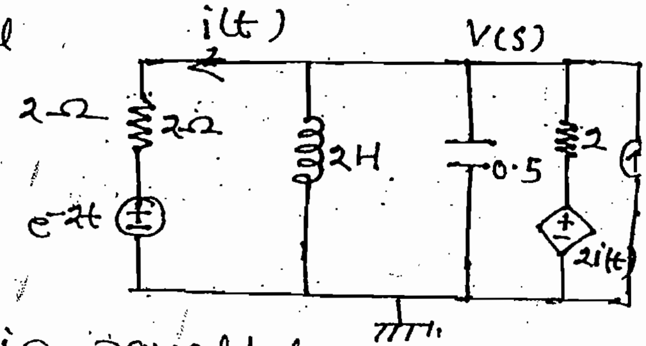


$$\boxed{Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs} + \frac{V(0^+)}{sI(s)}}$$

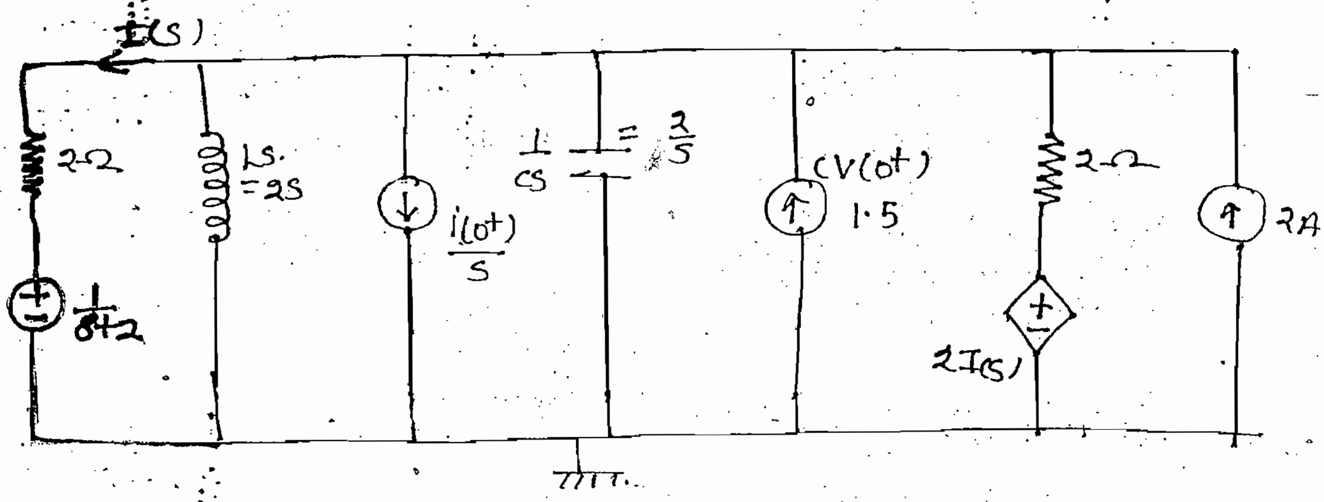
$V(0^+) = 0 \longrightarrow$



Ques:- Find V(s) when initial **current** of the inductor is **2A** and initial voltage of **the capacitor** is 3V



Soln:- When elements are in parallel then for simple calculation convert elements into current sources

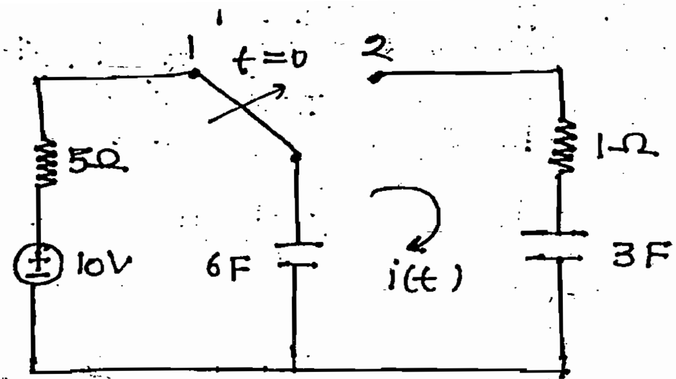


$$\frac{V(s) - \frac{1}{s+2}}{2} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{V(s)}{\frac{1}{Cs}} + \frac{V(s) - 2I(s)}{2} = 1.5 + 2$$

—(1)

$$I(s) = \frac{V(s) - \frac{1}{s+2}}{2}$$

Ques:- Find $i(t)$ for $t>0$

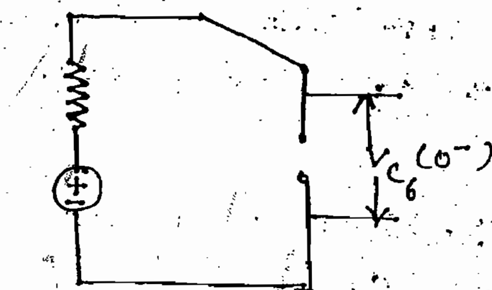


Soln:-

At $t=0^-$

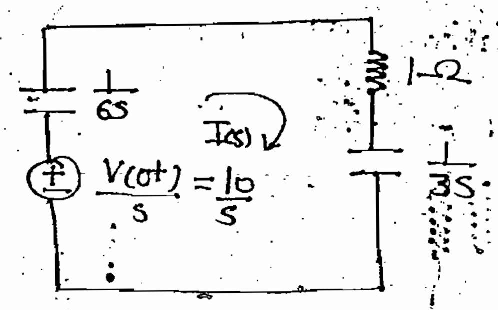
$V_{c_6}(0^-) = 10V$

$V_{c_3}(0^-) = 0V$



At $t>0$

$$I(s) = \frac{10/s}{\frac{1}{6s}+\frac{1}{3s}+1}$$



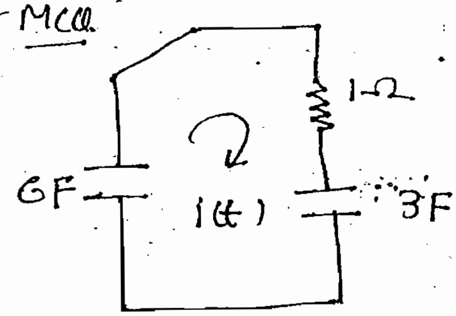
$i(t) = L^{-1} I(s)$

$\Rightarrow \quad i(t) = 10\, e^{-t/2}$

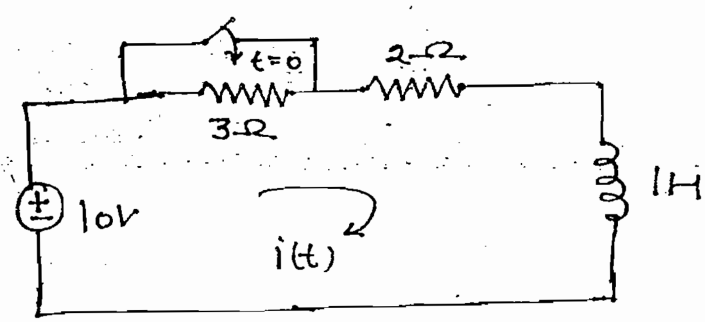
Verification :- → Method for MCQ

$$i(t) = \frac{V_0}{R}\, e^{-t/RC}$$

$$\boxed{i(t) = \frac{10}{1}\, e^{-t/2}}$$



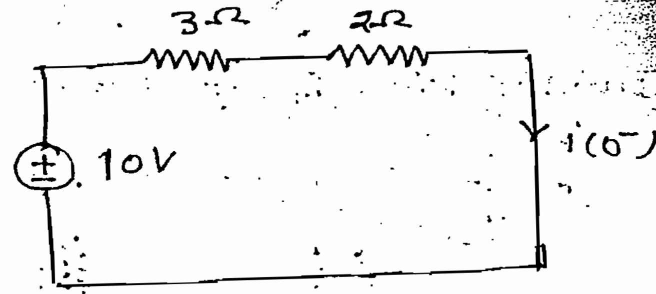
Ques:- Find $i(t)$ for $t>0$



Soln:-

At $t = 0^-$

$$i(0^-) = \frac{10}{3+2} = 2A.$$

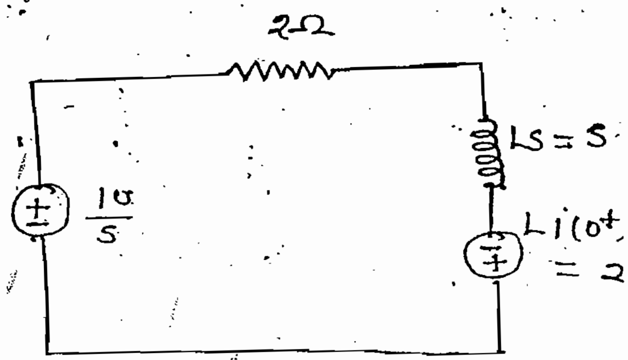


For $t > 0$

$$I(S) = \frac{\frac{10}{s} + 2}{2 + s}$$

$$i(t) = L^{-1} I(S)$$

$$\boxed{i(t) = 5 - 3e^{-2t}}$$



Alternate way!-

$$i(t) = [i(0^+) - i(\infty)] e^{-\frac{R}{L}t} + i(\infty)$$

$$i(0^+) = i(0^-) = 2$$

$$i(t) = [2 - 5] e^{-2t} + 5$$