CBSE Test Paper 05

Chapter 3 Matrices

1. If A and B are any two square matrices of the same order, then
   a. adj(AB) = adj(A) adj (B)
   b. $(AB)^t = B^tA^t$
   c. AB = O
   d. $(AB)^t = A^tB^t$
2. A square matrix A = $[a_{ij}]_{nxn}$ is called an upper triangular if a_{ij} = 0 for
   a. none of these
   b. i is less than j
   c. i = j
   d. i is greater than j
3. If A = $\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -7 \\ -3 & 7 & 0 \end{bmatrix}$, then, which of the following is true:
   a. None of these
   b. A = -A'
   c. A = -A
   d. A = A'
4. If the system of equations x + 4ay + az = 0, x + 3by + bz = 0 and x + 2 cy +cz = 0 have a non-zero solution,then a, b, c are in
   a. G.P.
   b. A.P.
   c. none of these
   d. H.P.
5. For what value of λ the following system of equations does not have a solution x + y + z = 6, 4x + λy - λz = 0, 3x + 2y – 4z = - 5 ?
   a. 1
   b. -3
   c. 0
   d. 3

6. Matrix multiplication is ________ over addition.
7. If A and B are symmetric matrices, then AB - BA is a ________ matrix.
8. If A and B are square matrices of the same order, then (AB)' = ________.

9. If a matrix has 8 elements, what are the possible orders it can have.?

10. If $\begin{bmatrix} 4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 4 & b+2 \\ 8 & a-8b \end{bmatrix}$, then write the value of a - 2b.

11. Solve the following matrix equation for x.
$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

12. Using elementary transformation, find the inverse of the matrix:
$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

13. If $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ Find A + A'.

14. Using elementary transformation, find the inverse of the matrices:
$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$.

15. If A is a square matrix such that $A^2 = A$, then prove that $(I + A)^3 - 7A$ is equal to I.

16. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$, $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$, then show that AB is a zero matrix if α and β differ by an odd multiple of $\frac{\pi}{2}$.

17. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$,Show that $(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the identify matrix of order 2 and $n \in N$.

18. For a matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that:

    i. (A + A’) is a symmetric matrix.
    ii. (A – A’) is a skew symmetric matrix.

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Solution

1. b. $(AB)^t = B^tA^t$, **Explanation:** By the property of transpose, (AB)' = B'A'
2. d. i is greater than j

   Explanation: Upper Triangular matrix is given by : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$. Here,

   $a_{ij} = 0$, if i is greater than j.and $a_{ij} \neq 0$, if i is less than j.
3. b. A = - A', **Explanation:** The given matrix is a skew – symmetric matrix.,therefore , A = - A'
4. d. H.P., **Explanation:** For a non trivial solution: $\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$

   $\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 0 & 3b-4a & b-a \\ 0 & 2c-4a & c-a \end{vmatrix} = 0 \Rightarrow bc + ab - 2ac = 0 \Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$.

   Therefore , a , b, c are in H.P.
5. d. 3, **Explanation:** The given system of equations does not have solution if

   $\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4+\lambda & 2\lambda & -\lambda \\ 7 & 6 & 4 \end{vmatrix} = 0$
6. distributive
7. skew-symmetric
8. B'A'
9. $1 \times 8,\ 8 \times 1,\ 4 \times 2,\ 2 \times 4$
10. According to the question,We are given that, $\begin{bmatrix} 4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 4 & b+2 \\ 8 & a-8b \end{bmatrix}$

    Equating the corresponding elements,

    $3b = b + 2$ ......(i)

    $-6 = a - 8b$...(ii)

On solving the Eqs. (i) and (ii) , we get,

a = 2 and b = 1 .

Now, a - 2b = 2 - 2(1) = 2 - 2 = 0

11. According to the question, $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$

Using matrix multiplication,

$\Rightarrow \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

Equating the corresponding elements,

$\Rightarrow x - 2 = 0$

$\Rightarrow$ x = 2

12. Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Since A = IA $\Rightarrow \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2$,

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 + R_1$,

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2$,

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

13. $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

14. Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Since A = IA $\Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$,

$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$,

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$,

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \ [R_1 \to R_1 - R_2]$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

15. $(I + A)^3 - 7A = I^3 + A^3 + 3IA(I + A) - 7A$

$= I + A^3 + 3I^2A + 3IA^2 - 7A$

$= I + A^3 + 3A + 3A^2 - 7A$

$= I + A^3 + 3A + 3A - 7A$ {since $A^2 = A$}

$= I + A^3 - A$

$= I + A.A^2 - A$

$= I + A.A - A[A^2 = A]$

$= I + A^2 - A$

$= I + A - A[A^2 = A]$

$= I$

16. $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}, B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$

$$AB = \begin{bmatrix} \cos^2\alpha\cos^2\beta + \cos\alpha\cos\beta\sin\alpha\sin\beta & \cos^2\alpha\cos\beta\sin\beta + \cos\alpha\sin\alpha\sin^2\beta \\ \cos\alpha\cos^2\beta\sin\alpha + \sin^2\alpha\cos\beta\sin\beta & \cos\alpha\sin\alpha\cos\beta\sin\beta + \sin^2\alpha\sin^2\beta \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \cos\alpha\sin\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix}$$

AB = O if $\cos(\alpha - \beta) = 0$

$\Rightarrow \alpha - \beta = (2n+1)\frac{\pi}{2}$

or $\alpha - \beta$ is the odd multiple of $\frac{\pi}{2}$

17. When n = 1

$(aI+bA)^1 = a^1I + 1.a^{1-1}bA$

$aI + bA = aI + bA$

L.H.S = R.H.S

The result is true for n = 1.

When n = k

$(aI + bA)^K = a^K I + Ka^{K-1}bA$..........; (i)

Assume that the result is true for n = k

When n = k + 1

$(aI + bA)^{k+1} = (aI + bA).\ (aI + bA)^k$

$= (aI + bA).\ (a^k I + ka^{k-1}bA)$ [From (i)]

$= a^{k+1}I + ka^k bA + a^k bA + ka^{k-1} b^2A^2 \begin{bmatrix} \because II = I \\ IA = A = AI \end{bmatrix}$

$= a^{k+1} I + (k+1)\ a^k bA \left[\because A^2 = 0\right]$

Hence result is true for n = k+1, when ever it is true for n = k

Hence ,by the principle of mathematical induction the result is true for all n in N.

18. i. Given: $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

Let B = A + A' $= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 5+6 \\ 6+5 & 7+7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\therefore B' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = B$

$\therefore$ B = A + A' is a symmetric matrix.

ii. Given: $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

Let B = A – A' $= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1-1 & 5-6 \\ 6-5 & 7-7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\therefore B' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Taking (-1) common, $-\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -B$

$\therefore$ B = A – A' is a skew-symmetric matrix.