6. PRESSURE VESSELS

TYPES OF PRESSURE VESSELS

Pressure vessels are mainly of two type:

(i) Thin shells

If the thickness of the wall of the shell is less than 1/10 to 1/15 of its diameter, then shell is called thin shells.

$$t < \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

> **Remember**
> For thin shell, it is assumed that the normal stresses, which may be ***either*** tensile or compressive are ***uniformly*** distributed through the thickness of wall.

(ii) Thick Shells

If the thickness of the wall of the shell is greater than 1/10 to 1/15 of its diameter, then shell is called thick shells.

$$t > \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

NATURE OF STRESS IN THIN CYLINDRICAL SHELL SUBJECTED TO INTERNAL PRESSURE

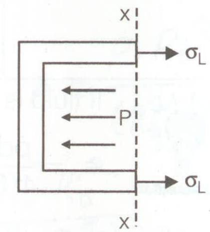
(i) Hoop stress /circumferential stress will be tensile in nature.

(ii) Longitudinal stress/axial stress will be tensile in nature.

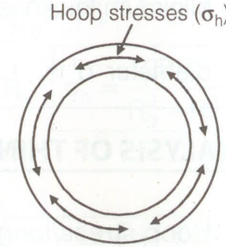
(iii) Radial stress will be compressive in nature.

> **Remember**
> Radial compressive stress varies from a value at the inner surface equal to pressure 'P' to the atmospheric pressure at the outside surface.
>
> If internal pressure in thin cylinders is low, the radial stress is negligible compared with axial stress and hoop stress. This ***radial stress*** is neglected.

ANALYSIS OF THIN CYLINDER

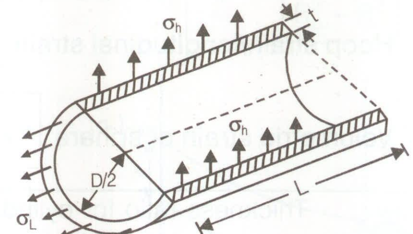


- Longitudinal Stress $\sigma_L = \frac{pd}{4t}$



- Hoop Stress $\sigma_h = \frac{pd}{2t}$

- Longitudinal Strain



$$\epsilon_L = \frac{pd}{4t\,E}(1-2\mu)$$

- Hoop Strain $\epsilon_h = \frac{pd}{4tE}(2-\mu)$

Here, p = Pressure of fluid, t = Thickness of cylinder
d = Inside diameter, μ = Poisson's ratio

- Ratio of Hoop Strain to Longitudinal Strain $\frac{\epsilon_h}{\epsilon_L} = \frac{2-\mu}{1-2\mu}$

- Volumetric Strain (ϵ_v) of Cylinder $\epsilon_v = \frac{pd}{4t\,E}(5-4\mu)$

- Max shear stress in the plane of metal (x-y plane) or $\sigma_h - \sigma_L$ plane

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_h - \sigma_L}{2}$$

$$\tau_{max} = \frac{PD}{8t}$$

- Absolute max shear stress

$$\tau_{obs.max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{PD/2t - 0}{2} = \frac{PD}{4t}$$

> If fluid is compressible, volumetric strain will be
>
> $$\epsilon_V = \frac{pd}{4t\,E}(5 - 4\mu) + \frac{p}{K}$$ k = Bulk modulus of fluid
>
> P = Pressure of fluid
>
> Minimum thickness of cylinder required for a given pressure 'P' and diameter 'd' is $t \geq \frac{pd}{2\sigma}$

ANALYSIS OF THIN SPHERE

- Hoop stress/longitudinal stress $\sigma_L = \sigma_h = \frac{pd}{4t}$
- Hoop strain/longitudinal strain $\epsilon_L = \epsilon_h = \frac{pd}{4t\,E}(1 - \mu)$
- Volumetric strain of sphere $\epsilon_V = \frac{3pd}{4t\,E}(1 - \mu)$

> Thickness ratio for cylindrical shell (t_c) and sphere (t_s), for *same strain* in both side.
>
> $$\frac{t_c}{t_s} = \frac{2 - \mu}{1 - \mu}$$
>
> Thickness ratio for cylindrical shell (t_c) and sphere (t_s), for *same maximum stress* in both side.
>
> $$\frac{t_c}{t_s} = 2$$
>
> Auto frittage is used for prestressing the cylinder.
>
> *Wire winding* is done for *strenghting thin shell. Compounding* is done for *thick* shell cylinders.

ANALYSIS OF THICK CYLINDERS/LAME'S THEOREM

- Lame's Assumption
  (i) Material of shell is homogeneous, isotropic and linear elastic.
  (ii) Plane section of cylinder, perpendicular to longitudinal axis remains plane under pressure.

- Lame's equations
  (i) Hoop stress: $\sigma_x = \frac{B}{x^2} + A$ (tensile)
  (ii) Radial stress: $P_x = \frac{B}{x^2} - A$ (compressive)

  Where, B and A are Lame's constant

- **Subjected to internal pressure**

  (i) At $x = R_i$, $\sigma_h = \frac{P[R_o^2 + R_i^2]}{R_o^2 - R_i^2}$ (ii) At $x = R_o$, $\sigma_h = \frac{2PR_i^2}{R_o^2 - R_i^2}$

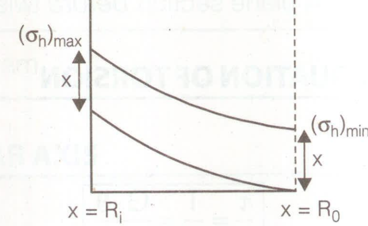
- **Subjected to external pressure**

  (i) At $x = R_i$, $\sigma_h = \frac{-2PR_o^2}{R_o^2 - R_i^2}$ (ii) At $x = R_o$, $\sigma_h = \frac{-P[R_o^2 + R_i^2]}{[R_o^2 - R_i^2]}$

Here, R_i = Inner radius

R_0 = Outlet radius

$$(\sigma_h)_{max} = p + (\sigma_h)_{min}$$



$$\sigma_x - P_x = \left(\frac{B}{x^2} + A\right) - \left(\frac{B}{x^2} - A\right) = 2A$$

> Radial and hoop compression vary *hyperbolically*.

ANALYSIS OF THICK SPHERES

- Lame's equation:

$$\sigma_x = \frac{2B}{x^3} + A\,(\text{Tensile}) \qquad P_x = \frac{2B}{x^3} - A\,(\text{compressive})$$

■■■