Chapter 3

Powers and Exponents

3.1 In earlier class, we have learnt about exponential form of numbers. Let us recall these numbers:

$$10^3, 2^{10}, 5^5$$

How to express these numbers in extended form? Let us try to do this.

$10^3 = 10 \times 10 \times 10$

$2^{10} = \dots\dots\dots\dots$

$5^5 = \dots\dots\dots\dots$

With this, we also learned that $10^2 \times 10^4 = 10^{2+5} = 10^7$

and $\dfrac{2^5}{2^3} = 2^{5-3} = 2^2$

i.e., when two numbers, having the common base, are multiplied then powers of their bases are added and on dividing, powers are subtracted.

$$a^m \times a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \text{ when } m > n$$

and $(a^m)^n = a^{mn}$

In this chapter, we shall study about the other problems related to the exponents.

3.2 Exponent (Integers), Base (Rational numbers ≠ 0)

Consider the following exponent of rational numbers.

1. $\left(\frac{5}{7}\right)^4 = \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}$

   $\frac{5 \times 5 \times 5 \times 5}{7 \times 7 \times 7 \times 7} = \frac{5^4}{7^4}$

2. $\left(\frac{-3}{11}\right)^5 = \left\{(-1) \times \left(\frac{3}{11}\right)\right\}^5 = (-1)^5 \times \left(\frac{3}{11}\right)^5$

   $= (-1) \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \quad [\because (-1)^5 = -1]$

   $= -\frac{3^5}{11^5}$

3. $\left(\frac{-4}{3}\right)^6 = (-1)^6 \times \left(\frac{4}{3}\right)^6$

$= \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3}$ $\quad [\because (-1)^6 = 1]$

$= \frac{4^6}{3^6}$

So, if we have any rational number $\left(\frac{5}{4}\right)^m$, then

$$\left(\frac{5}{4}\right)^m = \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \text{----------} \text{ (m times)}$$

$$= \frac{5 \times 5 \times 5 \times \text{----------} \text{ m times}}{4 \times 4 \times 4 \times \text{----------} \text{ m times}} = \frac{5^m}{4^m}$$

Do and Learn: Extend the following

$\left(\frac{3}{2}\right)^3, \left(\frac{9}{4}\right)^5, \left(-\frac{4}{7}\right)^6, \left(-\frac{2}{5}\right)^3, \left(\frac{2}{3}\right)^p$

If $\left(\frac{p}{q}\right)$ is any rational number raised to the power m, where $q \neq 0$, then

$$\left(\frac{p}{q}\right)^m = \frac{p}{q} \times \frac{p}{q} \times \frac{p}{q} \times \text{------} \text{ (m times)}$$

$$= \frac{p \times p \times p \times \text{-----} \text{(m times)}}{q \times q \times q \times \text{-----} \text{(m times)}} = \frac{p^m}{q^m}$$

i.e., $\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$ where, p and q are integers and $q \neq 0$

Now, if the power of rational number is negative then what would be the condition? Let us consider the following examples:

(i) $\left(\frac{5}{4}\right)^{-2}$

$= \frac{5^{-2}}{4^{-2}}$

$= \frac{\frac{1}{5^2}}{\frac{1}{4^2}}$

$= \frac{4^2}{5^2}$

$= \left(\frac{4}{5}\right)^2$

(ii) $\left(\frac{3}{7}\right)^{-4}$

$= \frac{3^{-4}}{7^{-4}}$

$= \frac{\frac{1}{3^4}}{\frac{1}{7^4}}$

$= \frac{7^4}{3^4}$

$= \left(\frac{7}{3}\right)^4$

(iii) $\left(\frac{2}{5}\right)^{-m}$

$= \frac{2^{-m}}{5^{-m}}$

$= \frac{\frac{1}{2^m}}{\frac{1}{5^m}}$

$= \frac{5^m}{2^m}$

$= \left(\frac{5}{2}\right)^m$

$\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ and } a^{-m} = \frac{1}{a^m}\right]$

Do and Learn: Express the following with positive exponent-

$$\left(\frac{7}{5}\right)^{-5}, \left(\frac{14}{13}\right)^{-9}, \left(\frac{15}{6}\right)^{-4}, \left(\frac{113}{53}\right)^{-11}, \left(\frac{5}{7}\right)^{-7}$$

Rethink on $\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^{m}}{a^{m}} = \left(\frac{b}{a}\right)^{m}$

Similarly, it is clear that

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$$

Here a and b are integers. And

$$a \neq 0 , b \neq 0$$

Look at the following actions

$$5^4 \div 5^4 = 5^{4-4} = 5^0$$

$$5^4 \div 5^4 = \frac{5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 1$$

so $5^0 = 1$

Thus, any base with power 0 (zero) the result is always 1. For example,

i.e. (i) $(3)^4 \div (3)^4 = 3^{4-4} = 3^0 = 1$

(ii) $(-5)^6 \div (-5)^6 = (-5)^{6-6} = (-5)^0 = 1$

(iii) $\left(\frac{2}{5}\right)^3 \div \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^{3-3} = \left(\frac{2}{5}\right)^0 = 1$

From the above, it is clear that any number (except 0) with raised to the power 0, result is always 1.

If a is any rational number, then $a^0 = 1, (a \neq 0)$

Do and Learn:
Simplify the following-

(i) $\left(\frac{2}{7}\right)^{-3}$ (ii) $\left(\frac{3}{10}\right)^{-2}$ (iii) $\left(\frac{5}{12}\right)^{-3}$

(iv) $(3)^2 \div (3)^2$ (v) $(2)^5 \div (2)^5$

Ex. 1 : Find the value of $7^8 \div 7^8$

Sol. : $7^{8-8} = 7^0 = 1$

Ex. 2 : Find the value of $\left(\frac{4}{7}\right)^5 \div \left(\frac{4}{7}\right)^5$

Sol.

$\left(\frac{4}{7}\right)^5 \div \left(\frac{4}{7}\right)^5$

$\left(\frac{4}{7}\right)^{5-5} = \left(\frac{4}{7}\right)^0 = 1$

Ex. 3 : Find the value of $(2^3)^2$

Sol. $(2^3)^2$

$= 2^{3\times 2}$

$= 2^6$

$(2^3)^2$	$(2^3)^2$
$= 2^3 \times 2^3 = 2^{3+3} = 2^6 = 64$	$= 2^{3 \times 3} = 2^9 = 512$

Ex. 4: Simplify the following:

1. $\left(\frac{5}{7}\right)^4 \times \left(\frac{7}{5}\right)^2$

Sol.

$= \left(\frac{5}{7}\right)^4 \times \left(\frac{7}{5}\right)^2$

$= \left(\frac{5}{7}\right)^4 \times \left(\frac{5}{7}\right)^{-2}$

$= \left(\frac{5}{7}\right)^{4+(-2)}$

$= \left(\frac{5}{7}\right)^2$

$= \frac{5^2}{7^2}$

$= \frac{25}{49}$

2. $\left(-\frac{2}{9}\right)^{-4} \times \left(\frac{9}{2}\right)^2$

Sol.

$= \left(-\frac{2}{9}\right)^{-4} \times \left(\frac{9}{2}\right)^2$

$= \left(-\frac{9}{2}\right)^4 \times \left(\frac{9}{2}\right)^2$

$= (-1)^4 \times \left(\frac{9}{2}\right)^4 \times \left(\frac{9}{2}\right)^2$

$= 1 \times \left(\frac{9}{2}\right)^{4+2}$

$= \left(\frac{9}{2}\right)^6$

$= \frac{531441}{64}$

Exercise 3.1

1. Simplify the following:

(i) $\left(\frac{2}{7}\right)^3 \times \left(\frac{1}{2}\right)^3$ (ii) $\left(\frac{4}{5}\right)^4 \times \left(\frac{5}{4}\right)^2$

(iii) $(-5)^3 \times \left(-\frac{1}{5}\right)^2$ (iv) $\left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^{-5}$

2. Find the value.

(i) $(-5)^3$ (ii) $\left(\frac{1}{2}\right)^3$ (iii) $\left(-\frac{2}{3}\right)^4$

3. With the help of prime factor, change the following into exponent form.

(i) $\frac{1}{64}$ (ii) $\frac{16}{125}$ (iii) $-\frac{8}{27}$

(iv) $-\frac{1}{8}$ (v) $-\frac{25}{49}$

4. Find the value.

(i) $3^2 \times 3^3$ (ii) $\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$ (iii) $\left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^3$

(iv) $\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{2}\right)^4$ (v) $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{2}{5}\right)^3$

5. Answer in exponent form.

(i) $4^5 \div 4^2$ (ii) $(-5)^7 \div (-5)^4$

(iii) $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^4$ (iv) $\left(-\frac{1}{5}\right)^{11} \div \left(-\frac{1}{5}\right)^6$

6. Find the value.

(i) $(3^2)^3$ (ii) $(2^3)^2$ (iii) $(5^2)^2$

(iv) $(-2^4)^2$ (v) $\left[\left(\frac{1}{2}\right)^2\right]^4$ (vi) $\left[\left(-\frac{1}{3}\right)^3\right]^2$

7. Find the value.

(i) 3^0 (ii) 7^{5-5} (iii) $(-2)^{3-3}$

(iv) $\left(\frac{2}{5}\right)^{2+3-5}$ (v) $2^0 \times 3^0$ (vi) $2^0 + 5^0$

(vii) $\left(\frac{7}{15}\right)^0 + \left(\frac{1}{7}\right)^{3-3}$

8. Change into positive exponent numbers.

(i) 2^{-3} (ii) 3^{-5} (iii) a^{-4} (iv) $(-2)^{-5}$

(v) $(-x)^{-3}$ (vi) $\frac{1}{5^{-3}}$ (vii) $\frac{1}{y^{-3}}$ (viii) $\frac{1}{\left(\frac{2}{3}\right)^{-3}}$

9. Simplify the following in form of exponent.

(i) $(2^2 \times 3^3)^2$ (ii) $\left(\frac{15}{16}\right)^3 \div \left(\frac{9}{8}\right)^2$ (iii) $\left(\frac{4}{9}\right)^2 \div \left(\frac{28}{27}\right)^3$

(iv) $\left(\frac{2}{3}\right)^2 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^2$ (v) $\left(\frac{5^2}{3^2}\right)^2$ (vi) $\left[\frac{2^2 \times 3^2}{2^3 \times 6^2}\right]^2$

10. Find the value of $\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

3.3 Questions of more than one operations:

Ex. 5 Solve $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

Sol. $\left\{\left(\frac{3}{1}\right)^{2} - \left(\frac{2}{1}\right)^{3}\right\} \div \left(\frac{4}{1}\right)^{2}$

$= (3^2 - 2^3) \div 4^2$

$= (9 - 8) \div 16$

$= \frac{1}{16}$

Ex. 6 Solve $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Sol. $= \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \left(\frac{2+1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \frac{\cancel{3}^{1}}{\cancel{8}_{4}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = \frac{1}{4}$

Ex. 7 If $(-2)^{x+1} \times (-2)^3 = (-2)^5$ then find the value of x.

Sol. $(-2)^{x+1} \times (-2)^3 = (-2)^5$

Or $(-2)^{x+1+3} = (-2)^5$

Or $(-2)^{x+4} = (-2)^5$

Since, the bases are same. Therefore, powers are put equivalent.

$x + 4 = 5$

Or $x = 5 - 4 = 1$

Ex. 8 Solve $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Sol.

$$\frac{3^{-5} \times (2 \times 5)^{-5} \times (5 \times 5 \times 5)}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$= \frac{5^{-5} \text{ x } 5^3}{5^{-7}}$$

$$= \frac{5^{-5+3}}{5^{-7}} = \frac{5^{-2}}{5^{-7}} = 5^{-2+7}$$

$$= 5^5 = 3125$$

Ex. 9 Find the value of $\left(\frac{9}{8}\right)^{-3} \times \left(\frac{8}{9}\right)^{-2}$

Sol.

$$\left(\frac{9}{8}\right)^{-3} \times \left(\frac{8}{9}\right)^{-2}$$

$$= \left(\frac{8}{9}\right)^{3} \times \left(\frac{9}{8}\right)^{2}$$

$$= \frac{8^3}{9^3} \times \frac{9^2}{8^2}$$

$$= \frac{8^3}{8^2} \times \frac{9^2}{9^3}$$

$$= \frac{8^{3-2}}{9^{3-2}} = \frac{8}{9}$$

One more method

$$\left(\frac{8}{9}\right)^{3} \times \left(\frac{8}{9}\right)^{-2}$$

$(a^m \times a^n = a^{m+n})$

$$= \left(\frac{8}{9}\right)^{3-2} = \left(\frac{8}{9}\right)^{1} = \frac{8}{9}$$

Exercise 3.2

1. Find the value

(i) $(5^{-1} \times 2^{-1}) \div 6^{-1}$ (ii) $\left(\frac{5}{6}\right)^{6} \times \left(\frac{5}{6}\right)^{-4}$ (iii) $\left(\frac{5}{8}\right)^{-2} \times \left(\frac{8}{5}\right)^{-5}$ (iv) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{0}$

2. Simplify

(i) $\frac{16^{-1} \times 5^3}{2^{-4}}$ (ii) $\frac{25 \times t^{-4}}{5^{-3} \times 5 \times t^{-8}}$, $(t \neq 0)$

(iii) $\frac{6^3 \times 7^4 \times 8^5}{4^3 \times 9^2 \times 16}$ (iv) $\frac{15^3 \times 18^2}{3^5 \times 5^4 \times 12^2}$ (v) $\left(\frac{6}{15}\right)^3 \div \left(\frac{25}{32}\right)^2 \times \left(\frac{45}{16}\right)^3$

3. Find the value of x

(i) $\left(\frac{4}{3}\right)^{-4} \times \left(\frac{4}{3}\right)^{-5} = \left(\frac{4}{3}\right)^{-3x}$ (ii) $7^x \div 7^3 = 7^5$ (iii) $4^{2x+1} \div 16 = 64$

4. Find the value

(i) $\frac{3125 \times 1296}{6561 \times 1875}$ (ii) $\frac{1536 \times 972}{486 \times 1152}$

$\left[\text{Hint } \frac{3125 \times 1296}{6561 \times 1875} = \frac{5^5 \times 2^4 \times 3^4}{3^8 \times 3 \times 5^4}\right]$

3.4.1 Scientific Notation

In the previous classes, we have studied how large numbers are written in its standard form.

(i) $3{,}00{,}000 = 3 \times 1{,}00{,}000 = 3 \times 10^5$

(ii) $15{,}00{,}00{,}000 = 15 \times 1{,}00{,}00{,}000 = 1.5 \times 10^8$

(iii) $78{,}00{,}00{,}00{,}000 = 78 \times 1{,}00{,}00{,}000 = 7.8 \times 10^{10}$

Similarly,

(iv) $0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$

(v) $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

(vi) $0.0001 = \frac{1}{10000} = \frac{1}{10^4} = 10^{-4}$

As in previous class, we represented large numbers in standard form easily, can small numbers be written in standard form like this?

Ex. -Diameter of Red Blood Cell = 0.0000007 metre

Diameter of Wire of computer chip = 0.0000003 metre

In above example we have seen $0.0001 = \frac{1}{10000} = \frac{1}{10^4} = 10^{-4}$

Like this $0.0000007 = \frac{7}{10000000} = \frac{7}{10^7} = 7 \times 10^{-7}$

$0.0000003 = \frac{3}{10000000} = \frac{3}{10^7} = 3 \times 10^{-7}$

Other example $0.0000058 = \frac{58}{10000000} = \frac{58}{10^7} = \frac{5.8 \times 10}{10^7}$

$= 5.8 \times 10^1 \times 10^{-7} = 5.8 \times 10^{-6}$

Similarly, small numbers can easily be expressed in standard form.

Ex. 10 Express 150000000 in standard form.

Sol. $15 \times 10^7 = \frac{15}{10} \times 10^1 \times 10^7$

$= 1.5 \times 10^8$

Note:
In large number
1.50000000 decimal shift to 8 place on left side.

In small number
0.0000005 decimal shift to 7 place on right side.

Ex. 11: Express the measurement of virus 0.0000005 metre in standard form.

$= \frac{5}{10000000} = \frac{5}{10^7}$

$= 5 \times 10^{-7}$

Ex. 12 Express the following numbers in normal form :-

(i) 2.43×10^6 (ii) 9.3×10^{-5} (iii) 3×10^{-6}

Sol. (i) $2.43 \times 10^6 = 2.43 \times 10{,}00{,}000 = 2430000$

(ii) $9.3 \times 10^{-5} = \frac{9.3}{10^5} = \frac{9.3}{100000} = 0.000093$

(iii) $3 \times 10^{-6} = \frac{3}{10^6} = \frac{3}{1000000} = 0.000003$

Do and Learn

1. Express the following numbers in standard form

(i) 20700000 (ii) 0.000000154 (iii) 0.000095
(iv) 28400000 (v) 0.00002459

2. Express the following numbers in normal form

(i) 1.5×10^5 (ii) 2.78×10^3 (iii) 3.9×10^{-5}

3.4.2 Comparison of largest numbers with smallest numbers

If the mass of Earth and the Moon is 5.97×10^{24} kg and 7.35×10^{22} kg respectively, then how much mass of the earth is more than the moon.

On subtracting $= (5.97 \times 10^{24}\,kg) - (7.35 \times 10^{22}\,kg)$

$= (5.97 \times 100 \times 10^{22}\,kg) - (7.35 \times 10^{22}\,kg)$

$= 10^{22}(597 - 7.35)\,kg$ (by taking 10^{22} as common factor)

$= 10^{22} \times 589.65\,kg$

The Earth has $10^{22} \times 589.65$ kg mass more.

Similarly, if the distance between the Sun and the Earth is 1.496×10^{11} meter and the distance between the Earth and the Moon is 3.84×10^{8} meter, then the difference between these distances is

$= (1.496 \times 10^{11}\,m) - (3.84 \times 10^{8}\,m)$

$= (1.496 \times 1000 \times 10^{8}\,m) - (3.84 \times 10^{8}\,m)$

$= (1.496 \times 1000 - 3.84)\,10^{8}\,m$

$= (1496 - 3.84)\,10^{8}m$

$= 1492.16 \times 10^{8}\,m$

Note: When we subtract the numbers written in standard form then we change these in equal power of 10.

Comparison of the small numbers:

Size of a red blood cell $= 0.0000007\,m = 7 \times 10^{-6}\,m$

Size of a plant cell $= 0.00001275\,m = 1.1275 \times 10^{-5}\,m$

Difference between these $= (1.275 \times 10^{-5} - 7 \times 10^{-6})\,m$

$= (1.275 \times 10^{-5} - 7^{-1} \times 10^{-5})\,m$

$= (1.275 \times 0.7) \times 10^{-5}\,m$

$= 0.575 \times 10^{-5}\,m = 5.75 \times 10^{-6}\,m$

Compare it by dividing =

$$\frac{\text{Size of a red blood cell}}{\text{Size of a plant cell}} = \frac{1.275 \times 10^{-5}\,m}{7 \times 10^{-6}\,m}$$

$$\frac{1.275 \times 10^{-5-(-6)}}{7} = \frac{1.275 \times 10^{1}}{7} = \frac{12.75}{7} \cong 2 \text{ (approx. less than 2)}$$

Comparison of large number by division.

Diameter of the sun is (1.4×10^9)m and the diameter of the earth is (1.2756×10^7) m. Let us compare their diameters

$$\frac{\text{Diameter of the sun}}{\text{Diameter of the earth}} = \frac{(1.4\times10^9)\text{ m}}{(1.2756\times10^7)\text{ m}} = \frac{1.4\times10^{9-7}}{1.2756} = \frac{1.4 \text{ x } 10^2}{1.2756}$$

$$= \frac{1.4 \text{ x } 100}{1.2756} \text{ (which is approx 100 times)}$$

Exercise 3.3

1. Change in standard form
   (i) 128000000 (ii) 1680000000 (iii) 0.0005
   (iv) 0.00000017 (v) 0.000000000397 (vi) 0.00000004358

2. Express the following numbers in normal form
   (i) $4 \text{ x } 10^9$ (ii) $245 \text{ x } 10^7$ (iii) $5.61729 \text{ x } 10^7$
   (iv) $8.5 \text{ x } 10^{-6}$ (v) $3.02 \text{ x } 10^{-6}$ (vi) $7 \text{ x } 10^{-4}$
3. Change the numbers in standard form of the following statements -
   (i) Diameter of the thickness of human hair is 0.0002cm approx.
   (ii) Charge on an electron is 0.000,000,000,000,000,00016 coulomb.
   (iii) 1 Micron $= \frac{1}{100000}$ meter.
   (iv) Thickness of a paper = 0.0016 cm

We Learnt

1. $(-1)^{\text{even no}} = 1$ and $(-1)^{\text{odd no}} = -1$.
2. If $\frac{p}{q}$ is any rational number, then $\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$
3. If $\frac{a}{b}$ is any rational number, then $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
4. If a is a rational number other than 0, then $a^0 = 1$
5. By using the negative exponent, the smallest numbers can be expressed in standard form.