Linear Equations In one Variable

FUNDAMENTAL

In the GMO Class VII Excellence Book, Chapter 4, we had learnt about linear equations and their solutions. However, we will review these facts again and also take up new types of linear equations.

Equation: An equation is a statement of equality of two algebraic expressions involving one or more unknown quantities called variables.

An equation involving only linear polynomials is called a linear equation.

Some example of linear equations are given below:

(i) $2x-3=6-2x$ (ii) $2(y-3)=10$

(iii) $\frac{7}{3}m=14$ (iv) $91z=182$

Rules for solving a linear equation

(i) We can add the same number on both sides of the equation.

For e.g., We can add 3 on both sides of $2x-3=6-2x$ (See examples above) to get $2x=9-2x$

(ii) We can subtract the same number from both sides of the equation. .

For e.g., We can subtract 10 from both sides of $2(y-3)=10$ (See examples above) to get

$2(y-3)-10=10-10$ or, $2y-6-10=0$ or, $2y-16=0$

(iii) We can multiply both sides of the equation by the same nonzero number.

For e.g.. We can multiply both sides of $\frac{7}{3}m=14$ by $\frac{7}{3}$ to get, $\frac{7}{3}m\times\frac{7}{3}=14\times\frac{7}{3}$

or, m = 6

(iv) We can divide both sides of the equation by the same non-zero number.

For e.g., we can divide both sides by $91z=182$ by 91 to get z = 2

(v) if $\frac{ax+b}{cx+d}=\frac{m}{n}$ then n (ax +b)= m(cx +d).

(by cross-multiplication)

(vi) **Transposition:** when a term of an equation is taken to the other side, its sign gets changed..

This process is known as transposition.

Examples:

(i) $7x+3=4x+5 \Rightarrow 7x-4x=5-3$

(Transporting by adding or subtracting)

(ii) $6xyz=7x^2 \Rightarrow yz=\frac{7x^2}{6x}=\frac{7x}{6}$

(Transportation by multiplying or dividing)

Elementary question-1

Solve: $10x=5x+25$

Solution: $10x=5x+25 \Rightarrow 10x-5x=25 \Rightarrow 5x=25 \Rightarrow x=\frac{25}{5}=5$

Elementary question-2

Solve: $\frac{5y}{6}-1=\frac{7}{3}$

Solution: $\frac{5y}{6}-1=\frac{7}{3} \Rightarrow \frac{5y}{6}=\frac{7}{3}+1 \Rightarrow \frac{5y}{6}=\frac{10}{3} \Rightarrow y=\frac{10}{3}\times\frac{6}{5}=4$

Elementary question-3

Solution: $\frac{a+5}{3}+\frac{a+6}{5}=\frac{2a+3}{4}$

Multiplying throughout by LCM of 3, 4, 5 (LCM = 60),

We get, $20(a+5)+12(a-6)=15(2a+3)$

$\Rightarrow 20a+100+12a-72=30a+45$

$\Rightarrow (20a+12a-30a)=(45-100+72)$

$\Rightarrow 2a=17 \Rightarrow a=\frac{17}{2}$

Elementary question 4

Solve: $\frac{2x+3}{3x-1}=\frac{1}{2}$

Solution: By cross - multiplying, we get,

$4x+6=3x-1 \Rightarrow (4x-3x)=-1-6 \Rightarrow x=-7$

Elementary question 5

Solve: $\frac{5x-4}{2x-7}=\frac{10x+3}{4x-7}$

Solution: Cross-Multiplying,

$$(5x-4)(4x-7)=(2x-7)(10x+3)$$

$$\Rightarrow \mathbf{20x^2}-\mathbf{16x}-35x+28=\mathbf{20x^2}-\mathbf{70x}+\mathbf{6x}-21 \Rightarrow -51x+28=-64x-21$$

$$\Rightarrow -51x+64x=-21-28$$

$$\Rightarrow 13x=-49 \Rightarrow x=\frac{-49}{13}$$

Application Based Questions for concept-Building

1. Given that two numbers are in the ratio 2:3. If the difference of number is 17, find the numbers.

Ans. You should carefully think about the wording of question for a while. Rather than choosing numbers as x & y, you should choose them as $2x \,\&\, 3x$. Thus, ratio $\frac{2x}{3x}=\frac{2}{3}$ is maintained and it is easy to solve the problem. As given in question,

Difference $=3x-2x=17 \Rightarrow x=17$

$\therefore 2x=34$ and $3x=51$ are the numbers.

2. The three digits of a three digit number is such that face value of first digit (at hundreds place) is twice that of third digit (at units place). Also, sum of three digits is 14. Also, upon reversing the digits of the number, the value of number decreases by 198. Find the number.

Ans. Let the number be 'abc'. 'abc' can be written as $2x(14-3x)$ x (where x = digit at units place) Upon reversing, the number becomes, $x(14-3x)2x$

$$\Rightarrow 100x+10(14-3x)+2x$$

$$=100\times 2x+10(14-3x)+x-198$$

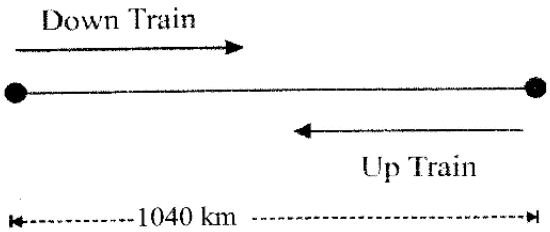
$$100x+2x=200x+x-198$$

$$\Rightarrow 198-201x-102x \Rightarrow 198-99x$$

$$\Rightarrow x=\frac{198}{99}=2$$

3. New Delhi and Patna sahib stations are 1080 km apart. Magadh express (Down train) starts from new Delhi at 8 P.M at some speed. Whereas Magadh express (UP train) starts form patna Sahib at the same time, 8 P.M, but at a speed higher by 10 km / hr. If after 8 hrs, they cross each other, find their speeds.

Ans. Diagrammatically, it can be shown as



Let speed of down train be x

$\therefore$ Let speed of Up train $= (x + 10)$

$\therefore$ Distances travelled are $8x$ and $8(x+10)$ respectively and, since they just cross each other,

$\therefore 8x + 8(x+10) = 1040 \Rightarrow 16x + 80 = 1040$

$\Rightarrow 6x = 960$

$\Rightarrow x = \frac{960}{16} = 60 km/hr; x + 10 = 70 km/hr.$