24 PRACTICE PAPER

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. If $A=\begin{bmatrix}1 & -2\\ 3 & 0\end{bmatrix}, B=\begin{bmatrix}-1 & 4\\ 2 & 3\end{bmatrix}, C=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ then $5A-3B-2C$ is

(*a*) $\begin{bmatrix}8 & 20\\ 7 & 9\end{bmatrix}$ (*b*) $\begin{bmatrix}8 & -20\\ 7 & -9\end{bmatrix}$ (*c*) $\begin{bmatrix}-8 & 20\\ -7 & 9\end{bmatrix}$ (*d*) $\begin{bmatrix}8 & 7\\ -20 & -9\end{bmatrix}$

2. The number of distinct real roots of $\begin{vmatrix}\sin x & \cos x & \cos x\\ \cos x & \sin x & \cos x\\ \cos x & \cos x & \sin x\end{vmatrix}=0$ in the interval $\frac{-\pi}{4}\le x\le\frac{\pi}{4}$ is

(*a*) 0 (*b*) 2 (*c*) 1 (*d*) 3

3. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ then A^{-1} is

(*a*) $\begin{bmatrix}2 & -3\\ 5 & 2\end{bmatrix}$ (*b*) $\begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}$ (*c*) $\frac{1}{19}\begin{bmatrix}3 & -2\\ 5 & 3\end{bmatrix}$ (*d*) $\frac{1}{19}A$

4. If $y=x^x$ then $\frac{d^2y}{dx^2}$ equal to

(*a*) $x^x\left\{(1+\log x)^2-\frac{1}{x}\right\}$ (*b*) $x^x\left\{(1+\log x)^2+\frac{1}{x}\right\}$ (*c*) 0 (*d*) $x^x\left\{(1-\log x)^2+\frac{1}{x}\right\}$

5. A ladder, 5 m long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downward at the rate of 10 cm/s, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 m from the wall is

(*a*) $\frac{1}{10}$ rad/s (*b*) $\frac{1}{20}$ rad/s (*c*) 20 rad/s (*d*) 10 rad/s

6. The value of $\int\frac{e^{5\log x}-e^{3\log x}}{e^{4\log x}-e^{2\log x}}dx$ equals

(*a*) $x+C$ (*b*) $\frac{x^2}{2}+C$ (*c*) $\frac{x^3}{3}+C$ (*d*) $\log x+C$

7. The value of $\int \frac{\sin^4 x}{\cos^8 x}\,dx$ equals

(a) $\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$ (b) $\frac{\tan^5 x}{5} - \frac{\tan^7 x}{7} + C$ (c) $\frac{\tan^5 x}{5} - \frac{\cot^7 x}{7} + C$ (d) $\frac{\tan^7 x}{7} - \frac{\tan^5 x}{5} + C$

8. The value of $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$ is

(a) $\frac{\pi}{4}\log 2$ (b) $\frac{\pi}{8}\log 2$ (c) $\frac{\pi}{8}\log 3$ (d) $\frac{\pi}{8}\log 4$

9. $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^2 x\,dx$ is equal to

(a) 0 (b) –1 (c) $x \sin x$ (d) x

10. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

(a) 2 sq. units (b) 4 sq. units (c) 3 sq. units (d) 1 sq. unit

11. The family $y = Ax + A^3$ of curves is represented by differential equation of degree

(a) 1 (b) 2 (c) 3 (d) 4

12. The general equation of $e^x \cos y\,dx - e^x \sin y\,dy = 0$ is

(a) $e^x \cos y = k$ (b) $e^x \sin y = k$ (c) $e^x = k \cos y$ (d) $e^x = k \sin y$

13. Which of the following statements is correct?

(a) Every LPP admits an optimal selection.

(b) A LPP admits unique optimal solution.

(c) If a LPP admits two optimal solutions then it has an infinite solution.

(d) The set of all feasible solutions of a LPP is not a convex set.

14. The probability distribution of a discrete random variable X is given below:

X	1	2	3	4
$P(X)$	1/3	K	1/3	1/3

then the value of K is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 0 (d) None of these

15. For binomial distribution $B\left(5, \frac{1}{3}\right)$ the mean is

(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\sqrt{\frac{5}{3}}$

Section–B (B1)

16. If $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then, R is

(a) Reflexive but not symmetric (b) Reflexive but not transitive

(c) Symmetric and transitive (d) Neither Symmetric nor transitive

17. Let * be binary operation on $\mathbb{R}$ defined by $a * b = ab + 1$ is

(a) Associative (b) Associative and Commutative

(c) Commutative (d) None of these

18. Let the function f be defined by $f(x) = \frac{2x+1}{1-3x}$, then $f^{-1}(x)$ is

(a) $\frac{x-1}{3x+2}$ (b) $\frac{3x+2}{x-1}$ (c) $\frac{x+1}{3x-2}$ (d) $\frac{2x+1}{1-3x}$

19. $A = \{x \in \mathbb{R} \ -1 \leq x \leq 1\} = B$. The function $f : A \longrightarrow B$ be given by $f(x) = x\,|x|$ is

(a) injective but not surjective (b) surjective but not injective

(c) bijective (d) none of these

20. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} 5 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{cases}$ where $[x]$ denotes greatest integer less than or equal to x. Then for all x, $f(g(x))$ is equal to

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

21. The value of $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

22. The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$ is

(a) 0 (b) 1 (c) 2 (d) ∞

23. The value of $2\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right) - \cos^{-1}(-1)$ is

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

24. If $p > q > 0$ and $pr < -1 < qr$, then

$$\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right) \text{ is equal to}$$

(a) 0 (b) $-\pi$ (c) π (d) $\frac{\pi}{2}$

25. For any square matrix A, AA' is a

(a) Unit matrix (b) Symmetric matrix (c) Skew Symmetric matrix (d) Diagonal matrix

26. A square matrix $A = [a_{ij}]$ in which $a_{ij} = 0$ for $i \neq j$ and $a_{ij} = k$ (Constant) for $i = j$ is called a

(a) Unit matrix (b) Scalar matrix (c) Null matrix (d) Diagonal matrix

27. If x, y, z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then value of $x^{-1} + y^{-1} + z^{-1}$ is

(a) xyz (b) $x^{-1}y^{-1}z^{-1}$ (c) $-x-y-z$ (d) -1

28. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then the value of $A^T A^{-1}$ is

(a) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ (b) $\begin{bmatrix} \cos x & \sin x \\ 1 & 0 \end{bmatrix}$ (c) A' (d) Zero matrix

29. If $f(x) = \begin{cases} 1 & , \text{ if } x \leq 3 \\ ax + b & , \text{ if } 3 < x < 5 \\ 7 & , \text{ if } 5 \leq x \end{cases}$ then the values of a and b so that $f(x)$ is continuous are

(a) $a = 3, b = 3$ (b) $a = 3, b = 4$ (c) $a = 3, b = -8$ (d) None of these

30. **Function $f(x) = e^{-|x|}$ is**

(a) continuous everywhere but not differentiable at $x = 0$

(b) continuous and differentiable everywhere

(c) not continuous at $x = 0$

(d) none of these

31. **Let $f(x) = |\cos x|$. Then,**

(a) $f(x)$ is everywhere differentiable.

(b) $f(x)$ is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.

(c) $f(x)$ is everywhere continuous but not differentiable at $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$.

(d) None of these

32. **If $x^3 + y^3 = 3axy$, then $\frac{dy}{dx}$ equal to**

(a) $\frac{ay - x^2}{y^2 + ax}$ (b) $\frac{ay - x^2}{y - a}$ (c) 0 (d) $\frac{ay - x^2}{y^2 - ax}$

33. **The curve $y = x^{\frac{1}{5}}$ has _____ at (0, 0).**

(a) a vertical tangent (parallel to y-axis) (b) a horizontal tangent (parallel to x-axis)

(c) an oblique tangent (d) no tangent

34. **Integration of $\int \frac{dx}{3x^2 + 13x - 10}$ is**

(a) $\frac{1}{17}\log\left|\frac{x+5}{3x-2}\right| + C$ (b) $\frac{1}{17}\log\left|\frac{3x-2}{x+5}\right| + C$ (c) $\frac{1}{5}\log\left|\frac{x+5}{3x-2}\right| + C$ (d) $\frac{-1}{17}\log\left|\frac{x+5}{3x-2}\right| + C$

35. **The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$ is**

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) -1

36. **Integration of $\int (\cos^2 x - \sin^2 x)\, dx$ is**

(a) $\frac{\sin 2x}{2} + C$ (b) $\sin 2x + C$ (c) $\sin x + C$ (d) $\cos x + C$

37. **The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is**

(a) $\frac{4}{3}$ sq. units (b) 1 sq. unit (c) $\frac{2}{3}$ sq. unit (d) $\frac{1}{3}$ sq. unit

38. **$\tan^{-1}x + \tan^{-1}y = C$ is general solution of the differential equation**

(a) $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$ (b) $\frac{dy}{dx} = \frac{1 + x^2}{1 + y^2}$

(c) $(1 + x^2)dy + (1 + y^2)\, dx = 0$ (d) $(1 + x^2)dx + (1 + y^2)\, dy = 0$

39. **The integrating factor of differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is**

(a) $\cos x$ (b) $\sec x$ (c) $e^{\cos x}$ (d) $e^{\sec x}$

40. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ respectively is

(*a*) 1/2 (*b*) 1 (*c*) 2 (*d*) 4

41. The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is

(*a*) 0 (*b*) –1 (*c*) 1 (*d*) 3

42. If the vectors $\sec^2 A\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\sec^2 B\,\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\sec^2 C\,\hat{k}$ are coplanar then the value of $\operatorname{cosec}^2 A + \operatorname{cosec}^2 B + \operatorname{cosec}^2 C$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 9

43. The vector $\hat{i}+a\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4a-2)\hat{j}+2\hat{k}$. The value of a is

(*a*) 3, 5 (*b*) 2/3, 2 (*c*) 1/3, 3 (*d*) –2/3, 2

44. The distance of the plane $\vec{r}.\left(\frac{2\hat{i}}{7}+\frac{3\hat{j}}{7}-\frac{6\hat{k}}{7}\right)=1$ from the origin is

(*a*) 1 (*b*) 7 (*c*) $\frac{2}{7}$ (*d*) $\frac{3}{7}$

45. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$ is

(*a*) $\frac{10}{6\sqrt{5}}$ (*b*) $\frac{4}{5\sqrt{2}}$ (*c*) $\frac{2\sqrt{3}}{5}$ (*d*) $\frac{\sqrt{2}}{10}$

46. The reflection of the point (α, β, γ) in the XY-plane is

(*a*) $(\alpha, \beta, 0)$ (*b*) $(0, 0, \gamma)$ (*c*) $(-\alpha, -\beta, \gamma)$ (*d*) $(\alpha, \beta, -\gamma)$

47. The locus represented by $xy + yz = 0$ is

(*a*) a pair of perpendicular lines (*b*) a pair of parallel lines

(*c*) a pair of perpendicular planes (*d*) a pair of parallel planes

48. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, then probability that both are dead is

(*a*) $\frac{31}{56}$ (*b*) $\frac{19}{24}$ (*c*) $\frac{1}{14}$ (*d*) $\frac{3}{28}$

49. Which one is not a requirement of a binomial distribution?

(*a*) There are 2 outcomes for each trial.

(*b*) There is a fixed number of trials.

(*c*) The outcomes must be dependent on each other.

(*d*) The probability of success must be the same for all the trials.

50. If two cards are drawn from a well shuffled deck of 52 playing cards with replacement, then the probability that both cards are queen, is

(*a*) $\frac{1}{13}\cdot\frac{1}{13}$ (*b*) $\frac{1}{13}+\frac{1}{13}$ (*c*) $\frac{1}{13}\cdot\frac{1}{17}$ (*d*) $\frac{1}{13}\cdot\frac{4}{51}$