8. Division of Algebraic Expressions

Exercise 8.1

1. Question

Write the degree of each of the following polynomials:

(i) $2x^3 + 5x^2 - 7$

(ii) $5x^2 - 3x + 2$

(iii) $2x + x^2 - 8$

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$

(v) $3x^3 + 1$

(vi) 5

(vii) $20x^3 + 12x^2y^2 + 20$

Answer

(i) $2x^3 + 5x^2 - 7$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 3.

Therefore degree of the polynomial is 3.

(ii) $5x^2 - 3x + 2$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 2.

Therefore degree of the polynomial is 2.

(iii) $2x + x^2 - 8$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 2.

Therefore degree of the polynomial is 2.

(iv) $\frac{1}{2}y^7 - 12y^5 + 48y^5 - 10$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 7.

Therefore degree of the polynomial is 7.

(v) $3x^3 + 1$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 3.

Therefore degree of the polynomial is 3.

(vi) 5

Degre is the highest power of the variable of a polynomial. In the given polynomial there is no variable term.

Therefore degree of the polynomial is 0.

(vii) $20x^3 + 12x^2y^2 + 20$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 4.

Therefore degree of the polynomial is 4.

2. Question

Which of the following expressions are not polynomiasl?

(i) $x^2 + 2x^{-2}$

(ii) $\sqrt{a}x + x^2 - x^3$

(iii) $3y^3 - \sqrt{5}y + 9$

(iv) $ax^{1/2} + ax + 9x^2 + 4$

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

Answer

(i) $x^2 + 2x^{-2}$

A polynomial never has negative or fractional power. In the given expression x has negative power.

Therefore it is not a polynomial.

(ii) $\sqrt{a}x + x^2 - x^3$

A polynomial always has positive power.

Therefore the given expression is a polynomial.

(iii) $3y^3 - \sqrt{5}y + 9$

A polynomial always has positive power.

Therefore the given expression is a polynomial.

(iv) $ax^{1/2} + ax + 9x^2 + 4$

A polynomial never has negative or fractional power. In the given expression x has fractional power.

Therefore it is not a polynomial.

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

A polynomial never has negative or fractional power. In the given expression x has negative power.

Therefore it is not a polynomial.

3. Question

Write each of the following polynomicals in the standard from. Also, write their drgree:

(i) $x^2 + 3 + 6x + 5x^4$

(ii) $a^2 + 4 + 5a^6$

(iii) $(x^3 - 1)(x^3 - 4)$

(iv) $(y^3 - 2)(y^3 + 11)$

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

Answer

(i) $x^2 + 3 + 6x + 5x^4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5x^4 + x^2 + 6x + 3$ or $3 + 6x + x^2 + 5x^4$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 4

(ii) $a^2 + 4 + 5a^6$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5a^6 + a^2 + 4$ or $4 + a^2 + 5a^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(iii) $(x^3 - 1)(x^3 - 4)$

$(x^3 - 1)(x^3 - 4) = x^6 - 4x^3 - x^3 + 4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $x^6 - 5x^3 + 4$ or $4 - 5x^3 + x^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(iv) $(y^3 - 2)(y^3 + 11)$

$(y^3 - 2)(y^3 + 11) = y^6 + 11y^3 - 2y^3 - 22$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $y^6 + 9y^3 - 22$ or $-22 + 9y^3 + y^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

$\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right) = a^6 + \frac{16a^3}{17} - \frac{3a^3}{8} - \frac{6}{17}$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^6 + \frac{11a^3}{136} - \frac{6}{17}$ or $-\frac{6}{17} + \frac{11a^3}{136} + a^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

$\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right) = a^2 + \frac{4a}{3} + \frac{3a}{4} + 1$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^2 + \frac{25a}{12} + 1$ or $1 + \frac{25a}{12} + a^2$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 2

Exercise 8.2

1. Question

Divide:

$6x^3y^3z^2$ by $3x^2yz$

Answer

$\frac{6x^3y^3z^2}{3x^2yz} = (\frac{6}{3}x^{3-2}y^{3-1}z^{2-1}) = 2xy^2z$ [Using $a^n \div a^m = a^{n-m}$]

2. Question

Divide:

$15m^3n^3$ by $5m^2n^2$

Answer

$\frac{15m^3n^3}{5m^2n^2} = (\frac{15}{5}m^{3-2}n^{3-2}) = 3mn$ [Using $a^n \div a^m = a^{n-m}$]

3. Question

Divide:

$24a^3b^3$ by $-8ab$

Answer

$\frac{24a^3b^3}{-8ab} = (\frac{24}{-8}a^{3-1}b^{3-1}) = -3a^2b^2$ [Using $a^n \div a^m = a^{n-m}$]

4. Question

Divide:

$-21abc^2$ by $-7abc$

Answer

$\frac{-21abc^2}{-7abc} = (\frac{-21}{-7}a^{1-1}b^{1-1}c^{2-1}) = 3a^\circ b^\circ c = 3c$ [Using $a^n \div a^m = a^{n-m}$] and [$a^\circ = 1$]

5. Question

Divide:

xyz^2 by $-9xz$

Answer

$\frac{xyz^2}{-9xz} = (\frac{1}{-9}x^{1-1}yz^{2-1}) = -\frac{1yz}{9} = -\frac{yx}{9}$ [Using $a^n \div a^m = a^{n-m}$] and [$a^\circ = 1$]

6. Question

Divide:

$-72a^4b^5c^8$ by $-9a^2b^2c^3$

Answer

$\frac{-72a^4b^5c^8}{-9a^2b^2c^3}$

$= 8a^2b^3c^5$

7. Question

Simplify:

$\frac{16m^3y^2}{4m^2y}$

Answer

$\frac{16m^3y^2}{4m^2y} = \left(\frac{16}{4}m^{3-2}y^{2-1}\right) = 4my$ [Using $a^n \div a^m = a^{n-m}$]

8. Question

Simplify:

$\frac{32m^2n^2p^2}{4mnp}$

Answer

$\frac{32m^2n^2p^2}{4mnp} = (\frac{32}{4}m^{2-1}n^{2-1}p^{2-1}) = 8mnp$ [Using $a^n \div a^m = a^{n-m}$]

Exercise 8.3

1. Question

Divide:

$x + 2x^2 + 3x^4 - x^5$ by $2x$

Answer

$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^4}{2x} - \frac{x^5}{2x} = \frac{1}{2} + x + \frac{3x^3}{2} - \frac{x^4}{5}$ [Using $a^n \div a^m = a^{n-m}$]

2. Question

Divide:

$y^4 - 3y^3 + \frac{1}{2}y^2$ by $3y$

Answer

$\frac{y^4-3y^3+\frac{1y^2}{2}}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{y^2}{6y} = \frac{y^3}{3} - y^2 + \frac{y}{6}$ [Using $a^n \div a^m = a^{n-m}$]

3. Question

Divide:

$-4a^3 + 4a^2 + a$ by $2a$

Answer

$-\frac{4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a} = -2a^2 + 2a + \frac{1}{2} =$ [Using $a^n \div a^m = a^{n-m}$]

4. Question

Divide:

$-x^6 + 2x^4 + 4x^3 + 2x^2$ by $\sqrt{2}x^2$

Answer

$-\frac{x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} = -\frac{x^4}{\sqrt{2}} + \frac{2x^2}{\sqrt{2}} + \frac{4x}{\sqrt{2}} = -\frac{x^4}{\sqrt{2}} + \sqrt{2}x^2 + \sqrt{2}x$ [Using $a^n \div a^m = a^{n-m}$]

5. Question

Divide:

$5z^3 - 6z^2 + 7z$ by $2z$

Answer

$\frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z} = = \frac{5z^2}{2} - 3z + \frac{7}{2}$ [Using $a^n \div a^m = a^{n-m}$]

6. Question

Divide:

$\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3^2 - 6a$ by $3a$

Answer

$\frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{9}{3a} - \frac{6a}{3a} = \frac{\sqrt{3}a^3}{3} + \frac{2\sqrt{3}a^2}{3} + \frac{3}{a} - 2$ [Using $a^n \div a^m = a^{n-m}$]

Exercise 8.4

1. Question

Divide:

$5x^3 - 15x^2 + 25x$ by $5x$

Answer

$\frac{5x^3}{5x} - \frac{15x^2}{5x} + \frac{25x}{5x} = = 5x^2 - 3x + 5$ [Using $a^n \div a^m = a^{n-m}$]

2. Question

Divide:

$4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

Answer

$\frac{2\times 4z^3}{-1z} + \frac{2\times 6z^2}{-1z} - \frac{2\times z}{-1z} = = -8z^2 - 12z + 2$ [Using $a^n \div a^m = a^{n-m}$]

3. Question

Divide:

$9x^2y - 6xy + 12xy^2$ by $-\frac{3}{2}xy$

Answer

$\frac{2\times 9x^2y}{-3xy} - \frac{2\times 6xy}{-3xy} + \frac{2\times 12xy^2}{-3xy} = = -6x^2y + 4y - 8y$ [Using $a^n \div a^m = a^{n-m}$]

4. Question

Divide:

$3x^3y^2 + 2x^2y + 15xy$ by $3xy$

Answer

$\frac{3x^3y^2}{3xy} + \frac{2x^2y}{3xy} + \frac{15xy}{3xy} = = x^2y + \frac{2x}{3} + 5$ [Using $a^n \div a^m = a^{n-m}$]

5. Question

Divide:

$x^3 + 7x + 12$ by $x + 4$

Answer

$x+4 \overline{) x^2+7x+12 } (x+3$

x^2+4x

$- \quad -$

$3x+12$

$3x+12$

$- \quad -$

0

Ans: x+3

6. Question

Divide:

$4y^2+3y+\frac{1}{2}$ by $2y+1$

Answer

$2y+1 \overline{) 4y^2+3y+\frac{1}{2} } (2y+\frac{1}{2}$

$4y^2+2y$

$- \quad - \quad -$

$y+\frac{1}{2}$

$y+\frac{1}{2}$

$- \quad -$

0

7. Question

Divide:

$3x^3+4x^2+5x+18$ by $x+2$

Answer

$x+2 \overline{) 3x^3+4x^2+5x+18 } (3x^2-2x+9$

$3x^3+6x^2$

$- \quad - \quad -$

$-2x^2+5x$

$-2x^2-4x$

$+ \quad +$

$9x+18$

$9x+18$

$- \quad -$

0

8. Question

Divide:

$14x^2-53x+45$ by $7x-9$

Answer

$7x-9 \overline{) 14x^2-53x+45 } (2x-5$

$14x^2-18x$

$- \quad +$

$-35x+45$

$-35x+45$

$+ \quad -$

0

9. Question

Divide:

$-21+71x-31x^2-24x^3$ by $3-8x$

Answer

$$-8x+3\,\big)\,-24x^3-31x^2+71x-21\,\big(\,3x^2+5x-7$$
$$-24x^3+9x^2$$
$$+\quad -$$
$$-40x^2+71x$$
$$-40x^2+15x$$
$$+\quad -$$
$$56x-21$$
$$56x-21$$
$$-\quad +$$
$$0$$

10. Question

Divide:

$3y^4-3y^3-4y^2-4y$ by y^2-2y

Answer

$$y^2-2y\,\big)\,3y^4-3y^3-4y^2-4y\,\big(\,3y^2+3y+2$$
$$3y^4-6y^3$$
$$-\quad +$$
$$3y^3-4y^2$$
$$3y^3-6y^2$$
$$-\quad +$$
$$2y^2-4y$$
$$2y^2-4y$$
$$-\quad +$$
$$0$$

11. Question

Divide:

$2y^5+10y^4+6y^3+y^2+5y+3$ by $2y^3+1$

Answer

$$2y^3+1\,\big)\,2y^5+10y^4+6y^3+y^2+5y+3\,\big(\,y^2+5y+3$$
$$2y^5 \qquad +y^2$$
$$- \qquad -$$
$$10y^4+6y^3+5y+3$$
$$10y^4 \qquad +5y$$
$$- \qquad -$$
$$6y^3 \qquad +3$$
$$6y^3 \qquad +3$$
$$- \qquad -$$
$$0$$

12. Question

Divide:

$x^4-2x^3+2x^2+x+4$ by x^2+x+1

Answer

$$x^2+x+1\,\big)\,x^4-2x^3+2x^2+x+4\,\big(\,x^2-3x+4$$
$$x^4+x^3+x^2$$
$$-\quad -\quad -$$
$$-3x^3+x^2+x+4$$
$$-3x^3-3x^2-3x$$
$$+\quad +\quad +$$
$$4x^2+4x+4$$
$$4x^2+4x+4$$
$$-\quad -\quad -$$
$$0$$

13. Question

Divide:

$m^3 - 14m^2 + 37m - 26$ by $m^2 - 12m + 13$

Answer

$m^2 - 12m + 13 \,\overline{)\; m^3 - 14m^2 + 37m - 26\;}\,(m - 2$

$m^3 - 12m^2 + 13m$

$- \quad - \quad -$

$-2m^2 + 24m - 26$

$-2m^2 + 24m - 26$

$+ \quad + \quad +$

0

14. Question

Divide:

$x^4 + x^2 + 1$ by $x^2 + x + 1$

Answer

$x^2 + x + 1 \,\overline{)\; x^4 + 0x^3 + x^2 + 1\;}\,(x^2 - x + 1$

$x^4 + x^3 + x^2$

$- \quad - \quad -$

$-x^3 + 1$

$-x^3 \quad - x^2 - x$

$+ \quad + \quad +$

$x^2 + x + 1$

$x^2 + x + 1$

$- \quad - \quad -$

0

15. Question

Divide:

$x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$

Answer

$x^3 + 1 \,\overline{)\; x^5 + x^4 + x^3 + x^2 + x + 1\;}\,(x^2 + x + 1$

$x^5 \quad + x^2$

$- \quad -$

$x^4 + x^3 + x + 1$

$x^4 \quad + x$

$- \quad -$

$x^3 \quad + 1$

$x^3 \quad + 1$

$- \quad -$

0

16. Question

Divide each of the following and find the quotient and remainder:

$14x^3 - 5x^2 + 9x - 1$ by $2x - 1$

Answer

$2x - 1 \,\overline{)\; 14x^3 - 5x^2 + 9x - 1\;}\,(7x^2 + x + 5$

$14x^3 - 7x^2$

$- \quad +$

$2x^2 + 9x - 1$

$2x^2 - x$

$- \quad +$

$10x - 1$

$10x - 5$

$- \quad +$

4

Quotient: $7x^2 + x + 5$

Remainder: 4

17. Question

Divide each of the following and find the quotient and remainder:

$3x^3 - x^2 - 10x - 3$ by $x - 3$

Answer

$$x - 3 \overline{)\,3x^3 - x^2 - 10x - 3\,(}\; 3x^2 + 8x + 14$$
$$3x^3 - 9x^2$$
$$- \quad +$$
$$8x^2 - 10x - 3$$
$$8x^2 - 24x$$
$$- \quad +$$
$$14x - 3$$
$$14x - 42$$
$$- \quad +$$
$$39$$

Quotient: $3x^2 + 8x + 14$

Remainder: 39

18. Question

Divide each of the following and find the quotient and remainder:

$6x^3 + 11x^2 - 39x - 65$ by $3x^2 + 13x + 13$

Answer

$$3x^2 + 13x + 13 \overline{)\,6x^3 + 11x^2 - 39x - 65\,(}\; 2x - 5$$
$$6x^3 + 26x^2 + 26x$$
$$- \quad - \quad -$$
$$-15x^2 - 65x - 65$$
$$-15x^2 - 65x - 65$$
$$+ \quad + \quad +$$
$$0$$

Quotient: $2x - 5$

Remainder: 0

19. Question

Divide each of the following and find the quotient and remainder:

$30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$

Answer

$$3x^2 + 2x - 4 \overline{)\,30x^4 + 11x^3 - 82x^2 - 12x + 48\,(}\; 10x^2 - 3x - 12$$
$$30x^4 + 20x^3 - 40x^2$$
$$- \quad - \quad +$$
$$-9x^3 - 42x^2 - 12x + 48$$
$$-9x^3 - 6x^2 + 12x$$
$$+ \quad + \quad -$$
$$-36x^2 - 24x + 48$$
$$-36x^2 - 24x + 48$$
$$+ \quad + \quad -$$
$$0$$

Quotient: $10x^2 - 3x - 12$

Remainder: 0

20. Question

Divide each of the following and find the quotient and remainder:

$9x - 4x^2 + 4$ by $3x^2 - 4x + 2$

Answer

$$3x^2 - 4x + 2 \overline{)\ 9x^4 + 0x^3 - 4x^2 + 4\ (} 3x^2 + 4x + 2$$
$$9x^4 - 12x^3 + 6x^2$$
$$- \quad + \quad -$$
$$12x^3 - 10x^2 + 4$$
$$12x^3 - 16x^2 + 8x$$
$$- \quad + \quad -$$
$$6x^2 - 8x + 4$$
$$6x^2 - 8x + 4$$
$$- \quad + \quad -$$
$$0$$

Quotient: $3x^2 + 4x + 2$

Remainder: 0

21. Question

Verify division algorithm i.e. Dividend=Divisor $\times$ Quotient + Remainder, in each of the following. Also, write the quotient and remainder;

Dividend	Divisor
(i) $14x^2 + 13x - 15$	$7x - 4$
(ii) $15z^3 - 20z^2 + 13z - 12$	$3z - 6$
(iii) $6y^5 - 28y^3 + 3y^2 + 30y - 9$	$2x^2 - 6$
(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75$	$3x + 7$
(v) $15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6$	$3y - 2$
(vi) $4y^3 + 8y + 8y^2 + 7$	$2y^2 - y + 1$
(vii) $6y^4 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$	$2y^3 + 1$

Answer

(i)

$$7x - 4 \overline{)\ 14x^2 + 13x - 15\ (} 2x + 3$$
$$14x^2 - 8x$$
$$- \quad +$$
$$21x - 15$$
$$21x - 12$$
$$- \quad +$$
$$-3$$

Dividend = Divisor $\times$ Quotient + Remainder

$14x^2 + 13x - 15 = (7x - 4)\times(2x + 3) + (-3)$

$14x^2 + 13x - 15 = 14x^2 + 21x - 8x - 12 - 3$

$14x^2 + 13x - 15 = 14x^2 + 13x - 15$

(ii)

$$3z-6\,\overline{)\,15z^3-20z^2+13z-12\,(}\;5z^2+\frac{10z}{3}+11$$

$$15z^3-30z^2$$
$$-\quad+$$
$$10z^2+13z-12$$
$$10z^2-20z$$
$$-\quad+$$
$$33z-12$$
$$33z-66$$
$$-\quad+$$
$$54$$

Dividend = Divisor × Quotient + Remainder

$$15z^3-20z^2+13z-12=(3z-6)\times\left(5z^2+\frac{10z}{3}+11\right)+54$$

$$15z^3-20z^2+13z-12=15z^3+10z^2+33z-30z^2-20z+54$$

$$15z^3-20z^2+13z-12=15z^3-20z^2+13z-12$$

(iii)

$$2y^2-6\,\overline{)\,6y^5-28y^3+3y^2+30y-9\,(}\;3y^3-5y+\frac{3}{2}$$

$$6y^5-18y^3$$
$$-\quad+$$
$$-10y^3+3y^2+30y-9$$
$$-10y^3\qquad+30y$$
$$+\qquad-$$
$$3y^2\qquad-9$$
$$3y^2\qquad-9$$
$$-\qquad+$$
$$0$$

Dividend = Divisor × Quotient + Remainder

$$6y^5-28y^3+3y^2+30y-9=(2y^2-6)\times\left(3y^3-5y+\frac{3}{2}\right)+0$$

$$6y^5-28y^3+3y^2+30y-9=6y^5-10y^3+3y^2-18y^3+30y-9$$

$$6y^5-28y^3+3y^2+30y-9=6y^5-28y^3+3y^2+30y-9$$

(iv)

$$3x+7\,\overline{)\,-12x^4-22x^3-10x^2+34x-75\,(}-4x^3+2x^2-8x+30$$

$$-12x^4-28x^3$$
$$+\quad+$$
$$6x^3-10x^2+34x-75$$
$$6x^3+14x^2$$
$$-\quad-$$
$$-24x^2+34x-75$$
$$-24x^2-56x$$
$$+\quad+$$
$$90x-75$$
$$90x+210$$
$$-\quad-$$
$$-285$$

Dividend = Divisor × Quotient + Remainder

$$-12x^4-22x^3-10x^2+34x-75=(3x+7)\times(-4x^3+2x^2-8x+30)-285$$

$$-12x^4-22x^3-10x^2+34x-75=-12x^4+6x^3-24x^2-28x^3+14x^2+90x-56x+210-285$$

$$-12x^4-22x^3-10x^2+34x-75=-12x^4-22x^3-10x^2+34x-75$$

(v)

$$3y-2\overline{)\,15y^4-16y^3+9y^2-\frac{10y}{3}+6\,(}\,5y^3-2y^2+\frac{5y}{3}$$

$15y^4-10y^3$

$-\quad +$

$-6y^3+9y^2-\frac{10y}{3}+6$

$-6y^3+4y^2$

$+\quad -$

$5y^2-\frac{10y}{3}+6$

$5y^2-\frac{10y}{3}$

$+\quad +$

6

Dividend = Divisor × Quotient + Remainder

$$15y^4-16y^3+9y^2-\frac{10y}{3}+6=(3y-2)\times\left(5y^3-2y^2+\frac{5y}{3}\right)+6$$

$$15y^4-16y^3+9y^2-\frac{10y}{3}+6=15y^4-6y^3+5y^2-10y^3+4y^2-\frac{10y}{3}+6$$

$$15y^4-16y^3+9y^2-\frac{10y}{3}+6=15y^4-16y^3+9y^2-\frac{10y}{3}+6$$

(vi)

$$2y^2-y+1\overline{)\,4y^3+8y^2+8y+7\,(}\,2y+5$$

$4y^3-2y^2+2y$

$-\quad +\quad -$

$10y^2+6y+7$

$10y^2-5y+5$

$-\quad +\quad -$

$11y+2$

Dividend = Divisor × Quotient + Remainder

$$4y^3+8y^2+8y+7=(2y^2-y+1)\times(2y+5)+11y+2$$

$$4y^3+8y^2+8y+7=4y^3+10y^2-2y^2-5y+2y+5+11y+2$$

$$4y^3+8y^2+8y+7=4y^3+8y^2+8y+7$$

(vii)

$$2y^3+1\overline{)\,6y^5+4y^4+4y^3+7y^2+27y+6\,(}\,3y^2+2y+2$$

$6y^5 \qquad +3y^2$

$- \qquad -$

$4y^4+4y^3+4y^2+27y+6$

$4y^4 \qquad +2y$

$- \qquad -$

$4y^3+4y^2+25y+6$

$4y^3 \qquad +2$

$- \qquad -$

$4y^2+25y+4$

Dividend = Divisor × Quotient + Remainder

$$6y^5+4y^4+4y^3+7y^2+27y+6=(2y^3+1)\times(3y^2+2y+2)+4y^2+25y+4$$

$$6y^5+4y^4+4y^3+7y^2+27y+6=6y^5+4y^4+4y^3+3y^2+2y+2+4y^2+25y+4$$

$$6y^5+4y^4+4y^3+7y^2+27y+6=6y^5+4y^4+4y^3+7y^2+27y+6$$

22. Question

Divide $15y^4+16y^3+\frac{10}{3}y-9y^2-6$ by $3y-z$ Write down the coeficients of the terms in the quotient.

Answer

$$3y-2\ \overline{)\ 15y^4+16y^3-9y^2+\frac{10y}{3}-6\ }\ \left(5y^3+\frac{26y^2}{3}+\frac{25y}{9}+\frac{80}{27}\right.$$

$$15y^4-10y^3$$
$$-\quad +$$
$$26y^3-9y^2+\frac{10y}{3}-6$$
$$26y^3-\frac{52y^2}{3}$$
$$-\quad +$$
$$\frac{25y^2}{3}+\frac{10y}{3}-6$$
$$\frac{25y^2}{3}-\frac{50y}{9}$$
$$-\quad +$$
$$\frac{80y}{9}-6$$
$$\frac{80y}{9}-\frac{160}{27}$$
$$-\quad +$$
$$-\frac{2}{27}$$

Quotient: $5y^3+\frac{26y^2}{3}+\frac{25y}{9}+\frac{80}{27}$

Coefficient of y^3 = 5; Coefficient of y^2 $=\frac{26}{3}$; Coefficient of y $=\frac{25}{9}$; Constant term = Coefficient of y^2 $=\frac{80}{27}$

23. Question

Using division of polynomials state whether

(i) $x+6$ is a factor of x^2-x-42 3

(ii) 4x-1 is a factor of $4x^2-13x-12$

(iii) 2y-5 is a factor of $4y^4-10y^3-10y^2+30y-15$

(iv) $3y^2+5$ is a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

(v) z^2+3 is a factor of z^5-9z

(vi) $2x^2-x+3$ is a factor of $6x^5-x^4+4x^3-5x^2-x-15$

Answer

(i) $x+6$ is a factor of x^2-x-42

$$x+6\ \overline{)\ x^2-x-42\ }\ \left(x-7\right.$$
$$x^2+6x$$
$$-\quad -$$
$$-7x-42$$
$$-7x-42$$
$$+\quad +$$
$$0$$

Quotient: $x-7$

Remainder: 0

Since remainder is 0 therefore $(x+6)$ is a factor of x^2-x-42

(ii) 4x-1 is a factor of $4x^2-13x-12$

$$4x-1\ \overline{)\ 4x^2-13x-12\ }\ \left(x-3\right.$$
$$4x^2-x$$
$$-\quad +$$
$$-12x-12$$
$$-12x+3$$
$$+\quad -$$
$$15$$

Quotient: $x-3$

Remainder: 15

Since remainder is 15 therefore $(4x-1)$ is **NOT** a factor of $4x^2-13x-12$

(iii) 2y-5 is a factor of $4y^4-10y^3-10y^2+30y-15$

$$2y-5\ \overline{)\ 4y^4-10y^3-10y^2+30y-15\ }\ \left(2y^3-5y+\frac{5}{2}\right.$$
$$\underset{-\quad +}{4y^4-10y^3}$$
$$-10y^2+30y-15$$
$$\underset{+\quad -}{-10y^2+25y}$$
$$5y-15$$
$$\underset{-\quad +}{5y-\frac{25}{2}}$$
$$-\frac{5}{2}$$

Quotient: $2y^3-5y+\frac{5}{2}$

Remainder: $-\frac{5}{2}$

Since remainder is $-\frac{5}{2}$ therefore $(2y-5)$ is **NOT** a factor of $4y^4-10y^3-10y^2+30y-15$

(iv) $3y^2+5$ is a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

$$3y^2+5\ \overline{)\ 6y^5+15y^4+16y^3+4y^2+10y-35\ }\ \left(2y^3+2y+\frac{4}{3}\right.$$
$$\underset{-\qquad -}{6y^5\qquad +10y^3}$$
$$6y^3+4y^2+10y-35$$
$$\underset{-\qquad -}{6y^3\qquad +10y}$$
$$4y^2\qquad -35$$
$$\underset{-\qquad -}{4y^2\qquad +\frac{20}{3}}$$
$$-\frac{125}{3}$$

Quotient: $2y^3+2y+\frac{4}{3}$

Remainder: $-\frac{125}{3}$

Since remainder is $-\frac{125}{3}$ therefore $(3y^2+5)$ is **NOT** a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

(v) z^2+3 is a factor of z^5-9z

$$z^2+3\ \overline{)\ z^5-9z\ }\ \left(z^3-3z\right.$$
$$\underset{-\quad -}{z^5+3z^3}$$
$$-3z^3-9z$$
$$\underset{+\quad +}{-3z^3-9z}$$
$$0$$

Quotient: z^3-3z

Remainder: 0

Since remainder is 0 therefore (z^3-3z) is a factor of z^5-9z

(vi) $2x^2-x+3$ is a factor of $6x^5-x^4+4x^3-5x^2-x-15$

$$2x^2 - x + 3 \,\big)\, 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 \,\big(\, 3x^3 + x^2 - 2x - 3$$
$$6x^5 - 3x^4 + 9x^3$$
$$- \quad + \quad -$$
$$2x^4 - 5x^3 - 5x^2 - x - 15$$
$$2x^4 - x^3 + 3x^2$$
$$- \quad + \quad -$$
$$-4x^3 - 8x^2 - x - 15$$
$$-4x^3 - 2x^2 - 6x$$
$$+ \quad + \quad +$$
$$-6x^2 + 5x - 15$$
$$-6x^2 + 3x - 9$$
$$+ \quad - \quad +$$
$$2x - 6$$

Quotient: $3x^3 + x^2 - 2x - 3$

Remainder: $2x^2 - x + 3$

Since remainder is $2x - 6$ therefore $(3y^2 + 5)$ is **NOT** a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

24. Question

Find the value of a, if x+2 is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$.

Answer

$x + 2 \text{ } is \text{ } a \text{ } factor \text{ } of \text{ } 4x^4 + 2x^3 - 3x^2 + 8x + 5a$

$x + 2 = 0$

$x = -2$ Therefore substitute x = -2 in the given equation we get,

$4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$

$64 - 16 - 12 - 16 + 5a = 0$

$20 + 5a = 0$

$5a = -20$

$a = -\frac{20}{5} = -4$

$\boldsymbol{a = -4}$

25. Question

What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polymonial is exactly divible by $x^2 + 2x - 3$.

Answer

$$x^2 + 2x - 3 \,\big)\, x^4 + 2x^3 - 2x^2 + x - 1 \,\big(\, x^2 + 1$$
$$x^4 + 2x^3 - 3x^2$$
$$- \quad - \quad +$$
$$x^2 + x - 1$$
$$x^2 + 2x - 3$$
$$- \quad - \quad +$$
$$-x + 2$$

Quotient: $x^2 + 1$

Remainder: $-x + 2$

Therefore $x - 2$ to be added.

Exercise 8.5

1. Question

Divide the first polynomial by the second polynomial in each of the following Also write the quotient and remainder:

(i) $3x^2 + 4x + 5, x - 2$

(ii) $10x^2 - 7x + 8, 5x - 3$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

(iv) $x^4 - x^3 + 5x, x - 1$

(v) $y^4 + y^2, y^2 - 2$

Answer

(i) $3x^2 + 4x + 5, x - 2$

$$x - 2 \overline{)\,3x^2 + 4x + 5\,(}\ 3x + 10$$
$$3x^2 - 6x$$
$$- \quad +$$
$$10x + 5$$
$$10x - 20$$
$$- \quad +$$
$$25$$

Quotient: $3x + 10$

Remainder: 25

(ii) $10x^2 - 7x + 8, 5x - 3$

$$5x - 3 \overline{)\,10x^2 - 7x + 8\,(}\ 2x - \frac{1}{5}$$
$$10x^2 - 6x$$
$$- \quad +$$
$$-x + 8$$
$$-x + \frac{3}{5}$$
$$+ \quad -$$
$$8 - \frac{3}{5} = \frac{37}{5}$$

Quotient: $2x - \frac{1}{5}$

Remainder: $\frac{37}{5}$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

$$5y - 1 \overline{)\,5y^3 - 6y^2 + 6y - 1\,(}\ y^2 - y + 1$$
$$5y^3 - y^2$$
$$- \quad +$$
$$-5y^2 + 6y - 1$$
$$-5y^2 + y$$
$$+ \quad -$$
$$5y - 1$$
$$5y - 1$$
$$- \quad +$$
$$0$$

Quotient: $y^2 - y + 1$

Remainder: 0

(iv) $x^4 - x^3 + 5x, x - 1$

$$x-1 \overline{) x^4 - x^3 + 5x} \left(x^3 + 5 \right.$$
$$x^4 - x^3$$
$$- \quad +$$
$$5x$$
$$5x - 5$$
$$- \quad +$$
$$5$$

Quotient: $x^3 + 5$

Remainder: 5

(v) $y^4 + y^2, y^2 - 2$

$$y^2 - 2 \overline{) y^4 - y^2} \left(y^2 + 1 \right.$$
$$y^4 - 2y^2$$
$$- \quad +$$
$$y^2$$
$$y^2 - 2$$
$$- \quad +$$
$$2$$

Quotient: $y^2 + 1$

Remainder: 2

2. Question

Find Whether or not the first polynomial is a factor of the second:

(i) $x + 1, 2x^2 + 5x + 4$

(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$

(iii) $4x^2 - 5, 4x^4 + 7x^2 + 15$

(iv) $4 - z, 3z^2 - 13z + 4$

(v) $2a - 3, 10a^2 - 9a - 5$

(vi) $4y + 1, 8y^2 - 2y + 1$

Answer

(i) $x + 1, 2x^2 + 5x + 4$

$$x + 1 \overline{) 2x^2 + 5x + 4} \left(2x + 3 \right.$$
$$2x^2 + 2x$$
$$- \quad -$$
$$3x + 4$$
$$3x + 3$$
$$- \quad -$$
$$1$$

Quotient: $2x + 3$

Remainder: 1

Since remainder is 1 therefore the first polynomial is **NOT** a factor of the second polynomial.

(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$

$$y-2\,\overline{)\,3y^3+5y^2+5y+2\,(}\,3y^2+11y+27$$
$$3y^3-6y^2$$
$$-\quad+$$
$$11y^2+5y+2$$
$$11y^2-22y$$
$$-\quad+$$
$$27y+2$$
$$27y-54$$
$$-\quad+$$
$$56$$

Quotient: $3y^2+11y+27$

Remainder: 56

Since remainder is 56 therefore the first polynomial is **NOT** a factor of the second polynomial.

(iii) $4x^2-5, 4x^4+7x^2+15$

$$4x^2-5\,\overline{)\,4x^4+7x^2+15\,(}\,x^2+3$$
$$4x^4-5x^2$$
$$-\quad+$$
$$12x^2+15$$
$$12x^2-15$$
$$-\quad+$$
$$30$$

Quotient: x^2+3

Remainder: 30

Since remainder is 30 therefore the first polynomial is **NOT** a factor of the second polynomial.

(iv) $4-z, 3z^2-13z+4$

$$-z+4\,\overline{)\,3z^2-13z+4\,(}\,-3z+1$$
$$3z^2-12z$$
$$-\quad+$$
$$-z+4$$
$$-z+1$$
$$+\quad-$$
$$0$$

Quotient: $-3z+1$

Remainder: 0

Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a-3, 10a^2-9a-5$

$$2a-3\,\overline{)\,10a^2-9a-5\,(}\,5a+3$$
$$10a^2-15a$$
$$-\quad+$$
$$6a-5$$
$$6a-9$$
$$-\quad+$$
$$4$$

Quotient: $5a+3$

Remainder: 4

Since remainder is 4 therefore the first polynomial is **NOT** a factor of the second polynomial.

(vi) $4y+1, 8y^2-2y+1$

$$4y+1 \;\big)\; 8y^2-2y+1 \;\big(\; 2y-1$$
$$8y^2+2y$$
$$-\quad -$$
$$-4y+1$$
$$-4y-1$$
$$+\quad +$$
$$2$$

Quotient: $2y-1$

Remainder: 2

Since remainder is 2 therefore the first polynomial is **NOT** a factor of the second polynomial.

Exercise 8.6

1. Question

Divide:

$x^2 - 5x + 6$ by $x - 3$

Answer

$$x-3 \;\big)\; x^2-5x+6 \;\big(\; x-2$$
$$x^2-3x$$
$$-\quad +$$
$$-2x+6$$
$$-2x+6$$
$$+\quad -$$
$$0$$

Quotient: $x-2$

Remainder: 0

2. Question

Divide:

$ax^2 - ay^2$ by $ax + ay$

Answer

$$ax-ay \;\big)\; ax^2-ay^2 \;\big(\; x-y$$
$$ax^2 \quad -axy$$
$$-\quad +$$
$$-ay^2+axy$$
$$-ay^2+axy$$
$$+\quad -$$
$$0$$

Quotient: $x-y$

Remainder: 0

3. Question

Divide:

$x^4 - y^4$ by $x^2 - y^2$

Answer

$$x^2-y^2 \;\big)\; x^4-y^4 \;\big(\; x^2+y^2$$
$$x^4 \quad -x^2y^2$$
$$-\quad +$$
$$-y^4+x^2y^2$$
$$-y^4+x^2y^2$$
$$+\quad -$$
$$0$$

Quotient: $x^2 + y^2$

Remainder: 0

4. Question

Divide:

$acx^2 + (bc + ad)x + bd \text{ by } (ax + b)$

Answer

$$
\begin{array}{r|l|l}
ax + b & acx^2 + bcx + adx + bd & cx + d \\
 & acx^2 + bcx & \\
 & - \quad - & \\
\hline
 & adx + bd & \\
 & adx + bd & \\
 & + \quad - & \\
\hline
 & 0 &
\end{array}
$$

Quotient: $cx + d$

Remainder: 0

5. Question

Divide:

$(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2) \text{ by } 2a + b + c$

Answer

$$
\begin{array}{r|l|l}
2a + b + c & 2ab - 2ac + b^2 - c^2 & b - c \\
 & 2ab \quad + b^2 \quad + bc & \\
 & - \quad - \quad - & \\
\hline
 & -2ac \quad - c^2 - bc & \\
 & -2ac \quad - c^2 - bc & \\
 & + \quad + \quad + & \\
\hline
 & 0 &
\end{array}
$$

Quotient: $b - c$

Remainder: 0

6. Question

Divide:

$\frac{1}{4}x^2 - \frac{1}{2}x - 12 \text{ by } \frac{1}{2}x - 4$

Answer

$$
\begin{array}{r|l|l}
\frac{x}{2} - 4 & \frac{x^2}{4} - \frac{x}{2} - 12 & \frac{x}{2} + 3 \\
 & \frac{x^2}{4} - 2x & \\
 & - \quad + & \\
\hline
 & \frac{3x}{2} - 12 & \\
 & \frac{3x}{2} - 12 & \\
 & - \quad + & \\
\hline
 & 0 &
\end{array}
$$

Quotient: $\frac{x}{2} + 3$

Remainder: 0