GATE 2023
Electrical Engineering
Exam Held on : 05-02-2023
Forenoon Session

SECTION - A GENERAL APTITUDE

Q.1 Rafi told Mary, "I am thinking of watching a film this weekend."

The following reports the above statement in indirect speech :

Rafi told Mary that he _____ of watching a film that weedkend.

(a) thought
(b) is thinking
(c) am thinking
(d) was thinking

Ans. (d)

End of Solution

Q.2 Permit : _____ :: Enforce : Relax (by word meaning)

(a) Allow
(b) Forbid
(c) License
(d) Reinforce

Ans. (b)

FORBID is antonym of PERMIT.

End of Solution

Q.3 Given a fair six-faced dice where the faces are labelled '1', '2', '3', '4', '5' and '6', what is the probability of getting a '1' on the first roll of the dice and a '4' on the second roll?

(a) $\frac{1}{36}$
(b) $\frac{1}{6}$
(c) $\frac{5}{6}$
(d) $\frac{1}{3}$

Ans. (a)

$$P(1) = \frac{1}{6}$$

$$P(4) = \frac{1}{6}$$

$$\text{Required probability} = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

End of Solution

Q.4 A recent survey shows that 65% of tobacco users were advised tos top consuming tobacco. The survey also shows that 3 out of 10 tobacco users attempted to stop using tobacco.

Based only on the information in the above passage, which one of the following options can be logically inferred with *certainty*?

(a) A majority of tobacco users who were advised to stop consuming tobacco made an attempt to do so.

(b) A majority of tobacco users who were advised to stop consuming tobacco did not attempt to do so.

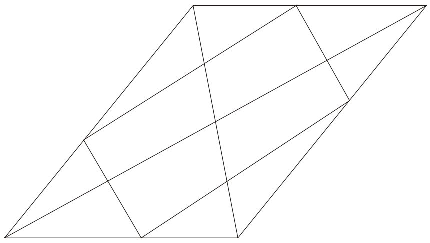
(c) Approximately 30% of tobacco users successfully stopped consuming tobacco.

(d) Approximately 65% of tobacco users successfully stopped consuming tobacco.

Ans. (b)

End of Solution

Q.5 How many triangles are present in the given figure?



(a) 12 (b) 16

(c) 20 (d) 24

Ans. (c)

Total number of triangles is 20.

End of Solution

Q.6 Students of all the departments of a college who have successfully completed the registration process are eligible to vote in the upcoming college elections. However, by the time the due date for registration was over, it was found that surprisingly none of the students from the Department of Human Sciences had completed the registration process.

Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with *certainty*?

(i) All those students who would not be eligible to vote in the college elections would certainly belong to the Department of Human Sciences.

(ii) None of the students from departments other than Human Sciences failed to complete the registration process within the due time.

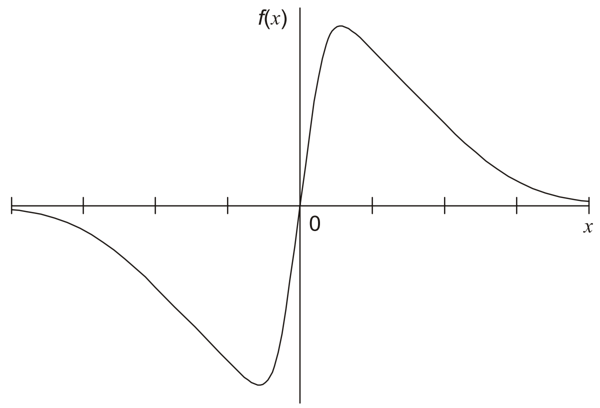
(iii) All the eligible voters would certainly be students who are not from the Department of Human Sciences.

(a) (i) and (ii)
(b) (i) and (iii)
(c) Only (i)
(d) Only (iii)

Ans. (d)

End of Solution

Q.7 Which one of the following options represents the given graph?



(a) $f(x) = x^2 2^{-|x|}$
(b) $f(x) = x2^{-|x|}$
(c) $f(x) = |x|2^{-x}$
(d) $f(x) = x2^{-x}$

Ans. (b)
Odd symmetry function.

End of Solution

Q.8 Which one of the options does NOT describe the passage below or follow from it?

We tend to think of cancer as a 'modern' illness because its metaphors are so modern. It is a disease of overproduction, of sudden growth, a growth that is unstoppable, tipped into the abyss of no control. Modern cell biology encourages us to imagine the cell as a molecular machine. Cancer is that machine unable to quench its initial command (to grow) and thus transform into an indestructibel, self-propelled automation.

(Adapted from *The Emperor of All Maladies* by Siddhartha Mukherjee)

(a) It is a reflection of why cancer seems so modern to most of us.
(b) It tells us that modern cell biology uses and promotes metaphors of machinery.
(c) Modern cell biology encourages metaphors of machinery, and cancer is often imagined as a machine.
(d) Modern cell biology never uses figurative langauge, such as metaphors, to describe or explain anyting.

Ans. (d)

End of Solution

Q.9 The digit in the unit's place of the product $3^{999} \times 7^{1000}$ is _____.

(a) 7 (b) 1

(c) 3 (d) 9

Ans. (a)

$\Rightarrow 3^{999} \times 7^{1000}$

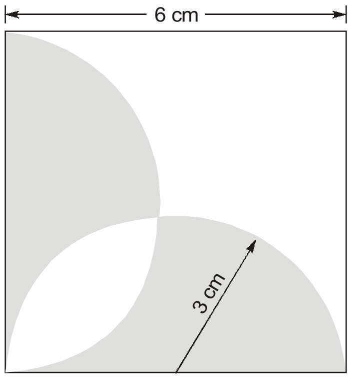
$= 3^{4(249)} \times 3^3 \times 7^{4(250)}$

$= 3^0 \times 3^3 \times 7^0 = 1 \times 7 \times 1 = 7$

Unit pace = 7

End of Solution

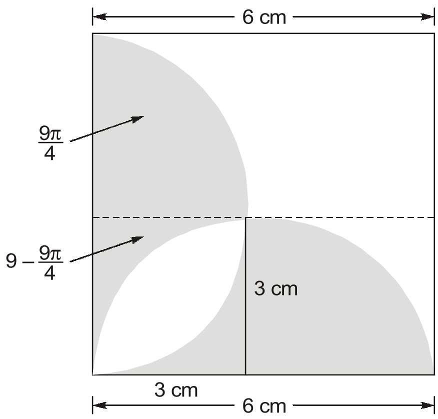
Q.10 A square with sides of length 6 cm is given. The boundary of the shaded region is defined by two semi-circles whose diameters are the sides of the square, as shown. The area of the shaded region is _____ cm^2.



(a) 6π (b) 18

(c) 20 (d) 9π

Ans. (b)



$$= 2\left\{\left(9 - \frac{9\pi}{4}\right) + \frac{9\pi}{4}\right\} = 18$$

End of Solution

■■■■

SECTION - B TECHNICAL

Q.11 For a given vector $\mathbf{w} = [1\ 2\ 3]^T$, the vector normal to the plane defined by $\mathbf{w}^T\mathbf{x} = 1$ is

(a) $[-2\quad -2\quad 2]^T$
(b) $[3\quad 0\quad -1]^T$
(c) $[3\quad 2\quad 1]^T$
(d) $[1\quad 2\quad 3]^T$

Ans. (d)

Given vector : $W = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Given plane : $W^T x = 1$

$$\Rightarrow \quad [1\ \ 2\ \ 3]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

ϕ : $x + 2y + 3z = 1$

Normal vector to plane ϕ : $W^T x = 1$ is $\overrightarrow{\nabla\phi}$

$$\overrightarrow{\nabla\phi} = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial z}$$

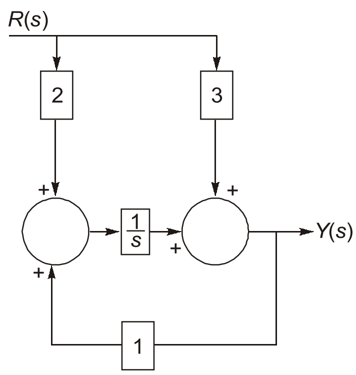
$$= i(1) + 2j + 3k$$

$$\nabla\phi = i + 2j + 3k$$

$\therefore$ Normal vector to given plane is $[1\ \ 2\ \ 3]^T$

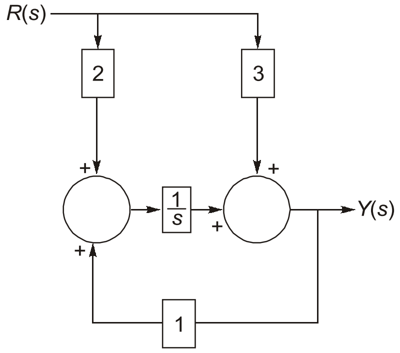
End of Solution

Q.12 For the block diagram shown in the figure, the transfer function $\dfrac{Y(s)}{R(s)}$ is



(a) $\dfrac{2s+3}{s+1}$
(b) $\dfrac{3s+2}{s-1}$
(c) $\dfrac{s+1}{3s+2}$
(d) $\dfrac{3s+2}{s+1}$

Ans. **(b)**



By signal flow graph

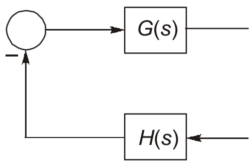
$$\frac{Y(s)}{R(s)} = \frac{3+\frac{2}{s}}{1-\left(\frac{1}{s}\right)} = \frac{3s+2}{(s-1)}$$

End of Solution

Q.13 In the Nyquist plot of the open-loop transfer function

$$G(s)H(s) = \frac{3s+5}{s-1}$$

corresponding to the feedback loop shown in the figure, the infinite semi-circular arc of the Nyquist contour in s-plane is mapped into a point at



(a) $G(s)H(s) = \infty$

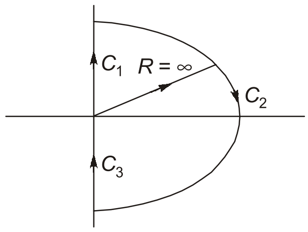
(b) $G(s)H(s) = 0$

(c) $G(s)H(s) = 3$

(d) $G(s)H(s) = –5$

Ans. **(c)**

Given : OLTF, $GH = \frac{3s+5}{(s-1)}$



Here for mapping C_2,

$$GH = \lim_{R\to\infty} \frac{3Re^{j\theta}+5}{Re^{j\theta}-1}$$

$$GH = 3$$

End of Solution

Q.14 Consider a unity-gain negative feedback system consisting of the plant *G(s)* (given below) and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

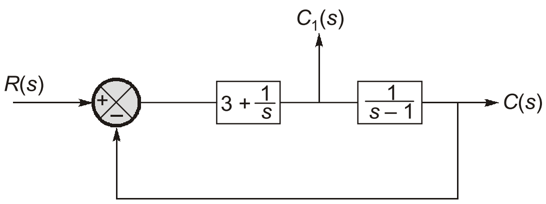
$$G(s) = \frac{1}{s-1}$$

(a) ∞, ∞
(b) 1, 0
(c) 1, –1
(d) –1, 1

Ans. (d)

$$G(s) = \frac{1}{s-1}$$

$$H(s) = 1$$



$$\frac{C(s)}{R(s)} = \frac{3s+1}{s^2+2s+1}$$

∵ $$R(s) = \frac{1}{s}$$

$$C_{ss} = \lim_{s\to 0} sC(s)$$

$$C_{ss} = \lim_{s\to 0} s\times\frac{3s+1}{s^2+2s+1}\times\frac{1}{s} = +1$$

As now, $$C(s) = C_1(s)\times\frac{1}{s-1}$$

$$C_1(s) = (s-1)C(s)$$

$$C_1(s) = (s-1)\times\left[\frac{3s+1}{s(s^2+2s+1)}\right]$$

$$(C_1)_{ss} = \lim_{s\to 0} sC_1(s) = -1$$

End of Solution

Q.15 The following columns present various modes of induction machine operation and the ranges of slip

(A) Mode of Operation	(B) Range of Slip
(a) Running in generator mode	(p) From 0.0 to 1.0
(b) Running in motor mode	(q) From 1.0 to 2.0
(c) Plugging in motor mode	(r) From –1.0 to 0.0

The correct matching between the elements in column **A** with those of column **B** is

(a) a-r, b-p, and c-q
(b) a-r, b-q, and c-p
(c) a-p, b-r, and c-q
(d) a-q, b-p, and c-r

Ans. **(a)**

Mode	Slip Range
Generator Mode	–1.0 to 0.0
Motor Mode	0.0 to 1.0
Plugging Mode	1.0 to 2.0

End of Solution

Q.16 A 10-pole, 50 Hz, 240 V, single phase induction motor runs at 540 RPM while driving rated load. The frequency of induced rotor currents due to backward field is

(a) 100 Hz
(b) 95 Hz
(c) 10 Hz
(d) 5 Hz

Ans. **(b)**

The slip for backward slip,

$$s_b = 2 - s_f = 2 - s$$

The slip,

$$s = \frac{N_s - N}{N_s} = \frac{600 - 540}{600} = 0.1$$

So,

$$s_b = 2 - 0.1 = 1.9$$

So, frequency due to backward field,

$$s_b f = 1.9 \times 50 = 95 \text{ Hz}$$

End of Solution

Q.17 A continuous-time system that is initially at rest is described by

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$

where $x(t)$ is the input voltage and $y(t)$ is the output voltage. The impulse response of the system is

(a) $3e^{-2t}$
(b) $\frac{1}{3}e^{-2t}u(t)$
(c) $2e^{-3t}u(t)$
(d) $2e^{-3t}$

Ans. **(c)**

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t) \quad ...(1)$$

For impulse response

$$x(t) = \delta(t)$$

$$\therefore \quad y(t) = h(t)$$

So, taking laplace transform of equation (1)

$$sH(s) + 3H(s) = 2(1)$$

$$H(s) = \frac{2}{s+3}$$

$$\upharpoonleft\downharpoonright$$

$$h(t) = 2e^{-3t}u(t)$$

End of Solution

Q.18 The Fourier transform $X(\omega)$ of the signal $x(t)$ is given by

$$X(\omega) = 1, \quad \text{for } |\omega| < W_o$$
$$= 0, \quad \text{for } |\omega| > W_o$$

Which one of the following statements is true?

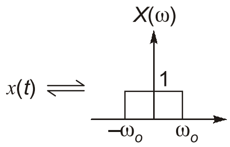
(a) $x(t)$ tends to be an impulse as $W_0 \to \infty$.

(b) $x(0)$ decreases as W_0 increases.

(c) At $t = \frac{\pi}{2W_0}, x(t) = -\frac{1}{\pi}$

(d) At $t = \frac{\pi}{2W_0}, x(t) = \frac{1}{\pi}$

Ans. **(a)**

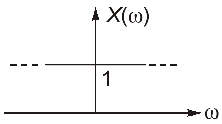


$$\therefore \quad x(t) = \frac{\sin \omega_o t}{\pi t}$$

At $t = \frac{\pi}{2\omega_o}$, $\quad x\left(\frac{\pi}{2\omega_o}\right) = \frac{\sin \frac{\pi}{2}}{\pi \cdot \frac{\pi}{2\omega_o}} = \frac{2\omega_o}{\pi^2}$

$\therefore$ Option (c) and (d) are wrong.

If $\omega_o \to \infty$, then



i.e., $X(\omega) = \text{DC-signal} = 1$

$\therefore$ $x(t) = \delta(t)$

So, option (b) is correct.

Now, $x(0) = \dfrac{\text{Area of } X(\omega)}{2\pi} = \dfrac{2\omega_o}{2\pi} = \dfrac{\omega_o}{\pi}$

$\therefore$ $x(0)$ will increase if ω_o increases.

So, option (a) is wrong.

End of Solution

Q.19 The Z-transform of a discrete signal $x[n]$ is

$$X(z) = \frac{4z}{\left(z-\frac{1}{5}\right)\left(z-\frac{2}{3}\right)(z-3)} \text{ with ROC } = R$$

Which one of the following statements is true?

(a) Discrete-time Fourier transform of $x[n]$ converges if R is $|z| > 3$

(b) Discrete-time Fourier transform of $x[n]$ converges if R is $\frac{2}{3} < |Z| < 3$

(c) Discrete-time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a left-sided sequence

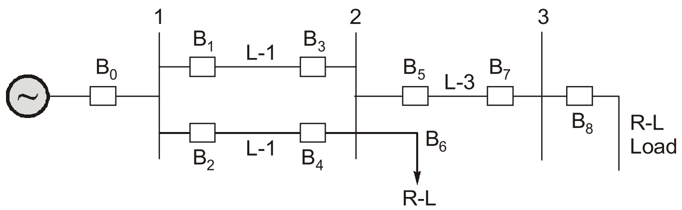
(d) Discrete-time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a right-sided sequence

Ans. **(b)**

If ROC is $\frac{2}{3} < |Z| < 3$ then it is including $Z = 1$ circle or unit-circle. So, DTFT will converge.

End of Solution

Q.20 For the three-bus power system shown in the figure, the trip signals to the circuit breakers B_1 to B_9 are provided by overcurrent relays R_1 to R_9, respectively, some of which have directional properties also. The necessary condition for the system to be protected for short circuit fault at any part of the system between bus 1 and the R-L loads with isolation of minimum portion of the network using minimum number of directional relays is



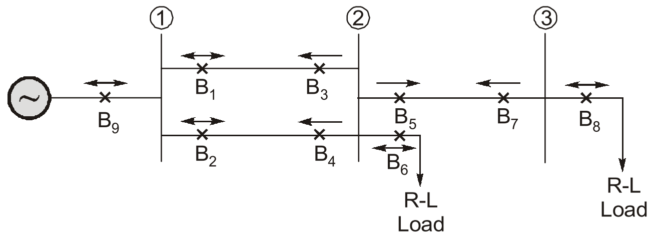
(a) R_3 and R_4 are directional overcurrent relays blocking faults towards bus 2.

(b) R_3 and R_4 are directional overcurrent relays blocking faults towards bus 2 and R_7 is directional overcurrent relay blocking faults towards bus 3.

(c) R_3 and R_4 are directional overcurrent relays blocking faults towards Line 1 and Line 2, respectively, R_7 is directional overcurrent relay blocking faults towards Line 3 and R_5 is directional overcurrent relay blocking faults towards bus 2.

(d) R_3 and R_4 are directional overcurrent relays blocking faults towards Line 1 and Line 2, respectively.

Ans. (a)



B_3, B_4 and B_5, B_7 are directional over current relays.
All remaining are (B_1, B_2, B_6, B_8, B_9)
Now directional over current relays.

End of Solution

Q.21 The expressions of fuel cost of two thermal generating units as a function of the respective power generation P_{G1} and P_{G2} are given as :

$$F_1(P_{G1}) = 0.1aP_{G1}^2 + 40P_{G1} + 120 \text{ Rs/hour} \qquad 0 \text{ MW} \le P_{G1} \le 350 \text{ MW}$$

$$F_2(P_{G2}) = 0.2aP_{G2}^2 + 30P_{G2} + 100 \text{ Rs/hour} \qquad 0 \text{ MW} \le P_{G2} \le 300 \text{ MW}$$

where a is a constant. For a given value of a, optimal dispatch requires the total load of 290 MW to be shared as P_{G1} = 175 MW and P_{G2} = 115 MW. With the load remaining unchanged, the value of a is increased by 10% and optimal dispatch is carried out. The changes in P_{G1} and the total cost of generation, F (= F_1 + F_2) in Rs/hour will be as follows :

(a) P_{G1} will decrease and F will increase.

(b) Both P_{G1} and F will increase.

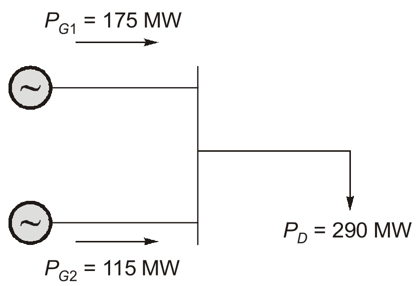
(c) P_{G1} will increase and F will decrease.

(d) Both P_{G1} and F will decrease.

Ans. (a)

$$C_1(P_{G1}) = 0.10P^2_{G1} + 40P_{G1} + 120 \text{ Rs/hr} \qquad 0 \text{ MW} \le P_{G1} \le 350 \text{ MW}$$

$$C_2(P_{G2}) = 0.10P^2_{G2} + 30P_{G2} + 100 \text{ Rs/hr} \qquad 0 \text{ MW} \le P_{G2} \le 300 \text{ MW}$$



$$\Rightarrow \quad IC_1(P_{G1}) = (0.2aP_{G1} + 40) \text{ Rs/Mwhr}$$

$$IC_2(P_{G2}) = (0.4P_{G2} + 30) \text{ Rs/Mwhr}$$

For economic scheduling of P_o = 290 MW, it is given that

$$P_{G1} = 175 \text{ MW}$$

$$P_{G2} = 115 \text{ MW}$$

Hence, $\quad I_{C1}(P_{G1} = 175) = I_{C2}(P_{G2} = 115)$

$$\Rightarrow \quad 0.2a \times 175 + 40 = 0.4 \times 115 + 30$$

$$a = 1.02857$$

$$\Rightarrow \quad F = E_1(P_{G1} = 175) + C_2(P_{G2} = 115)$$

$$= 10169.99 + 6195 = 16365 \text{ Rs/hr.}$$

Now if a is increased by 10%

$$a' = 1.1a = 1.1314$$

For economic scheduling of some demand

$$P_D = 290 \text{ MW}$$

$$P_{G1} + P_{G2} = 290 \quad \text{...(1)}$$

$$I_{C1} = I_{C2}$$

$$0.2 \times 1.1314 \times P_{G1} + 40 = 0.4P_{G2} + 30$$

$$0.2262P_{G1} - 0.4P_{G2} = -10 \quad \text{...(2)}$$

Solving eqn. (1) and (2)

$$P_{G1} = 169.27 \text{ MW}; \; P_{G2} = 120.70 \text{ MW}$$

We can see P_{G1} has decreased.

$$F = C_1(P_{G1} = 169.27) + C_2(P_{G2} = 120.70)$$

$$= 10132.52 + 6634.7$$

$$= 16767.21 \text{ Rs/hr.}$$

We can see total cost has increased.

So, correct answer is option (a).

End of Solution

Q.22 The four stator conductors (A, A', B and B') of a rotating machine are carrying DC currents of the same value, the directions of which are shown in the figure (i). The rotor coils a-a' and b-b' are formed by connecting the back ends of conductors 'a' and 'a'' and 'b' and 'b'', respectively, as shown in figure (ii). The e.m.f. induced in coil a-a' and coil b-b' are denoted by $E_{a\text{-}a'}$ and $E_{b\text{-}b'}$, respectively. If the rotor is rotated at uniform angular speed ω rad/s in the clockwise direction then which of the following correctly describes the $E_{a\text{-}a'}$ and $E_{b\text{-}b'}$?

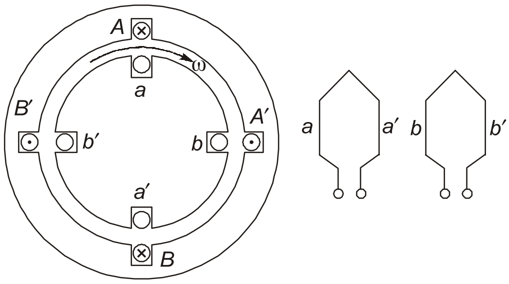


fig. (i) cross-section view

fig. (ii) rotor winding connection diagram

(a) $E_{a\text{-}a'}$ and $E_{b\text{-}b'}$ have finite magnitudes and are in the same phase.
(b) $E_{a\text{-}a'}$ and $E_{b\text{-}b'}$ have finite magnitudes with $E_{b\text{-}b'}$ leading $E_{a\text{-}a'}$
(c) $E_{a\text{-}a'}$ and $E_{b\text{-}b'}$ have finite magnitudes with $E_{a\text{-}a'}$ leading $E_{b\text{-}b'}$
(d) $E_{a\text{-}a'} = E_{b\text{-}b'} = 0$

Ans. (d)

End of Solution

Q.23 The chopper circuit shown in figure (i) feeds power to a 5 A DC constant current source. The switching frequency of the chopper is 100 kHz. All the components can be assumed to be ideal. The gate signals of switches S_1 and S_2 are shown in figure (ii). Average voltage across the 5 A current source is

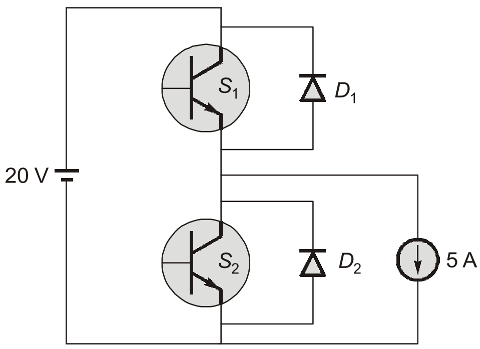


Fig. (i)

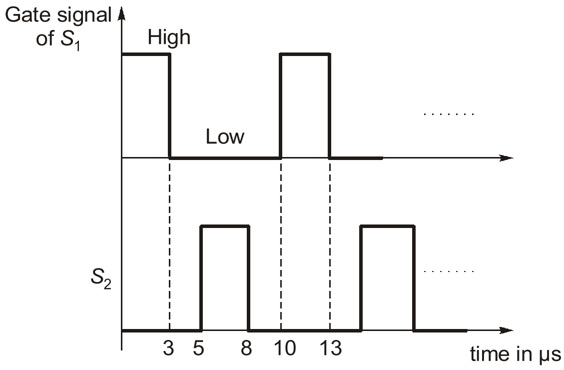
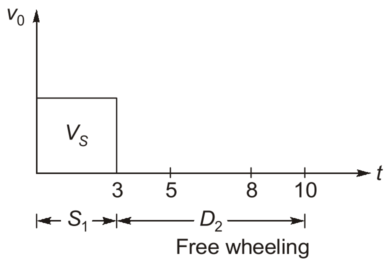


Fig. (ii)

(a) 10 V
(b) 6 V
(c) 12 V
(d) 20 V

Ans. (b)

Let voltage across 5 Amp current is v_0

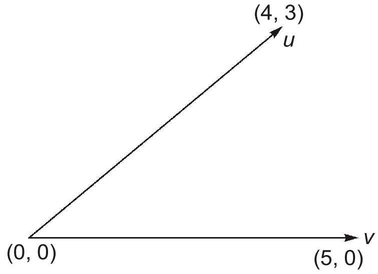


So, average voltage,

$$V_{0\,(avg)} = \frac{20 \times 3}{10} = 6 \text{ V}$$

End of Solution

Q.24 In the figure, the vectors **u** and **v** are related as: **Au** = **v** by a transformation matrix **A.** The correct choice of **A** is



(a) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

Ans. **(a)**

By verifying $Au = V$ with given options, we get

$$A = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$$

End of Solution

Q.25 One million random numbers are generated from a statistically stationary process with a Gaussian distribution with mean zero and standard deviation σ_o.

The σ_o is estimated by randomly drawing out 10,000 numbers of samples (x_n). The estimates $\hat{\sigma}_1, \hat{\sigma}_2$ are computed in the following two ways.

$$\hat{\sigma}_1^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2$$

$$\hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2$$

(a) $E\left(\hat{\sigma}_2^2\right) = \sigma_o^2$ (b) $E(\hat{\sigma}_2) = \sigma_o$

(c) $E\left(\hat{\sigma}_1^2\right) = \sigma_o^2$ (d) $E(\hat{\sigma}_1) = E(\hat{\sigma}_2)$

Ans. (c)

σ_o^2 is the variance of population of 1000 samples given by

$$\sigma_o^2 = \frac{1}{10000}\sum_{n=1}^{10000}(X-\bar{X})^2$$

Given $\bar{X} = 0$

$$\sigma_o^2 = \frac{1}{10000}\sum X_n^2$$

We know that $E\left[\sigma_1^2\right] = \sigma_1{}^2 = \sigma_0{}^2$

Hence, option (c) is the correct answer.

End of Solution

Q.26 A semiconductor switch needs to block voltage V of only one polarity ($V > 0$) during OFF state as shown in figure (i) and carry current in both directions during ON state as shown in figure (ii). Which of the following switch combination(s) will realize the same?

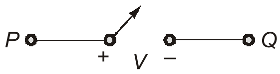


Fig. (i)

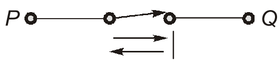
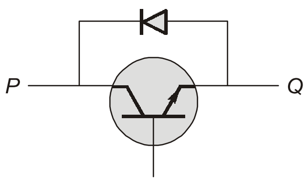
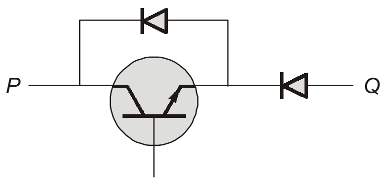
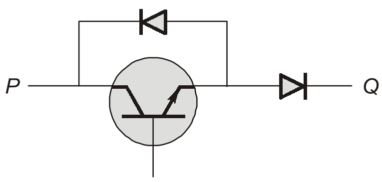
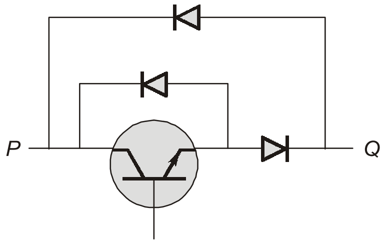


Fig. (ii)

(a) 

(b) 

(c) 

(d) 

Ans. (a, d)

From given configuration, the current flows in both direction (Bidirectional). The switch also has on drop voltage.

The switch configuration in option (a) and (d) can provide bidirectional current.

End of Solution

Q.27 Which of the following statement(s) is/are true?

(a) If an LTI system is causal, it is stable.

(b) A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.

(c) If a discrete time LTI system has an impulse response $h[n]$ of finite duration the system is stable.

(d) If the impulse response $0 < |h[n]| < 1$ for all n, then the LTI system is stable.

Ans. (b)

As we know for causal LTI-system :

$$h(n) = 0, \; n < 0$$

If input is $u(n)$, then step-response $S(n)$ will start either from $n = 0$ or right-side of $n = 0$.

i.e.,
$$S(n) = 0, \; n < 0$$

End of Solution

Q.28 The bus admittance (Y_{bus}) matrix of a 3-bus power system is given below.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j13.5 & j4 \\ j5 & j4 & -j8 \end{bmatrix} \end{array}$$

Considering that there is no shunt inductor connected to any of the buses, which of the following can NOT be true?

(a) Line charging capacitor of finite value is present in all three lines.

(b) Line charging capacitor of finite value is present in line 2-3 only.

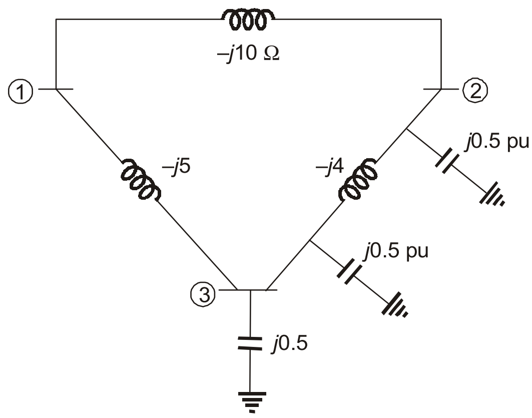
(c) Line charging capacitor of finite value is present in line 2-3 only and shunt capacitor of finite value is present in bus 1 only.

(d) Line charging capacitor of finite value is present in line 2-3 only and shunt capacitor of finite value is present in bus 3 only.

Ans. (a, c)

$$[Y_{Bus}] = j\begin{bmatrix} -15 & 10 & 5 \\ 10 & -13.5 & 4 \\ 5 & 4 & -8 \end{bmatrix} \begin{array}{l} \text{Sum} \\ = 0 \\ = j0.5 \\ = j1 \end{array}$$

No shunt branch is present at 1st bus.

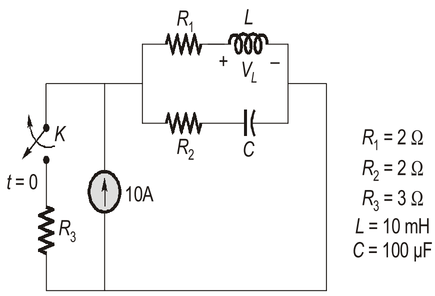


End of Solution

Q.29 The value of parameters of the circuit shown in the figure are :

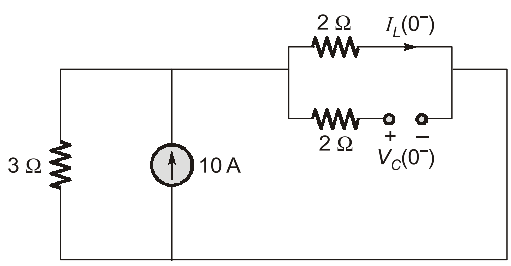
$$R_1 = 2\ \Omega,\ R_2 = 2\ \Omega,\ R_3 = 3\ \Omega,\ L = 10\ \text{mH},\ C = 100\ \mu\text{F}$$

For time $t < 0$, the circuit is at steady state with the switch 'K' in closed condition. If the switch is opened at $t = 0$, the value of the voltage across the inductor (V_L) at $t = 0^+$ in Volts is ____________ (Round off to 1 decimal place).



Ans. **(8)**

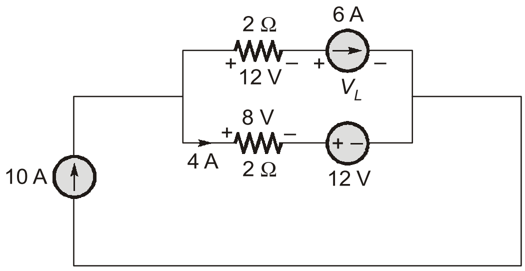
For $t = 0^-$



$$I_L(0^-) = 10 \times \frac{3}{3+2} = 6\ \text{A}$$

$$V_C(0^-) = 6 \times 2 = 12\ \text{V}$$

For $t = 0^+$

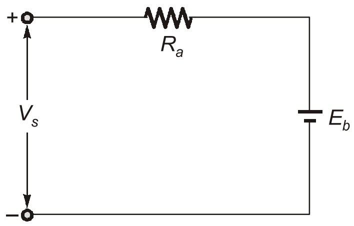


$$20 = 12 + V_L$$
$$V_L = 8\ \text{V}$$

End of Solution

Q.30 A separately excited DC motor rated 400 V, 15 A, 1500 RPM drives a constant torque load at rated speed operating from 400 V DC supply drawing rated current. The armature resistance is 1.2 Ω. If the supply voltage drops by 10% with field current unaltered then the resultant speed of the motor in RPM is ________ (Round off to the nearest integer).

Ans. (1343)



Given torque is constant, $T = K\phi I_a$ = Constant

Field is also unchanged, ϕ = Constant

So, I_a = Constant

The back emf with rated voltage

$$E_{b1} = V_s - I_a R_a = 400 - 1.2 \times 15$$
$$E_{b1} = 382\ \text{V}$$

and back emf with 10% drop in supply voltage

$$E_{b2} = V_s - I_{a2} R_a = 400 \times 0.90 - 1.2 \times 15$$
$$E_{b2} = 342\ \text{V}$$

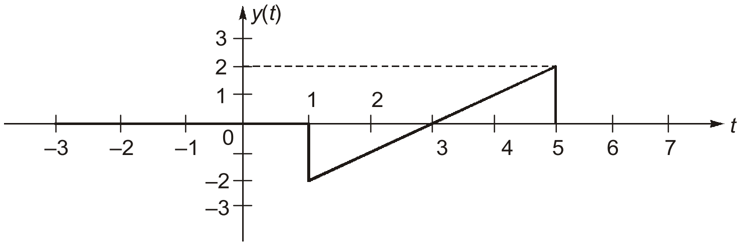
We know $E_b \propto N$ (for separately excited motor)

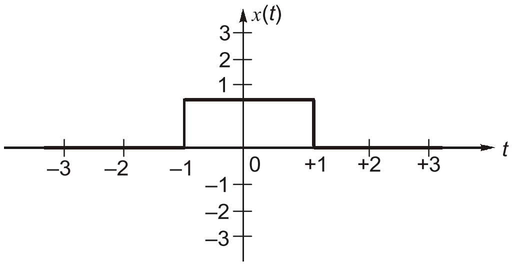
So, $$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$N_2 = \frac{342}{382} \times 1500 = 1342.93$$
$$N_2 \approx 1343\ \text{rpm}$$

End of Solution

Q.31 For the signals $x(t)$ shown in the figure, $z(t) = x(t) * y(t)$ is maximum at $t = T_1$. Then T_1 in seconds is ________ (Round off to the nearest integer).

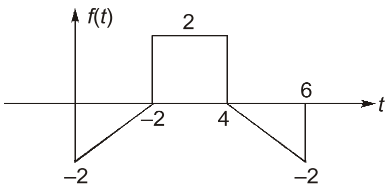




Ans. **(4)**

$$z(t) = y(t) * x(t) = y(t) * [u(t+1) - u(t-1)]$$
$$= y(t) * u(t) * [\delta(t+1) - \delta(t-1)]$$
$$= u(t) * [y(t) * \delta(t+1) - y(t) * \delta(t-1)]$$
$$= u(t) * [y(t+1) - y(t-1)]$$
$$= u(t) * f(t) \text{ where } f(t) = y(t+1) - y(t-1)$$

$\Rightarrow$
$$z(t) = \int_{-\infty}^{t} f(\tau)d\tau$$

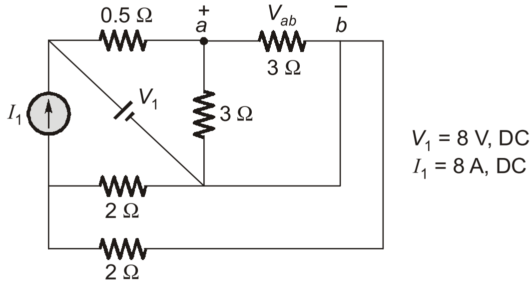


Now,
$$z(t)|_{t=0} = 0$$
$$z(t)|_{t=2} = \text{Area of } f(t) \text{ upto "} t = 2\text{"}$$
$$= -\left(\frac{1}{2} \times 2 \times 2\right) = -2$$
$$z(t)|_{t=4} = \text{Area of } f(t) \text{ upto "} t = 4\text{"} = -2 + 4 = 2$$
$$z(t)|_{t=6} = \text{Area of } f(t) \text{ upto "} t = 6\text{"} = 0$$

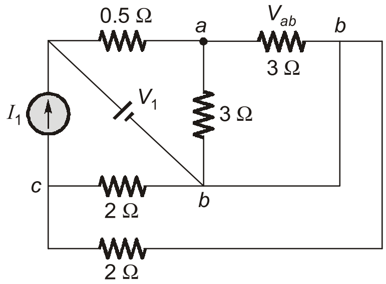
$\therefore$ $z(t)$ is maximum at $t = 4$.

End of Solution

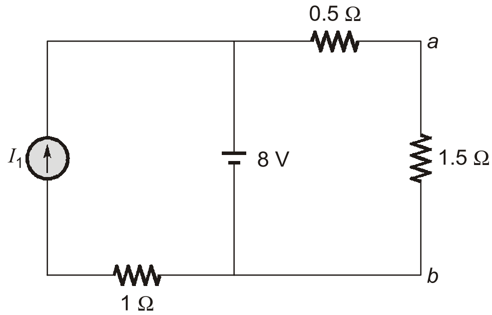
Q.32 For the circuit shown in the figure, V_1 = 8 V, DC and I_1 = 8 A, DC. The voltage V_{ab} in Volts is ________ (Round off to 1 decimal place).



Ans. **(6)**



Circuit can be redrawn



By applying voltage division rule, we can get

$$V_{ab} = 8 \times \frac{1.5}{1.5 + 0.5} = 6 \text{ V}$$

End of Solution

Q.33 A 50 Hz, 275 kV line of length 400 km has the following parameters:

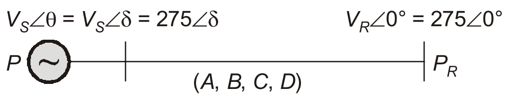
Resistance, $R = 0.035\ \Omega$/km;
Inductance, $L = 1$ mH/km;
Capacitance, $C = 0.01\ \mu$F/km;

The line is represented by the nominal-π model. With the magnitudes of the sending end and the receiving end voltages of the line (denoted by V_S and V_R, respectively) maintained at 275 kV, the phase angle difference (θ) between V_S and V_R required for maximum possible active power to be delivered to the receiving end, in degree is ________ (Round off to 2 decimal places).

Ans. **(83.64)**

For nominal π model :

$$B = Z = [R + j(2\pi + L)]l$$
$$= [0.035 + j(2\pi \times 50 \times 1 \times 10^{-3})] \times 400$$
$$= 126.44\angle 83.642° = |B|\angle\beta$$



$$P_R = \left|\frac{V_S V_R}{B}\right|\cos(\beta-\delta) - \left|\frac{AV_R^2}{B}\right|\cos(\beta-\alpha)$$

$$\frac{2P_R}{2\delta} = 0 \Rightarrow \qquad \delta = \beta$$

At $\delta = \beta \Rightarrow$ $$(P_R)_{max} = \left|\frac{V_S V_R}{B}\right| - \left|\frac{AV_R^2}{B}\right|\cos(\beta-\alpha)$$

For maximum PR occur at

$$\delta = \beta = 83.64° = \theta$$

End of Solution

Q.34 In the following differential equation, the numerically obtained value of $y(t)$, at $t = 1$, is ________________ (Round off to 2 decimal places).

$$\frac{dy}{dt} = \frac{e^{-\alpha t}}{2+\alpha t}, \ \alpha = 0.01 \text{ and } y(0) = 0$$

Ans. **(0.50)**

Given : $$\frac{dy}{dt} = \frac{e^{-at}}{2+at}, \ a = 0.01, \ y(0) = 0$$

By Taylor's series :

$$y(x) = y(0) + xy'(0) + \frac{x^2}{\angle 2}y''(0) + ...$$

$$= 0 + x\left(\frac{e^{-a(0)}}{2+a(0)}\right) + \frac{x^2}{2}\left(\frac{-3a}{4}\right)$$

$$y(x) = \frac{x}{2} - \frac{3}{8}ax^2$$

$$y(1) = \frac{1}{2} - \frac{3}{8}(0.01)1^2$$

$$y(1) = 0.5 - 0.00375$$
$$y(1) = 0.496$$
$$y(1) = 0.49 \text{ or } 0.50$$

End of Solution

Q.35 Three points in the x-y plane are (–1, 0.8), (0, 2.2) and (1, 2.8). The value of the slope of the best fit straight line in the least square sense is ________ (Round off to 2 decimal places).

Ans. (1)

Given that :

X	–1	0	1
Y	0.8	2.2	2.8

Best fit of straight line ($y = ax + b$) by least square.

Approximation given by normal equations

$$\Sigma y_i = a\,\Sigma x_i + b\,n \quad \text{...(i)}$$

$$\Sigma x_i y_i = a\,\Sigma x_i^2 + b\,\Sigma x_i \quad \text{...(ii)}$$

x	y	x_i^2	xy
–1	0.8	1	–0.8
0	2.2	0	0
1	2.8	1	2.8
$\Sigma x_i = 0$	$\Sigma y_i = 5.8$	$\Sigma x^2 = 2$	$\Sigma xy = 2$

Substituting values in eqn. (1) and (2)

$$5.8 = a(0) + 3b \quad \text{...(iii)}$$

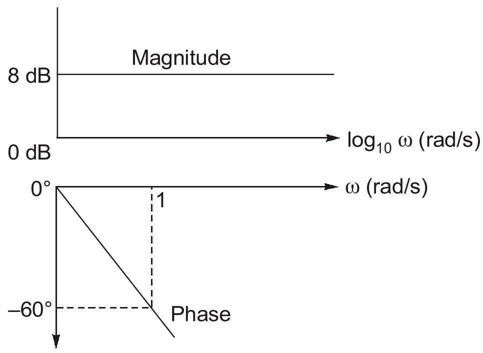
$$2 = a(2) + 0(b) \quad \text{...(iv)}$$

Solving we get, $a = 1,\ b = \dfrac{5.8}{3} = 1.93$

$\therefore$ Slope of best fit $y = ax + b$ is 'a' = 1.

End of Solution

Q.36 The magnitude and phase plots of an LTI system are shown in the figure. The transfer function of the system is



(a) $2.515e^{-0.032s}$ (b) $\dfrac{e^{-2.514s}}{s+1}$

(c) $1.04e^{-2.514s}$ (d) $2.51e^{-1.047s}$

Ans. (d)

The transfer function, TF = Ke^{-sT_d} (tranportation lag)

Given magnitude, M = 8 dB = 20 log [K]

K = 2.511

and angle at ω = 1 rad/sec = –60°

$$\text{Angle, } \phi = -\omega T_d \times \frac{180°}{\pi}$$

$$60° = -1 \times T_d \times \frac{180°}{\pi}$$

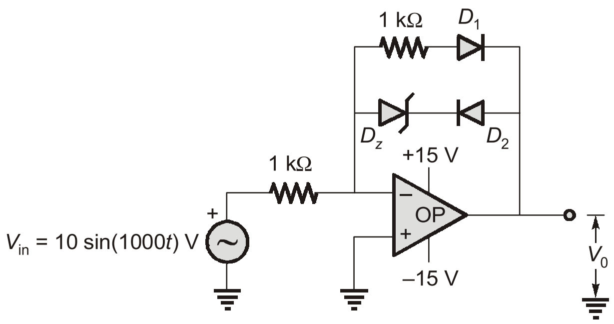
$$T_d = 1.047$$

So, required transfer function

$$TF = 2.511e^{-1.047s}$$

End of Solution

Q.37 Consider the OP AMP based circuit shown in the figure. Ignore the conduction drops of diodes D_1 and D_2. All the components are ideal and the breakdown voltage of the Zener is 5 V. Which of the following statements is true?



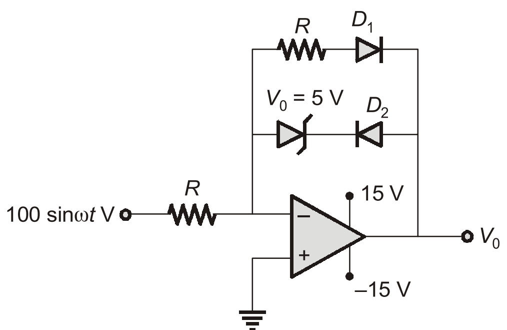
(a) The maximum and minimum values of the output voltage V_O are +15 V and –10 V, respectively.

(b) The maximum and minimum values of the output voltage V_O are +5 V and –15 V, respectively.

(c) The maximum and minimum values of the output voltage V_O are +10 V and –5 V, respectively.

(d) The maximum and minimum values of the output voltage V_O are +5 V and –10 V, respectively.

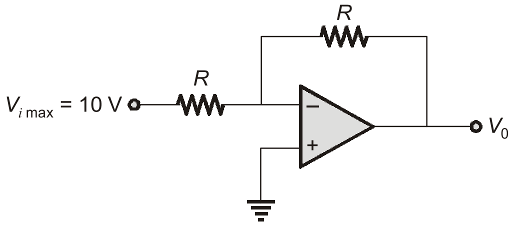
Ans. (d)



$V_{0\,min}$ and $V_{0\,max}$

$$V_{i\,max} = 10\text{ V}$$

D_1 on, D_2 off

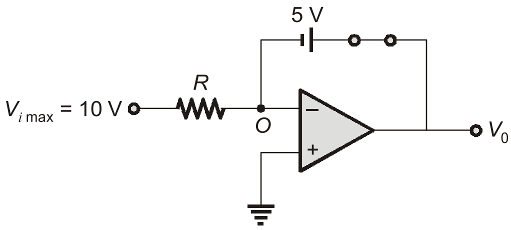


$$V_0 = -V_i$$

$$\text{Minimum } V_0 = -10\text{ V}$$

$$V_{i\,min} = -10\text{ V}$$

$D_{1\,off}$, $D_{2\,on}$, zerer on



$$V_0 = +5\text{ V } (V_{0\,max})$$

$$V_{0\,max} = +5\text{ V},\ V_{0\,min} = -10\text{ V}$$

End of Solution

Q.38 Consider a lead compensator of the form

$$K(s) = \frac{1+\dfrac{s}{\alpha}}{1+\dfrac{s}{\beta\alpha}}, \beta > 1, \alpha > 0$$

The frequency at which this compensator produces maximum phase lead is 4 rad/s. At this frequency, the gain amplification provided by the controller, assuming asymptotic Bode-magnitude plot of $K(s)$, is 6 dB. The values of α, β, respectively, are

(a) 1, 16
(b) 2, 4
(c) 3, 5
(d) 2.66, 2.25

Ans. **(b)**

$$\text{TF} = \frac{1+\dfrac{s}{\alpha}}{1+\dfrac{s}{\alpha\beta}}, \alpha > 0, \beta > 1$$

$$\text{TF} = \frac{s+\alpha}{(s+\alpha\beta)}$$

Given : $\omega_m = \sqrt{\alpha(\alpha\beta)} = 4$

$$\alpha\sqrt{\beta} = 4 \quad ...(1)$$

Given at $\omega = \omega_m$

$$x = \frac{1}{\beta}$$

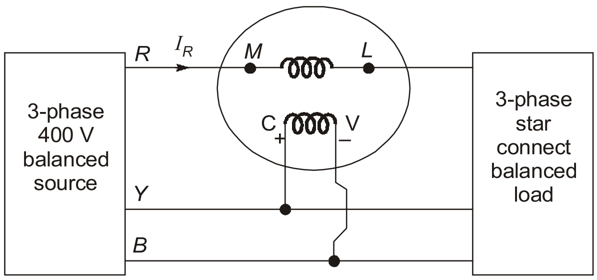
Amplification, $M = 10\log_{10}\frac{1}{x} = 6$

$$10 \log_{10} (\beta) = 6$$
$$\beta = 4$$

and $\alpha = \frac{4}{\sqrt{4}} = 2$

End of Solution

Q.39 A 3-phase, star-connected, balanced load is supplied from a 3-phase, 400 V (rms), balanced voltage source with phase sequence R-Y-B, as shown in the figure. If the wattmeter reading is –400 W and the line current is I_R = 2 A (rms), then the power factor of the load per phase is



(a) Unity
(b) 0.5 leading
(c) 0.866 leading
(d) 0.707 lagging

Ans. **(c)**

Given : $V_{line} = 400$ Volt $\Rightarrow$ Y-connected

$$V_{phase} = \frac{V_{line}}{\sqrt{3}} = \frac{400}{\sqrt{3}}$$

$$I_{line} = I_{phase} = 2 \text{ Amp}$$

Wattmeter reading = –400 Watt

This Wattmeter connection is related to reactive power measurement by single wattmeter method.

So, Wattmeter reading = $\sqrt{3} \cdot V_{ph} \cdot I_{ph} \sin(\phi)$

$$-400 \text{ Watt} = \sqrt{3} \times \frac{400}{\sqrt{3}} \times 2 \times \sin(\phi)$$

$$\sin(\phi) = -\frac{1}{2}$$

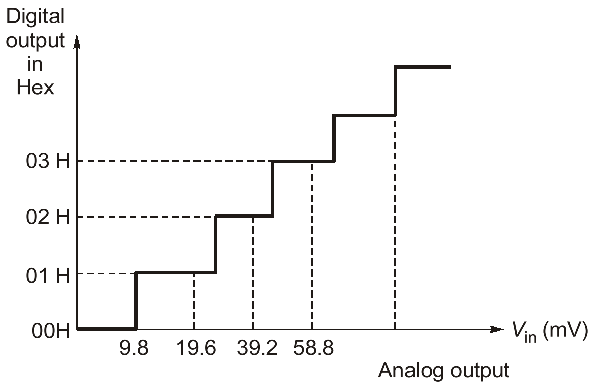
$$\Rightarrow \qquad \phi = \sin^{-1}\left(-\frac{1}{2}\right) = -30° \Rightarrow \text{Leading}$$

$$\text{P.f. of load} = \cos(\phi) = \cos(-30°)$$

$$\text{P.f.} = 0.866 \text{ leading}$$

End of Solution

Q.40 An 8-bit ADC converts analog voltage in the range of 0 to +5 V to the corresponding digital code as per the conversion characteristics shown in figure. For V_{in} = 1.9922 V, which of the following digital output, given in hex, is true?



(a) 64H
(b) 65H
(c) 66H
(d) 67H

Ans. **(c)**

$$V_{in} = 1.992 \text{ V},$$

$$n = 8,$$

$$V_{fs} = 5 \text{ V}$$

We know, from graph

$$\text{Step size} = \frac{V_{fs}}{2^n - 1}$$

$$\text{Step size} = \frac{5}{2^8 - 1} = \frac{5}{255}$$

Analog input = Step size × (decimal equivalent of binary code)

$$1.992 = \frac{5}{255} \times D$$

$$D = \frac{1.992 \times 255}{5} = (101.592)_{10}$$

For decimal equivalent of 101,

$$\text{Analog input} = \frac{5}{255} \times 101 = 1.980$$

Here 1.98 is less than 1.992 so, we have to take decimal equivalent as 102

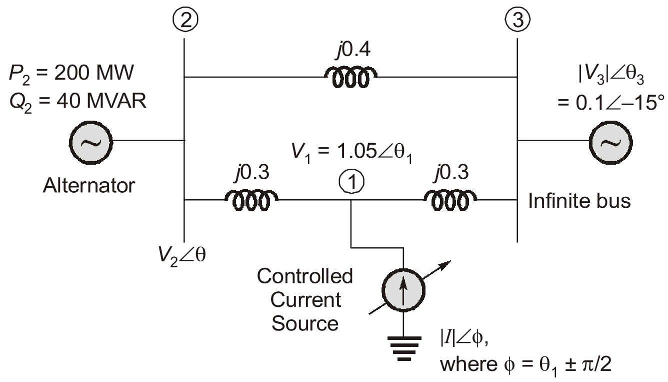
For $(102)_{10}$ Analog input $= \frac{5}{255} \times 102 = 2$

2 is near about 1.992 so, decimal equivalent will 102

In hexadecimal, $(102)_{10} = 66$ H

End of Solution

Q.41 The three-bus power system shown in the figure has one alternator connected to bus 2 which supplies 200 MW and 40 MVAr power. Bus 3 is infinite bus having a voltage of magnitude $|V_3| = 1.0$ p.u. and angle of $-15°$. A variable current source, $|I|\angle\phi$ is connected at bus 1 and controlled such that the magnitude of the bus 1 voltage is maintained at 1.05 p.u. and the phase angle of the source current, $\phi = \theta_1 \pm \frac{\pi}{2}$, where θ_1 is the phase angle of the bus 1 voltage. The three buses can be categorized for load flow analysis as



(a) Bus 1 Slack Bus; Bus 2 $P-|V|$ Bus; Bus 3 $P-Q$ Bus

(b) Bus 1 $P-|V|$ Bus; Bus 2 $P-|V|$ Bus; Bus 3 Slack Bus

(c) Bus 1 $P-Q$ Bus; Bus 2 $P-Q$ Bus; Bus 3 Slack Bus

(d) Bus 1 $P-|V|$ Bus; Bus 2 $P-Q$ Bus; Bus 3 Slack Bus

Ans. **(d)**

At Bus (1), voltage magnitude is maintained also specified active power is zero. So, it is a PV Bus.

At Bus (2), P_G and Q_G are specified and $|V|$, δ are unknown. Hence, it is PQ Bus.

At Bus (3), $|V|$ and δ are specified. So it is slack bus.

End of Solution

Q.42 Consider the following equation in a 2-D real-space.

$$|x_1|^p + |x_2|^p = 1 \text{ for } p > 0$$

Which of the following statement(s) is/are true.

(a) When $p = 2$, the area enclosed by the curve is π.

(b) When p tends to ∞, the area enclosed by the curve tends to 4.

(c) When p tends to 0, the area enclosed by the curve is 1.

(d) When $p = 1$, the area enclosed by the curve is 2.

Ans. **(a, b, d)**

Given space $|x_1|^P + |x_2|^P = 1$, for $P > 0$

(a) For $P = 2$,

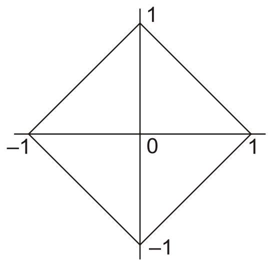
Eqn. : $|x_1|^q + |x_2|^2 = 1$ is a unit circle.

$\therefore$ Area $= \pi(1)^2 = \pi$

$\therefore$ It is true.

(d) For $P = 1$,

Eqn. : $|x_1| + |x_2| = 1$ is a square.

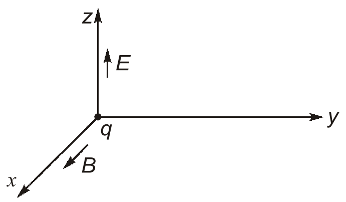


$$\text{Area of square} = \frac{d^2}{2}$$

$$= \frac{2^2}{2} = 2$$

End of Solution

Q.43 In the figure, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ point to x and z directions, respectively, and have constant magnitudes. A positive charge 'q' is released from rest at the origin. Which of the following statement(s) is/are true.



(a) The charge will move in the direction of **z** with constant velocity.

(b) The charge will always move on the **y**-**z** plane only.

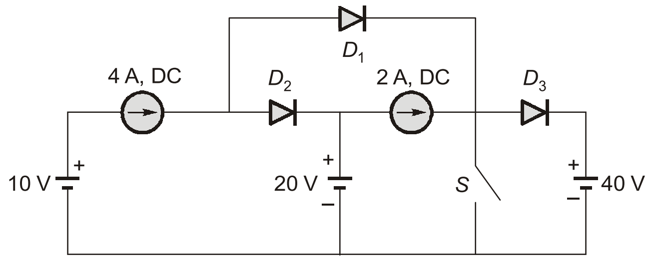
(c) The trajectory of the charge will be a circle.

(d) The charge will progress in the direction of **y**.

Ans. **(*)**

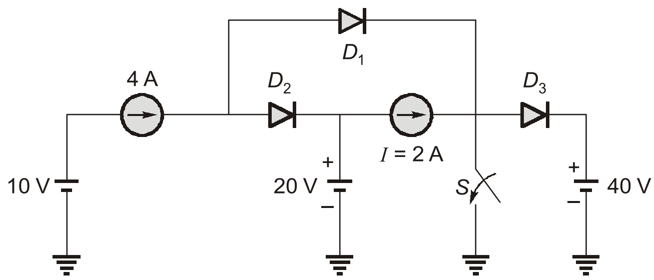
End of Solution

Q.44 All the elements in the circuit shown in the following figure are ideal. Which of the following statements is/are true?

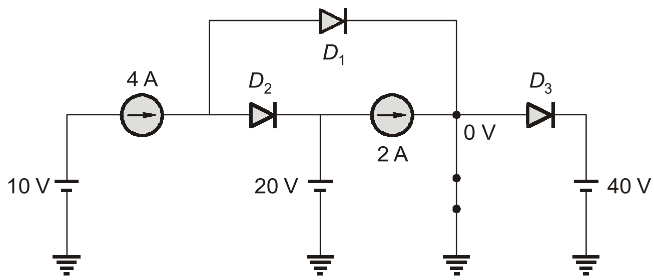


(a) When switch S is ON, both D_1 and D_2 conducts and D_3 is reverse biased.
(b) When switch S is ON, D_1 conducts and both D_2 and D_3 are reverse biased.
(c) When switch S is OFF, D_1 is reverse biased and both D_2 and D_3 conduct.
(d) When switch S is OFF, D_1 conducts, D_2 is reverse biased and D_3 conducts.

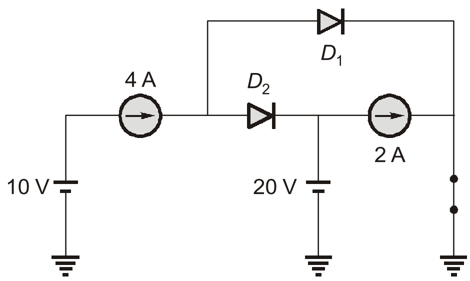
Ans. **(b, c)**



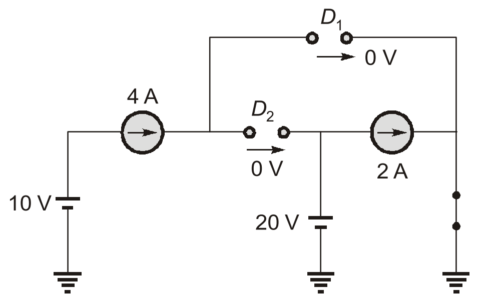
When switch on,



D_3 off by observation

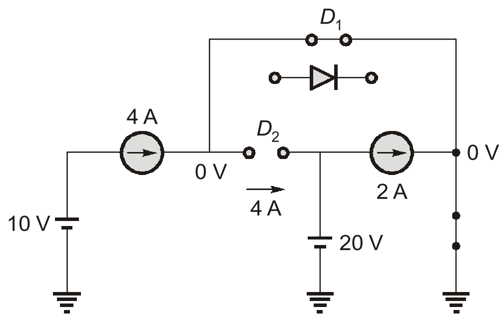


Assume D_1 and D_2 off



KCL validates not possible

Assume D_1 on, D_2 off



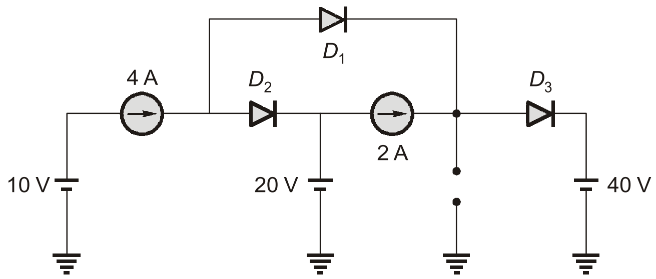
When D_1 on D_2 off

This assumption when switched:

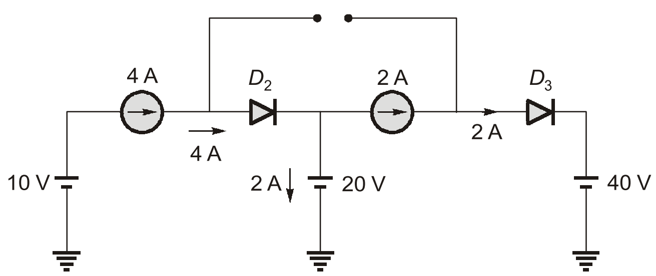
D_1 on , D_2 off, D_3 off correct

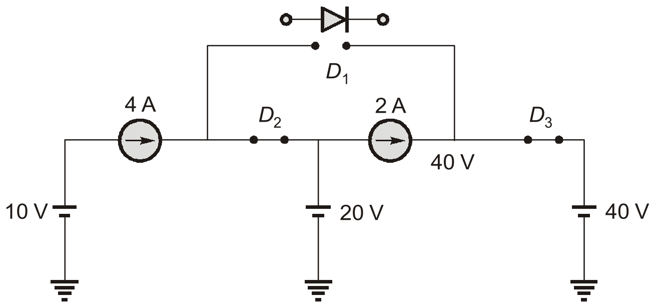
W

hen switch is off,



Assume D_1 off





D_1 off

Switch off : D_1 off, D_2 and D_3 on.

End of Solution

Q.45 The expected number of trials for first occurrence of a "head" in a biased coin is known to be 4. The probability of first occurrence of a "head" in the second trial is ____________ (Round off to 3 decimal places).

Ans. (0.188)

By Geometrical distribution,

$$E(x) = \frac{1}{P}$$

$$V(x) = \frac{q}{P^2}$$

$$E(x) = 4 = \frac{1}{P}$$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(H) = \frac{1}{4},$$

$$P(T) = \frac{3}{4}$$

Required probability, $P(E) = P(T\ H)$

$$= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} = 0.1875$$

End of Solution

Q.46 Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 \end{bmatrix}, \quad D = 1$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is __________ (Round off to the nearest integer).

Ans. (2)

Given :
$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = [3, \quad -2]$$
$$D = 1$$

The transfer function
$$TF = C[sI - A]^{-1}B + D$$
$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^{-1}$$
$$= \frac{1}{(s^2+2s+1)}\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}$$
$$TF = \frac{1}{(s^2+2s+1)}[3 \quad -2]\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}+1$$
$$TF = \frac{1}{(s^2+2s+1)}[3 \quad -2]\begin{bmatrix} 1 \\ s \end{bmatrix}+1$$
$$TF = \frac{(3-2s)}{(s^2+2s+1)}+1 = \frac{s^2+4}{(s^2+2s+1)}$$
$$H(j\omega) = \frac{4-\omega^2}{(1-\omega^2+2j\omega)}$$

The output will be zero, for $\omega^2 - 4 = 0$
$$\omega = 2 \text{ rad/sec}$$

End of Solution

Q.47 A three-phase synchronous motor with synchronous impedance of $0.1+j0.3$ per unit per phase has a static stability limit of 2.5 per unit. The corresponding excitation voltage in per unit is ________ (Round off to 2 decimal places).

Ans. (1.6022)

Given :
$$Z_s = (0.1 + j0.3) = \sqrt{0.1^2+0.3^2}\angle\tan^{-1}\left(\frac{0.3}{0.1}\right)$$
$$Z = 0.3162\angle 71.56° \text{ pu}$$

The developed power will be maximum, if $\delta = \theta$
$$P_m = \frac{E_f V}{Z_s} - \frac{E_f^2}{Z_s}\cos\theta$$
$$2.5 = \frac{E_f(1)}{0.3162} - \frac{E_f^2}{0.3162}\cos(71.56)$$

$$E_f^2 - 3.162E_f + 2.5 = 0$$
$$E_f = 1.6022 \text{ pu}$$

End of Solution

Q.48 A three phase 415 V, 50 Hz, 6-pole, 960 RPM, 4 HP squirrel cage induction motor drives a constant torque load at rated speed operating from rated supply and delivering rated output. If the supply voltage and frequency are reduced by 20%, the resultant speed of the motor in RPM (neglecting the stator leakage impedance and rotational losses) is ________ (Round off to the nearest integer).

Ans. (760)

Method 1 :

$$N_s = 120 \times \frac{50}{6} = 1000 \text{ rpm}$$

$\Rightarrow$ Slip speed = 40 rpm

Here, V/f = Constant. So, slip speed remains constant.

For $f = 50 \times 0.80 = 40$ Hz, the synchronous speed

$$N_{s2} = 120 \times \frac{40}{6}$$
$$N_{s2} = 800 \text{ rpm}$$

So, motor speed $N_2 = N_{s2} - \text{slip speed}$

$$= 800 - 40 = 760 \text{ rpm}$$

Method 2 :

Torque, $T \propto \dfrac{sV^2}{f}$

$$\frac{T_2}{T_1} = \frac{s_2}{s_1}\left(\frac{V_2}{V_1}\right)^2\left(\frac{f_2}{f_1}\right)$$

$$1 = \frac{s_2}{0.04}(0.8)^2 \times \frac{1}{0.8}$$

$$s_2 = 0.05$$

So, $N_2 = N_{s2}(1 - s_2) = \dfrac{120 \times f_2}{P}(1 - s_2)$

$$N_2 = \frac{120 \times 40}{6}(1 - 0.05) = 760 \text{ rpm}$$

End of Solution

Q.49 The period of the discrete-time signal $x[n]$ described by the equation below is N = _____ (Round off to the nearest integer).

$$x[n] = 1 + 3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

Ans. **(48)**

$$x[n] = 1+3\sin\left(\frac{15\pi}{8}n+\frac{3\pi}{4}\right)-5\sin\left(\frac{\pi}{3}n-\frac{\pi}{4}\right)$$

$$= 1+3\sin\left(\omega_1 n+\frac{3\pi}{4}\right)-5\sin\left(\omega_2 n-\frac{\pi}{4}\right)$$

where, $\omega_1 = \frac{15\pi}{8}$, $w_2 = \frac{\pi}{3}$

Now, $N_1 = \frac{2\pi}{\omega_1}K_1 = \frac{2\pi}{\frac{15\pi}{8}}K_1 = \frac{16}{15}K_1$

$= 16$ (for $K_1 = 15$)

$N_2 = \frac{2\pi}{\omega_2}K_2 = \frac{2\pi}{\frac{\pi}{3}}K_2 = 6K_2 = 6$ (for $K_2 = 1$)

Time-period of $x(n)$ is :

$N = \text{LCM}[N_1, N_2] = \text{LCM}[16, 6] = 48$

End of Solution

Q.50 The discrete-time Fourier transform of a signal $x[n]$ is $X(\Omega) = (1 + \cos\Omega)e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

$$x_p[n] = x[n], \text{ for } n = 0, 1, 2$$
$$= 0, \text{ for } n = 3, 4$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is ______________ (Round off to 3 decimal places).

Ans. **(0.038)**

Given that :

$$x(n) \Longleftrightarrow X(e^{j\Omega}) = (1+\cos\Omega)e^{-j\Omega}$$

and $x_p(n)$ = Periodic signal $\Longleftrightarrow$ a_K = DFS-coefficient with $N = 5$

where $x_p(n) = \begin{cases} x(n), & \text{for } n = 0,1,2 \\ 0, & \text{for } n = 3, 4 \end{cases}$

Shortcut : $a_K = \frac{X(e^{jK\Omega_o})}{N}$, where $\Omega_o = \frac{2\pi}{N} = \frac{2\pi}{5}$

$$= \frac{1}{5}(1+\cos K\Omega_o)\cdot e^{-jK\Omega_o}$$

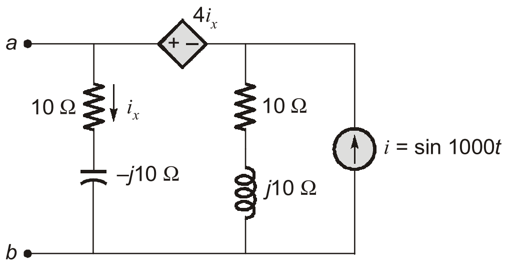
$$= \frac{1}{5}\left[1+\cos\frac{2\pi}{5}K\right]\cdot e^{-j\frac{2\pi}{5}K}$$

Now, $|a_K| = \frac{1}{5}\left[1+\cos\frac{2\pi}{5}K\right]$

Put $K = 3$; $|a_3| = \frac{1}{5}\left[1+\cos\frac{6\pi}{5}\right] = \frac{1}{5}(1-0.809) = 0.038$

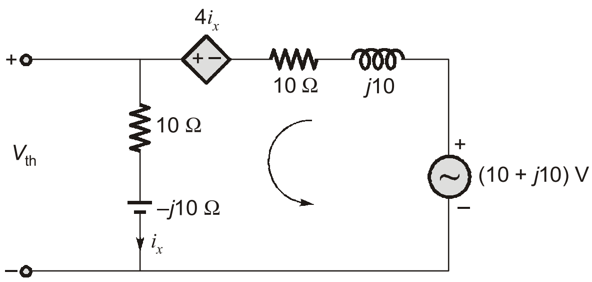
End of Solution

Q.51 For the circuit shown, if $i = \sin 1000t$, the instantaneous value of the Thevenin's equivalent voltage (in Volts) across the terminals a-b at time t = 5 ms is __________ (Round off to 2 decimal places).



Ans. **(–11.98)**

Applying source transformation



$$V_{th} = i_x(40 - j10)$$

Applying KVL

$$10 + j10 = (10 + j10)i_x - 4i_x + (10 - j10)i_x$$

$$i_x = \frac{10+j10}{16}$$

$$V_{th} = i_x(10 - j10)$$

$$= \frac{100+100}{16} = 12.5 \text{ Volts}$$

$$V_{th} = 12.5 \sin 1000t$$

$$= 12.5 \sin 1000 \times 5 \times 10^{-3}$$

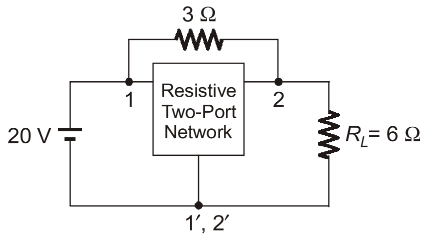
$$= -11.98 \text{ V}$$

End of Solution

Q.52 The admittance parameters of the passive resistive two-port network shown in the figure are :

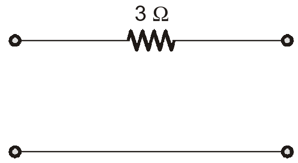
$$y_{11} = 5S,\ y_{22} = 1S,\ y_{12} = y_{21} = -2.5S$$

The power delivered to the load resistor R_L in Watt is ________ (Round off to 2 decimal places).



Ans. **(238)**

$$Y = [Y]_A + [Y]_B$$



$$Y_A = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$Y_B = \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix} S$$

$$Y = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} + \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{16}{3} & -\frac{8.5}{3} \\ -\frac{8.5}{3} & \frac{4}{3} \end{bmatrix}$$

$$I_1 = \frac{16}{3}V_1 - \frac{8.5}{3}V_2 \quad \text{...(1)}$$

$$I_2 = -\frac{8.5}{3}V_1 + \frac{4}{3}V_2 \quad \text{...(2)}$$

Put $I_2 = 0$ and $V_1 = 20$ V

$$0 = -\frac{8.5}{3} \times 20 + \frac{4}{3}V_2$$

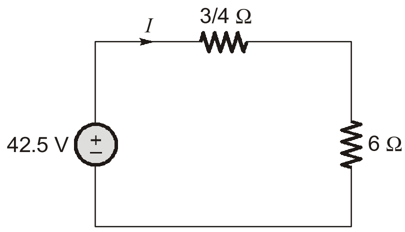
$$V_{th} = V_2 = \frac{8.5 \times 20}{4} = 42.5 \text{ V}$$

$$R_{th} = \frac{V_2}{I_2}$$

$$I_2 = \frac{4}{3} V_2$$

$$\frac{V_2}{I_2} = \frac{3}{4} \Omega$$

Equivalent circuit

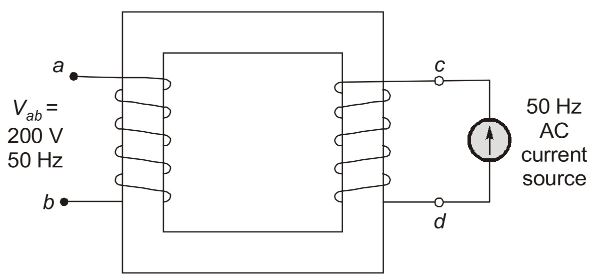


$$I = \frac{42.5}{\frac{3}{4} + 6} = 6.296 \text{ A}$$

$$P = I^2 R_L = (6.296)^2 \times 6$$
$$= 238 \text{ W}$$

End of Solution

Q.53 When the winding *c-d* of the single-phase, 50 Hz, two winding transformer is supplied from an AC current source of frequency 50 Hz, the rated voltage of 200 V (rms), 50 Hz is obtained at the open-circuited terminals *a-b*. The cross sectional area of the core is 5000 mm^2 and the average core length traversed by the mutual flux is 500 mm. The maximum allowable flux density in the core is B_{max} = 1 Wb/m^2 and the relative permeability of the core material is 5000. The leakage impedance of the winding *a-b* and winding *c-d* at 50 Hz are $(5 + j100\pi \times 0.16)$ Ω and $(11.25 + j100\pi \times 0.36)$ Ω, respectively. Considering the magnetizing characteristics to be linear and neglecting core loss, the self-inductance of the winding *a-b* in millihenry is __________ (Round off to 1 decimal place).



Ans. **(2218.43)**

Given :

$l = 500 \text{ mm} = 0.5 \text{ m}$

$A = 5000 \text{ mm}^2 = 5 \times 10^{-3} \text{ m}^2$

$\mu_r = 5000$

$E = 200 \text{ V}$

So,
$$R = \frac{l}{\mu_o \mu_R A} = \frac{0.5}{4\pi \times 10^{-7} \times 5000 \times 5 \times 10^{-3}} = 15915.49$$

and
$$E = 4.44 f N B_m A$$

$$N = \frac{E}{4.44 f N B_m A} = \frac{200}{4.44 \times 50 \times 1 \times 5 \times 10^{-3}}$$

$$N \cong 181$$

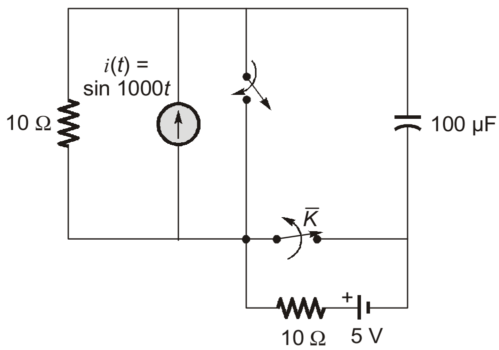
$$L = \frac{N^2}{R} = \frac{181}{15915.49} = 2058.43 \text{ mH}$$

The self-inductance of the winding a-b = L + 0.16 H = (2058.43 + 160) mH

= 2218.43

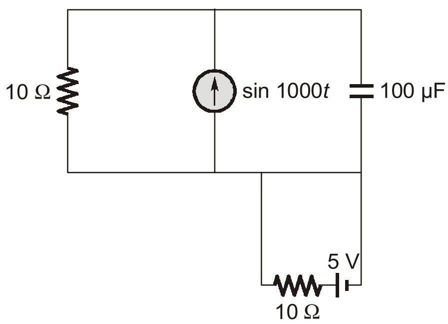
End of Solution

Q.54 The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds, such that there is no transient in the voltage across the 100 μF capacitor, is ____________ (Round off to 2 decimal places).



Ans. **(1.57)**

$t = 0^-$



$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 100 \times 10^{-6}} \Rightarrow 10\ \Omega$$

$$V_C = 1\angle 0° \times \frac{10}{10 - j10} * -j10$$

$$= \frac{(10)(-j10)}{10 - j10} \times \frac{10 + j10}{10 + j10} \Rightarrow 5 - j5$$

$$V_c = 7.07\angle{-45°}$$

$$V_c(t) = 7.07 \sin(1000t - 45°)$$

At $t = t_1$

$$V_c(t) = 7.07 \sin(1000t_1 - 45°)$$

$$V_c(\infty) = 5 \text{ V};\ \tau = RC \Rightarrow 10 \times 100 \times 10^{-6} \Rightarrow 10^{-3}$$

$$V_c(t) = 5 + (7.07 \sin(1000t_1 - 45) - 5)e^{-(t/10^{-3})}$$

$$V_c(t) = 5 + (7.07 \sin(1000t_1 - 45) - 5)e^{-1000(t - t_1)}$$

$$7.07 \sin(1000t_1 - 45°) = 5$$

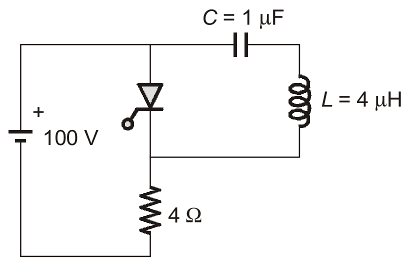
$$\sin(1000t_1 - 45°) = \frac{5}{7.07}$$

$$1000t_1 - 45° = 45°$$

$$t_1 = 1.57 \text{ msec}$$

End of Solution

Q.55 The circuit shown in the figure has reached steady state with thyristor 'T' in OFF condition. Assume that the latching and holding currents of the thyristor are zero. The thyristor is turned ON at $t = 0$ sec. The duration in microseconds for which the thyristor would conduct, before it turns off, is _____ (Round off to 2 decimal places).



Ans. **(7.33)**

Case-1 :

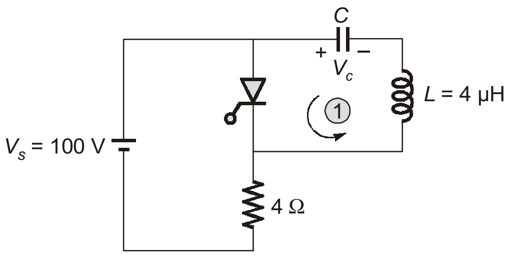
Steady state condition before $t = 0$ sec

The capacitor is charged already with supply voltage $V_S = 100$ V

Case-2 :

Now thyristor T is turned on

Mode-1:



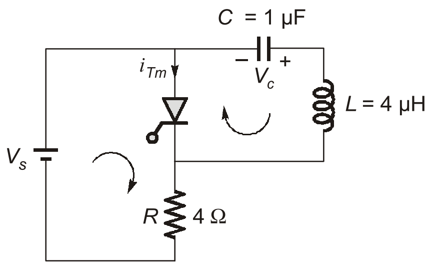
At starting V_C = 100 V

The capacitor will discharge through LC circuit

at the end, $t_1 = \pi\sqrt{LC}\,\text{sec}$,

the capacitor voltage will become V_C = –100 V (polarity is changed)

Mode-2:



The thyristor current, $i_{Tm} = \dfrac{V_s}{R} - I_p \sin\omega_0 t$

at the end,

$$I_0 = \frac{V_s}{R} - I_p \sin\omega_0 t$$

$$i_{Tm} = 0 \qquad (\therefore\ T_m \to \text{off})$$

So, $$\omega_0 t_2 = \sin^{-1}\left(\frac{I_0}{I_p}\right)$$

$$I_p = V_s\sqrt{\frac{C}{L}} = 100\sqrt{\frac{1}{4}} = 50\text{ A}$$

and $$I_0 = \frac{V_s}{R} = \frac{100}{4} = 25\text{ A}$$

So, $$t_2 = \sqrt{LC}\sin^{-1}\left(\frac{25}{50}\right) = \frac{\pi}{6}\sqrt{LC}$$

So total time for conduction,

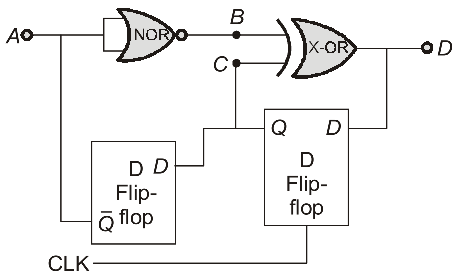
$$t_1 + t_2 = \pi\sqrt{LC} + \frac{\pi}{6}\sqrt{LC}$$

$$= \frac{7\pi}{6}\sqrt{1\times 10^{-6}\times 4\times 10^{-6}}$$

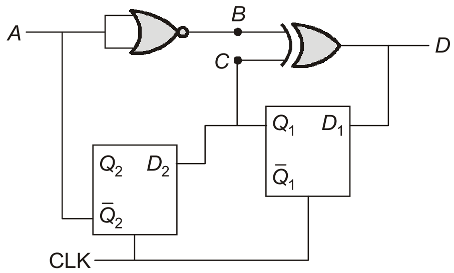
$$t_1 + t_2 = 7.33\ \mu\text{sec}$$

End of Solution

Q.56 Neglecting the delays due to the logic gates in the circuit shown in figure, the decimal equivalent of the binary sequence [ABCD] of initial logic states, which will not change with clock, is ___________.



Ans. (8)



From circuit,

$$D_1 = B \oplus C = Q_2 \oplus Q_1 \text{ ,}$$
$$D_2 = Q_1$$

Let, $Q_2Q_1 = 00$

CLK	D_2 D_1	Q_2 Q_1	
		0 0	No change
⎍	0 0	0 0	

Sequence :

$ABCD$

$\bar{Q}_2 Q_2 Q_1 (Q_1 \oplus Q_2)$

1 0 0 0

The decimal equivalent of 1000 is $(8)_{10}$.

End of Solution

Q.57 In a given 8-bit general purpose micro-controller there are following flags.
C-Carry, A-Auxiliary Carry, O-Overflow flag, P-Parity (0 for even, 1 for odd)
R0 and R1 are the two general purpose registers of the micro-controller.
After execution of the following instructions, the decimal equivalent of the binary sequence of the flag pattern [CAOP] will be __________.

MOV R0, +0x60
MOV R1, +0x46
ADD R0, R1

Ans. (2)

MOV R_0, 0 × 60 ; $R_0 \leftarrow$ 60 H
MOV R_1, +0 × 46 ; $R_1 \leftarrow$ 46 H
ADD R_0, R_1 ; $R_0 \leftarrow [R_0] + [R_1]$

```
     D4   D3
   0110 0000
   0100 0110
   1
   ---------
   1010 0110
```

60H + 46H = A6H, i.e., 10100110

Overflow (O) $\rightarrow$ 1 ; Since if the two 8-bit data were considered as signed data then the result shows negative, i.e., Msb = 1 in A6H but both data bytes are positive.

Parity (P) $\rightarrow$ Even, as there are 'four' binary '1's in result A6H.

$\therefore$ P $\rightarrow$ 0

For Carry Flag (C $\rightarrow$ 0) .... No carry bit out of Mantisa.

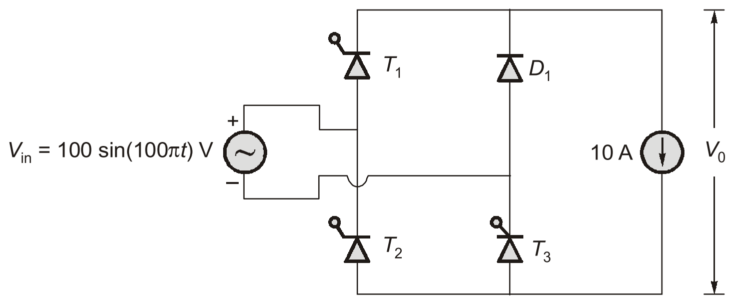
For auxiliary carry (AC $\rightarrow$ 0)

No carry from D_3 to D_4 bit.

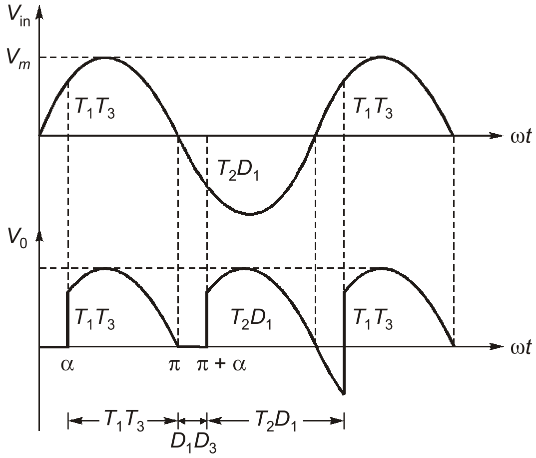
$\therefore$ [CAOP] $\rightarrow [0010]_2 = (2)_{10}$

End of Solution

Q.58 The single phase rectifier consisting of three thyristors T_1, T_2, T_3 and a diode D_1 feed power to a 10 A constant current load. T_1 and T_3 are fired at α = 60° and T_2 is fired at α = 240°. The reference for α is the positive zero crossing of V_{in}. The average voltage V_O across the load in volts is _____ (Round off to 2 decimal places).



Ans. (39.78)



The average output voltage,

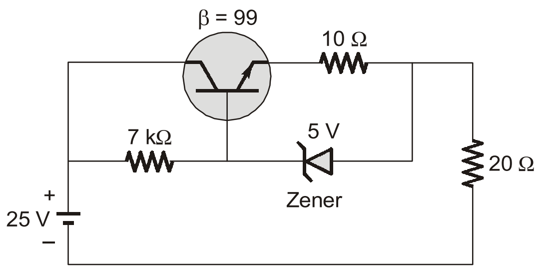
$$V_0 = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} V_m \sin\omega t\, d(\omega t) + \int_{\pi+\alpha}^{2\pi+\alpha} -V_m \sin\omega t\, d(\omega t)\right]$$

$$V_0 = \frac{V_m}{2\pi}[1+3\cos\alpha]$$

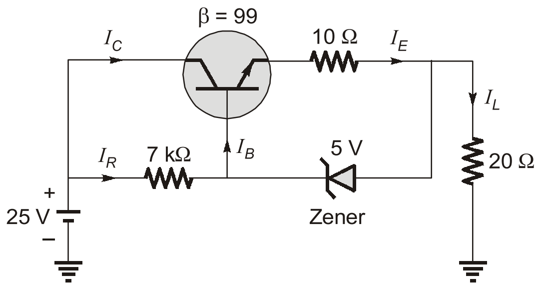
$$= \frac{100}{2\pi}[1+3\cos 60°] = 39.78 \text{ V}$$

End of Solution

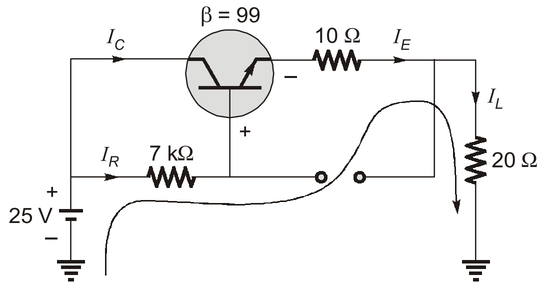
Q.59 The Zener diode in circuit has a breakdown voltage of 5 V. The current gain β of the transistor in the active region in 99. Ignore base-emitter voltage drop V_{BE}. The current through the 20 Ω resistance in milliamperes is ________(Round off to 2 decimal places).



Ans. (250)



Assume zener off



$$25 = 7K \times I_B + I_E\,(10\Omega + 20\Omega)$$
$$25 = 7K \times I_B + I(1 + \beta)\, I_B\,(30\Omega)$$
$$25 = 7K \times I_B + 3000\Omega \times I_B$$
$$25 = 7K \times I_B + 3K\, I_B$$
$$25 = 10K \times I_B$$
$$I_B = \frac{25}{10K} = 2.5\,\text{mA}$$
$$I_E = (1 + \beta)I_B$$
$$= (1 + 99) \times 2.5 \text{ mA}$$
$$= 100 \times 2.5 \text{ mA}$$
$$= 250 \text{ mA}$$

End of Solution

Q.60 The two-bus power system shown in figure (i) has one alternator supplying a synchronous motor load through a Y-Δ transformer. The positive, negative and zero-sequence diagrams of the system are shown in figures (ii), (iii) and (iv), respectively. All reactances in the sequence diagrams are in p.u. For a bolted line-to-line fault (fault impedance = zero) between phases '*b*' and '*c*' at bus 1, neglecting all pre-fault currents, the magnitude of the fault current (from phase '*b*' to '*c*') in p.u. is ____________ (Round off to 2 decimal places).

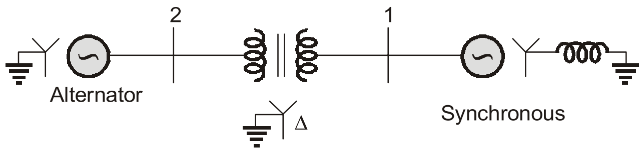


Fig. (i) : Single-line diagram of the power system

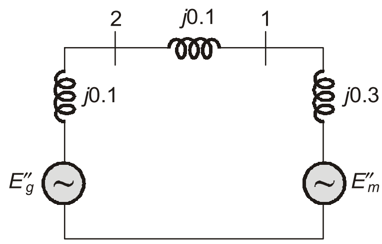


Fig. (ii) : Positive-sequence network

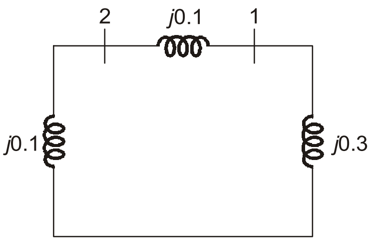


Fig. (iii) : Negative-sequence network

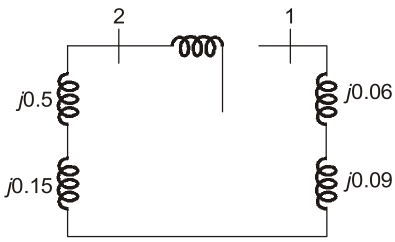


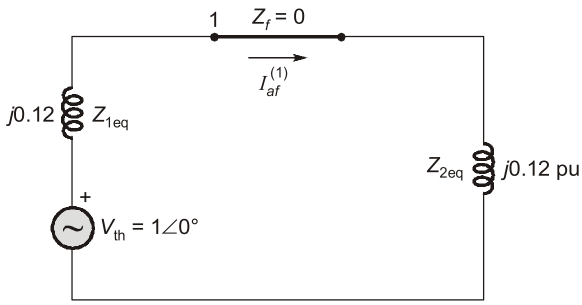
Fig. (iv) : Zero-sequence network

Ans. (7.21)

$$(Z_{1eq})_{bus\ 1} = j0.2 \parallel j0.3 = \frac{j0.2 \times 0.3}{0.5} = j0.12 \text{ pu}$$

$$(Z_{2eq})_{bus\ 1} = j0.2 \parallel j0.3 = j0.12 \text{ pu}$$

The per phase equivalent circuit for LLG fault.

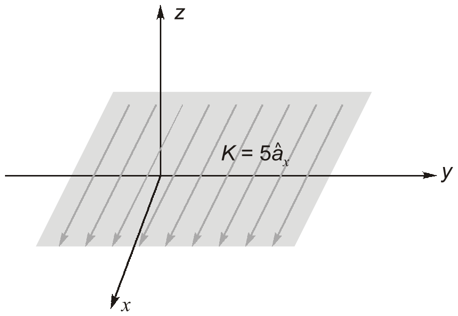


$$I_{af}^{(1)} = \frac{1\angle 0^\circ}{j0.12 + j0.12} = 4.16 \text{ pu}$$

So, fault current, $I_F = \sqrt{3} \times I_{of}^{(1)} = 7.21 \text{ pu}$

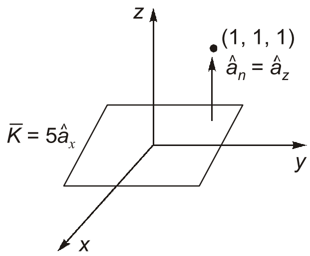
End of Solution

Q.61 An infinite surface of linear current density $\hat{K} = 5\hat{a}_x$ A/m exists on the x-y plane, as shown in the figure. The magnitude of the magnetic field intensity (**H**) at a point (1,1,1) due to the surface current in Ampere/meter is _______ (Round off to 2 decimal places).



Ans. **(2.5)**

Magnetic field due to sheet current is



$$\bar{H} = \frac{1}{2}(\bar{K} \times \hat{a}_n)\ \text{A/m}$$

Here, $\bar{K} = 5\hat{a}_x$

$\hat{a}_n = \hat{a}_z$

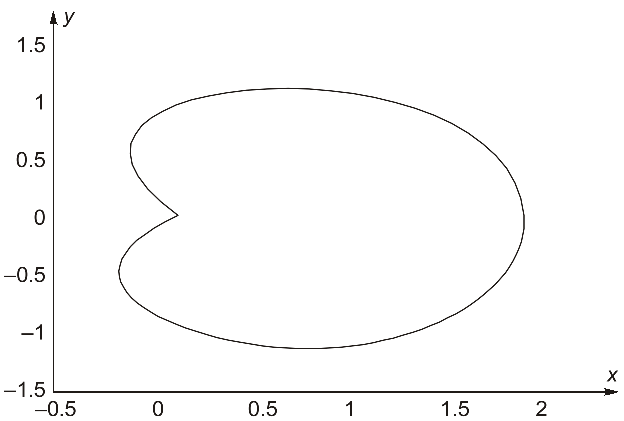
$\therefore$ $\bar{H} = \frac{1}{2}(5\hat{a}_x \times \hat{a}_z) = 2.5(-\hat{a}_y)$

Here, $\bar{H} = -2.5\hat{a}_y$ A/m

So, $|\bar{H}| = 2.5$ A/m

End of Solution

Q.62 The closed curve shown in the figure is described by $r = 1 + \cos\theta$, where r = $\sqrt{x^2 + y^2}$; $x = r\cos\theta$, $y = r\sin\theta$. The magnitude of the line integral of the vector field F $= -y\hat{i} + x\hat{j}$ around the closed curve is ___________ (Round off to 2 decimal places).



Ans. **(9.42)**

$$C : r = (1 + \cos\theta)$$

$$\oint_C \bar{F}\cdot\overline{dr} = \oint_C -ydx + xdy$$

By Green's Theorem

$$= 2\iint_R dydx$$

= 2[Area of region bounded by curve 'C']

$$= 2\times\frac{1}{2}\int_{\theta=0}^{2\pi} r^2 d\theta$$

$$= \int_0^{2\pi}(1+\cos\theta)^2 d\theta$$

$$= \int_0^{2\pi}(1+\cos^2\theta+2\cos\theta)d\theta$$

$$= (\theta)_0^{2\pi} + 4\int_0^{\pi/2}\cos^2\theta\, d\theta + 4\int_0^{\pi}\cos\theta\, d\theta$$

$$= 2\pi + 4\times\frac{1}{2}\times\frac{\pi}{2}+0$$

$$= 2\pi + \pi$$

$$= 3\pi = 3\times 3.14$$

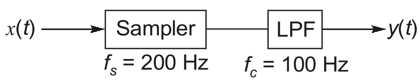
$$= 9.42$$

End of Solution

Q.63 A signal $x(t) = 2\cos(180\pi t)\cos(60\pi t)$ is sampled at 200 Hz and then passed through an ideal low pass filter having cut-off frequency of 100 Hz. The maximum frequency present in the filtered signal in Hz is ____________ (Round off to the nearest integer).

Ans. **(80)**

Given that :



$$x(t) = 2\cos(180\pi t)\cos(60\pi t)$$

$$= \cos(240\pi t) + \cos(120\pi t)$$

where $f_1 = 120$ Hz, $f_2 = 60$ Hz

Frequency components present at sampler output

: $f_1,\ f_s \pm f_1,\ 2f_s \pm f_1,\ .....$

$f_2,\ f_s \pm f_2,\ 2f_s \pm f_2,\ .....$

: 120, 200 ± 120, ...

60, 200 ± 60, ...

: 120, 80, 320, ...

60, 140, 260, ...

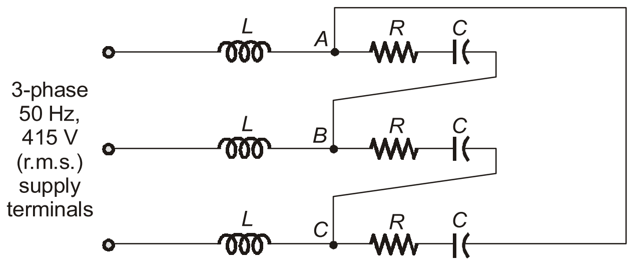
Now, LPF will pass only 60 Hz and 80 Hz because these frequencies are less than cut-off frequency 100 Hz.

i.e., LPF-output : 60 Hz, 80 Hz.

So, maximum frequency available at LPF o/p is 80 Hz.

End of Solution

Q.64 A balanced delta connected load consisting of the series connection of one resistor ($R = 15\ \Omega$) and a capacitor ($C = 212.21\ \mu F$) in each phase is connected to three-phase, 50 Hz, 415 V supply terminals through a line having an inductance of $L = 31.83$ mH per phase, as shown in the figure. Considering the change in the supply terminal voltage with loading to be negligible, the magnitude of the voltage across the terminals V_{AB} in Volts is ____________ (Round off to the nearest integer).

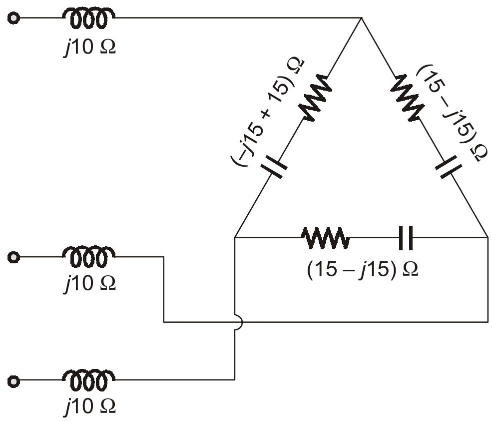


$R = 15\ \Omega$, $L = 31.783$ mH, $C = 212.21\ \mu F$

Ans. **(415)**

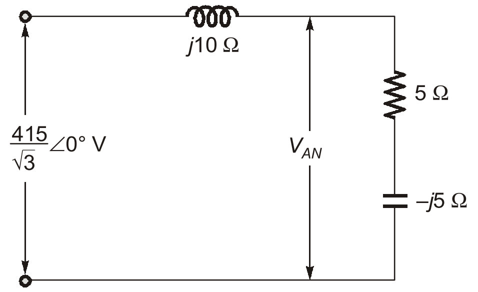
$$X_L = 2\pi fL = 2\pi \times 50 \times 31.83 \times 10^{-3} = 10\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 212.21 \times 10^{-6}} = 15\ \Omega$$



Star to delta ⇒ $$Z_Y = \frac{Z_\Delta}{3} = \frac{15 - j15}{3} = (5 - j5)\ \Omega$$

The per phase equivalent circuit



Per Phase Equivalent Circuit

$$V_{AN} = \frac{(5-j5)}{(5-j5+j10)} \times \frac{415}{\sqrt{3}} = \frac{(5-j5)}{(5+j5)} \times \frac{415}{\sqrt{3}}$$

$$|V_{AN}| = \frac{415}{\sqrt{3}} \text{ V}$$

So, $V_{AB} = 415$ V

End of Solution

Q.65 A quadratic function of two variables is given as

$$f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$$

The magnitude of the maximum rate of change of the function at the point (1,1) is _________ (Round off to the nearest integer).

Ans. (10)

Let, $\phi = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$

$$\vec{\nabla}\phi = i\,\phi_{x1} + j\phi_{x2}$$

$$= i(2x_1 + 3 + x_2) + j(4x_2 + 3 + x_1)$$

$$\vec{\nabla}\phi\Big|_{(1,1)} = 6\hat{i} + 8\hat{j}$$

$$|\vec{\nabla}\phi| = \sqrt{6^2 + 8^2} = 10$$

End of Solution

■■■■