Physics
(Chapter 13)(Nuclei)
(Class 12)
Exercises

Question 13.1:

Obtain the binding energy (in MeV) of a nitrogen nucleus $^{14}_{7}N$, given $m(^{14}_{7}N)$ =14.00307 u.

Answer 13.1:

Atomic mass of $^{14}_{7}N$ nitrogen, m = 14.00307 u

A nucleus of $^{14}_{7}N$ nitrogen contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n= 1.008665 u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

$= 7.054775 + 7.06055 - 14.00307 = 0.11236$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.11236 \times 931.5$ MeV/c^2

Hence, the binding energy of the nucleus is given as:

$E_b = \Delta mc^2$ Where, c = Speed of light

$\therefore E_b = 0.11236 \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 104.66334$ MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

Question 13.2:

Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:

$m(^{56}_{26}Fe) = 55.934939\, u,$ $m(^{209}_{83}Bi) = 208.980388\, u$

Answer 13.2:

Atomic mass of $^{56}_{26}Fe$, m_1 = 55.934939 u

$^{56}_{26}Fe$ nucleus has 26 protons and (56 − 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

$= 26.20345 + 30.25995 - 55.934939 = 0.528461$u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.528461 \times 931.5$ MeV/c^2

The binding energy of this nucleus is given as:

$E_{b1} = \Delta mc^2$ Where, c = Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 492.26$ MeV

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79\, MeV$

Atomic mass of $^{209}_{83}Bi$, m_2 = 208.980388 u

$^{209}_{83}Bi$ nucleus has 83 protons and (209 – 83) 126 neutrons.
Hence, the mass defect of this nucleus is given as: $\Delta m' = 83 \times m_H + 126 \times m_n - m_2$
Where, Mass of a proton, $m_H = 1.007825$ u
Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 83.649475 + 127.091790 - 208.980388 = 1.760877$ u

But 1u = 931.5 MeV/c^2

$\therefore \Delta m' = 1.760877 \times 931.5$ MeV/c^2

Hence, the binding energy of this nucleus is given as: $E_{b2} = \Delta m'c^2$

$= 1.760877 \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 1640.26$ MeV

Average binding energy per nucleon $= \frac{1640.26}{209} = 7.848\ MeV$

Question 13.3:

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

Answer 13.3:

Mass of a copper coin, m' = 3 g
Atomic mass of $^{63}_{29}Cu$ atom, m = 62.92960 u
The total number of $^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass Number}}$
Where,
N_A = Avogadro's number = 6.023×10^{23} atoms /g and mass number = 63 g

$$N = \frac{6.022 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms}$$

$^{63}_{29}Cu$ nucleus has 29 protons and (63 – 29) 34 neutrons.

$\therefore$ Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$
Where, Mass of a proton, $m_H = 1.007825$ u
Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296 = 0.591935$ u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = \Delta mc^2 = 1.69766958 \times 10^{22} \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 1.581 \times 10^{25}$ MeV

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13} = 2.5296 \times 10^{12}$ J

This much energy is required to separate all the neutrons and protons from the given coin.

Question 13.4:

Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Answer 13.4:

Nuclear radius of the gold isotope $^{197}_{79}Au$ = R_{Au}

Nuclear radius of the silver isotope $^{107}_{47}Ag$ = R_{Ag}

Mass number of gold, A_{Au} = 197

Mass number of silver, A_{Ag} = 107

The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

Question 13.5:

The Q value of a nuclear reaction A + b → C + d is defined by

$Q = [m_A + m_b - m_C - m_d]c^2$, where the masses refer to the respective nuclei.

Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) $^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$

(ii) $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

Atomic masses are given to be

$$m\left(^2_1H\right) = 2.014102 \text{ u}$$

$$m\left(^3_1H\right) = 3.016049 \text{ u}$$

$$m\left(^{12}_6C\right) = 12.000000 \text{ u}$$

$$m\left(^{20}_{10}Ne\right) = 19.992439 \text{ u}$$

Answer 13.5:

(i) The given nuclear reaction is:

It is given that:

$$^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$$

Atomic mass $m(^1_1H) = 1.007825$ u

Atomic mass $m(^3_1H) = 3.016049$ u

Atomic mass $m(^2_1H) = 2.014102$ u

According to the question, the Q-value of the reaction can be written as:

$$Q = [m\,(^1_1H) + m(^3_1H) - 2m(^2_1H)]c^2 = [1.007825 + 3.016049 - 2 \times 2.014102]c^2$$

$$Q = (-0.00433\ c^2)\ u$$

But 1 u = 931.5 MeV/c^2, ∴ Q = 0.00433 × 931.5 = – 4.0334 MeV

The negative Q-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

It is given that:

$${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$$

Atomic mass of $m({}^{12}_{6}\text{C})$= 12.0 u

Atomic mass of $m({}^{20}_{10}Ne)$= 19.992439 u

Atomic mass of $m({}^{4}_{2}\text{He})$= 4.002603 u

The Q-value of this reaction is given as:

$Q = [2m({}^{12}_{6}\text{C}) - m({}^{20}_{10}\text{Ne}) - m({}^{4}_{2}\text{He})]c^2$

$= [2 \times 12.0 - 19.992439 - 4.002603]c^2$

$= (0.004958\text{c}^2)u$

$= 0.004958 \times 931.5 = 4.618377$ MeV

The positive Q-value of the reaction shows that the reaction is exothermic.

Question 13.6:

Suppose, we think of fission of a ${}^{56}_{26}Fe$ nucleus into two equal fragments, ${}^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given $m({}^{56}_{26}Fe) = 55.93494\,u$ and $m({}^{28}_{13}Al) = 27.98191\,u$.

Answer 13.6:

The fission of ${}^{56}_{26}Fe$ can be given as:

${}^{56}_{13}\text{Fe} \rightarrow 2\,{}^{28}_{13}\text{Al}$

It is given that:

Atomic mass of $m({}^{56}_{26}Fe) = 55.93494\,u$

Atomic mass of $m({}^{28}_{13}Al) = 27.98191\,u$

The Q-value of this nuclear reaction is given as:

$Q = [m({}^{56}_{26}\text{Fe}) - 2m({}^{28}_{13}\text{Al})]c^2$

$= [55.93494 - 2 \times 27.98191]c^2$

$= (0.02888c^2)\,u$

But $1\text{u} = 931.5\ \text{MeV}/c^2$

$\therefore \text{Q} = -0.02888 \times 931.5 = -26.902$ MeV

The Q-value of the fission is negative.

Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the Q-value must be positive.

Question 13.7:

The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?

Answer 13.7:

Average energy released per fission of $^{239}_{94}Pu$, $E_{av} = 180\ MeV$

Amount of pure $^{239}_{94}Pu$, m = 1 kg = 1000 g

N_A= Avogadro number = 6.023×10^{23}

Mass number of $^{239}_{94}Pu$ = 239 g

1 mole of $^{239}_{94}Pu$ contains N_A atoms.

$\therefore$ mg of $^{239}_{94}Pu$ contains $\left(\frac{N_A}{Mass\ number} \times m\right)$ atoms

$\therefore$ 1000 g of $^{239}_{94}Pu$ contains $\left(\frac{N_A}{Mass\ number} \times 1000\right)$ atoms $= 2.52 \times 10^{24}$ atoms

$\therefore$ Total energy released during the fission of 1 kg of $^{239}_{94}Pu$ is calculated as:

$$E = E_{av} \times 2.52 \times 10^{24}$$
$$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26}\ \text{MeV}$$

Hence, $4.536 \times 10^{26}\ MeV$ is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission.

Question 13.8:

How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as:

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27\ \text{MeV}$$

Answer 13.8:

The given fusion reaction is:

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27\ \text{MeV}$$

Amount of deuterium, m = 2 kg

1 mole, *i.e.*, 2 g of deuterium contains 6.023×10^{23} atoms.

$\therefore$ 2.0 kg of deuterium contains $\frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

$\therefore$ Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26}\ \text{MeV}$$
$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$
$$= 1.576 \times 10^{14}\text{J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576 \times 10^{14}}{100}\text{s} = \frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^{4}$$

Question 13.9:

Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Answer 13.9:

When two deuterons collide head-on, the distance between their centres, d is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$\therefore$ d = $2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = e = 1.6×10^{-19} C Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi\varepsilon_0 d}$$

Where, ε_0 = permittivity of free space.

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{N m}^2\text{C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10} 9 \times 10^9 \text{N m}^2\text{C}^{-2}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15} \times (1.6 \times 10^{-19})} \text{Ev}$$

= 360 keV

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

Question 13.10:

From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Answer 13.10:

We have the expression for nuclear radius as:

$R = R_0 A^1/^3$

Where, R_0 = Constant

A = Mass number of the nucleus

$$\text{Nuclear matter density}, \rho = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$$

Let m be the average mass of the nucleus. Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.