Sample Question Paper - 20
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

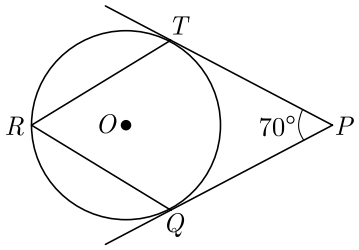
1. If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

OR

Find the positive root of $\sqrt{3x^2 + 6} = 9$.

2. The fifth term of an AP is 26 and its 10^{th} term is 51. Find the AP

3. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



4. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

5. Given below is a frequency distribution showing the marks by 50 students of a class :

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the distribution table for the above data.

6. Find the mode of the following frequency distribution :

Classes	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

OR

If median is 137 and mean is 137.05, then what is the value of mode ?

Section B

7. The 13^{th} term of an AP is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

8. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

9. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

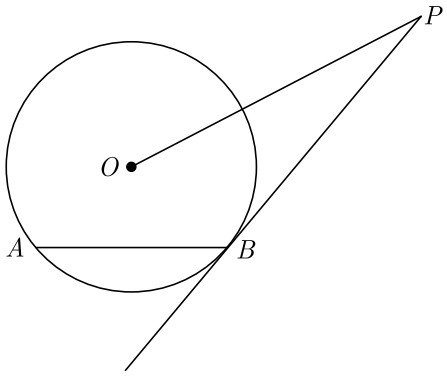
10. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$.

OR

A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. Use $\pi = \frac{22}{7}$

Section C

11. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



12. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0- 10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

OR

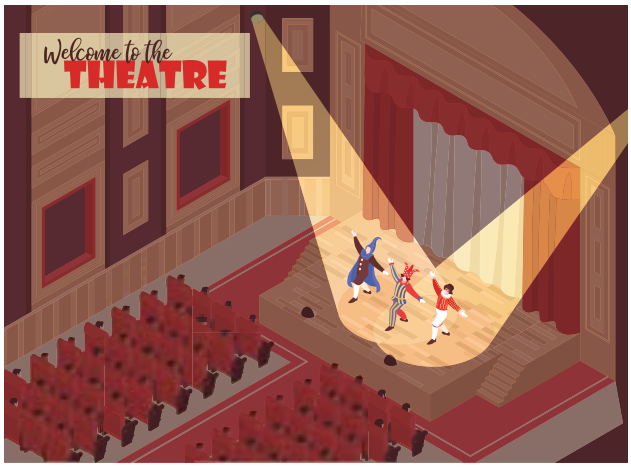
The median of the following data is 16. Find the missing frequencies a and b, if the total of the frequencies is 70.

Class	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	12	'a'	12	15	b	6	6	4

13. Drama Ticket : Last year, 300 people attended the Ravindra Manch Drama Club's winter play. The ticket price was Rs 70. The advisor estimates that 20 fewer people would attend for each Rs 10 increase in ticket price.

(i) What ticket price would give the most income for the Drama Club?

(ii) If the Drama Club raised its tickets to this price, how much income should it expect to bring in?



14. Statue of Unity : It is a colossal statue of Indian statesman and independence activist Sardar Vallabh bhai Patel, who was the first Deputy Prime Minister and Home minister of independent India.

Patel was highly respected for his leadership in uniting the 562 princely states of India to form the single Union of India. It is located in the state of Gujarat and it is the world's tallest statue.

(i) For a person standing 240 m from the center of the base of the statue, the angle of elevation to the top of the statue is 45°. How tall is the statue?

(ii) A cop in helicopter near the top of the statue, notices a car wreck some distance from the statue. If the angle of depression from the cop's eyes to the wreck is 60°, how far away is the accident from the centre of base of the statue?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
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3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. If 2 is a root of the equation $x^2+kx+12=0$ and the equation $x^2+kx+q=0$ has equal roots, find the value of q.

Ans :

We have $x^2+kx+12=0$

If 2 is the root of above equation, it must satisfy it.

$$(2)^2+2k+12=0$$

, $$2k+16=0$$

$$k=-8$$

Substituting $k=-8$ in $x^2+kx+q=0$

we have

$$x^2-8x+q=0$$

For equal roots,

$$(-8)^2-4(1)q=0$$

$$64-4q=0$$

$$4q=64 \Rightarrow q=16$$

OR

Find the positive root of $\sqrt{3x^2+6}=9$.

Ans :

We have $\sqrt{3x^2+6}=9$

$$3x^2+6=81$$

$$3x^2=81-6=75$$

$$x^2=\frac{75}{3}=25$$

Thus $x=\pm 5$

Hence 5 is positive root.

2. The fifth term of an AP is 26 and its 10^{th} term is 51. Find the AP

Ans :

Let the first term be a, common difference be d and nth term be a_n.

$$a_5=a+4d=26 \quad ...(1)$$

$$a_{10}=a+9d=51 \quad ...(2)$$

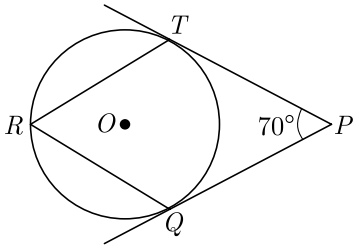
Subtracting (1) from (2) we have

$$5d=25 \Rightarrow d=5$$

Substituting this value of d in equation (1) we get $a=6$.

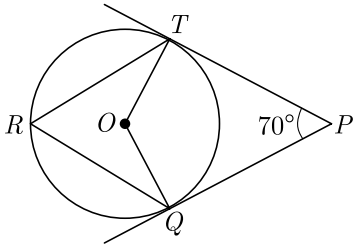
Hence, the AP is 6, 11, 16, ...

3. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ=70^\circ$, find $\angle TRQ$.



Ans :

We redraw the given figure by joining O to T and Q as shown below.



Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $\angle TOQ = 180^\circ - \angle TPQ$

$= 180^\circ - 70^\circ = 110^\circ$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle TRQ = \frac{1}{2}\angle TOQ$$

$$= \frac{1}{2} \times 110^\circ = 55^\circ$$

4. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Ans :

Volume of of any cylinder shape is $\pi r^2 h$.

Volume of coin $= \pi(0.75)^2 \times 0.2$ cm^3

Volume of cylinder $= \pi(2.25)^2 \times 10$ cm^3

$$\text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\pi(2.25)^2 \times 10}{\pi(0.75)^2 \times 0.2}$$

$$= \frac{(3)^2 \times 10}{0.2}$$

$$= 450$$

5. Given below is a frequency distribution showing the marks by 50 students of a class :

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the distribution table for the above data.

Ans :

Class	Frequency
0-20	17
20-40	5
40-60	7
60-80	8
80-100	13
Total	50

6. Find the mode of the following frequency distribution :

Classes	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

Ans :

Class 18-24 has the maximum frequency 12, therefore this is model class.

Now $l = 18,\ f_1 = 12,\ f_0 = 10,\ f_2 = 6,\ h = 6$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 18 + \frac{12 - 10}{24 - 10 - 6} \times 6$$

$$= 18 + 1.5 = 19.5$$

OR

If median is 137 and mean is 137.05, then what is the value of mode ?

Ans :

$M_o = 3M_d - 2M$

$= 3(137) - 2(137.05)$

$= 411 - 274.10 = 136.90$

Section B

7. The 13^{th} term of an AP is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n.

Here $a_{13} = 4a_3$

$a + 12d = 4(a + 2d)$

$3a = 4d$ ...(1)

and $a_5 = 16$

$a + 4d = 16$ ...(2)

Substituting the value of $a = \frac{4}{3}d$ in (2) we have

$\frac{4}{3}d + 4d = 16$

$16d = 48 \Rightarrow d = 3$

Thus $a = 4$ and $d = 3$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3]$

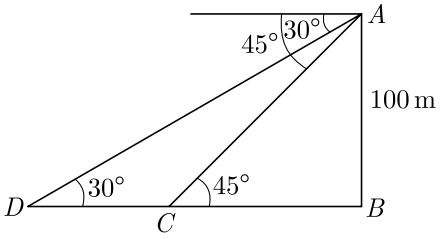
$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

8. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Ans :

Let AB be the tower and ships are at points C and D. As per question statement we have shown digram below.



Now in ΔABC we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\frac{AB}{AC} = 1 \Rightarrow AB = BC$$

Now in ΔABD we have

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$AB + CD = \sqrt{3}\, AB$$

$$CD = AB(\sqrt{3} - 1)$$

$$= 100 \times (1.732 - 1) = 73.2 \text{ m}$$

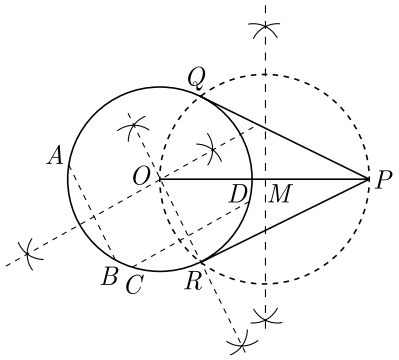
Distance between two ships is 73.2 m.

9. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

Ans :

Steps of Construction :

1. Draw a circle with the help of a bangle.
2. Take two non-parallel chords AB and CD of this circle.
3. Draw the perpendicular bisectors of AB and CD. Let these intersect at O. Then, O is the centre of the circle drawn.
4. Take a point P outside the circle.
5. Join PO and bisect it. Let M be the mid-point of PO.



6. Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q and R.
7. Join PQ and PR.

Then, PQ and PR are the required two tangents.

Justification :

If we join OQ and OR, $\angle PQO$ is an angle in the semicircle and, therefore,

$$\angle PQO = 90^\circ$$

$$PQ \perp OQ$$

Since OQ is a radius of the given circle, PQ has to be a tangent to the circle. Similarly, PR is also a tangent to the circle.

10. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$.

Ans :

Volume of water in cylinder is equal to the volume of cylinder. Thus

Volume of water in cylinder = Volume of cylinder

$$\pi r^2 h = \pi(60)^2 \times 180$$

$$= 648000\pi \text{ cm}^3$$

Water displaced on dropping cone is equal to the volume of solid cone, which is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (30)^2 \times 60$$

$$= 18000\pi \text{ cm}^3$$

Volume of water left in cylinder

= Volume of cylinder − Volume of cone

$= 648000\pi - 18000\pi = 630000\pi$ cm^3

$= \frac{630000 \times 22}{1000000 \times 7}$ m$^3 = 1.98$ m^3

OR

A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. Use $\pi = \frac{22}{7}$

Ans :

Radius of the glass $r = 7$ cm

Height of the glass $h = 10$ cm

Volume of 1 glass,

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 10$$

$$= 1540 \text{ cm}^3$$

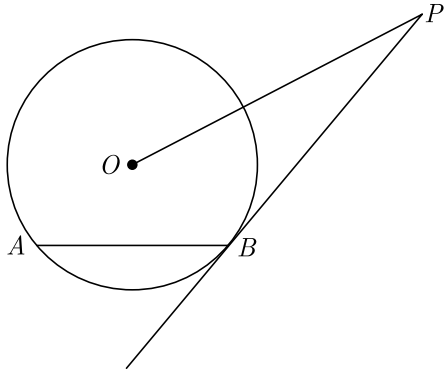
Volume of juice to fill 6 glasses,

$$6\pi r^2 h = 6 \times 1540 = 9240 \text{ cm}^3$$

$$\text{Volume in litre} = \frac{9240}{1000} = 9.240 \text{ litre.}$$

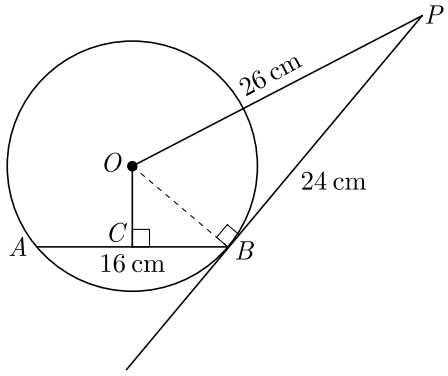
Section C

11. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Ans :

We redraw the given figure by joining O to B as shown below.



Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle,

We have $PB = 24$ cm, $OP = 26$ cm.

Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90^\circ$.

In right angled ΔOPB, we have

$$OB = \sqrt{OP^2 - BP^2} = \sqrt{26^2 - 24^2}$$

$$= \sqrt{676 - 576} = \sqrt{100} = 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2} AB = \frac{16}{2} = 8 \text{ cm}$$

Now in $\Delta OBC, OC^2 = OB^2 - BC^2$

$$= 10^2 - 8^2 = 36$$

$$OC = 6 \text{ cm}$$

Thus distance of the chord from the centre is 6 cm.

12. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0 - 10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Ans :

Class	Frequency (f)	Cumulative Frequency (cf)
0-10	f_1	f_1
10-20	5	$f_1 + 5$
20-30	9	$f_1 + 14$
30-40	12	$f_1 + 26$
40-50	f_2	$f_1 + f_2 + 26$
50-60	3	$f_1 + f_2 + 29$
60-70	2	$f_1 + f_2 + 31$
	$N = \sum f = 40$	

Now, $f_1 + f_2 + 31 = 40$

$$f_1 + f_2 = 9$$

$$f_2 = 9 - f_1 \quad \ldots(1)$$

Since median is 32.5, which lies in 30-40, median class is 30-40.

Here $l = 30$, $\frac{N}{2} = \frac{40}{2} = 20$, $f = 12$ and $F = 14 + f_1$

Now, Median $= 3.25$

$$l+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h = 32.5$$

$$30+\left(\frac{20-(14+f_1)}{12}\right)\times 10 = 32.5$$

$$\left(\frac{6-f_1}{12}\right)\times 10 = 2.5$$

$$\frac{60-10f_1}{12} = 2.5$$

$$60-10f_1 = 30$$

$$10f_1 = 30 \Rightarrow f_1 = 3$$

From equation (1), we get $f_2 = 9-3 = 6$

Hence, $f_1 = 3$ and $f_2 = 6$

OR

The median of the following data is 16. Find the missing frequencies a and b, if the total of the frequencies is 70.

Class	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	12	'a'	12	15	b	6	6	4

Ans :

Cumulative frequency table is given as follow

Class	Frequency	Cumulative Frequency
0-5	12	12
5-10	a	$12+a$
10-15	12	$24+a$
15-20	15	$39+a$
20-25	b	$39+a+b$
25-30	6	$45+a+b$
30-35	6	$51+a+b$
35-40	4	$55+a+b$
	$\sum f_i = 70$	

We have, $\sum f_i = 70$

$$55+a+b = 70$$

$$a+b = 15 \quad ...(1)$$

Now Median $= 16$

So median class will be 15-20 as 16 lies in 15-20

We have $l = 15$, $F = 24+a$, $f = 15$, $N = 70$, $h = 5$

$$\text{Median} = l+\left(\frac{\frac{N}{2}-F}{f}\right)\times h$$

$$16 = 15+\frac{35-(24+a)}{15}\times 5$$

$$1 = \frac{35-24-a}{3}$$

$$3 = 11-a \Rightarrow a = 8$$

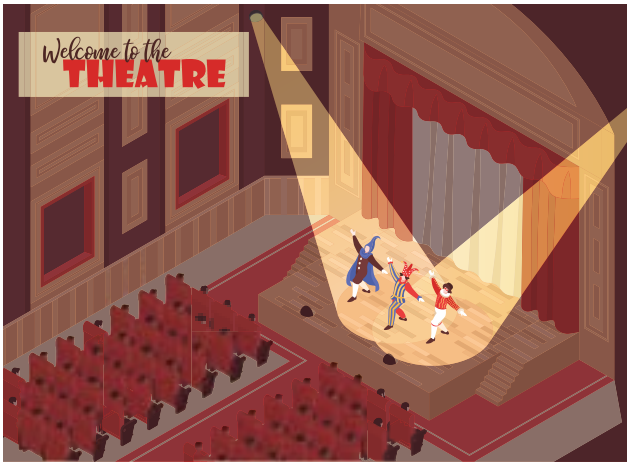
Substituting $a = 8$ in equation (1), we get

$$b = 15-a = 15-8 = 7$$

13. Drama Ticket : Last year, 300 people attended the Ravindra Manch Drama Club's winter play. The ticket price was Rs 70. The advisor estimates that 20 fewer people would attend for each Rs 10 increase in ticket price.

(i) What ticket price would give the most income for the Drama Club?

(ii) If the Drama Club raised its tickets to this price, how much income should it expect to bring in?



Ans :

(i) Let x represent the number of Rs 10 fare increases. Then $70+10x$ is the price per passenger and $300-20x$ is the number of passengers.

The income is the number of passengers multiplied by the price per ticket. Let $I(x)$ represent income as a function of x.

Now $I(x) = (300-20x)(70+10x)$

$$= 20(15-x)(10)(7+x)$$

$$= 200(105+8x-x^2)$$

$$= -200(x^2-8x-105)$$

$$= -200(x^2-8x+16-16-105)$$

$$= -200(x^2-8x+16-121)$$

$$= -200(x^2-8x+16)+24200$$

$$= -200(x-4)^2+24200$$

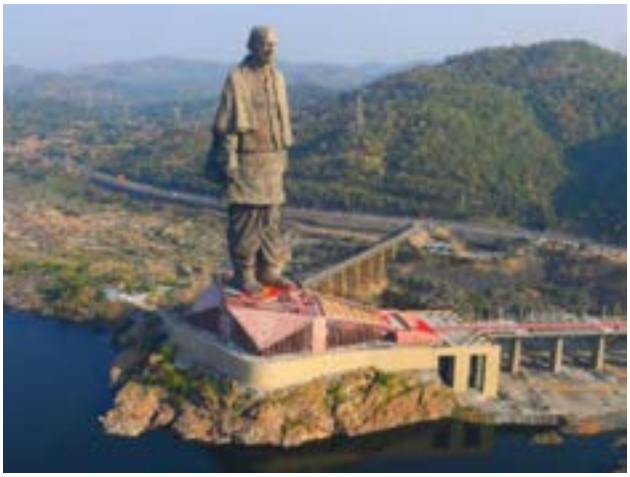
(ii) From above equation it is clear that $I(x)$ is maximum at $x = 4$ and this maximum value is 24200. This means the company should make 4 fare increases of Rs 10 to maximize

its income. Thus, the ticket price should be $70 + 4 \times 10 = 110$

14. Statue of Unity : It is a colossal statue of Indian statesman and independence activist Sardar Vallabh bhai Patel, who was the first Deputy Prime Minister and Home minister of independent India. Patel was highly respected for his leadership in uniting the 562 princely states of India to form the single Union of India. It is located in the state of Gujarat and it is the world's tallest statue.

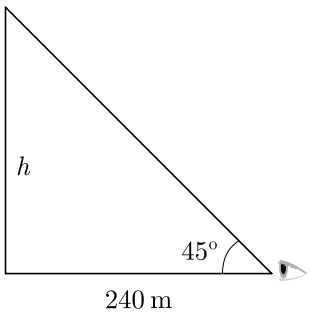
(i) For a person standing 240 m from the center of the base of the statue, the angle of elevation to the top of the statue is 45°. How tall is the statue?

(ii) A cop in helicopter near the top of the statue, notices a car wreck some distance from the statue. If the angle of depression from the cop's eyes to the wreck is 60°, how far away is the accident from the centre of base of the statue?



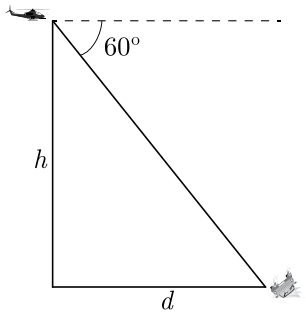
Ans :

Let h be the height of statue. We draw a diagram of the situation as shown below.



Now $$\tan 45^\circ = \frac{h}{240}$$
$$1 = \frac{h}{240}$$
$$h = 240 \text{ m}$$

Let d be the distance of car wreck from he centre of base of the statue. We draw a diagram of the situation as shown below.



Now $$\tan 30^\circ = \frac{d}{h}$$
$$\frac{1}{\sqrt{3}} = \frac{d}{240}$$
$$d = \frac{240}{\sqrt{3}} = 80\sqrt{3}$$
$$= 80 \times 1.732 = 138.56 \text{ m}$$

(i) The height of statue is 240 m.

(ii) Accident is 138.56 metre away from the centre of statue.