Long Answer Type Questions

[4 MARKS]

Que 1. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Sol. Here, we use step deviation method to find mean.

Let assumed mean A = 70 and class size h = 10

So, $u_i = \frac{x_i - 70}{10}$

Now, we have

Literacy rate (in %)	**Frequency (f_i)**	**Class mark (x_i)**	$\boldsymbol{u_i = \frac{x_i - 70}{10}}$	$\boldsymbol{f_i u_i}$
45 – 55	3	50	– 2	– 6
55 – 65	10	60	– 1	– 10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	$\boldsymbol{\Sigma f_i = 35}$			$\boldsymbol{\Sigma f_i u_i = -2}$

$\therefore \quad Mean\ (\overline{x}) = A + h \times \frac{\Sigma f_i u_i}{\Sigma f_i} = 70 + 10 \times \frac{-2}{35} = 70 - 0.57 = 69.43\%$

Que 2. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹18. Find the missing frequency f.

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	f	5	4

Sol. Let the assumed mean A = 16 and class size h = 2, here we apply step deviation method.

So, $u_i = \frac{x_i - A}{h} = \frac{x_i - 16}{2}$

Class interval	**Frequency (f_i)**	**Class mark (x_i)**	$\boldsymbol{u_i = \frac{x_i - 16}{2}}$	$\boldsymbol{f_i u_i}$
11 – 13	7	12	–2	–14
13 – 15	6	14	–1	–6

15 – 17	9	16	0	0
17 – 19	13	18	1	13
19 – 21	f	20	2	2f
21 – 23	5	22	3	15
23 – 25	4	24	4	16
Total	$\boldsymbol{\Sigma f_i = f + 44}$			$\boldsymbol{\Sigma f_i u_i = 2f + 24}$

Now, we have,

We have, Mean $(\overline{x}) = 18, A = 16 \; and \; h = 2$

$\therefore \quad \overline{x} = A + h \times \frac{\Sigma f_i u_i}{\Sigma f_i}$

$18 = 16 + 2 \times \left(\frac{2f+24}{f+44}\right) \;\Rightarrow\; 2 = 2 \times \left(\frac{2f+24}{f+44}\right)$

$\Rightarrow \quad 1 = \frac{2f+24}{f+44} \qquad \Rightarrow f + 44 = 2f + 24$

$\Rightarrow \quad f = 44 - 24$

$\Rightarrow \quad f = 20$

Hence, the missing frequency is 20.

Que 3. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 20
Frequency	5	8	x	12	7	8

Sol. We have

Class interval	**Frequency**	**Class mark** (x_i)	$\boldsymbol{f_i x_i}$
0 – 20	5	10	50
20 – 40	8	30	240
40 – 60	x	50	50x
60 – 80	12	70	840
80 – 100	7	90	630
100 – 120	8	110	880
Total	$\boldsymbol{\Sigma f_i = 40 + x}$		$\boldsymbol{\Sigma f_i x_i = 2640 + 50x}$

Here, $\Sigma f_i x_i = 2640 + 50x, \Sigma f_i = 40 + x, \overline{x} = 62.8$

$\therefore \quad Mean(\overline{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 62.8 = \frac{2640 + 50x}{40 + x}$

$\Rightarrow \quad 2512 + 62.8x = 2640 + 50x$

$\Rightarrow \quad 62.8x - 50x = 2640 - 2512$

$\Rightarrow \quad 12.8x = 128$

$\therefore \quad x = \frac{128}{12.8} = 10$

Hence, the missing frequency is 10.

Que 4. The distribution below gives the marks of 100 students of a class.

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of students	4	6	10	10	25	22	18	5

Draw a less than type and a more than type ogive from the gives data. Hence, obtain the median marks from the graph.

Sol.

Marks	**Cumulative Frequency**	**Marks**	**Cumulative Frequency**
Less than 5	4	More than 0	100
Less than 10	10	More than 5	96
Less than 15	20	More than 10	90
Less than 20	30	More than 15	80
Less than 25	55	More than 20	70
Less than 30	77	More than 25	45
Less than 35	95	More than 30	23
Less than 40	100	More than 35	5

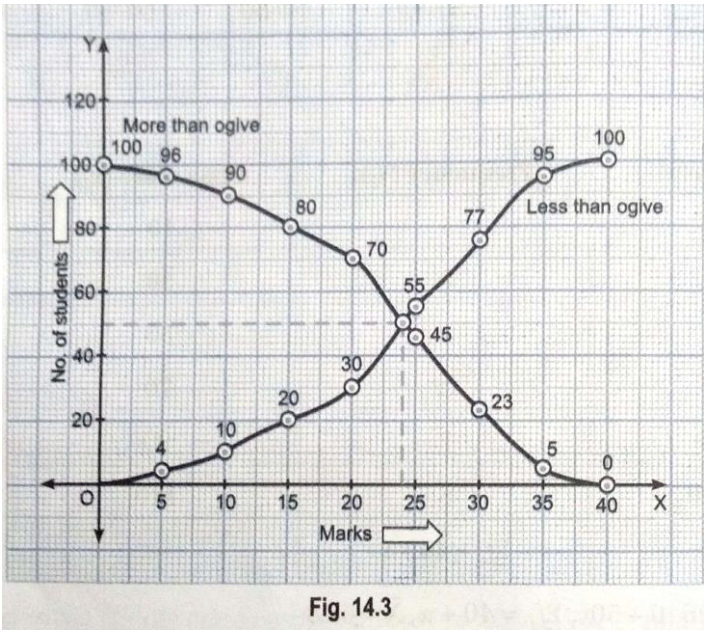

Fig. 14.3

Hence, median marks = 24

Que 5. During the medical check-up of 35 students of a class, their weight were recorded as follows:

Weight	Number of students	Weight (in kg)	Number of students
Less than 38	0	Less than 46	14
Less than 40	3	Less than 48	28
Less than 42	5	Less than 50	32
Less than 44	9	Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

Sol. To represent the data in the table graphically, we mark the upper limits of the class interval on x-axis and their corresponding cumulative frequency on y-axis choosing a convenient scale.

Now, let us plot the points corresponding to the ordered pair given by (38,0), (40,3), (42,5), (44,9), (46,14), (48,28), (50,32) and (52,35) on a graph paper and join them by a freehand smooth curve.

Thus, the curve obtained is the less than type ogive.

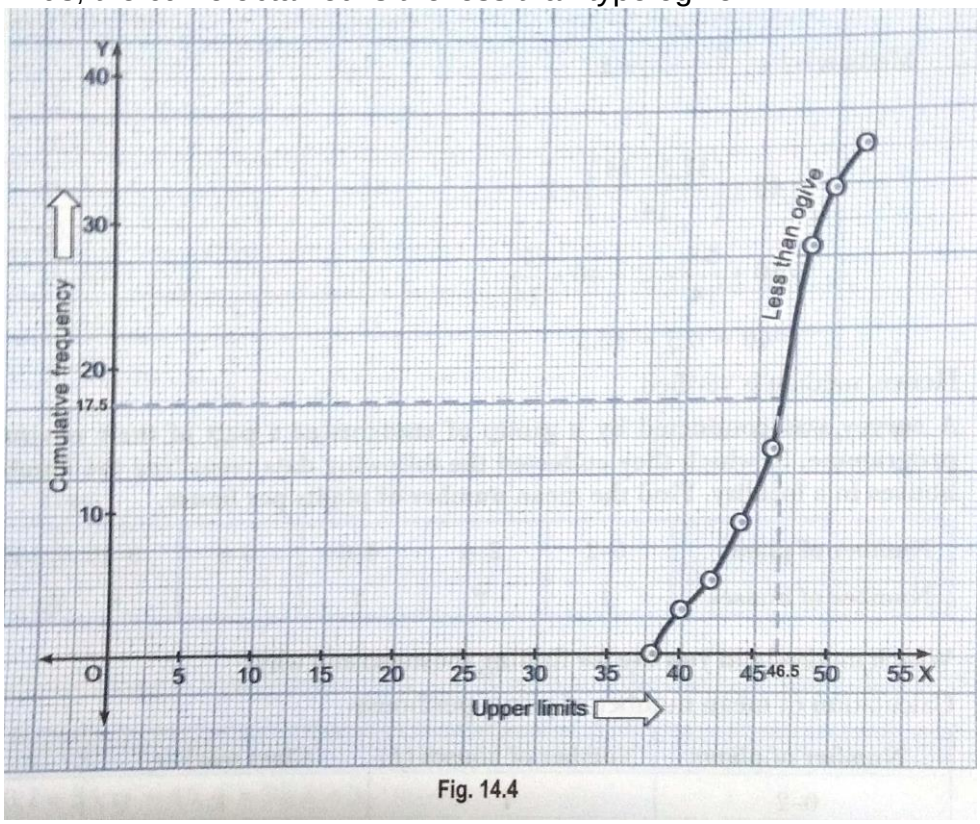

Fig. 14.4

Now, locate $\frac{n}{2} = \frac{35}{2= 17.5}$ on the y-axis,

We draw a line from this point parallel to x-axis cutting the curve at a point. From this point, draw a perpendicular line to the x-axis. The point of intersection of this perpendicular with the x-axis gives the median of the data. Here it is 46.5.
Let us make the following table in order to find median by using formula.

Weight (in kg)	**No. of students (frequency)**	**Cumulative frequency (cf)**
36 – 38	0	0
38 – 40	3	3
40 – 42	2	5
42 – 44	4	9
44 – 46	5	14
46 – 48	14	28
48 – 50	4	32
50 – 52	3	35
Total	$\Sigma f_i = 35$	

Here, n =35, $\frac{n}{2} = \frac{35}{2} = 17.5$, cumulative frequency greater than $\frac{n}{2} = 17.5$ is 28 and corresponding class is 46 – 48. So median class is 46 – 48.

Now, we have $l = 46, \frac{n}{2} = 17.5, cf = 14, f = 14, h = 2$

$$\therefore \quad Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5-14}{14}\right) \times 2$$

$$= 46 + \frac{3.5}{14} \times 2 = 46 + \frac{7}{14}$$

$$= 46 + 0.5 \ = 46.5$$

Hence, median is verified.

Que 6. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean and why?

Sol. Calculation of mean number of plants per house.

Number of plants	**Number of houses** (f_i)	**Class mark** (x_i)	$f_i x_i$
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	5	7	35
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
Total	$\boldsymbol{\Sigma f_i = 20}$		$\boldsymbol{\Sigma f_i x_i = 162}$

Hence, Mean$(\overline{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{162}{20} = 8.1$

Here, we used direct method to find mean because numerical values of x_i and f_i are small.

Age (in years)	**Number of policy holders**	**Age (in years)**	**Number of policy holders**
Below 20	2	Below 45	89
Below 25	6	Below 50	92
Below 30	24	Below 55	98
Below 35	45	Below 60	100
Below 40	78		

Que 7. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Sol. We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median.

Class interval	**Frequency** (f_i)	**Cumulative frequency (cf)**
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45
35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100
Total	$\boldsymbol{\Sigma f_i = 100}$	

Here, n = 100

$\Rightarrow \quad \frac{n}{2} = 50$

And, cumulative frequency just greater than $\frac{n}{2} = 50$ is 78 and the corresponding class is 35 – 40. So 35 – 40 is the median class.

$\therefore \quad \frac{n}{2} = 50, l = 35, cf = 45, f = 33, h = 5$

$\therefore \quad Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 35 + \left(\frac{50-45}{33}\right) \times 5 = 35 + \frac{5}{33} \times 5$

$= 35 + \frac{25}{33} = 35 + 0.76 = 35.76$

Hence, the median age is 35.76 years.

Que 8. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Sol. Converting gives distribution to a less than type cumulative frequency distribution, we have,

Daily income (in ₹)	**Cumulative frequency**
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Now, let us plot the points corresponding to the ordered pairs (120, 12), (140, 26), (160, 34), (180, 40), (200, 50) on a graph paper and join them by a freehand smooth curve.

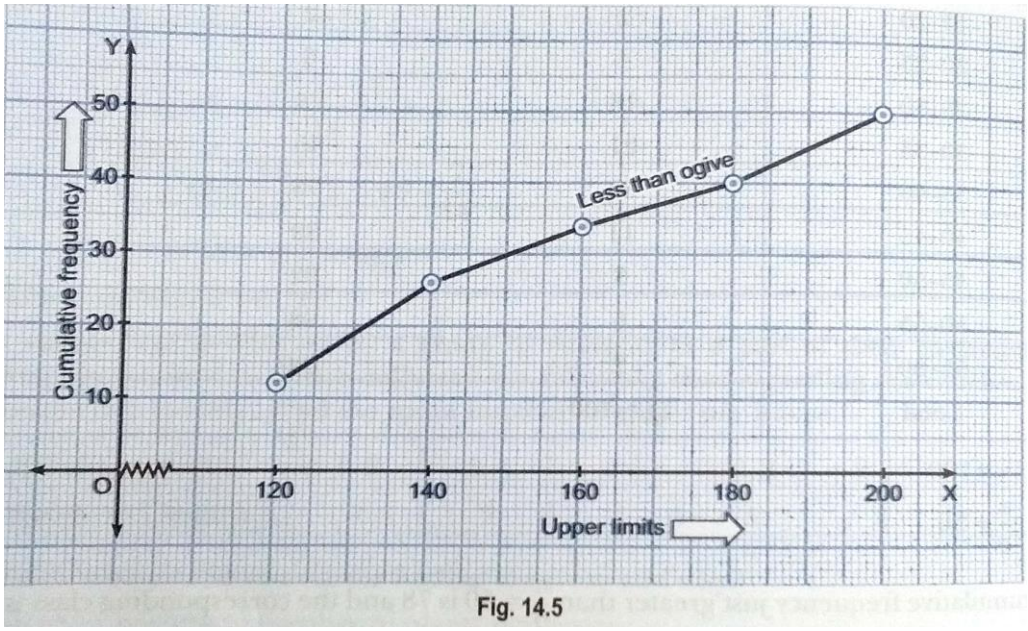

Fig. 14.5

Thus, obtained curve is called the less than type ogive.

Que 9. Find the mean of the following frequency distribution:

Class interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of workers	15	18	21	29	17

Sol. Calculation of mean

We have, $A = 50, h = 20, \Sigma f_i = 100 \; and \; \Sigma f_i u_i = 15.$

Class interval	**Class mark** (x_i)	**Frequency** (f_i)	$u_i = \frac{x_i - A}{20} = \frac{x_i - 50}{20}$	$f_i u_i$
0 – 20	10	15	– 2	–30
20 – 40	30	18	–1	–18
40 – 60	50	21	0	0
60 – 80	70	29	1	29
80 – 100	90	17	2	34
Total		$\Sigma f_i = 100$		$\Sigma f_i u_i = 15$

$$\therefore \quad Mean\ (\overline{x}) = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 50 + 20 \times \frac{15}{100}$$

$$= 50 + 3 = 53.$$