LOGARITHMS
16

INTRODUCTION

Logarithms is an important chapter with questions being frequently asked in all important Management entrance exams, from this chapter. In exams, most problems that have used the concept of logs have been of an applied nature. However, the aspirants should know the basic concepts of logarithms to ensure there are no surprises in the paper.

While studying this chapter, the student should pay particular attention to the basic rules of logarithms as well as develop an understanding of the range of the values of logs.

THEORY

Let a be a positive real number, $a \neq 1$ and $a^x = m$. Then x is called the logarithm of m to the base a and is written as $\log_a m$, and conversely, if $\log_a m = x$, then $a^x = m$.

Note: Logarithm to a negative base is not defined.

Also, logarithm of a negative number is not defined. Hence, in the above logarithmic equation, $\log_a m = x$, and we can say that $m > 0$ and $a > 0$.

Thus, $a^x = m \Rightarrow x = \log_a m$ and $\log_a m = x \Rightarrow a^x = m$.

In short, $a^x = m \Rightarrow x = \log_a m$.

$x = \log_a m$ is called the logarithmic form and $a^x = m$ is called the exponential form of the equation connecting a, x and m.

Two Properties of Logarithms

1. $\log_a 1 = 0$ for all $a > 0$, $a \neq 1$

   That is, log 1 to any base is zero.

   Let $\log_a 1 = x$. then by definition, $a^x = 1$.

   But $a^0 = 1 \therefore a^x = a^0 \times x = 0$.

   Hence, $\log_a 1 = 0$ for all $a > 0$, $a \neq 1$.

2. $\log_a a = 1$ for all $a > 0$, a π 1

   That is, log of a number to the same base is 1.

   Let $\log_a a = x$. then by definition, $a^x = a$.

   But $a^1 = a \therefore a^x = a^1 \Rightarrow x = 1$.

   Hence, $\log_a a = 1$ for all $a > 0$, $a \neq 1$.

Laws of Logarithms

First law: $\log_a (mn) = \log_a m + \log_a n$

That is, log of product = sum of logs

Second law: $\log_a (m/n) = \log_a m - \log_a n$

That is, log of quotient = difference of logs

Note: The first theorem converts a problem of multiplication into a problem of addition and the second theorem converts a problem of division into a problem of subtraction, which are far easier to perform than multiplication or division. That is why, logarithms are so useful in all numerical calculations.

Third Law: $\log_a m^n = n \log_a m$

Generalisation

1. $\log (mnp) = \log m + \log n + \log p$
2. $\log (a_1 a_2 a_3 \ldots a_k) = \log a_1 + \log a_2 + \ldots + \log a_k$

> ***Note:*** *Common logarithms:* We shall assume that the base $a = 10$ whenever it is not indicated. Therefore, we shall denote $\log_{10} m$ by $\log m$ only. The logarithms calculated to base 10 are called common logarithms.

Characteristic and Mantissa of a Logarithm

The logarithm of a number consists of two parts: the *integral* part and the *decimal* part. The integral part is known as the *characteristic* and the decimal part is called the *mantissa*.

For example,

In log 3257 = 3.5128, the integral part is 3 and the decimal part is 0.5128; therefore, characteristic = 3 and mantissa = 0.5128.

It should be remembered that the mantissa is always written as positive.

Rule: To make the mantissa positive (in case the value of the logarithm of a number is negative), subtract 1 from the integral part and add 1 to the decimal part.

Thus, $-3.4328 = -(3 + .4328) = -3 - 0.4328$

$= (-3 - 1) + (1 - 0.4328)$

$= -4 + 0.5672$

So the mantissa is = 0.5672.

Note: The characteristic may be positive or negative. When the characteristic is negative, it is represented by putting a bar on the number.

Thus, instead of −4, we write $\bar{4}$.

Hence, we may write −4 + .5672 as $\bar{4}.5672$.

Base Change Rule

Up till now all rules and theorems you have studied in logarithms have been related to operations on logs with the same bases. However, there are a lot of situations in logarithm problems where you have to operate on logs having different bases. The base change rule is used in such situations.

This rule states that

(i) $\log_a (b) = \log_c (b)/\log_c (a)$

It is one of the most important rules for solving logarithms.

A corollary of this rule is:

(ii) $\log_a (b) = 1/\log_b (a)$

(iii) log c to the base a^b is equal to $\frac{\log_a c}{b}$

Results on Logarithmic Inequalities

(a) If $a > 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 > x_2$

(b) If $a < 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 < x_2$

Applied conclusions for logarithms

1. The characteristic of common logarithms of any positive number less than 1 is negative.
2. The characteristic of common logarithm of any number greater than 1 is positive.

3. If the logarithm to any base a gives the characteristic n, then we can say that the number of integers possible is given by $a^{n+1} - a^n$.

Example: $\log_{10} x = 2.bcde\ldots$, then the number of integral values that x can take is given by: $10^{2+1} - 10^2 = 900$. This can be physically verified as follows.

log to the base 10 gives a characteristic of 2 for all three digit numbers with the lowest being 100 and the highest being 999. Hence, there are 900 integral values possible for x.

4. If $-n$ is the characteristic of $\log_{10} y$, then the number of zeros between the decimal and the first significant number after the decimal is $n - 1$.

Thus, if the log of a number has a characteristic of –3, then the first two decimal places after the decimal point will be zeros.

Thus, the value will be $-3.00ab\ldots$

WORKED-OUT PROBLEMS

Problem 16.1 Find the value of x in $3^{|3x-4|} = 9^{2x-2}$.

(a) 8/7

(b) 7/8

(c) 7/4

(d) 16/7

Solution: Take the log of both sides, then we get,

$$\begin{aligned} |3x - 4| \log 3 &= (2x - 2) \log 9 \\ &= (2x - 2) \log 3^2 \\ &= (4x - 4) \log 3 \end{aligned}$$

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Dividing both sides by log 3, we get

$|3x-4| = (4x-4)$ (1)

Now, $|3x-4| = 3x-4$ if $x > 4/3$

so if $x > 4/3$

$3x-4 = 4x-4$

or $3x = 4x$

or $3 = 4$

but this is not possible.

Let's take the case of $x < 4/3$

Then $|3x-4| = 4-3x$

Therefore, $4-3x = 4x-4$ from (1)

or $7x = 8$

or $x = 8/7$

Problem 16.2 Solve for x.

$$\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$$

Solution: Now, $\log_{10}\sqrt{x} = \frac{1}{2} \times \log_{10}x$

Therefore, the equation becomes

$$\log_{10}x - \frac{1}{2}\log_{10}x = 2\log_x 10$$

or $\frac{1}{2}\log_{10}x = 2\log_x 10 \quad (1)$

Using base change rule ($\log_b a = 1/\log_a b$)

Therefore, equation (1) becomes

$$\frac{1}{2}\log_{10}x = 2/\log_{10}x$$

$\Rightarrow (\log_{10}x)^2 = 4$

or $\log_{10}x = 2$

Therefore, $x = 100$

Problem 16.3 If $7^{x+1} - 7^{x-1} = 48$, find x.

Solution: Take 7^{x-1} as the common term. The equation then reduces to

$$7^{x-1}(7^2 - 1) = 48$$

or $7^{x-1} = 1$

or $x - 1 = 0$ or $x = 1$

Problem 16.4 Calculate: $\log_2(2/3) + \log_4(9/4)$.

$= \log_2(2/3) + (\log_2(9/4)/\log_2 4$

$= \log_2(2/3) + 1/2\log_2(9/4)$

$= \log_2 2/3 + \log_2 3/2 = \log_2 1 = 0$

Problem 16.5 Find the value of the expression

$1/\log_3 2 + 2/\log_9 4 - 3/\log_{27} 8$

Passing to base 2.

we get

$$
\begin{aligned}
&\log_2 3 + 2\log_{2^2}9 - 3\log_{2^3}27 \\
&= \log_2 3 + \frac{4\log_2 3}{2} - \frac{9\log_2 3}{3} \\
&= 3\log_2 3 - 3\log_2 3 \\
&= 0
\end{aligned}
$$

Problem 16.6 Solve the inequality.

(a) $\log_2 (x + 3) < 2$

$\Rightarrow 2_2 > x + 3$

$\Rightarrow 4 > x + 3$

$1 > x$

or $x < 1$

But log of negative number is not possible.

Therefore, $x + 3 \geq 0$

That is, $x \geq -3$

Therefore, -3 £ $x < 1$

(b) $\log_2 (x_2 - 5x + 5) > 0$

$= x_2 - 5x + 5 > 1$

$\rightarrow x_2 - 5x + 4 > 0$

$\rightarrow (x - 4)(x - 1) > 0$

Therefore, the value of x will lie outside 1 and 4.

That is, $x > 4$ or $x < 1$.

LEVEL OF DIFFICULTY (I)

1. log 32700 = ?

   (a) log 3.27 + 4

   (b) log 3.27 + 2

(c) 2 log 327

(d) 100 × log 327

2. log 0.0867 = ?

(a) log 8.67 + 2

(b) log 8.67 – 2

(c) $\frac{\log 867}{1000}$

(d) –2 log 8.67

3. If $\log_{10}2 = 0.301$, find $\log_{10}125$.

(a) 2.097

(b) 2.301

(c) 2.10

(d) 2.087

4. $\log_{32} 8$ = ?

(a) 2/5

(b) 5/3

(c) 3/5

(d) 4/5

Find the value of x in equations 5–6.

5. $\log_{0.5}x = 25$

(a) 2^{-25}

(b) 2^{25}

(c) 2^{-24}

(d) 2^{24}

6. $\log_3 x = \frac{1}{2}$

(a) 3

(b) $\sqrt{3}$

(c) $\frac{3}{2}$

(d) $\frac{2}{3}$

7. $\log_{15} 3375 \times \log_4 1024 = ?$

(a) 16

(b) 18

(c) 12

(d) 15

8. $\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$. Then, $a = ?$

(a) 4

(b) 2

(c) 8

(d) 5

9. $\log_{625} \sqrt{5} = ?$

(a) 4

(b) 8

(c) 1/8

(d) 1/4

10. If $\log x + \log (x + 3) = 1$, then the value(s) of x will be, the solution of the equation

(a) $x + x + 3 = 1$

(b) $x + x + 3 = 10$

(c) $x (x + 3) = 10$

(d) $x (x + 3) = 1$

11. If $\log_{10} a = b$, find the value of 10^{3b} in terms of a.

(a) a^3

(b) $3a$

(c) $a \times 1000$

(d) $a \times 100$

12. $3 \log 5 + 2 \log 4 - \log 2 = ?$

(a) 4

(b) 3

(c) 200

(d) 1000

Solve equations 13–25 for the value of x.

13. $\log (3x - 2) = 1$

(a) 3

(b) 2

(c) 4

(d) 6

14. $\log(2x - 3) = 2$

(a) 103

(b) 51.5

(c) 25.75

(d) 26

15. $\log(12 - x) = -1$

(a) 11.6

(b) 12.1

(c) 11

(d) 11.9

16. $\log(x^2 - 6x + 6) = 0$

(a) 5

(b) 1

(c) Both (a) and (b)

(d) 3 and 2

17. $\log 2x = 3$

(a) 9.87

(b) 3 log 2

(c) 3/log 2

(d) 9.31

18. $3^x = 7$

(a) $1/\log_7 3$

(b) $\log_7 3$

(c) $1/\log_3 7$

(d) $\log_3 7$

19. $5^x = 10$

(a) log 5

(b) log 10/log 2

(c) log 2

(d) 1/log 5

20. Find x, if $0.01^x = 2$.

(a) log 2/2

(b) 2/log 2

(c) –2/log 2

(d) 4

22. Find x, if $\log x = \log 1.5 + \log 12$.

(a) 12

(b) 8

(c) 18

(d) 15

23. Find x, if $\log x = 2 \log 5 + 3 \log 2$.

(a) 50

(b) 100

(c) 150

(d) 200

24. $\log (x - 13) + 3 \log 2 = \log (3x + 1)$

(a) 20

(b) 21

(c) 22

(d) 24

25. $\log (2x - 2) - \log (11.66 - x) = 1 + \log 3$

(a) 452/32

(b) 350/32

(c) 11

(d) 11.33

LEVEL OF DIFFICULTY (II)

1. Express $\log \frac{\sqrt[3]{a^2}}{b^5 \sqrt{c}}$ or $\frac{a^{2/3}}{b^5 \sqrt{c}}$ in terms of $\log a$, $\log b$ and $\log c$.

(a) $\frac{3}{2} \log a + 5 \log b - 2 \log c$

(b) $\frac{2}{3}\log a - 5\log b - \frac{1}{2}\log c$

(c) $\frac{2}{3}\log a - 5\log b + \frac{1}{2}\log c$

(d) $\frac{3}{2}\log a + 5\log b - \frac{1}{2}\log c$

2. If $\log 3 = 0.4771$, find $\log(.81)_2 \times \log\left(\frac{27}{10}\right)^{\frac{2}{3}} \prod \log 9$.

(a) 2.689

(b) – 0.0552

(c) 2.2402

(d) 2.702

3. If $\log 2 = 0.301$, $\log 3 = 0.477$, find the number of digits in $(108)_{10}$.

(a) 21

(b) 27

(c) 20

(d) 18

4. If $\log 2 = 0.301$, find the number of digits in $(125)_{25}$.

(a) 53

(b) 50

(c) 25

(d) 63

5. Which of the following options represents the value of log $\sqrt{128}$ to the base 0.625?

(a) $\frac{2+\log_8 2}{\log_8 5-1}$

(b) $\frac{\log_8 128}{2\log_8 0.625}$

(c) $\frac{2+\log_8 2}{2(\log_8 5-1)}$

(d) Both (b) and (c)

6-8. Solve for x:

$$\log\frac{75}{35}+2\log\frac{7}{5}-\log\frac{105}{x}-\log\frac{13}{25}=0.$$

(a) 90

(b) 65

(c) 13

(d) 45

7. $2\log\frac{4}{3}-\log\frac{x}{10}+\log\frac{63}{160}=0$

(a) 7

(b) 14

(c) 9

(d) 3

8. $\log\frac{12}{13}-\log\frac{7}{25}+\log\frac{91}{3}=x$

(a) 0

(b) 1

(c) 2

(d) 3

Directions for Questions 9 and 11: Which one of the following is true?

9. (a) $\log_{17}275 = \log_{19}375$

(b) $\log_{17}275 < \log_{19}375$

(c) $\log_{17}275 > \log_{19}375$

(d) Cannot be determined

10. (a) $\log_{11}1650 > \log_{13}1950$

(b) $\log_{11}1650 < \log_{13}1950$

(c) $\log_{11}1650 = \log_{13}1950$

(d) None of these

11. (a) $\frac{\log_2 4096}{3} = \log_8 4096$

(b) $\frac{\log_2 4096}{3} < \log_8 4096$

(c) $\frac{\log_2 4096}{3} > \log_8 4096$

(d) Cannot be determined

12. $\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log x,\ x = ?$

(a) 2

(b) 3

(c) 0

(d) None of these

If log 2 = 0.301 and log 3 = 0.4771, then find the number of digits in the following.

13. 60_{12}

(a) 25

(b) 22

(c) 23

(d) 24

14. 72_{9}

(a) 17

(b) 20

(c) 18

(d) 15

15. 27_{25}

(a) 38

(b) 37

(c) 36

(d) 35

Directions for Questions 16 and 18: Find the value of the logarithmic expression in the questions below.

16. $\dfrac{\log\sqrt{27}+\log 8-\log\sqrt{1000}}{\log 1.2}$

where, $\log_{10} 2 = 0.30103$, $\log_{10} 3 = 0.4771213$

(a) 1.77

(b) 1.37

(c) 2.33

(d) 1.49

17. $\frac{1}{\log_{xy}(xyz)}+\frac{1}{\log_{yz}(xyz)}+\frac{1}{\log_{zx}(xyz)}$ = ? = ?

(a) 1

(b) 2

(c) 3

(d) 4

18. $\log a^n/b^n + \log b^n/c^n + \log c^n/a^n$ = ?

(a) 1

(b) n

(c) 0

(d) 2

19. $\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$, then x = ?

(a) 50

(b) 100

(c) 150

(d) 200

20. $\left(\frac{21}{10}\right)^x = 2$. Then x = ?

(a) $\frac{\log 2}{\log 3+\log 7-1}$

(b) $\frac{-2-\sqrt{2}}{2}$

(c) Both (a) and (b)

(d) None of these

22. $(a4 - 2a2b2 + b4)x - 1 = (a - b)-2\ (a + b)-2$, then x = ?

(a) 1

(b) 0

(c) None of these

(d) 2

23. If $\log_{10} 242 = a$, $\log_{10} 80 = b$ and $\log_{10} 45 = c$, express $\log_{10} 36$ in terms of a, b and c.

(a) $\frac{(c-1)(3c+b-4)}{2}$

(b) $\frac{(c-1)(3c+b-4)}{3}$

(c) $\frac{(c-1)(3c-b-4)}{2}$

(d) None of these

24. For the above problem, express $\log_{10} 66$ in terms of a, b and c.

(a) $\frac{(c-1)(3c+b-4)}{8}$

(b) $\frac{3(a+c)+(2b+-5)}{6}$

(c) $\frac{3(a+c)+(2b-5)}{6}$

(d) $\frac{3(c-1)(3c+b-4)}{6}$

25. $\log_2 (9 - 2x) = 10\log (3 - x)$. Solve for x.

(a) 0

(b) 3

(c) Both (a) and (b)

(d) 0 and 6

26. If $\frac{\log x}{(b-c)} = \frac{\log y}{(c-a)} = \frac{\log z}{(a-b)}$. Mark all the correct options.

(a) $xyz = 1$

(b) $x^a y^b z^c = 1$

(c) $x^{b+c} y^{c+a} z^{a+b} = 1$

(d) All the options are correct.

27. What will be the value of x, if it is given that

$$\log_x\left[\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\ldots+\infty \text{ terms}\right]^2 = 2$$

28. $(\log_4 x^2)(x\log_{27} 8)(\log_x 243)$ is equal to

(a) $2x$

(b) $5x$

(c) $3x$

(d) 1

29. For how many real values of x will the equation $\log_3 \log_6 (x^3 - 18x^2 + 108x) = \log_2 \log_4 16$ be satisfied?

30. If $n = 12\sqrt{3}$, then

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}$$

$$+\frac{1}{\log_6 n}+\frac{1}{\log_8 n}$$
$$+\frac{1}{\log_9 n}+\frac{1}{\log_{18} n}=?$$

31. $(\log_2 x)^2 + 2\log_2 x - 8 = 0$,where x is a natural number. If $xp = 64$, then what is the value of $x + p$?

Directions for 32 and 34: a =

$\sum_{i=2}^{a} \log_3(i)$, $B = \sum_{j=2}^{b} \log_3(j)$ & C
$= \sum_{k=2}^{(a-b)} \log_3 \log_3 k$, where $a \geq b$. If
$D = A - B - C$. then answer the following questions.

32. If $a = 10$, then for what value of b, D is minimum
33. For $a = 6$, D is maximum for b = ?
34. If 'p' and 'q' are integers and $\log_p(-q^2 + 6q - 8) + \log_q(-2p^2 + 20p - 48) = 0$ then $p \times q$ = ?

ANSWER KEY

Level of Difficulty (I)

1. (a)
2. (b)
3. (a)
4. (c)
5. (a)
6. (b)
7. (d)

8. (a)
9. (c)
10. (c)
11. (a)
12. (b)
13. (c)
14. (b)
15. (d)
16. (c)
17. (c)
18. (a)
19. (d)
20. (d)
21. (c)
22. (c)
23. (d)
24. (b)
25. (c)

Level of Difficulty (II)

1. (b)
2. (b)
3. (a)
4. (a)
5. (d)
6. (c)
7. (a)

8. (c)

9. (b)

10. (a)

11. (a)

12. (d)

13. (b)

14. (a)

15. (c)

16. (d)

17. (b)

18. (c)

19. (b)

20. (a)

21. (c)

22. (b)

23. (d)

24. (c)

25. (a)

26. (d)

27. 25/48

28. (b)

29. 1

30. 4

31. 7

32. 10

33. 3

34. 15

Solutions and Shortcuts

Level of Difficulty (I)

1. Log 32700 = log 3.27 + log 10000 = log 3.27 + 4

2. Log 0.0867 = log (8.67/100) = log 8.67 – log 100

   Log 8.67 – 2

3. $\log_{10} 125 = \log_{10}(1000/8) = \log 1000 - 3\log 2$

   $= 3 - 3 \times 0.301 = 2.097$

4. $\log_{32} 8 = \log 8/\log 32$ (by base change rule)

   $= 3 \log 2/5\log 2 = 3/5$

5. $\log_{0.5} x = 25 \Rightarrow x = 0.5^{25} = (½)^{25} = 2^{-25}$

6. $x = 3^{1/2} = \sqrt{3}$

7. $\log_{15}3375 \times \log_4 1024$

   $= 3 \log_{15} 15 \times 5 \log_4 4 = 3 \times 5 = 15$

8. The given expression is:

   $\log_a (4 \times 16 \times 64 \times 256) = 10$

   i.e. $\log_a 4^{10} = 10$

   Thus, $a = 4$.

9. $1/2 \log_{625} 5 = [1/(2 \times 4)] \log_5 5 = 1/8$

10. $\log x (x + 3) = 1 \Rightarrow 10^1 = x^2 + 3x$

    or $x(x + 3) = 10$

11. $\log_{10} a = b \Rightarrow 10^b = a \Rightarrow$ by definition of logs.

Thus, $10^{3b} = (10^b)^3 = a^3$.

12. $3 \log 5 + 2 \log 4 - \log 2$

$= \log 125 + \log 16 - \log 2$

$= \log (125 \times 16)/2 = \log 1000 = 3$

13. $10^1 = 3x - 2 \Rightarrow x = 4$

14. $10^2 = 2x - 3 \Rightarrow x = 51.5$

15. $1/10 = 12 - x \Rightarrow x = 11.9$

16. $x^2 - 6x + 6 = 10^\circ \Rightarrow x^2 - 6x + 6 = 1$

$\Rightarrow x^2 - 6x + 5 = 0$

Solving gives us $x = 5$ and 1.

17. $x \log 2 = 3$

$\log 2 = 3/x$

Therefore, $x = 3/\log 2$.

18. $3^x = 7 \Rightarrow \log_3 7 = x$

Hence, $x = 1/\log_7 3$.

19. $x = \log_5 10 = 1/\log_{10} 5 = 1/\log 5$

20. $x = \log_{0.01} 2 = -\log 2/2$

21. $\log x = \log (7.2/2.4) = \log 3 \Rightarrow x = 3$

22. $\log x = \log 18 \Rightarrow x = 18$

23. $\log x = \log 25 + \log 8 = \log (25 \times 8) = \log 200$

24. $\log (x - 13) + \log 8 = \log[3x + 1]$

$\Rightarrow \log (8x - 104) = \log (3x + 1)$

$\Rightarrow 8x - 104 = 3x + 1$

$5x = 105 \Rightarrow x = 21$

25. $\log (2x - 2)/(11.66 - x) = \log 30$

$\Rightarrow (2x - 2)/(11.66 - x) = 30$

$2x - 2 = 350 - 30x$

Hence, $32x = 352 \Rightarrow x = 11$.

Level of Difficulty (II)

1. $2/3 \log a - 5 \log b - 1/2 \log c$

2. $2 \log (81/100) \times 2/3 \log (27/10) \div \log 9$

$= 2 [\log 3_4 - \log 100] \times 2/3 [(\log 3_3 - \log 10)] \div 2 \log 3$

$= 2 [\log 3_4 - \log 100] \times 2/3 [(3\log 3 - 1)] \div 2 \log 3$

Substitute $\log 3 = 0.4771 \Rightarrow -0.0552$.

3. let the number be y.

$$y = 108^{10}$$

$$\Rightarrow \log y = 10 \log 108$$

$$\text{Log } y = 10 \log (27 \times 4)$$

$$\text{Log } y = 10 [3\log 3 + 2 \log 2]$$

$$\text{Log } y = 10 [1.43 + 0.602]$$

Hence, $\log y = 10[2.03] = 20.3$.

Thus, y has 21 digits.

4. $\log y = 25 \log 125$

   $= 25 [\log 1000 - 3 \log 2] = 25 \times (2.097)$

   $= 52 +$

   Hence, 53 digits.

5. $0.5 \log_{0.625} 128$

$$= 0.5\ [\log_8 128/\log_8 0.625]$$
$$= 1/2\ [\ \log_8 128/\log_8 0.625]$$

$$\frac{\text{Log}_8 128}{2(\log_8 5 - \log_8 8\ )} = \frac{\text{Log}_8 128}{2[\log_8 5 - 1]} = \frac{2 + \log_8 2}{2(\log_8 5 - 1)}$$

6. $(75/35) \times (49/25) \times (x/105) \times (25/13) = 1$

   $\Rightarrow x = 13$

7. $(16/9) \times (10/x) \times (63/160) = 1$
   $\Rightarrow x = 7$

8. Solve in similar fashion.

9. $\log_{17} 275 < \log_{19} 375$

   Because the value of $\log_{17} 275$ is less than 2, while $\log_{19} 375$ is greater than 2.

10. $\log_{11} 1650 > 3$

    $\log_{13} 1950 < 3$

    Hence, $\log_{11} 1650 > \log_{13} 1950$

11. $\frac{\log_2 4096}{3} = \log_8 4096$

12. $x = (16/15) \times (25_5/24_5) \times (81_3/80_3)$

None of these is correct.

Solutions for Questions 13 to 15:

Solve similarly as 3 and 4.

18. $\log (a_n b_n c_n / a_n b_n c_n) = \log 1 = 0$

19. $(1/2) \log x = 2\log_x 10$

$\Rightarrow \log x = 4 \log_x 10$

$\Rightarrow \log x = 4/\log_{10} x \Rightarrow (\log x)_2 = 4$

So $\log x = 2$ and $x = 100$.

20. $x = \log_{(21/10)} 2$

$$= \frac{\text{Log}\,2}{\text{Log}\,21 - \log 10} = \frac{\text{Log}\,2}{[\text{Log}\,3 + \log 7 - 1]}$$

21. $6x_2 + 12x + 3 = 0$ or $2x_2 + 4x + 1 = 0$

Solving, we get both the options (a) and (b) as correct. Hence, option (c) is the correct answer.

25. For $x = 0$, we have LHS

$\log_2 8 = 3$

RHS: $10_{\log 3} = 3$

We do not get LHS = RHS for either $x = 3$ or $x = 6$.

Thus, option (a) is correct.

26. $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k$

$\Rightarrow x = 10\ {}_{k(b-c)}, y = 10k(c-a), z = 10k(a-b)$

$\therefore xyz = 10^{k(b-c+c-a+a-b)} = 10^{0} = 1$

Therefore, option (a) is correct.

$x^{a} y^{b} z^{c} = 10^{k[a(b-c)+b(c-a)+c(a-b)]}$

$= 10^{k(ab-ac+bc-ab+ca-bc)}$

$= 10^{k.0} = 1$

Therefore option (b) is correct.

$x^{b+c} y^{c+a} z^{a+b} = 10^{k[(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)]}$

$= 10^{k.0} = 1$

Therefore, option (c) is also correct.

Since all the first three options are correct, we choose option (d) as the correct answer.

27. $\log_x\left[\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\ldots+\infty \text{ terms}\right]^2$

$= 2\log_x\left(\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\ldots+\infty \text{ terms}\right)$

Let $\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\ldots+\infty \text{ terms} = P$

$P = \frac{1}{4}\left[\frac{4}{1\times5}+\frac{4}{2\times6}+\frac{4}{3\times7}+\frac{4}{4\times8}+\frac{4}{5\times9}+\ldots\infty\right]$

$4P = \left[1-\frac{1}{5}+\frac{1}{2}-\frac{1}{6}+\frac{1}{3}-\frac{1}{7}+\frac{1}{4}-\frac{1}{8}+\frac{1}{5}-\frac{1}{9}\right]$

$4P = \left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right]$

$$4P = \frac{25}{12}$$

$$P = \frac{25}{48}$$

$$\log_x \frac{25}{48} = 1 \text{ or } x = 25/48$$

28. $\log_4 x^2 x \log_{27} 8 . \log_x 243 = \frac{2\log x}{\log 4} . \frac{x \log 8}{\log 27} . \frac{\log 243}{\log x}$

$$= \frac{\log x}{\log 2} . \frac{3x \log 2}{3 \log 3} . \frac{5 \log 3}{\log x} = 5x$$

29. $\log_2(\log_4 16) = \log_2 \log_4 4^2 = \log_2 2 = 1$

$\log_3 \log_6 (x^3 - 18x^2 + 108x) = 1$

$\log_6(x^3 - 18x^2 + 108x) = 3$

$x^3 - 18x^2 + 108x = 6^3$

$x^3 - 18x^2 + 108x - 216 = 0$

$(x - 6)^3 = 0$

$x = 6$ is the only value for which the above equation is true.

30. $n = 12\sqrt{3} = 2^2 \times 3^{1.5}$

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \frac{1}{\log_6 n}$$
$$+ \frac{1}{\log_8 n} + \frac{1}{\log_9 n} + \frac{1}{\log_{18} n}$$
$$= \log_n 2 + \log_n 3 + \log_n 4 + \log_n 6 + \log_n 8 + \log_n 9 + \log_n 18$$
$$= \log_n (2 \times 3 \times 4 \times 6 \times 8 \times 9 \times 18)$$
$$= \log_n (2^8 \times 3^6)$$
$$= \log_n (2^2 \times 3^{1.5})^4$$
$$= 4 \log_n (2^2 \times 3^{1.5})$$
$$= 4 \log_{2^2 \times 3^{1.5}} \left(2^2 \times 3^{1.5}\right) = 4$$

31. $(\log_2 x)^2 + 2\log_2 x - 8 = 0$

$(\log_2 x)^2 + 4\log_2 x - 2\log_2 x - 8 = 0$

$\log_2 x\,[\log_2 x + 4] - 2\,[\log_2 x + 4] = 0$

$[\log_2 x - 2]\,[\log_2 x + 4] = 0$

Since, x is a natural number hence, $[\log_2 x + 4]$ cannot be zero. Hence, $\log_2 x - 2 = 0$.

$\log_2 x = 2$

$x = 2^2 = 4$

We are given that: $x^p = 64$. Since x is 4, this means that:

$4^p = 64$

$p = 3$

Hence, the value of $(x + p) = 4 + 3 = 7$.

32. $A = \sum_{i=2}^{a} \log_3 i = \log_3 2 + \log_3 3 + \log_3 4 + \ldots + \log_3 a$

$= \log_3(2.3.4....a) = \log_3(a!)$

Similarly, $b = \log_3(b!)$, $C = \log_3(a - b)!$

$D = \log_3 a! - \log_3 b! - \log_3(a - b)!$

$= \log_3 \dfrac{a!}{b!(a-b)!}$

$= \log_3({}_aC_b)$

If $a = 10$, then D will be minimum when $b = 10$, since the smallest value of ${}_nC_r$ occurs when $n = r$.

33. If $a = 6$, then D will be maximum for $b = 3$ (since the value of ${}_nC_r$ attains its maximum when the value of r is half the value of n).

34. Both p, q must be greater than 0 as logarithms are not defined to negative bases. Now looking at the two parts of the expression, we see that both:

$-q^2 + 6q - 8 > 0$ and $-2p^2 + 20p - 48 > 0$. This leads us to the following conclusions:

$q^2 - 6q + 8 < 0$. Hence, $(q - 2)(q - 4) < 0$. The only integer value of q that satisfies this is $q = 3$.

Likewise, $2p^2 - 20p + 48 < 0$ means $2(p - 4)(p - 6) < 0$.

Only integer value of p which satisfies the above ineq uality is $p = 5$.

$\therefore p \times q = 3 \times 5 = 15$

TRAINING GROUND FOR BLOCK V

HOW TO THINK ON PROBLEMS ON BLOCK V?

1. Let $x_1, x_2, \ldots, x_{100}$ be positive integers such that $x_! + x_{!-1} + 1 = k$ for all !, where k is a constant. If $x_{10} = 1$, then the value of x_1 is

(a) k

(b) $k - 2$

(c) $k + 1$

(d) 1

Solution: Using the information in the expression which defines the function, we realise that if we use x_{10} as $x_!$, the expression gives us:

$x_{10} + x_9 + 1 = k \rightarrow x_9 = k - 2$; Further, using the value of x_9 to get the value of x_8 as follows:

$x_9 + x_8 + 1 = k \rightarrow k - 2 + x_8 + 1 = k \rightarrow x_8 = 1$;

Next: $x_8 + x_7 + 1 = k \rightarrow x_7 = k - 2$.

In this fashion, we can clearly see that x_6 would again be 1 and x_5 be $k - 2$; $x_4 = 1$

and $x_3 = k-2$; $x_2 = 1$ and $x_1 = k-2$. Option (b) is correct.

2. If $a_0 = 1$, $a_1 = 1$ and $a_n = a_{n-2}\, a_{n-1} + 3$ for $n > 1$, which of the following options would be true?

(a) a_{450} is odd and a_{451} is even

(b) a_{450} is odd and a_{451} is odd

(c) a_{450} is even and a_{451} is even

(d) a_{450} is even and a_{451} is odd

Solution: In order to solve this question, you need to think about how the initial values of a_x will behave in terms of being even and odd.

The value of $a_2 = 1 \times 1 + 3 = 4$ (this is necessarily even, since we have the construct as follows: Odd × Odd + Odd = Odd + Odd = Even.)

The value of $a_3 = 1 \times 4 + 3 = 7$ (this is necessarily odd, since we have the construct as follows: Odd × Even + Odd = Even + Odd = Odd.)

The value of $a_4 = 4 \times 7 + 3 = 31$ (this is necessarily odd, since we have the construct as follows: Odd × Even + Odd = Even + Odd = Odd.)

The value of $a_5 = 7 \times 31 + 3 = 220$ (this is necessarily even since we have the construct as follows: Odd × Odd + Odd = Odd + Odd = Even.)

The next two in the series values viz: a_6 and a_7 will be odd again since they would take the construct of Odd × Even + Odd = Even + Odd = Odd. Also, once, a_6 and a_7 turn out to be odd, it is clear that a_8 would be even and a_9 a_{10} would be odd again. Thus, we can understand that the terms a_2, a_5, a_8, a_{11}, a_{14} are even while all other terms in the series are odd. Thus, the even terms occur when we take a term whose number answers the description of a_{3n+2}. If you look at the options, all the four options in this question are asking about the value of a_{450} and a_{451}. Since, neither of these terms are in the series of a_{3n+2}, we can say that both of these are necessarily odd and hence, option (b) will be the correct answer.

3. If $\frac{a+b}{b+c}=\frac{c+d}{d+a}$ then

(a) $a = c$

(b) Either $a = c$ or $a + b + c + d = 0$

(c) $a + b + c + d = 0$

(d) $a = c$ and $b = d$

Solution: Such problems should be solved using values and also by doing a brief logical analysis of the algebraic equation. In this case, it is clear that if $a = c = k$ (say), the LHS of the expression would become equal to the RHS of the expression (and both would be equal to 1). Once we realise that the expression is satisfied for LHS = RHS, we have to choose between options *a*, *b* and *d*. However, a closer look at option (d) shows us that since it says $a = c$ and $b = d$, it is telling us that both of these (i.e. $a = c$ as well as $b = d$) get satisfied and we have already seen that even if only $a = c$ is true, the expression gets satisfied. Thus, there is no need to have $b = d$ as simultaneously true with $a = c$ as true. Based on this logic, we can reject option (d). To check for option (b), we need to see whether $a + b + c + d = 0$ would necessarily be satisfied if the expression is true. In order to check this, we can take a set of values for *a*, *b*, *c* and *d* such that their sum is equal to 0 and check whether the equation is satisfied. Taking, *a*, *b*, *c* and *d* as 1, 2, 3 and –6 respectively, we get the LHS of the expression as 3/(–1) = –3; the RHS of the expression would be (–3)/(–5) = 3/5 which is not equal to the LHS. Thus, we can understand that $a + b + c + d = 0$ would not necessarily satisfy the equation. Hence, option (a) is the correct answer.

4. If *a*, *b*, *c* and *d* satisfy the equations

$a + 7b + 3c + 5d = 0$

$8a + 4b + 6c + 2d = -32$

$2a + 6b + 4c + 8d = 32$

$5a + 3b + 7c + d = -32$

Then $(a + d)(b + c)$ equals

(a) 64

(b) – 64

(c) 0

(d) None of these

Solution: Adding each of the four equations, in the expression, we get: $16(a + d) + 20(b + c) = -32$.

Also, by adding the second and the third equations, we get: $a + b + c + d = 0$, which means that $(a + d) = -(b + c)$.

Then from: $16(a + d) + 20(b + c) = -32$, we have: $-16(b + c) + 20(b + c) = -32 \rightarrow 4(b + c) = -32$. Hence, $(b + c) = -8$ and $(a + d) = 8$. Hence, the multiplication of $(a + d)(b + c) = -64$.

5. For any real number x, the function $I(x)$ denotes the integer part of x – i.e. the largest integer less than or equal to x. At the same time, the function $D(x)$ denotes the fractional part of x. For arbitrary real numbers x, y and z, only one of the following statements is correct. Which one is it?

(a) $I(x + y) = I(x) + I(y)$

(b) $I(x + y + z) = I(x + y) + I(z) = I(x) + I(y + z) = I(x + z) + I(y)$

(c) $I(x + y + z) = I(x + y) + I(z + D(y + x))$

(d) $D(x + y + z) = y + z - I(y + z) + D(x)$

Solution: There are three principle, themes you need to understand in order to answer such questions.

1. **Thinking in language**. The above question is garnished with a plethora of mathematical symbols. Unless you are able to convert each of the situations given in the options into clear logical language, your mind cannot make sense of what is written in the options. The best mathematical brains work this way. Absolutely nobody has the mathematical vision to solve such problems by simply reading the notations in the problem and/or the options.

2. While thinking in language terms in the case of a question such as this, in order to understand and grasp the mathematical situation confronting us, the best thing to do is to put values into the situation. This is very critical in such problems because as you can yourself see – if you are thinking about say $I(x + y + z)$ and you keep the same notation to think through the problem, you would need to carry $I(x + y + z)$ throughout the thinking inside the problem. On the other hand, if you replace the $I(x + y + z)$ situation by replacing values for x, y and z, the expression would change to a single number. Thus, if you take $I(4.3 + 2.8 + 5.3) = I(12.4) = 12$. Obviously thinking further in the next steps with 12, as the handle, would be much easier than trying to think with $I(x + y + z)$.
3. Since the question here is asking us to identify the correct option which always gives us LHS = RHS, we can proceed further in the problem using the options given to us. While doing this, when you are testing an option, the approach has to be to try to think of values for the variables such that the option is rejected, i.e. we need to think of values such that LHS ± RHS.

In this fashion, the idea is to eliminate three options and identify the one option that cannot be eliminated because it cannot be disproved.

Keeping these principles in mind, if we try to look at the options in this problem, we have to look for the one correct statement.

Let us check option (a) to begin with:

The LHS can be interpreted as: $I(x + y)$ means the integer part of $x + y$. Suppose we use x as 4.3 and y as 4.2, we would get $I(x + y) = I(4.3 + 4.2) = I(8.5) = 8$.

The RHS in this case would be $I(4.3) + I(4.2) = 4 + 4 = 8$. This gives us LHS = RHS.

However, if you use $x = 4.3$ and $y = 4.8$, you would see that LHS = $I(4.3 + 4.8) = I(9.1) = 9$, while the RHS would be $I(4.3) + I(4.8) = 4 + 4 = 8$. This would clearly give us LHS ± RHS and hence, this option is incorrect.

The point to note here is that whenever you are solving a function-based question through the rejection of the options route, the vision about what kind of numbers would reject the case becomes critical. It is advisable that as you start solving questions through this route, you would need to improve your vision of what values to assume while rejecting an option. This is one key skill that differentiates the minds and the capacities of the top people from the average aspirants. Hence, if you want to compete against the best, you should develop this numerical vision. To illustrate what is meant by numerical vision, think of a situation where you are faced with the expression $(a + b) > (a \times b)$. Normally this does not happen, except when you are multiplying with numbers between 0 and 1.

Moving on with our problem. Let us look at option (b).

$I(x + y + z) = I(x + y) + I(z) = I(x) + I(y + z) = I(x + z) + I(y)]$

In order to reject this option, the following values would be used:

$x = 4.3$, $y = 5.1$, and $z = 6.7$

LHS = $I(4.3 + 5.1 + 6.7) = I(16.1) = 16$

When we try to see whether the first expression on the RHS satisfies this, we can clearly see it does not because:

$I(x + y) + I(z)$ would give us $I(4.3 + 5.1) + I(6.7) = I(9.4) + I(6.7) = 9 + 6 = 15$ in this case.

Thus, $I(x + y + z) = I(x + y) + I(z)$ is disproved and this option can be rejected at this point.

We, thus, move onto option (c), which states:

$I(x + y + z) = I(x + y) + I(z + D(y + x))$

Let us try this in the case of the values we previously took, i.e. $x = 4.3$, $y = 5.1$, and $z = 6.7$.

We get:

$I\,(4.3 + 5.1 + 6.7) = I\,(4.3 + 5.1) + I\,(6.7 + D\,(4.3 + 5.1))$

$\rightarrow I\,(16.1) = I\,(9.4) + I\,(6.7 + D\,(9.4))$

$\rightarrow I\,(16.1) = I\,(9.4) + I\,(6.7 + 0.4)$

$\rightarrow I\,(16.1) = I\,(9.4) + I\,(7.1)$

$\rightarrow 16 = 9 + 7$

Suppose we try 4.3, 4.9 and 5.9, we get:

$I\,(4.3 + 4.9 + 5.9) = I\,(4.3 + 4.9) + I\,(5.9 + D\,(4.3 + 4.9)$

$\rightarrow I\,(15.1) = I\,(9.2) + I\,(5.9 + D\,(9.2))$

$\rightarrow I\,(15.1) = I\,(9.4) + I\,(5.9 + 0.2)$

$\rightarrow I\,(15.1) = I\,(9.4) + I\,(6.1)$

$\rightarrow 15 = 9 + 6$

We can see that this option is proving to be difficult to shake off as a possible answer. However, this logic is not enough to select this as the correct answer. In order to make sure that you never err when you solve a question this way, you would need to either do one of two things at this point in the problem-solving approach.

Approach 1: Try to understand and explain to yourself the mathematical reason as to why this option should be correct.

Approach 2: Try to eliminate the remaining option/s at this point of time.

Amongst these, it is recommended to opt for approach 2 because that is likely to be easier than approach 1. Approach 1 is only to be used in case you have seen and understood during your checking of the option as to why the particular option is always guaranteed to be true. In case, you have not seen the mathematical logic for the same during your checking of the option, you typically should not try to search for the logic while trying to solve the problem. The quicker way to the correct answer would be to eliminate the remaining option/s.

(Of course, once you are done with solving the question, during your review of the question, you should ideally try to explain to yourself as to why one particular option worked – because that might become critical mathematical logic inside your mind for the next time you face a similar mathematical situation.)

In this case, let us try to do both. To freeze option (c) as the correct answer, you would need to look at option (d) and try to reject it.

Option (d) says:

$D(x + y + z) = y + z - I(y + z) + D(x)$

Say we take $x = 4.3$, $y = 5.1$ and $z = 6.7$, we can see that:

$D(4.3 + 5.1 + 6.7) = 5.1 + 6.7 - I(5.1 + 6.7) + D(4.3) \rightarrow$

$D(16.1) = 11.8 - I(11.8) + D(4.3) \rightarrow$

$0.1 = 11.8 - 11 + 0.3 \rightarrow$

$0.1 = 1.1$; this is clearly incorrect.

Hence, option (c) is the correct answer.

If we were to look at the mathematical logic for option (c) (for our future reference), we can think of why option (c) would always be true as follows:

One of the problems in these greatest integer problems is what can be described as the loss of value due to the greatest integer function.

Thus, $I(4.3 + 4.8) > I(4.3) + I(4.8)$ since the LHS is 9 and the RHS is only 8. What happens here is that the LHS gains by 1 unit because the 0.3 and the 0.8 in the two numbers add up to 1.1 and help the sum of the two numbers to cross 9. On the other hand, if you were to look at the RHS in this situation, you would realise that the decimal values of 4.3 and 4.8 are individually both lost.

In this context, when you look at the LHS of the equation given in option (c), you see that the value of the LHS would retain the integer values of x, y and z while the sum of the decimal values of x, y and z would get aggregated and combined into 1 number. This gives us three cases:

Case 1: When the addition of the decimal values of x, y and z is less than 1;

Case 2: When the addition of the decimal values of x, y and z is more than 1 but less than 2;

Case 3: When the addition of the decimal values of x, y and z is more than 2 but less than 3.

Each of these three cases would be further having a two-way fork – viz:

Case A: When the addition of the decimal values of x and y add up to less than 1;

Case B: When the addition of the decimal values of x and y add up to more than 1 but less than 2.

The readers are encouraged to take this case from this point and move it to a point where they can explore each of these six situations and see that for all these situations, the value of the LHS of the expression is equal to the value of the RHS of the equation.

6. During the reign of the great government in the country of Riposta, the government forms committees of ministers whenever it is faced with a problem. One particular year, there are x ministers in the government and they are organised into four committees such that:

(i) Each minister belongs to exactly two committees

(ii) Each pair of committees has exactly one minister in common

Then

(a) $x = 4$

(b) $x = 6$

(c) $x = 8$

(d) x cannot be determined from the given information

Solution: In order to think about this situation, you need to think of the number of unique people you need in order to make up the committees as defined in the problem. However, before you start to do this, a problem you need to solve is—how many members do you put in each committee?

Given the options for x, when we look at the options, it is clear that the number is unlikely to be larger than 4. Hence, suppose we try to think of committees with four members, we will get the following thought process:

First, we create the first two committees with exactly one member common between the two committees. We would get the following table at this point of time:

Committee 1	*Committee 2*	*Committee 3*	*Committee 4*
1	4		
2	5		

3	6		
4	7		

Here we have taken the individual members of the committees as 1, 2, 3, 4, 5, 6 and 7. We have obeyed the second rule for committee formation (i.e. each pair of committees has exactly one member in common) by taking only the member 4 as the common member.

From this point in the table, we need to try to fill in the remaining committees obeying the twin rules given in the problem.

So, each member should belong to exactly two committees and each pair of committees should have only 1 member in common.

When we try to do that in this table we reach the following point.

Committee 1	*Committee 2*	*Committee 3*	*Committee 4*
1	**4**	**7**	
2	5	**1**	
3	6	8	
4	**7**	9	

Here we have taken 7 common between the committees 2 and 3, while 1 is common between committees 1 and 3. This point freezes the individuals 1, 4 and 7 as they have been used twice (as required). However, this leaves us with 2, 3, 5, 6, 8 and 9 to be used once more and only committee 4 is left to fill in into the table. This is obviously impossible to do and hence, we are sure that each committee would not have had four members.

Obviously, if four members are too many, we cannot move to trying five members per committee. Thus, we should move trying to form committees with three members each.

When we do so, the following thought-process unfolds:

We first fill-in the first two committees by keeping exactly one person common between these committees. By taking the person '3' as a common member between committees 1 and 2, we reach the following table:

Committee 1	*Committee 2*	*Committee 3*	*Committee 4*
1	3		
2	4		
3	5		

We now need to fill in committee 3 with exactly one member from committee 1 and exactly one member from committee 2. Also, we cannot use the member number '3' as that has already been used twice. Thus, by repeating '5' from committee 2 and '1' from committee 1, we can reach the following table.

Committee 1	*Committee 2*	*Committee 3*	*Committee 4*
1	3	5	
2	4	1	
3	5		

Since the remaining person in committee 3 would have to be unique from members of committees 1 and 2, we would need to introduce a new member (say 6) in order to complete committee 3. Thus, our table evolves to the following situation.

Committee 1	*Committee 2*	*Committee 3*	*Committee 4*
1	**3**	**5**	

2	4	**1**	
3	**5**	6	

At this point, the members 2, 4 and 6 have not been used a second time. Also, the committee 4 has to have its three members filled such that it has exactly one member common with committees 1, 2 and 3 respectively.

This is easily achieved using the following structure.

Committee 1	*Committee 2*	*Committee 3*	*Committee 4*
1	**3**	**5**	2
2	4	**1**	4
3	**5**	6	6

Hence, we can clearly see that the value of x is 6, i.e. the government had six ministers.

7. During the IPL Season 14, the Mumbai Indians captained by a certain Sachin Tendulkar who emerged out of retirement, played 60 games in the season. The team never lost three games consecutively and never won five games consecutively in that season. If N is the number of games the team won in that season, then N satisfies

(a) $24 \le N \le 48$

(b) $20 \le N \le 48$

(c) $12 \le N \le 48$

(d) $20 \le N \le 42$

Solution: In order to solve this question, we need to see the limit of the minimum and maximum number of matches that the team could have won. Let us first think about the maximum number of matches the team could have won. Since the team 'never won five games consecutively' during that season, we would get the value for the maximum number of wins by trying to make the team win four games consecutively—as many times as we can. This can be thought of as follows:

WWWWLWWWWLWWWWL... and so on

From the above sequence, we can clearly see that with four wins consecutively, we are forming a block of five matches in which the team has won 4 and lost 1. Since, there are a total of 60 matches in all; there would be 12 such blocks of five matches each. The total number of wins in this case would amount to 12 × 4 = 48 (this is the highest number possible).

This eliminates option (d) as the possible answer.

If we think about the minimum number of wins, we would need to maximise the number of losses. In order to do so, we get the following thought-process:

Since the team never lost three games consecutively; for the maximum number of losses the pattern followed would be–

LLWLLWLLWLLW… and so on

Thus, there are two losses and one win in every block of three matches. Since, there would be a total of 20 such blocks; it would mean that there would be a total of 20 × 1 = 20 wins. This number would represent the minimum possible number of wins for the team.

Thus, *N* has to be between 20 and 48. Thus, option (b) is correct.

8. If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b?

(a) $-1/\sqrt{3}$

(b) – 1

(c) 0

(d) 1

(e) $1/\sqrt{3}$

Solution: Since the question represents a cubic expression, and we want the smallest possible value of *b*—keeping the constraint of their roots being three consecutive values—a little bit of guess estimation would lead you to think of –1, 0 and 1 as the three roots for minimising the value of *b*.

Thus, the expression would be $(x + 1)(x)(x – 1) = (x2 + x)(x – 1) = x3– x$. This gives us the value of *b* as –1.

It can be seen that changing the values of the roots from –1, 0 and 1 would result in increasing the coefficient of *x*–which is not what we want. Hence, the correct answer should be that the minimum value of *b* would be –1.

Note: For trial purposes, if you were to take the values of the three roots as 0, 1, and 2, the expressions would become $x(x - 1)(x - 2) = (x2 - x)(x - 2)$ which would lead to the coefficient of *x* being 2. This would obviously increase the value of the coefficient of *x* above −1.

You could also go for changing the three consecutive integral roots in the other direction to −2, −1 and 0. In such a case the expression would become: $x (x + 2)(x + 1) = (x2 + 2x)(x + 1)$ → which would again give us the coefficient of *x* as + 2.

The total solving time for this question would be 30 seconds, if you were to hit on the right logic for taking the roots as –1, 0 and 1. In case you had to check for the value of *b* in different situations by altering the values of the roots (as explained above), the time would still be less than two minutes.

9. A shop stores *x* kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x?

(a) $2 \leq x \leq 6$

(b) $5 \leq x \leq 8$

(c) $9 \leq x \leq 12$

(d) $11 \leq x \leq 14$

(e) $13 \leq x \leq 18$

Solution: This question is based on odd numbers as only with an odd value of x you would be getting integers; if you halved the value of rice and took out another half a kg from the shop store.

From the options, let us start from the second option. (**Note:** In such questions, one should make it a rule to start from one of the middle options only as the normal realisation we would get from checking one option would have been that more than one option gets removed if we have not picked up the correct option—as we would normally know whether the correct answer needs to be increased from the value we just checked or should be decreased.)

Thus, trying for $x = 7$ according to the second option, you would get

$7 \rightarrow 3 \rightarrow 1 \rightarrow 0$ (after three customers)

This means that $5 \leq x \leq 8$ is a valid option for this question. Also, since the question is definitive about the correct range, there cannot be two ranges. Hence, we can conclude that option (b) is correct.

Note: The total solving time for this question should not be more than 30 seconds. Even if you are not such an experienced solver through options, and you had to check 2–3 options in order to reach the correct option, you would still need a maximum of 90 seconds.

Directions for Questions 10 and 11: Let $f(x) = ax^2 + bx + c$, where a, b and c are certain constants and $a \pm 0$. It is known that $f(5) = -3f(2)$ and that 3 is a root of $f(x) = 0$.

10. What is the other root of $f(x) = 0$?

(a) – 7

(b) – 4

(c) 2

(d) 6

(e) Cannot be determined

11. What is the value of $a + b + c$?

(a) 9

(b) 14

(c) 13

(d) 37

(e) Cannot be determined

Solution: Since, 3 is a root of the equation, we have $9a + 3b + c = 0$ (Theory point —A root of any equation $f(x) = 0$ has the property that if it is used to replace 'x' in every part of the equation, then the equation $f(x) = 0$ should be satisfied.)

Also $f(5) = -3f(2)$ gives us that $25a + 5b + c = -3(4a + 2b + c) \rightarrow 37a + 11b + 4c = 0$.

Combining both equations, we can see that $37a + 11b + 4c = 4(9a + 3b + c) \rightarrow a - b = 0$, i.e. $a = b$.

Now, we know that the sum of roots of a quadratic equation is given by $-b/a$.

The answer to question 10 would be option (b).

For question 11, we need the sum of $a + b + c$. We know that $a + b = 0$. Also, product of roots is –12. One of the possible equations could be $(x - 3)(x + 4) = 0$ → $x^2 + x - 12 = 0$, which gives us the value of $a + b + c$ as –10. However, –10 is not in the options. This should make us realise that there is a possibility of another equation as: $(2x - 6)(2x + 8) = 0$ → $4x^2 + 4x - 48 = 0$ in which case the value of $a + b + c$ changes. Hence, the correct answer is 'cannot be determined'.

12. Suppose, the seed of any positive integer n is defined as follows:

Seed(n) = n, if $n < 10$

= seed($s(n)$), otherwise,

Where $s(n)$ indicates the sum of digits of n. For example,

seed(7) = 7, seed (248) = seed(2 + 4 + 8) = seed (14) = seed (1 + 4) = seed (5) = 5 etc. How many positive integers n, such that $n < 500$, will have seed (n) = 9?

(a) 39

(b) 72

(c) 81

(d) 108

(e) 55

Solution: The first number to have a seed of 9 would be the number 9 itself.

The next number whose seed is be 9 will be 18, then 27 and you should recognise that we are talking about numbers which are multiples of 9. Hence, the number of such numbers would be the number of numbers in the arithmetic progression:

9, 18, 27, 36, 45, ...495 = [(495 – 9)/9] + 1 = 55 such numbers.

13. Find the sum of $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots\ldots+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$

(a) $2008-\frac{1}{2008}$

(b) $2007-\frac{1}{2007}$

(c) $2007-\frac{1}{2008}$

(d) $2008-\frac{1}{2007}$

(e) $2008-\frac{1}{2009}$

Solution: Such questions are again solved through logical processes. If you were to try this problem by going through mathematical processes you would end up with a messy solution which is not going to yield any answer in any reasonable time frame.

Instead, look at the following process.

1. The first thing you should notice is that the value in the answer has got something to do with the number 2008. Suppose we were to look at only the first term of the expression, by analogy the value of the sum should have something to do with the number.
2. Accordingly, by looking at the value obtained, we can decide on which of the options fits the given answer.

So, for the first term, we see that the value is equal to the square root of 2.25 = 1.5.

By analogy, that in this case the value of 2008 is 2, the value of 2007 would be 1 and 2009 would be 3. Replacing these values, the options become

(a) $2 - \frac{1}{2}$

(b) $1 - \frac{1}{1}$

(c) $1 - \frac{1}{2}$

(d) $2 - \frac{1}{1}$

(e) $2 - \frac{1}{3}$

It can be easily verified that only option (a) gives a value of 1.5. Hence, that is the only possible answer as all other values are different. In case you need greater confirmation and surety, you can solve this for the first two terms too.

14. A function $f(x)$ satisfies $f(1) = 3600$, and $f(1) + f(2) + \ldots + f(n) = n^2 f(n)$, for all positive integers $n > 1$. What is the value of $f(9)$?

(a) 80

(b) 240

(c) 200

(d) 100

(e) 120

Solution: This question is based on chain functions where the value of the function at a particular point depends on the previous values.

$f(1) + f(2) = 4f(2) \rightarrow f(1) = 3f(2) \rightarrow f(2) = 1200$

Similarly, for $f(3)$, we have the following expression:

$f(1) + f(2) + f(3) = 9f(3) \rightarrow f(3) = 4800/8 = 600$

Further, $f(1) + f(2) + f(3) = 15f(4) \rightarrow f(4) = 5400/15 = 360$

Further, $f(1) + f(2) + f(3) + f(4) = 24f(5) \rightarrow 5760/24 = f(5) = 240$

If you were to pause a while at this point and try to look at the pattern of the numerical outcomes in the series we are getting, we get:

3600, 1200, 600, 360, 240, 1200/7

A little bit of perceptive analysis about the fractions used as multipliers to convert $f(1)$ to $f(2)$ and $f(2)$ to $f(3)$ and so on will tell us that the respective multipliers themselves are following a pattern, viz:

$f(1) \times 1/3 = f(2)$;

$f(2) \times 2/4 = f(3)$;

$f(3) \times 3/5 = f(4)$ and $f(4) \times 4/6 = f(5)$

Using this logic string, we can move onto the next values as follows:

$f(6) = 240 \times 5/7 = 1200/7$;

$f(7) = 1200/7 \times 6/8 = 900/7$;

$f(8) = 900/7 \times 7/9 = 100$ and

$f(9) = 100 \times 8/10 = 80$.

Thus, option (a) is the correct answer.

Directions for Questions 15 and 16: Let S be the set of all pairs (i, j) where $1 \le i < j \le n$, and $n \ge 4$. Any two distinct members of S are called "friends", if they have one constituent of the pairs in common and "enemies" otherwise. For example, if $n = 4$, then $S = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$. Here, (1, 2) and (1, 3) are friends, (1, 2) and (2, 3) are also friends, but (1, 4) and (2, 3) are enemies.

15. For general n, how many enemies will each member of S have?

(a) $n - 3$

(b) $(n2 - 3n - 2)/2$

(c) $2n - 7$

(d) $(n2 - 5n + 6)/2$

(e) $(n2 - 7n + 14)/2$

Solution: Solve by putting values: suppose we have $n = 5$; the members would be {(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)}.

In this case, any member can be found to have three enemies.

Thus, the answer to the above question should give us a value of 3 with $n = 5$.

Option (a): $n–3 = 5–3 = 2$. Hence, cannot be the answer.

Option (b): $8/2 = 4$. Hence, cannot be the answer.

Option (c): $10 – 7 = 3$. To be considered.

Option (d): $6/2 = 3$. To be considered.

Option (e): $4/2 = 2$. Hence, cannot be the answer.

We still need to choose one answer between options (c) and (d).

It can be seen that for $n = 6$, the values of options (c) and (d) will differ. Hence, we need to visualise how many enemies each member would have for $n = 6$.

The members would be {(1,2), (1,3), (1,4), (1,5), (1,6), (2,3),(2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6)}. It can be clearly seen that the member (1, 2) will have six enemies. Option (c) gives us a value of five and hence, can be eliminated while option (d) gives us a value of six leaving it as the only possible answer.

16. For general n, consider any two members of S that are friends. How many other members of S will be common friends of both these members?

(a) $(n2 – 5n + 8)/2$

(b) $2n – 6$

(c) $n(n–3)/2$

(d) $n – 2$

(e) $(n2 – 7n + 16)/2$

Solution: Again for this question, consider the following situation where $n = 6$.

The members would be {(1,2), (1,3), (1,4), (1,5), (1,6), (2,3),(2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6)}.

Suppose we consider the pair (1, 2) and (1, 3). Their common friends would be (1, 4), (1, 5), (1, 6) and (2, 3) .

Thus, there are four common friends for any pair of friendly members. (You can verify this by taking any other pair of friend members.)

Thus, for $n = 6$, the answer should be 4.

Checking the options, it is clear that only option (d) gives us a value of 4.

Maximum solving time: 60 – 90 seconds

17. In a tournament, there are n teams $T_1, T_2, \ldots, T_n$, with $n > 5$. Each team consists of k players, $k > 3$. The following pairs of teams have one player in common:

$$T_1 \,\&\, T_2,\ T_2 \,\&\, T_3,\ \ldots,\ T_{n-1} \,\&\, T_n, \text{ and } T_n \,\&\, T_1$$

No other pair of teams has any player in common. How many players are participating in the tournament, considering all the n teams together?

(a) $n(k - 2)$

(b) $k(n - 2)$

(c) $(n - 1)(k - 1)$

(d) $n(k - 1)$

(e) $k(n - 1)$

Thought-process:

If we take six teams and four players per team, we would get four players in T_1 (each one of them unique), three more players in T_2 (since one player of T_2 will be shared with T_1), three more players in T_3 (since one player of T_3 will be shared with T_2), three more players in T_4 (since one player of T_4 will be shared with T_3), three more players in T_5 (since one player of T_5 will be shared with T_4) and two more players in T_6 (since one player of T_6 will be shared with T_5 and one with T_1). Hence, there would be a total of 18 (4 + 3 + 3 + 3 + 3 + 2) players with $n = 6$ and $k = 4$. Checking from the options, we see that only option (d) gives us 18 as the solution.

Maximum solution time: 60 seconds.

18. Consider four digit numbers for which the first two digits are equal and the last two digits are also equal. How many such numbers are perfect squares?

(a) 4

(b) 0

(c) 1

(d) 3

(e) 2

Thought-process:

A lot of CAT takers got stuck on this question for over 5–7 minutes in the exam, since they tried to find out the squares of all two digit numbers starting from 32. However, if you are aware of the logic of finding squares of two digit numbers, you would realise that only three two-digit numbers after 32 have the last two digits in their squares equal (38, 62 and 88). Hence, you do not need to check any other number apart from these three. Checking these you would get the square of 88 as 7744. And hence, there is only one such number.

Note: Of course, you would ignore the values of squares with the last two digits as '00'.

19. A confused bank teller transposed the rupees and paisa when he cashed a cheque for Shailaja, giving her rupees instead of paisa and paisa instead of rupees. After buying a toffee for 50 paisa, Shailaja noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount?

(a) Over rupees 22 but less than rupees 23

(b) Over rupees 18 but less than rupees 19

(c) Over rupees 4 but less than rupees 5

(d) Over rupees 13 but less than rupees 14

(e) Over rupees 7 but less than rupees 8

Thought-process:

Deduction 1: Question interpretation: The solution language for this question requires you to think about what possible amount could be such that when its rupees and paisa value are interchanged, the resultant value is 50 paisa more than thrice the original amount.

Deduction 2: Option-checking process:

Armed with this logic, suppose we were to check for option (a), i.e. the value is above ₹ 22 but below ₹ 23. This essentially means that the amount must be approximately between ₹ 22.66 and ₹ 22.69. We get the paisa amount to be between 66 and 69, based on the fact that the relationship between the actual amount x and the transposed amount y is: y – 50 paisa = $3x$.

Hence, values below 22.66 and values above 22.70 are not possible.

- From this point onwards, we just have to check whether this relationship is satisfied by any of the values between ₹ 22.66 and ₹ 22.69.

- Also, realise the fact that in each of these cases the paisa value in the value of the transposed amount y would be 22. Thus, $3x$ should give us the paisa value as 72 (since we have to subtract 50 paisa from the value of 'y' in order to get the value of $3x$).

→ This also means that the unit digit of the paisa value of $3x$ should be 2.

→ It can be clearly seen that none of the numbers 66, 67, 68 or 69 when being multiplied by 3 give us a units digit of 2. Hence, this is not a possible answer.

Checking for option (b) in the same fashion:

You should realise that the outer limit for the range of values when the amount is between 18 and 19 is: 18.54 to 18.57. Also, the number of paisa in the value of the transposed sum 'y' would be 18. Hence, the value of $3x$ should give us a paisa value as 68 paisa. Again, using the units digit principle, it is clear that the only value where the units digit would be 8 would be for a value of 18.56.

Hence, we cheque for the check amount to be 18.56. Transposition of the rupee and paisa value would give us 56.18. When you subtract 50 paisa from this, you would get 55.68 which also happens to be thrice 18.56. Hence, the correct answer is option (b).

Notice here that if you can work out this logic in your reactions, the time required to checking each option would be no more than 30 seconds. Hence, the net problem solving time to get the second option as correct would not be more than one minute. Add the reading time and this problem should still not require more than two minutes.

20. How many pairs of positive integers m, n satisfy $1/m + 4/n = 1/12$ where n is an odd integer less than 60?

(a) 7

(b) 5

(c) 3

(d) 6

(e) 4

Thought-process:

Deduction 1: Since two positive fractions on the LHS equals $1/12$ on the right hand side; the value of both these fractions must be less than $1/12$. Hence, n can take only the values 49, 51, 53, 55, 57 and 59.

Deduction 2: We now need to check which of the possible values of n would give us an integral value of m.

The equation can be transformed to: $1/12 - 4/n = 1/m \rightarrow (n - 48)/12n = 1/m$. On reading this equation, you should realise that for m to be an integer, the LHS must be able to give you a ratio in the form of $1/x$. It can be easily seen that this occurs for $n = 49$, $n = 51$ and $n = 57$. Hence, there are only three pairs.

21. The price of Darjeeling tea (in rupees per kilogram) is $100 + 0.1\,n$, on the nth day of 2007 ($n = 1, 2, \ldots, 100$), and then remains constant. On the other hand, the price of Ooty tea (in rupees per kilogram) is $89 + 0.15n$, on the nth day of 2007 ($n = 1, 2, \ldots, 365$). On which date in 2007, will the prices of these two varieties of tea be equal?

(a) May 21

(b) April 11

(c) May 20

(d) April 10

(e) June 30

The gap between the two prices initially is of ₹11 or 1100 paisa. The rate at which the gap closes down is 5 paisa per day for the first hundred days. (The gap covered would be 500 paisa which would leave a residual gap of 600 paisa.) Then the price of Darjeeling tea stops rising and that of Ooty tea rises at 15 paisa per day. Hence, the gap of 600 paisa would get closed out in another 40 days. Hence, the prices of the two varieties would become equal on the 140th day of the year. 31 + 28 + 31 + 30 + 20 = 140, means May 20th is the answer.

22. Let a, b, m, n be positive real numbers, which satisfy the two conditions that

(i) If $a > b$ then $m > n$; and

(ii) If $a > m$ then $b < n$

Then one of the statements given below is a valid conclusion. Which one is it?

(a) If $a < b$ then $m < n$ (b) If $a < m$ then $b > n$

(c) If $a > b + m$ then $m < b$

(d) If $a > b + m$ then $m > b$

The best way to think about this kind of a question is to try to work-out a possibility matrix of the different possibilities that exist with respect to which of the values is at what position relative to each other. While making this kind of a figure for yourself, use the convention of keeping the higher number on top and the lower number below.

If we look at condition (i) as stated in the problem, it states that: if $a > b$ then $m > n$.

This gives us multiple possibilities for the placing of the four variables in relative order of magnitude. These relative positions of the variables can be visualised as follows for the case that $a > b$:

	Possibility 1	*Possibility 2*	*Possibility 3*
Largest number	a	a	a
2nd largest number	b	m	m
3rd largest number	m	b	n
Smallest number	n	n	b

Looking at the options, option (a) can be rejected because, when we use the condition If *X* then necessarily *Y*, it does not mean that If Not *X*, then not *Y*.

For instance, if for a statement like – "If the Jan Lokpal Bill is passed, corruption will be eradicated from the country"; this does not mean that if the Jan Lokpal Bill is not passed, then corruption would not be eradicated from the country.

Note: For this kind of reverse truth to exist, the pre-existing conditionality has to be of the form: only if *x*, then Y. In such a case, the conclusion will be: if not *x*, then not Y is valid.

For instance, for a statement like – "Only if the Jan Lokpal Bill is passed, will corruption be eradicated from the country"; this necessarily means the reverse – i.e. if the Jan Lokpal Bill is not passed then corruption would not be eradicated from the country.

This exact logic helps us eliminate the first option – which says that if $a < b$ then m should be less than n (this would obviously not happen just because if $a > b$, then m is greater than n – we would need the only 'If condition in order for this to work in the reverse fashion).

Option (b) has the same structure based on the condition (ii) in the problem – it tries to reverse "If *X*, then *Y* conditionality" into "If not *X*, then not *Y*" conclusion – which would only have been valid in the case of 'Only if *X*' as explained above.

This leaves us with options (c) and (d) to check. If you go through these options, you realise that they are basically opposite to each other.

The following thought-process will help you identify which of these is the correct answer.

When we say that $a > b + m$ where a, b and m are all positive, it obviously means that a must be greater than both b and m. Thus, in this situation, we have $a > b$ as well as $a > m$. In this case, both the conditions (i) and (ii) would activate themselves. It is at this point that the possibility matrix for the case of $a > b$ would become usable.

The possibility matrix that exists currently for $a > b$ is built using the following thought-chain:

First think of the various positions in which 'a' and 'b' can be put; with 'a' greater than 'b' given that we have to fix up four numbers in decreasing order. The following possibilities emerge when we do this.

	Possibility 1	*Possibility 2*	*Possibility 3*	*Possibility 4*	*Possibility 5*	*Possibility 6*
Largest number	a	a	a			
2nd largest number	b			a	a	
3rd largest number		b			b	a
Smallest number			b	b		b

When we add the fact that when $a > b$, then m is also greater than n to this picture, the complete possibility matrix emerges as below:

	Possibility 1	*Possibility 2*	*Possibility 3*	*Possibility 4*	*Possibility 5*	*Possibility 6*
Largest number	a	a	a	m	m	m
2nd largest number	b	m	m	a	a	n
3rd largest number	m	b	n	n	b	a
Smallest number	n	n	b	b	n	b

Now, for options (c) and (d), we know that the condition to be checked for is $a > b + m$, which means that $a > b$ and $a > m$ simultaneously. We have drawn above the possibility matrix for $a > b$. We also know from condition (ii) in the problem above, that when $a > m$, then b should be less than n. Looking at the possibility matrix, we need to search for cases where simultaneously each of the following is occurring- (i) $a > b$; (ii) $a > m$ and $b < n$. We can see that the possibilities 1, 2 and 5 will get rejected because in each of these cases b is not less than n. Similarly, possibilities 4 and 6 both do not have $a > m$ and hence can be rejected. Only possibility (iii) remains, and in that case, we can see than $m > b$.

Hence, option (d) is correct.

23. A quadratic function $f(x)$ attains a maximum of 3 at $x = 1$. The value of the function at $x = 0$ is 1.

What is the value of $f(x)$ at $x = 10$?

(a) –119

(b) –159

(c) –110

(d) –180

(e) –105

Let the equation be $ax_2 + bx + c = 0$. If it gains the maximum at $x = 1$, it means that 'a' is negative.

Also $2ax + b = 0 \rightarrow x = -b/2a$. So $-b/2a$ should be 1. So the expression has to be chosen from:

$-X_2 + 2x + c$

$-2X_2 + 4x + c$

$-3X_2 + 6x + c$ ....and so on (since we have to keep the ratio of $-b/2a$ constant at

1)

Also, it is given that the value of the function at $x = 0$ is 1. This means that c = 1. Putting this value of c in the possible expressions, we can see that at $x = 1$, the value of the function is equal to 3 in the case:

$-2X^2 + 4x + 1$

So the expression is $-2X^2 + 4x + 1$

At $x = 10$, the value would be $-200 + 40 + 1 = -159$.

Directions for Questions 24 and 25: Let $a_1 = p$ and $b_1 = q$, where p and q are positive quantities. Define

$a_n = pb_{n-1}$, $b_n = qb_{n-1}$, for even $n > 1$,

and $a_n = pa_{n-1}$, $b_n = qa_{n-1}$, for odd $n > 1$.

24. Which of the following best describes $a_n + b_n$ for even n?

(a) $q(pq)^{(n/2-1)}(p + q)$

(b) $(pq)^{(n/2-1)}(p + q)$

(c) $q(1/2)^n (p + q)$

(d) $q(1/2)^n(p + q)(1/2)^n$

(e) $q(pq)^{(n/2)-1}(p + q)(1/2)^n$

Again to solve this question, we need to use values.

Let $a_1 = p = 5$ and $b_1 = q = 7$ (any random values)

In such a case,

$a_2 = 5 \times 7 = 35$ and $b_2 = 7 \times 7 = 49$. So the sum of $a_2 + b_2 = 84$.

Checking the values, we get:

Option (a): $7 \times 1 \times (12) = 84$

Option (b): $1 \times (12)$

Option (c): 7 × (12)

Option (d): 7 × (12)

Option (e): 7 × (12)

Obviously apart from option (b), all other options have to be considered. So it is obvious that the question setter wants us to go at least till the value of n as 4 to move ahead.

$a_3 = 5 \times 35 = 175$, $b_3 = 7 \times 35 = 245$

$a_4 = 5 \times 245 = 1225$, $b_4 = 7 \times 245 = 1715$

Sum of $a_4 + b_4 = 2940$

Option (a): 7 × 35 × 12 = 2940

Option (c): 7 × 7 × 12 eliminated

Option (d): 7 × 7 × 12 × 12 eliminated

Option (e): 7 × 35 × 12 × 12 eliminated

Hence, only option (a) gives us a value of 2940 for $n = 4$. Thus, it has to be correct.

25. If $p = 1/3$ and $q = 2/3$, then what is the smallest odd n such that $a_n + b_n < 0.01$?

(a) 7

(b) 13

(c) 11

(d) 9

(e) 15

According to the question, $a_1 = p = 1/3$ and $b_1 = q = 2/3$.

$a_2 = 1/3 \times 2/3 = 2/9$, $b_2 = 2/3 \times 2/3 = 4/9$

$a_3 = 1/3 \times 2/9 = 2/27$, $b_3 = 2/3 \times 2/9 = 4/27$

$a_4 = 1/3 \times 4/27 = 4/81$, $b_3 = 2/3 \times 4/27 = 8/81$

In this way, you can continue to get to the value of n at which the required sum goes below 0.01 (it would happen at $n = 9$). However, if you are already comfortably placed in the paper, you can skip this process as it would be time-consuming and also there is a high possibility of silly errors being induced under pressure.

26. The number of ordered pairs of integers (x, y) satisfying the equation $x^2 + 6x + 2y^2 = 4 + y^2$ is

(a) 12

(b) 8

(c) 10

(d) 14

These kinds of questions and thinking are very common in examinations and hence, you need to understand how to solve such questions.

In order to think of such questions, you need to first 'read' the equation given. What does it mean by 'reading' the equation? Let us illustrate:

The first thing we do is to simplify the equation by putting all the variables on the LHS. This would give us the equation $x^2 + 6x + y^2$. When we have an equation like $x^2 + 6x + y^2 = 4$, we should realise that the value on the RHS is fixed at 4. Also, if we take a look at the LHS of the equation, we realise that the terms x^2 and y^2 would always be positive integers or 0 (given that x and y are integers). $6x$, on the other hand, could be positive, zero or negative depending on the value of x (if x is positive, $6x$ is also positive; if x is negative, $6x$ would be negative and if x is 0, $6x$ would also be zero).

Thus, we can think of the following structures to build a value of 4 on the LHS for the equation to get satisfied:

	Value of x^2	*Value of 6x*	*Value of y^2*
Case 1	0	0	+
Case 2	+	+	+
Case 3	+	–	+
Case 4	+	–	0

Once you have these basic structures in place, you can think of the cases one-by-one. Thinking in this structured fashion makes sure that you do not miss out on any possible solutions — and that, as you should realise, is critical for any situation where you have to count the number of solutions. You simply cannot get these questions correct without identifying each possible situation. Trying to do such questions without first structuring your thought-process this way would lead to disastrous results in such questions!! Hence, this thinking is very critical for your development of quantitative thinking.

Let us look at ***Case 1:*** In case 1, since the value of the first two components on the LHS is 0, it must mean that we are talking about the case of $x = 0$. Obviously, in this case the entire value of 4 for the LHS has to be created by using the term y^2.

Thus, to make $y^2 = 4$, we can take $y = +2$ or $y = -2$. This gives us two possible solutions: (0, 2) and (0, –2)

Case 2: The minimum positive value for $6x$ would be when $x = 1$. This value for $6x$ turns out to be 6 – which has already made the LHS larger than 4. To this, if we were to add two more positive integers for the values of x^2 and y^2, it would simply take the LHS further up from + 6. Hence, in case 2, there are no solutions.

Case 3: In this case the value of x has to be negative and y can be either positive or negative. Possible negative values of x as –1, – 2, – 3, – 4, – 5, etc., give us values for $6x$ as – 6, –12, –18, –24, – 30, etc.

We then need to fill-in the values of x^2 and y^2 and see whether it is possible to add an exact value to any of these and get + 4 as the final value of the LHS.

This thinking would go the following way:

If $6x = -6$: x must be -1 and hence, $x^2 = 1$. Thus, $x^2 + 6x = -5$, which means for $x^2 + 6x + y^2 = 4$, we would need $-5 + y^2 = 4 \rightarrow y^2 = 9 \rightarrow y = +3$ and $y = -3$

Thus, we have identified two more solutions as $(-1, 3)$ and $(-1, -3)$.

If $6x = -12$: x must be -2 and hence, $x^2 = 4$. Thus, $x^2 + 6x = -8$, which means for $x^2 + 6x + y^2 = 4$, we would need $-8 + y^2 = 4 \rightarrow y^2 = 12 \rightarrow y$ is not an integer in this case and hence, we get no new solutions for this case.

If $6x = -18$: x must be -3 and hence, $x^2 = 9$. Thus, $x^2 + 6x = -9$, which means for $x^2 + 6x + y^2 = 4$, we would need $-9 + y^2 = 4 \rightarrow y^2 = 13 \rightarrow y$ is not an integer in this case and hence, we get no new solutions for this case.

If $6x = -24$: x must be -4 and hence, $x^2 = 16$. Thus, $x^2 + 6x = -8$, which means for $x^2 + 6x + y^2 = 4$, we would need $-8 + y^2 = 4 \rightarrow y^2 = 12 \rightarrow y$ is not an integer in this case and hence, we get no new solutions for this case.

If $6x = -30$: x must be -5 and hence, $x^2 = 25$. Thus, $x^2 + 6x = -5$, which means for $x^2 + 6x + y^2 = 4$, we would need $-5 + y^2 = 4 \rightarrow y^2 = 9 \rightarrow y = +3$ and $y = -3$

Thus we have identified two more solutions as $(-5, 3)$ and $(-5, -3)$.

If $6x = -36$: x must be -6 and hence, $x^2 = 36$. Thus, $x^2 + 6x = 0$, which means for $x^2 + 6x + y^2 = 4$, we would need $0 + y^2 = 4 \rightarrow y^2 = 4 \rightarrow y = +2$ and $y = -2$.

Thus, we have identified two more solutions as $(-6, 2)$ and $(-6, -2)$.

If $6x = -42$: x must be -7 and hence, $x^2 = 49$. Thus, $x^2 + 6x = 7$, which means that $x^2 + 6x$ itself is crossing the value of 4 for the LHS. There is no scope to add any value of y^2 as a positive integer to get the LHS of the equation equal to 4. Thus, we can stop at this point.

Notice that we did not need to check for the case 4 because if there were a solution for case 4, we would have been able to identify it while checking for case 3 itself.

Thus, the equation has eight solutions.

27. The number of ordered pairs of integral solutions (m, n) which satisfy the equation $m \times n - 6(m + n) = 0$ with $m \leq n$ is

(a) 5
(b) 10

(c) 12

(d) 9

In order to solve a question of this nature, you again first need to 'read' the equation

The equation $m \times n - 6(m + n) = 0$ can be restructured as: $m \times n = 6(m + n)$.

When we read an equation of this form, we should be able to read this as:

The LHS is the product of two numbers, while the RHS is always going to be a multiple of 6. Further, we also know that the value of $m \times n$ would normally always be higher than the value of $m + n$. Thus, we are trying to look for situations where the product of two integers is six times their sum.

While looking for such solutions, we need to look for situations where either:

The RHS is non-negative – hence, 0, 6, 12, 18, 24, 30, 36…

The RHS is negative – hence, – 6, –12, –18…

We would now need to use individual values of $m + n$ in order to check for whether $m \times n = 6(m + n)$.

For $m + n = 0$; $6(m + n) = 0$. If we use m and n both as 0, we would get $0 = 0$ in the two sides of the equation. So this is obviously one solution to this equation.

For the RHS = – 6, we can visualise the product of $m \times n$ as – 6, if we use m as –3 and n as 2 or vice versa. Thus, we will get two solutions as (2, –3) and (–3, 2).

For the RHS = –12, $m + n = -2$, which means that m and n would take values like (– 4, 2); (–5, 3); (–6, 4). If you look inside the factor pairs of –12, there is no factor pair which has an addition of –2.

For RHS = –18, $m + n = -3$. Looking into the factor pairs of 18 (viz: 1 × 18, 2 × 9, 3 × 6), we can easily see –6 and 3 as a pair of factors of –18 which would add up to –3 as required. Thus, we get two ordered solutions for (m, n), viz: (–6, 3); (3, –6)

For RHS = –24, $m + n = -4$. If we look for factors of –24, which would give us a difference of –4, we can easily see that within the factor pairs of 24 (viz: 1 × 24, 2 × 12, 3 × 8 and 4 × 6), there is no opportunity to create a sum of $m + n = -4$.

For RHS = –30, $m + n = -5$ → factors of 30 are 2 × 15, 5 × 6; there is no opportunity to create $m + n = -5$ and $m \times n = -30$, simultaneously. Hence, there are no solutions in this case.

For RHS = –36, $m + n = -6$ → factors of 36 are 2 × 18, 3 × 12, 4 × 9 and we can stop looking further; there is no opportunity to create $m + n = -6$ and $m \times n = -36$, simultaneously. Hence, there are no solutions in this case.

For RHS = –42, $m + n = -7$ → factors of 42 are 2 × 21, 3 × 14, 6 × 7 and we can stop looking further; there is no opportunity to create $m + n = -42$ and $m \times n = -7$, simultaneously. Hence, there are no solutions in this case.

For RHS = – 48, $m + n = -8$ → factors of 48 are 2 × 24, 3 × 16, 4 × 12… 4 × 12 and give us the opportunity to create $m + n = -8$ and $m \times n = -48$, simultaneously. Hence, there are two solutions in this case, viz: (4, – 12) and (– 12, 4).

Note: While this process seems to be extremely long and excruciating, it is important to note that there are a lot of refinements you can make in order to do this fast. The 'searching inside the factors' shown above is itself a hugely effective short-cut in this case. Further, when you are looking for pairs of factors for any number, you need not look at the first few pairs because their difference would be very large. This point is illustrated below:

For RHS = –54, $m + n = -9$ → factors of 54 are 3 × 18, 6 × 9 and we can stop looking further; (notice here that we did not need to start with 1 × 54 and 2 × 27 because they are what can be called as 'too far apart' from each other). There is no opportunity to create $m + n = -6$ and $m \times n = -36$ simultaneously. Hence, there are no solutions in this case.

For RHS = – 60, $m + n = -10$ → relevant factor search for 60 are 4 × 15, 5 × 12 and we can stop looking further; there is no opportunity to create $m + n = -10$ and $m \times n = -60$ simultaneously. Hence, there are no solutions in this case.

For RHS = – 66, $m + n = -11$ → relevant factor search for 66 is 6 × 11 and we can stop looking further; there is no opportunity to create $m + n = -11$ and $m \times n = -66$ simultaneously. Hence, there are no solutions in this case.

Note: While solving through this route each value check should take not more than five seconds. The question that starts coming into one's mind is, how far does one need to go in order to check for values? Luckily, the answer is not too far.

As you move to the next values beyond – 66, – 72 (needs $m + n = -12$, while the factors of 72 do not present this opportunity), – 78 (needs $m + n = -13$, which does not happen again).

To move further, you need to start working out the logic when $6(m + n)$ is positive. In such a case, the value of m and n would both need to be positive for $m \times n$ also to be positive. If $m + n = 1$, we cannot get two positive values of m and n such that their product is 6. If $m + n = 2$, $m \times n = 12$ will not happen.

A little bit of logical thought will give you the first point at which this situation would get satisfied. It would be when $6(m + n) = 144$, which means that $m + n = 24$ and for $m \times n$ to be equal to 144, the value of each of m and n would be 12 each.

(A brief note about why it is not possible to get a value before this:

If we try $6(m + n) = 132$ (for instance), we would realise that $m + n = 22$. The highest product $m \times n$ with a limit of $m + n = 22$ would occur when each of m and n is equal and hence, individually equal to 11 each. However, the value of 11 for m and n gives us a product of 121 only, which is lower than the required product of 132. This will happen in all cases where $6(m + n)$ is smaller than 144.)

Checking subsequent values of $6(m + n)$, we will get the following additional solutions to this situation:

6(m + n)	*(m + n)*	*Relevant factor pairs for the value of 6(m + n)*	*Solutions*
150	25	10,15	10,15; 15,10
156	26	None	None
162	27	9,18	9,18; 18,9
168,174	28,29	None	None
180,186	30,31	None	None
192	32	8,24	8,24; 24,8
198,204	33,34	None	None
210,216	35,36		

Break-down of the above thought-process:

(**Note:** That while checking the factor pairs for a number like 216, if you were to list the entire set of factors along with the sum of the individual factors within the pairs, you will get a list as follows:

Factor pairs for 216:

Pair	*Sum of factors in the pair*
1 × 216	217

2 × 108	110
3 × 72	75
4 × 54	58
6 × 36	42
8 × 27	35
9 × 24	33
12 × 18	30

However, a little bit of introspection in the correct direction will show you that this entire exercise was not required in order to do what we were doing in this question – i.e. trying to solve for $m + n = 36$ and $m \times n = 216$.)

The first five pairs where the larger number itself was greater than or equal to 36 were irrelevant as far as searching for the correct pair of factors is concerned for this question. When we saw the sixth pair of 8 × 27, we should have realised that since the sum of 8 + 27 = 35, which is < 36, the subsequent pairs would also have a sum smaller than 36. Hence, you can stop looking for more factors for 216.

Thus, effectively to check whether a solution exists for $6(m + n) = 216$, in this question, all we needed to identify was the 8 × 27 = 216 pair and we can reject this value for $6(m + n)$ giving us an integral solution in this case.

This entire exercise can be completed in one 'five second thought' as follows:

If $6(m + n) = 216 \rightarrow m + n = 36$, then one factor pair is 6 × 36 itself whose sum obviously is more than 36. So, looking for the next factor pair where the smaller number is > 6, we see that 8 × 27 gives us a factor pair sum of 8 + 27 = 35 <36 and hence we reject this possibility.

Also, you should realise that we do not need to look further than 216 – as for values after 216, when we go to the next factor pair after $6 \times (m + n)$, we would realise that the sum of the factor pair would be lower than the required value for the immediately next factor pair.

Hence, the following solutions exist for this question: 0,0; 2,–3; –3,2; 3,– 6; –6,3; 4,–12; –12,4; 12,12; 15,10; 10,15; 9,18; 18,9; 8,24; 24,8, i.e.

a total of 14 solutions.

Author's Note: Doubtless this question is very long, but if you are able to understand the thought-process to adopt in such situations, you would do yourself a big favour in the commonly-asked questions of finding number of integral solutions.

28. How many positive integral solutions exist for the expression $a^2 - b^2 = 666$?

In order to solve this question, we need to think of the expression $(a - b)(a + b) = 666$. Obviously, the question is based on factor pairs of 666. If we look at the list of factor pairs of 666, we get:

1	666
2	333
3	222
6	111
9	74
18	37

Since, 37 is a prime number, we will get no more factor pairs.

Now, if we look at trying to fit in the expression $(a - b)(a + b)$ for any of these values, we see the following occurring:

Example: $(a - b) = 9$ and $(a + b) = 74$. If we try to solve for a and b, we get: $2a = 83$ (by adding the equations) and 'a' would not be an integer. Consequently, b would also not be an integer and we can reject this possibility as giving us a solution of $a^2 - b^2 = 666$.

Armed with this logic, if we were to go back to each of the factor pairs in the table above, we realise that the sum of the two factors within a factor pair is always odd and hence, none of these factor pairs would give us a solution for the equation.

Thus, the correct answer would be 0.

29. How many positive integral solutions exist for the expression $a^2 - b^2 = 672$?

In order to solve this question, we need to think of the expression $(a - b)(a + b) = 672$ Obviously, the question is based on factor pairs of 672. If we look at the list of factor pairs of 672, we get:

		Sum of factors
1	672	Odd
2	336	Even
3	224	Odd
4	168	Even
6	112	Even
8	84	Even
12	56	Even
16	42	Even
21	32	Odd
24	28	Even

Based on our understanding of the logic in the previous question, we should realise that this works for all situations where the sum of factors is even. Hence, there are seven positive integral solutions to the equation $a^2 - b^2 = 672$.

30. The function a_n is defined as $a_n - a_{n-1} = 2n$ for all $n \geq 2$. $a_1 = 2$.

Find the value of $a_1 + a_2 + a_3 + \dots a_{12}$.

In order to solve such questions, the key is to be able to identify the pattern of the series. A little bit of thought would give you the following structure:

$$a_1 = 2$$
$$a_2 - a_1 = 4 \rightarrow a_2 = 6$$
$$a_3 - a_2 = 6 \rightarrow a_3 = 12$$
$$a_4 - a_3 = 8 \rightarrow a_4 = 20$$
$$a_5 - a_4 = 10 \rightarrow a_5 = 30$$

If we look for the pattern in these numbers, we should be able to see the following:

2 + 6 + 12 + 20 + 30...

= 2(1 + 3 + 6 + 10 + 15 + ...)

Looking at it this way shows us that the numbers in the brackets are consecutive triangular numbers.

Hence, for the sum till a_{12}, we can do the following:

2(1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 + 66 + 78) = 728.

31. Two brothers Binnu and Kinnu have been challenged by their father to solve a mathematical puzzle before they are allowed to go out to play. Their father has asked them "Imagine two integers a and b, such that $1 \leq b \leq a \leq 10$. Can you correctly find out the value of the expression $\sum ab$?" Can you help them identify the correct value of the foregoing expression?

(a) 1155

(b) 1050

(c) 1705

(d) None of these

This question is again based on a pattern-recognition principle. The best way to approach the search of the pattern is to start by working-out a few values of the given expression. The following pattern would start emerging:

Value of a	*Possible values of b*	*Value of the sum of the possible products 'a × b'*	*Explanation*
1	1	1	With '*a*' as 1, the only value for *b* is 1 itself
2	1, 2	2 × 1 + 2 × 2 = 2 × 3	With '*a*' as 2, *b* can take the values of 1 and 2 – and 2 × 1 + 2 × 2 can be written as 2 × 3
3	1, 2, 3	3 × 1 + 3 × 2 + 3 × 3 = 3 × 6	With '*a*' as 3, *b* can take the values of 1, 2 and 3 – and 3 × 1 + 3 × 2 + 3 × 3 can be written as 3 × 6
4	1, 2, 3, 4	4 × 1 + 4 × 2 + 4 × 3 + 4 × 4 = 4 × 10	
At this point, you should realise that the values are following a certain pattern – the series of values for '*a*' are multiplied by a series of consecutive triangular numbers (1, 3, 6, 10 and so on.) Thus, we can expect the subsequent numbers to be obtained by multiplying 5 × 15, 6 × 21, 7 × 28, 8 × 36, 9 × 45 and 10 × 55.			

Hence, the answer can be obtained by:

1 × 1 + 2 × 3 + 3 × 6 + 4 × 10 + 5 × 15 + 6 × 21 + 7 × 28 + 8 × 36 + 9 × 45 + 10 × 55=

$1 + 6 + 18 + 40 + 75 + 126 + 196 + 288 + 405 + 550 = 1705$

32. For the above question, what would be the answer in case the inequality is expressed as: $1 \le b < a \le 10$?

In this case, the solution will change as follows:

Value of a	*Possible values of b*	*Value of the sum of the possible products 'a × b'*	*Explanation*
1	No possible values for '*b*', since *b*" has to be greater than or equal to 1 but less than '*a*' at the same time	0	With '*a*' as 1, *b* has no possible values that it can take
2	1	2 × 1	With '*a*' as 2, *b* can only take the value of 1
3	1, 2	3 × 1 + 3 × 2 = 3 × 3	With '*a*' as 3, *b* can take the values of 1 and 2 – and 3 × 1 + 3 × 2 can be written as 3 × 3
4	1, 2, 3	4 × 1 + 4 × 2 + 4 × 3 = 4 × 6	With '*a*' as 4, *b* can take the values of 1,2 and 3 – and 4 × 1 + 4 × 2 + 4 × 3 can be written as 4 × 6
At this point, you should realise that the values are following a certain pattern (just like in the previous question) – the series of values for '*a*' are multiplied by a series of consecutive triangular numbers (1, 3, 6, 10 and so on). The only difference, in this case, is that the multiplication is what can			

be described as 'one removed', i.e. it starts from the value of '*a*' as 2. Thus, we can expect the subsequent numbers to be obtained by multiplying 5 × 10, 6 × 15, 7 × 21, 8 × 28 and 9 × 36 and 10 × 45. Notice here that the values of '*a*' can go all the way up till 10 in this case as the inequality on the rightmost side of the expression is $a \le 10$.

Hence, the answer can be obtained by:

1 × 0 + 2 × 1 + 3 × 3 + 4 × 6 + 5 × 10 + 6 × 15 + 7 × 21 + 8 × 28 + 9 × 36 + 10 × 45 =

0 + 2 + 9 + 24 + 50 + 90 + 147 + 224 + 324 + 450 = 1320

33. Consider the equation of the form $x2 + bx + c = 0$. The number of such equations that have real roots and have coefficients b and c in the set {1, 2, 3, 4, 5, 6, 7), is

(a) 20

(b) 25

(c) 27

(d) 29

We know, that in order to have the roots of an equation to be real, we should have the values of the discriminant of the quadratic equation (defined as the value $b2 – 4ac$ for a standard quadratic equation $ax2 + bx + c = 0$) to be non-negative.

In the context of the given equation in this problem, since the value of the coefficient of $x2$ is 1, it means that we need to have $b2 – 4c$ to be positive or 0.

The only thing to be done from this point is to look for possible values of a and b which fit this requirement. In order to do this, assume a value for 'b' from the set {1, 2, 3, 4, 5, 6, and 7} and try to see which values of b and c satisfy $b\ 2 – 4c > 0$.

When $b = 1$, c can take none of the values between 1 and 6, since "$b2– 4c$" would end up being negative.

When $b = 2$, c can be 1;

When $b = 3$, c can be 1 or 2;

When $b = 4$, c can be 1 or 2 or 3 or 4;

When $b = 5$, c can take any value between 1 and 6;

When $b = 6$, c can take any value between 1 and 7;

When $b = 7$, c can take any value between 1 and 7.

Thus, there are a total of 27 such equations with real roots.

34. The number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$ and where a, b and c belong to {1, 2, 8), is

(a) 1

(b) 8

(c) 9

(d) 10

For a polynomial $P(x)$ to be divisible by another polynomial $D(x)$, there needs to be a third polynomial $Q(x)$ which would represent the quotient of the expression $P(x)/D(x)$. In other words, this also means that the product of the polynomials $D(x) \times Q(x)$ should equal the polynomial $P(x)$.

In simpler words, we are looking for polynomials that would multiply $(x^2 + 1)$ and give us a polynomial in the form of $x^3 + ax^2 + bx + c$. The key point in this situation is that the expression that would multiply $(x^2 + 1)$ to give an expression of the form $x^3 + ...$ will necessarily be of the form (x + constant). It is only in such a case that we would get an expanded polynomial starting with x^3. Further, the value of the constant has to be such that the product of 1 × constant will give a value for 'c' that would belong to the set {1, 2, 3, 4, 5, 6, 7, or 8). Clearly, there are only eight such values possible for the constant – viz. 1, 2, 3,...8 and hence, the required polynomials that would be divisible by $x^2 + 1$ will be obtained by the expansion of the following expressions: $(x^2 + 1)(x + 1)$; $(x^2 + 1)(x + 2)$; $(x^2 + 1)(x + 3)$;... ; $(x^2 + 1)(x + 8)$. Thus, there would be a total of eight such expressions which would be divisible by $x^2 + 1$. Hence, option (b) is the correct answer.

35. A point P with coordinates (x, y) is such that the product of the coordinates $xy = 144$. How many possible points exist on the $X–Y$ plane such that both x and y are integers?

(a) 15

(b) 16

(c) 30

(d) 32

The number of values for (x, y) such that both are integers and their product is equal to 144 is dependent on the number of factors of 144. Every factor pair would give us four possible solutions for the ordered pair (x, y). For instance, if we were to consider 1×144 as one ordered pair, the possible values for (x, y) would be (1,144); (144, 1); (–1,–144); (–144,–1). This would be true for all factor pairs except the factor pair 12×12. In this case, the possible pairs of (x, y) would be (12, 12) and (–12,–12).

If we find out the factor pairs of 144, we will get the following list:

Factor pair	*Number of solutions for (x, y)*
1 × 144	4 solutions – as explained above
2 × 72	4 solutions
3 × 48	4 solutions
4 × 36	4 solutions

6 × 24	4 solutions
8 × 18	4 solutions
9 × 16	4 solutions
12 × 12	2 solutions

Thus, there are a total of 30 solutions in this case.

36. Let $x_1, x_2, \ldots\ldots, x_{40}$ be forty non-zero numbers such that $x_i + x_{i+1} = k$ for all i, $1 \le i \le \infty$. If $x_{14} = a$, $x_{27} = b$, then $x_{30} + x_{39}$ equals

(a) $3(a + b) - 2k$

(b) $k + a$

(c) $k + b$

(d) None of the foregoing expressions

Since, $x_{14} = a$, x_{15} would equal $(k - a)$ [we get this by equating $x_{14} + x_{15} = k \rightarrow a + x_{15} = k \rightarrow x_{15} = (k - a)$]. By using the same logic on the equation $x_{15} + x_{16} = k$, we will get $x_{16} = a$. Consequently, $x_{17} = k - a$, $x_{18} = a$, $x_{19} = k - a$, $x_{20} = a$. Thus, we see that every odd term is equal to '$k - a$' and every even term is equal to 'a'. Further, we can develop a similar logic for x_i in the context of 'b'. Since, $x_{27} = b$, $x_{28} = k - b$, $x_{29} = b$ and so on. This series also follows a similar logic with x_i being equal to b, when 'i' is odd and being equal to $k - b$, when 'i' is even.

Thus, for every value of x_i, we have two ways of looking at its value—viz. either in terms of a or in terms of b. Thus, for any $x$even, we have for instance $x_{20} = a = k - b$.

Solving $a = k - b$, we get $a + b = k$.

Further, when we look at trying to solve for the specific value of what the question has asked us, i.e. the value of $x_{30} + x_{39}$, we realise that we can either solve it in terms of 'a' or in terms of 'b'. If we try to solve it in terms of 'a', we would see the following happening:

$X_{30} + x_{39} = a + k - a = k$

Similarly, in terms of 'b', $x_{30} + x_{39} = k - b + b = k$. Since we know that $k = a + b$ (deduced above), we can conclude $x_{30} + x_{39} = a + b$. However, if we look at the options, none of the options is directly saying that.

Options (b) and (c) can be rejected because their values are not equal to $a + b$. A closer inspection of option (a), gives us an expression: $3(a + b) - 2k$. This expression can be expressed as $3(a + b) - 2(a + b) = (a + b)$ and hence, this is the correct answer.

37. The great mathematician Ramanujam, once was asked a puzzle in order to test his mathematical prowess. He was given two sets of numbers as follows:

Set X is the set of all numbers of the form: $4^n - 3n - 1$, where $n = 1, 2, 3, \ldots$..

Set Y is the set of all numbers of the form $9n$, where $n = 0, 1, 2, 3, \ldots$……

Based on these two definitions of the set, can you help Ramanujam identify the correct statement from amongst the following options?

(a) Each number in Y is also in X

(b) Each number in X is also in Y

(c) Every number in X is in Y and every number in Y is in X

(d) There are numbers in X that are not in Y and vice versa

In order to solve such questions conveniently, you would need to first create a language representation for yourself with respect to the two sets.

Set X can be mentally thought of as:

A positive integral power of 4 – a multiple of 3 (with the multiplier being equal to the power of 4 used) – a constant value '1'

Thus, the set of values in X can be calculated as (0, 9, 54, 243 and so on).

Similarly, Set Y can be thought of as multiples of 9, starting from $9 \times 0 = 0$.

The numbers that would belong to the set Y will be: (0, 9, 18, 27, 36 and pretty much all multiples of 9).

It can be clearly seen that while all values in X are also in Y, the reverse is not true. Hence, the statement in option (b) is correct.

38. The number of real roots of the equation $\log_{2x}\left(\frac{2}{x}\right)(\log_2(x))^2 + (\log_2(x))^4 = 1$, for values of $x > 1$, is

(a) 0
(b) 1

(c) 2

(d) 27

The only way to handle such questions is to try to get a 'feel' of the equation by inserting a few values for x and trying to see the behaviour of the various terms in the equation.

Towards this end, let us start by trying to insert values for x in the given equation.

Because the problem tells us that $x > 1$, the first value of x which comes to mind is $x = 2$. At $x = 2$, the value of the LHS would become equal to 1. This can be thought of as follows:

LHS = $\log_{2x}(1)(\log_2(2))^2 + (\log_2(2))^4 = 0 + 1 = 1$

As we try to take higher values of x as 3, 4, and 5 and so on we realise first that for values like 3, 5, we will get terms like

$\log_6(2/3)(\log_2(3))^2 + (\log_2(3))^4$ which is clearly not going to be an integral value, because terms like $\log_2 3$, $\log_6 2$ and $\log_6 3$ would have irrational decimal values by themselves. In fact, we can see that for numbers of x that are not powers of 2, we will never get an integral value to the LHS of the expression.

Thus, we need to see the behaviour of the expression only for values of x like 4, 8 and so on before we can conclude about the number of real roots of the equation.

At $x = 4$, the expression becomes:

$\log_8(2/4)(\log_2(4))^2 + (\log_2(4))^4$

By thinking about this expression, it is again clear that there are going to be decimals in the first part of the expression—although they are going to be rational numbers and not irrational—and hence, we cannot summarily rule out the possibility of an integral value of this expression— without doing a couple of more calculations. However, there is another thought which can help us confirm that the equation will not get satisfied in this case because the LHS is much bigger than the RHS.

This thought goes as follows:

The LHS has two parts: The first part is $\log_8(2/4)(\log_2(4))^2$ while the second part is $(\log_2(4))^4$. The value of the second part is 16 while the value of the first part (though it is negative) is much smaller than the required, i.e. –15, which will make the LHS = 1.

Hence, we can reject this value.

39. The number of points at which the curve $y = x^6 + x^3 - 2$ cuts the x-axis is

(a) 1

(b) 2

(c) 4

(d) 6

By replacing $x^3 = m$, the equation given in the question, can be written in the form:

$y = m^2 + m - 2 \rightarrow$

$y = (m + 2)(m - 1) \rightarrow m = -2$ and $m = 1$

This gives $x^3 = -2$ and $x^3 = 1$. This gives us two clear values of x (these would be the roots of the equation) and hence x, the curve would cut the 'x' axis at two points exactly.

40. Number of real roots of the equation $8x^3 - 6x + 1 = 0$ lying between -1 and 1, is

(a) 0

(b) 1

(c) 2

(d) 3

In order to trace the number of real roots for any equation (cubic or larger), the only feasible way to look at it is to try to visualise how many times the graph of the parallel function (in this case: $8x^3 - 6x + 1$) cuts the X-axis.

The following thought-process would help you do this:

Since the question is asking us to find out the number of real roots of $8x^3 - 6x + 1 = 0$ between the range -1 and $+1$, we will need to investigate only the behaviour of the curve for the function $y = 8x^3 - 6x + 1$ between the values -1 and $+1$.

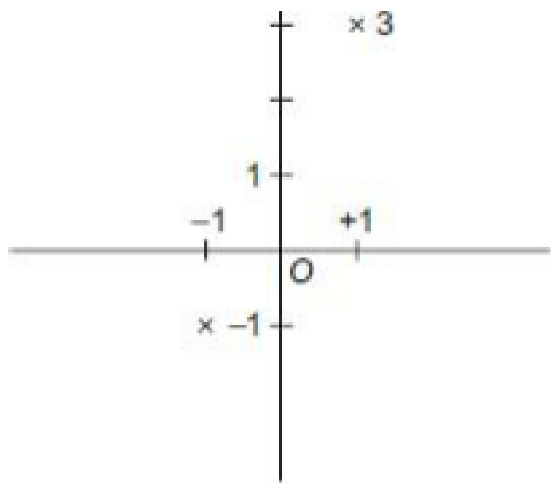
The first thing we do is to look at the values of the function at -1, 0 and $+1$.

At $x = -1$; the value of the function is $-8 + 6 + 1 = -1$.

At $x = 0$; the value of the function is 1.

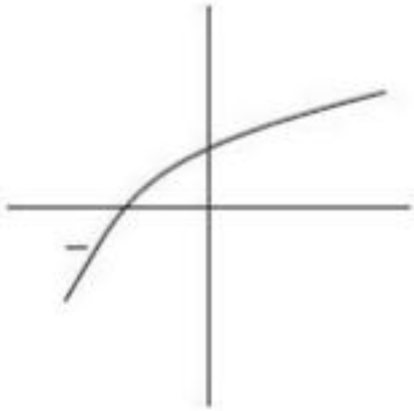
At $x = 1$; the value of the function is 3.

If plotted on a graph, we can visualise the three points as given here.

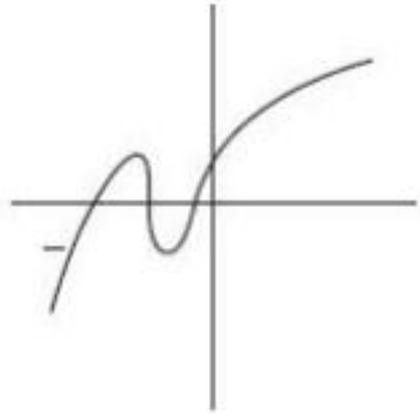


From this visualisation, it is clear that the value of the function is –1 at x = –1 and + 1 at x = 0 and + 3 at x = 1. It is clear that somewhere between –1 and + 1, the function would cut the x-axis at least once as it transits from a negative value to a positive value. Hence, the equation would necessarily have at least one root between –1 to 0. What we need to investigate in order to solve this question is specifically—does the function cut the x-axis more than once during this range of values on the x-axis? Also, we should realise that in case the graph will cut the x–axis more than once, it would cut it thrice—since if it goes from negative to positive once, and comes back from positive to negative, it would need to become positive again to go above the x-axis.

This can be visualised as the following possible shapes:

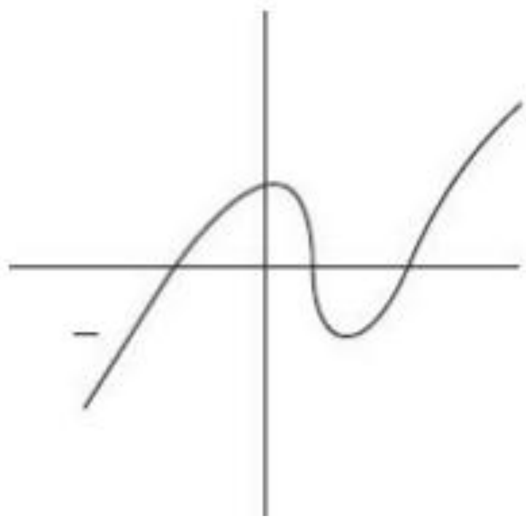


If the graph cuts the x-axis once (and it has one real root in this range)



If the graph cuts the x-axis thrice (and it has three real roots in this range)

Of course, the graph can technically also cut the x-axis twice between the values of $x = 0$ and $x = 1$. In such a case, the graph would look something like below:



Which of these graphs will be followed would depend on our analysis of the behaviour of the values of $8x^3 – 6x + 1$ between the values of x between –1 and 0 first and then between 0 and 1.

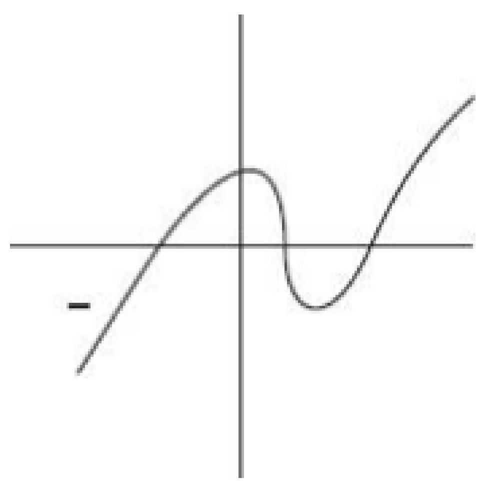
Between –1 and 0:

The value of $8x^3$ will remain negative, while $–6x$ would be positive and + 1 would always remain constant. If we look at the values of $8x^3$ as we increase the value of x from –1 to – 0.9 to – 0.8 to – 0.7 to … – 0.1, the negative impact of $8x^3$ reduces as its magnitude reduces. (Please understand the difference between magnitude and value when we are talking about a negative number. For instance, when we talk about increasing a negative number, its magnitude is decreased).

At the same time, the positive magnitude of $-6x$ also reduces but the rapidity with which the value of $-6x$ would decrease will be smaller than the rapidity with which the value of $8x^3$ would decrease. Hence, the positive parts of the expression would become 'more powerful' than the negative part of the expression and hence, the graph would not cut the x-axis more than once between –1 and 0.

The last part of our investigation, then would focus on the behaviour of the graph between the values of $x = 0$ and $x = 1$. When x moves into the positive direction (i.e. when we take $x > 0$), we realise that of the three terms $8x^3$ and $+1$ would be positive, while the value of $-6x$ would be negative.

It can be easily visualised that at $x = 1/4$, the value of the expression on the LHS of the equation becomes: $8/64 - 6/4 + 1$, which is clearly negative. Thus, after $x = 0$, when we move to the positive values of x, the value of the expression $8x^3 - 6x + 1$ becomes negative once more. Thus, the correct graph will look as given below:



Thus, the equation has three real roots between –1 and + 1.

REVIEW CAT SCAN

REVIEW CAT Scan 1

Directions for Questions 1 and 3: These are based on the functions defined below:

$Q(a, b)$ = quotient when a is divided by b

$R_2(a, b)$ = remainder when a is divided by b

$R(a, b) = a^2/b^2$

$SQ(a, b) = \sqrt{(a-1)(b-1)}$

1. $SQ(5, 10) - ? > 0$

   (a) $(8/3)\, R(5, 10)$

   (b) $R_2(5, 10) + Q(5, 10)$

   (c) $R_2(5, 10)/2$

   (d) None of these

2. $SQ(a, b)$ is same as

   (a) $bQ(a, b) + R_2(a)$

   (b) $\sqrt{R(a,b) - 1}$

   (c) $[R\{(a-1), (b-1)\}$

   (d) None of these

3. Which of the following relations cannot be false?

   (a) $R(a, b) = R_2(a, b) \cdot Q(a, b)$

   (b) $a^2 \cdot Q(a, b) = b^2 \cdot R_2(a, b)$

(c) $a = R2(a, b) + b \cdot Q(a, b)$

(d) $SQ(a, b) = R(a, b) \cdot R2(a, b)$

Directions for Questions 4 and 7: *Answer the questions based on the following information:*

$$W(a, b) = \text{least of } a \text{ and } b$$
$$M(a, b) = \text{greatest of } a \text{ and } b$$
$$N(a) = \text{absolute value of } a$$

4. Find the value of $1 + M[y + N\{-W(x, y)\}, N\{y + W(M(x, y), N(y))\}]$ given that $x = 2$ and $y = -3$.

   (a) 0

   (b) 1

   (c) 2

   (d) 3

5. Given that $x > y$, then the relation $M\{N(x), W(x, y)\} = W[x, N\{M(x, y)\}]$ does not hold if

   (a) $x > 0, y < 0, |x| > |y|$

   (b) $x > 0, y < 0, |y| > |x|$

   (c) $x > 0, y > 0$

   (d) $x < 0, y < 0$

6. Which of the following must be correct for $x, y < 0$?

   (a) $N(W(x, y)) \le W(N(x), N(y))$

   (b) $N(M(x, y)) > W(N(x), N(y))$

   (c) $N(M(x, y)) = W(N(x), N(y))$

(d) $N(M(x, y)) < M(N(x), N(y))$

7. For what value of x is $W(x2 + 2x, x + 2) < 0$?

(a) $-2 < x < 2$

(b) $-2 < x < 0$

(c) $x < -2$

(d) Both (b) and (c)

REVIEW CAT Scan 2

1. If three positive real numbers a, b and c ($c > a$) are in Harmonic Progression, then $\log(a + c) + \log(a - 2b + c)$ is equal to:

(a) $2 \log(c - b)$

(b) $2 \log(c - c)$

(c) $2 \log(c - a)$

(d) $\log a + \log b + \log c$

2. If $f = \dfrac{1}{\log_2 \neq} + \dfrac{1}{\log_{4.5} \neq}$, which of the following is true?

(a) $f > 4$

(b) $2 < f < 4$

(c) $1 < f < 2$

(d) $0 < f < 1$

3. If $\log 2x \cdot \log x/64 2 = \log x/16 2$. Then x is

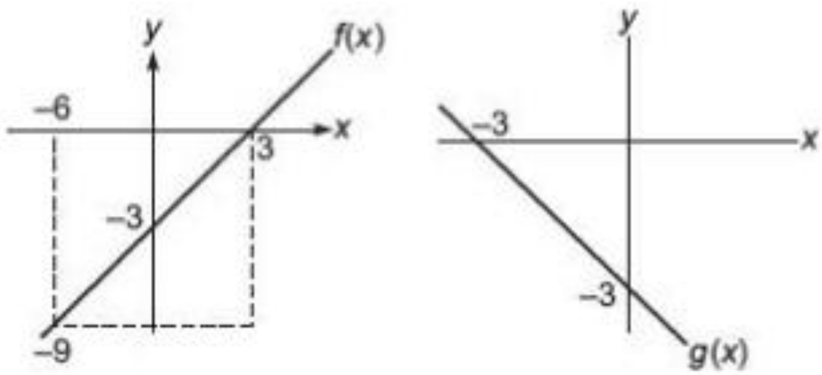
(a) 2

(b) 4

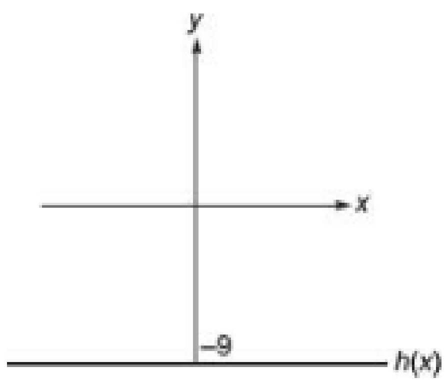
(c) 16

(d) 12

Directions for Questions 4 and 6: $f(x)$ *and* $g(x)$ are defined by the graphs shown below:



Each of the following questions has a graph of function $h(x)$ with the answer choices expressing $h(x)$ in terms of a relationship of $f(x)$ or/and $g(x)$. Choose the alternative that could represent the relationship appropriately.

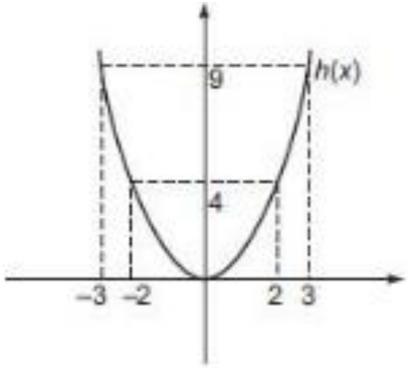
4. 

(a) $6f(x) + 6g(x)$

(b) $-1.5f(x) + 1.5g(x)$

(c) $1.5f(x) + 1.5g(x)$

(d) None of these

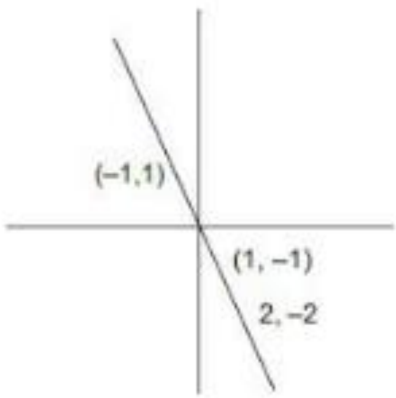
5. 

(a) $9 - f(x)\,g(x)$

(b) $[f(x) + g(x) + 4]_2 + [f(x) - 2]_2 + [g(x)]$

(c) $[f(x) - g(x) + 4]_2 - 2$

(d) $f(x)g(x) - 9$

6. 

(a) $2f(x) + g(x)$

(b) $f(x) + 2g(x) - 9$

(c) $\frac{3}{2} f(x) + \frac{g(x)}{2}$

(d) None of these

7. If $p_a = q_b = r_c$ and $\frac{p}{q} = \frac{q}{r}$, $\left(\frac{1}{a} + \frac{1}{c}\right) b = ?$

(a) 1

(b) 1/2

(c) 3/4

(d) 2

REVIEW CAT Scan 3

1. Given that $f(a) = a\,(a + 1)\,(a + 2)$ where $a = 1, 2, 3, ...$ Then find $S = f(1) + f(2) + f(3) + ... + f(10)$?

   (a) 4200

   (b) 4290

   (c) 4400

   (d) None of these

Directions for Questions 2 and 4: It is given that $f(x) = p^x$, $g(x) = (-p)^x$; $h(x) = (1/p)^x$, $k(x) = (-1/p)^x$

2. Think of a situation where a function is odd or even, if the function is odd it is given a weightage of 1; otherwise, it is given a weightage of 0. What is the result if the weightages of four functions are added?

   (a) 2

   (b) 0

   (c) 1

   (d) –1

3. If 'p' and 'x' are both whole numbers other than 0 and 1, which of the functions must have the highest value?

   (a) $g(x)$ only

   (b) $f(x)$ only

(c) $g(x)$ and $h(x)$ both

(d) $h(x)$ and $k(x)$ *both*

4. Which of the following is true if 'p' is a positive number and x is a real number?

(a) $\{f(x) - h(x)\} / \{g(x) - k(x)\}$ is always positive

(b) $f(x).\, g(x)$ is always negative

(c) $f(x).\, h(x)$ is always greater than one

(d) $g(x).\, h(x)$ could exist outside the real domain

5. If x, and $y \geq 1$ and belong to set of integers then which of the following is true about the function $(xy)_n$?

(a) The function is odd if 'x' is even and 'y'is odd

(b) The function is odd if 'x' is odd and 'y' is even

(c) The function is odd if 'x' and 'y' both are odd

(d) The function is even if 'x' and 'y' both are odd

6. If $f(a,b)$ = remainder left upon division of b by a, then the maximum value for $f(f(a, b), f(a + 1, b + 1)) \times f(f(a, b), 0)$ is (b and a are co-primes)

(a) $a - 1$

(b) a

(c) 0

(d) 1

7. We are given two variables x and y. The values of the variables are $x = \frac{1}{a+b}$ and $y = \frac{3}{c+x}$. Find the value of the expression $\frac{7y}{x}$.

(a) $\frac{21(a+b)^2}{ca+cb+1}$

(b) $\frac{3(a+b)}{7\,ab+ac}$

(c) $\frac{7}{3(ca+cb+1)}$

(d) None of these

REVIEW CAT Scan 4

1. The number of solutions of $\frac{\log 5+\log(y^2+1)}{\log(y-2)}=2$ is:

(a) 3

(b) 2

(c) 1

(d) None of these

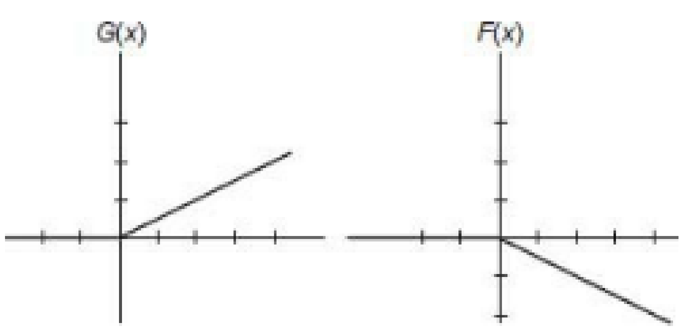
Directions for Questions 2 and 3: Given below are two graphs labelled $F(x)$ and $G(x)$. Compare the graphs and give the answer in accordance to the options given below:

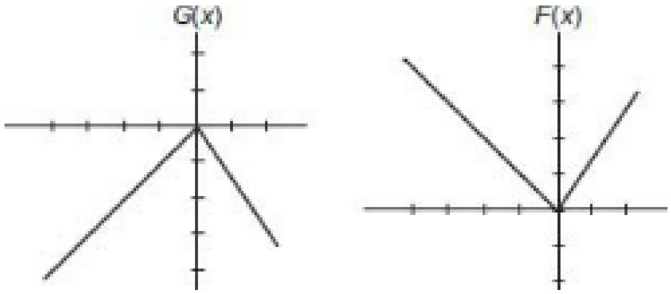
(a) $F(-x) = G(x) + x/3$

(b) $F(-x) = -G(x) - x/2$

(c) $F(-x) = -G(x) + x/2$

(d) None of these

2. 

3. 

4. If a is a natural number which of the following statements is always true?

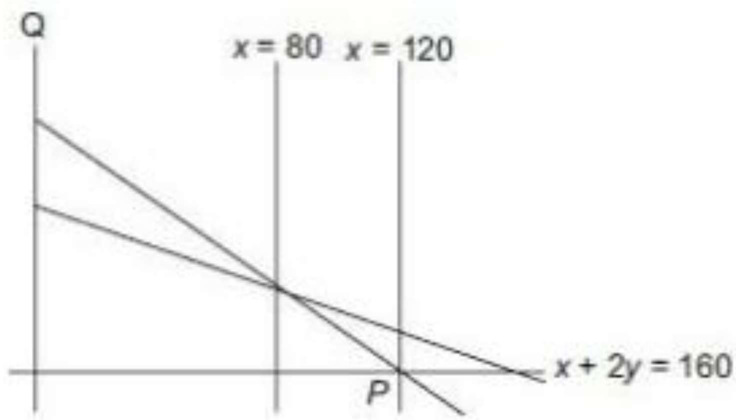
(a) $(a + 1)(a^2 + 1)$ is odd

(b) $9a^2 + 6a + 6$ is even

(c) $a^2 - 2a$ is even

(d) $a^2(a^2 + a) + 1$ is odd

5. In the figure below, equation of the line PQ is



(a) $x + y = 120$

(b) $2x + y = 120$

(c) $x + 2y = 120$

(d) $2x + y = 180$

6. For which of the following functions is $\frac{f(a) - f(b)}{a - b}$ constant for all the numbers 'a' and 'b', where $a \neq b$?

(a) $f(y) = 4y + 7$

(b) $f(y) = y + y2$

(c) $f(y) = \cos y$

(d) $f(y) = \log_e y$

7. Given that $f(a, b, c) = \frac{a+b+c}{3}$, then

(a) $f(a, b, c) \geq \frac{|a|+|b|+|c|}{3}$

(b) $f(a, b, c) \geq \max(a, b, c)$

(c) $|f(a, b, c)| \geq \frac{|a+b+c|}{3}$

(d) $|f(a, b, c)| \leq \frac{|a|+|b|+|c|}{3}$

REVIEW CAT Scan 5

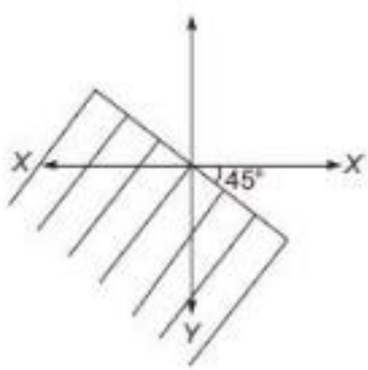
1. If $p = \frac{12-|x-3|}{12+|x-3|}$ the maximum value that 'p' can attain is

(a) 1

(b) 2

(c) 21

(d) 12

2. 

Refer to the graph. What does the shaded portion represent?

(a) $x + y \leq 0$

(b) $x \geq y$

(c) $x + 1 \geq y + 1$

(d) $x + y \geq 0$

3. If x satisfies the inequality $|x - 1| + |x - 2| + |x - 3| \geq 6$, then:

(a) $0 \leq x \leq 4$

(b) $x \leq 0$ or $x \geq 4$

(c) x £–2 or $x \geq 3$

(d) None of the above

4. The equation $7x - 1 + 11x - 1 = 170$ has

(a) no solution

(b) one solution

(c) two solutions

(d) three solutions

Directions for Questions 5 and 6: These questions are based on the relation given below:

$f_a(y) = f_{a-1}(y - 1)$ where $a > 1$ (integer values only) and $f_1(y) = 2/y$ if 'y' is positive or $f_1(y) = 1/(y^2 + 1)$ otherwise.

5. What is $f_a(a - 1)$?

(a) 0

(b) 1

(c) 2

(d) Indeterminate.

6. What is the value of $f_a(a + 1)$?

(a) 1

(b) a

(c) $2a$

(d) 2

7. Raman derived an equation to denote distance of Haley's comet (x) in the form of a quadratic equation. Distance is given by solution of quadratic $x^2 + Bx + c = 0$. To determine constants of the above equation for Haley's Comet, two separate series of experiments were conducted by Raman. Based on the data of first series, value of x obtained is (1, 8) and based on the second series of data, value of x obtained is (2, 10). Later on it is discovered that first series of data gave incorrect value of constant C while second series of data gave incorrect value of constant B. What is the set of actual distance of Haley's comet found by Raman?

(a) (11, 3)

(b) (6, 3)

(c) (4, 5)

(d) (3, 11)

REVIEW CAT Scan 6

1. 'a', 'b' and 'c' are three real numbers. Which of the following statements is/are always true?

(A) $(a - 1)(b - 1)(c - 1) < abc$.

(B) $(a^2 + b^2 + c^2)/2 \geq ca + cb - ab$

(C) $a^2b \div c$ is a real number

(a) Only A is true

(b) Only B and C are true

(c) Only B is true

(d) None is true

2. If we have f[g(y)] = g[f(y)], then which of the following is true?

(a) [f[f[g[g[g[g(y)]]]]]] = [f[g[g[f[f[g(y)]]]]]]

(b) [f[f[f[g[f[g(y)]]]]]] = f[f[g[g[g[f(y)]]]]]

(c) [g[f[g[g[g[f(y)]]]]]] = [f[g[g[f[f[f(y)]]]]]]

(d) [g[f[g[g[f[g[f(y)]]]]]]] = [f[g[g[g[g[g[f(y)]]]]]]]

3. $f(a) = \dfrac{a^8 - 1}{a^2 + 1}$ and $g(a) = \dfrac{a^4 - 3}{(a+1)^2}$, what is $f\left(\dfrac{1}{g(2)}\right)$?

(a) 0.652

(b) $\dfrac{1468}{2250}$

(c) $-\dfrac{734}{1625}$

(d) None of these

4. Dev was solving a question from his Mathematics book, when he encountered the expression $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$ then $a^a\, b^b\, c^c$ is

(a) –1

(b) 1

(c) 0.5

(d) 2

5. If log 3, log(3x – 2) and log(3x + 4) are in arithmetic progression, then x is equal to

(a) 8/3

(b) log38

(c) log23

(d) 4

Directions for Questions 6 and 7: We are given that $f(x) = f(y)$ and $f(x, y) = x + y$, if $x, y > 0$

$$f(x, y) = xy, \text{ if } x, y = 0$$
$$f(x, y) = x - y, \text{ if } x, y < 0$$
$$f(x, y) = 0, \text{ otherwise}$$

6. Find the value of the following function: $f[f(2, 0), f(-3, 2)] + f[f(-6, -3), f(2, 3)]$.

(a) 0

(b) 2

(c) –8

(d) None of these

7. Find the value of the following function:

$\{f[f(1, 2), f(2, 3)]\} \times \{f[f(1.6, 2.9), f(-1, -3)]\}$

(a) 12

(b) 36

(c) 48

(d) 52

REVIEW CAT Scan 7

Directions for Questions 1 and 3: Refer to the data given below and answer the questions.

Given $\frac{a}{b} = \frac{1}{2}$, $\frac{c}{d} = \frac{1}{3}$ and $z = \frac{a+c}{b+d}$, answer the questions below on limits of z.

1. If $b \geq 0$ and $c \geq 0$ then the limits of 'z' are:

    (a) $z \leq 0$ or $z \geq 1$

    (b) $\frac{1}{3} \leq z \leq \frac{1}{2}$

    (c) $z \geq \frac{1}{2}$ or $z \leq \frac{1}{3}$

    (d) $0 \leq z \leq 1$

2. $c \leq 0$ and $1/3 \leq z \leq ½$ only if

    (a) $a > 1.5c$

    (b) $c > -1$

    (c) $a \leq 0$

    (d) $a > -1.5c$

3. If $a = -31$, which of the following value of 'd' gives the highest value of 'z'?

    (a) $d = 72$

    (b) $d = 721$

(c) d = –31

(d) $d = 0.01$

4. Find the integral solution of: $5y - 1 < (y + 1)^2 < (7y - 3)$

(a) 2

(b) $2 < y < 4$

(c) $1 < y < 4$

(d) 3

5. If $f(a) = \frac{a-1}{a+1}$, $a \geq 0$ and if $y = f\left(\frac{1}{a}\right)$ then

(a) As 'a' decreases, 'y' decreases

(b) As 'a' increases, 'y' decreases

(c) As 'x' increases, 'y' increases

(d) As 'x' increases, 'y' remains unchanged

6. If f and g are real functions defined by $f(a) = a + 2$ and $g(a) = 2a^2 + 5$, then fog is equal to

(a) $2a^2 + 7$

(b) $2a^2 + 5$

(c) $2(a + 2)^2 + 5$

(d) $2a + 5$

7. If 'p' and 'q' are the roots of the equation $x^2 - 10x + 16 = 0$, the value of $(1 - p)(1 - q)$ is

(a) –7

(b) 7

(c) 16

(d) –16

REVIEW CAT Scan 8

1. Given that 'a' and 'b' are positive real numbers such that $a + b = 1$, then what is the minimum value of $\sqrt{12+\frac{1}{a^2}}+\sqrt{12+\frac{1}{b^2}}$?

   (a) 8

   (b) 16

   (c) 24

   (d) 4

2. Let p, q and r be distinct positive integers satisfying $p < q < r$ and $p + q + r = k$. What is the smallest value of k that does *not* determine p, q, and r uniquely?

   (a) 9

   (b) 6

   (c) 7

   (d) 8

3. Given odd positive integers p, q and r which of the following is not necessarily true?

   (a) $p^2 q^2 r^2$ is odd

   (b) $3(p^2 + q3)r^2$ is even

   (c) $5p + q + r4$ is odd

   (d) $r^2 (p4 + q4)/2$ is even

4. $f(a) = (a_2 + 1)(a_2 - 1)$ where $a = 1, 2, 3\ldots$ which of the following statement is correct about $f(a)$?

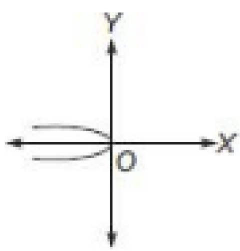
(a) $f(a)$ is always divisible by 5

(b) $f(a)$ is always divisible by 3

(c) $f(a)$ is always divisible by 30

(d) None of these

Directions for Questions 5 and 6: The following questions are based on the graph of parabola plotted on $x - y$ axes. Answer the questions according to given conditions if applicable and deductions from the graph.

5. 

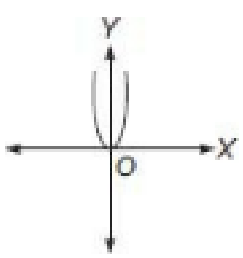
The above graph represents the equations

(a) $y_2 = kx, k > 0$

(b) $y_2 = kx, k < 0$

(c) $x_2 = ky - 1, k < 0$

(d) $x_2 = ky - 1, k > 0$

6. 

The above graph represents the equation

(a) $x^2 = ky, k < 0$

(b) $y^2 = kx + 1, k > 0$

(c) $y^2 = kx + 1, k < 0$

(d) None of these

7. Mala, while teaching her class on functions gives her students a question.

According to the question, the functions are $f(x) = -x, g(x) = x$. She also provides her students with following functions also.

$$f(x, y) = x - y \text{ and } g(x, y) = x + y$$

Since she wants to test the grasp of her students on functions she asks them a simple question "which of the following is not true?" and provides her students with the following options. None of her students were able to answer the question in single attempt. Can you answer her question?

(a) $f[f(g(x, y))] = g(x, y)$

(b) $g[f[g[f(x, y)]]] = f(x, y)$

(c) $f(x) + g(x) + f(x, y) + g(x, y) = g(x) - f(x)$

(d) None of these

ANSWER KEY

REVIEW CAT Scan 1

1. (d)
2. (d)
3. (c)
4. (c)
5. (d)

6. (c)
7. (d)

REVIEW CAT Scan 2

1. (c)
2. (c)
3. (b)
4. (c)
5. (a)
6. (d)
7. (d)

REVIEW CAT Scan 3

1. (b)
2. (b)
3. (b)
4. (d)
5. (c)
6. (c)
7. (a)

REVIEW CAT Scan 4

1. (d)
2. (d)
3. (d)
4. (d)
5. (a)
6. (a)
7. (d)

REVIEW CAT Scan 7

1. (b)
2. (c)
3. (d)
4. (b)
5. (b)
6. (a)
7. (b)

REVIEW CAT Scan 8

1. (a)
2. (d)
3. (d)
4. (d)
5. (b)
6. (d)
7. (b)

TASTE OF THE EXAMS—BLOCK V

CAT

Directions for Questions 1 and 4: Answer the questions based on the following information.

In each of the following questions, a pair of graphs $F(x)$ and $F1(x)$ is given. These are composed of straight line segments, shown as solid lines, in the domain $x \Rightarrow (-2, 2)$. **(CAT 1999)**

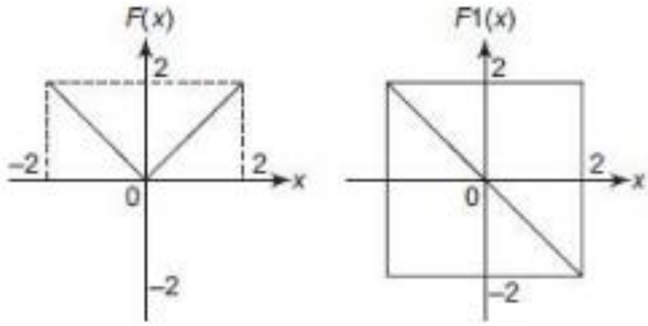
Choose the answer as

a. if $F1(x) = -F(x)$

b. if $F1(x) = F(-x)$

c. if $F1(x) = -F(-x)$

d. if none of the above is true

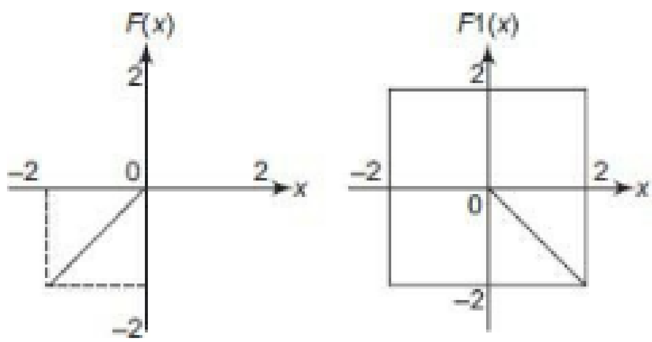
1. 

(a) a

(b) b

(c) c

(d) d

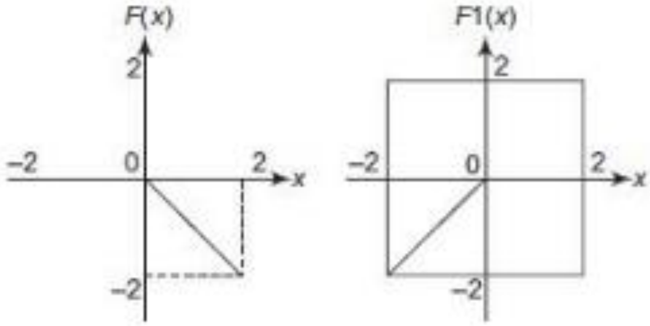
2. 

(a) a

(b) b

(c) c

(d) d

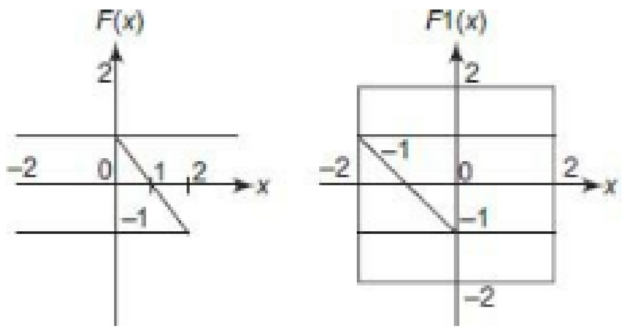
3. 

(a) a

(b) b

(c) c

(d) d

4. 

(a) a

(b) b

(c) c

(d) d

Directions for Question 5 to 7: These questions are based on the situation given below.

Let x and y be real numbers and let **(CAT 1999)**

$f(x, y) = |x + y|$, $F(f(x, y)) = -f(x, y)$ and $G(f(x, y)) = -F(f(x, y))$

5. Which of the following statements is true?

(a) $f(f(x, y)) \times G(f(x, y)) = -f(f(x, y)) \times G(f(x, y))$

(b) $F(f(x, y)) \times G(f(x, y)) > -F(f(x, y)) \times G(f(x, y))$

(c) $F(f(x, y)) \times G(f(x, y)) \neq G(f(x, y)) \times F(f(x, y))$

(d) $F(f(x, y)) \times G(f(x, y)) \leq G(f(x, y)) \times f(-x, -y)$

6. What is the value of $f(G(f(1, 0)), f(F(f(1, 2)), G(f(1, 2))))$?

   (a) 3

   (b) 2

   (c) 1

   (d) 0

7. Which of the following expressions yields $x2$ as its result?

   (a) $F(f(x, -x))) \times G(f(x, -x))$

   (b) $F(f(x, x)) \times G(f(x, x)) \times 4$

   (c) $-F(f(x, x) \times G(f(x, x))$ ∏ log216

   (d) $f(x, x) \times f(x, x)$

Directions for Questions 8 and 9: Answer the questions based on the following information. **(CAT 2000)**

A, B and C are three numbers. Let

@ (A, B) = Average of A and B,

/ (A, B) = Product of A and B, and

× (A, B) = The result of dividing A by B.

8. The sum of A and B is given by

(a) / (@ (A, B), 2)

(b) × (@ (A, B), 2)

(c) @ (/ A, B), 2)

(d) @ (× (A, B), 2)

9. Average of A, B and C is given by

(a) @ (/ (@ (/ (B, A), 2), C), 3)

(b) × (@ (/ (@ (B, A), 3), C), 2)

(c) / (×(× (@ (B, A), 2), C), 3)

(d) / (× (@ (/ (@ (B, A) 2), C), 3), 2)

Directions for Questions 10 and 11: Answer the questions based on the following information.

For real numbers x and y, let **(CAT 2000)**

$$f(x, y) = \begin{cases} \text{Positive square root of} \\ (x+y), \text{ if } (x+y)^{0.5} \text{ is real} \\ (x+y)^2 \text{ otherwise} \end{cases}$$

$$g(x, y) = \begin{cases} (x+y)^2, \text{ if } (x+y)^{0.5} \text{ is real} \\ -(x+y), \text{ otherwise} \end{cases}$$

10. Which of the following expressions yields a positive value for every pair of non-zero real numbers (x, y)?

(a) $f(x, y) - g(x, y)$

(b) $(f(x, y) - (g(x, y))2$

(c) $g(x, y) - (f(x, y))2$

(d) $f(x, y) + g(x, y)$

11. Under which of the following conditions is $f(x, y)$ necessarily greater than $g(x, y)$?

(a) Both x and y are less than -1

(b) Both x and y are positive

(c) Both x and y are negative

(d) $y > x$

Directions for Questions 12 and 14: Answer the questions based on the following information.

For three distinct positive real numbers x, y and z, let **(CAT 2000)**

$f(x, y, z) = \text{Min}(\text{Max}(x, y), \text{Max}(y, z), \text{Max}(z, x))$

$g(x, y, z) = \text{Max}(\text{Min}(x, y), \text{Min}(y, z), \text{Min}(z, x))$

$h(x, y, z) = \text{Max}(\text{Max}(x, y), \text{Max}(y, z), \text{Max}(z, x))$

$j(x, y, z) = \text{Min}(\text{Min}(x, y), \text{Min}(y, z), \text{Min}(z, x))$

$m(x, y, z) = \text{Max}(x, y, z)$

$n(x, y, z) = \text{Min}(x, y, z)$

12. Which of the following is necessarily greater than 1?

(a) $[h(x, y, z) - f(x, y, z)] / j(x, y, z)$

(b) $j(x, y, z) / h(x, y, z)$

(c) $f(x, y, z) / g(x, y, z)$

(d) $[f(x, y, z) + h(x, y, z) - g(x, y, z)] / j(x, y, z)$

13. Which of the following expressions is necessarily equal to 1?

(a) $[f(x, y, z) - m(x, y, z)] / [g(x, y, z) - h(x, y, z)]$

(b) $[m(x, y, z) - f(x, y, z)] / [g(x, y, z) - n(x, y, z)]$

(c) $[j(x, y, z) - g(x, y, z)] / h(x, y, z)$

(d) $[f(x, y, z) - h(x, y, z)] / f(x, y, z)$

14. Which of the following expressions is indeterminate?

(a) $[f(x, y, z) - h(x, y, z)] / [g(x, y, z) - j(x, y, z)]$

(b) $[f(x, y, z) + h(x, y, z) + g(x, y, z) + j(x, y, z)] / [j(x, y, z) + h(x, y, z) - m(x, y, z) - n(x, y, z)]$

(c) $[g(x, y, z) - j(x, y, z)] / [f(x, y, z) - h(x, y, z)]$

(d) $[h(x, y, z) - f(x, y, z)] / [n(x, y, z) - g(x, y, z)$

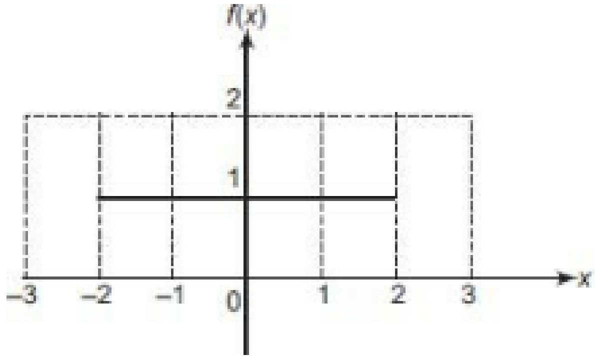
Directions for Questions 15 and 17: Given below are three graphs made up of straight line segments shown as thick lines. In each case choose the answer as **(CAT 2000)**

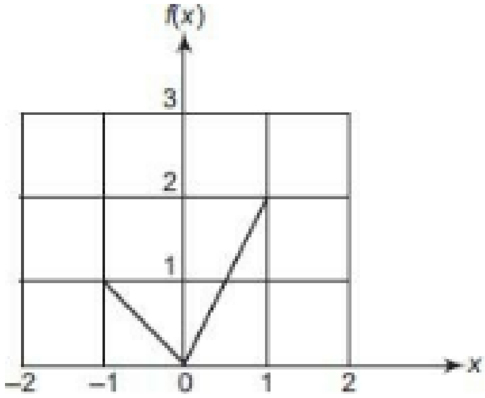
(a) if $f(x) = 3\,f(-x)$

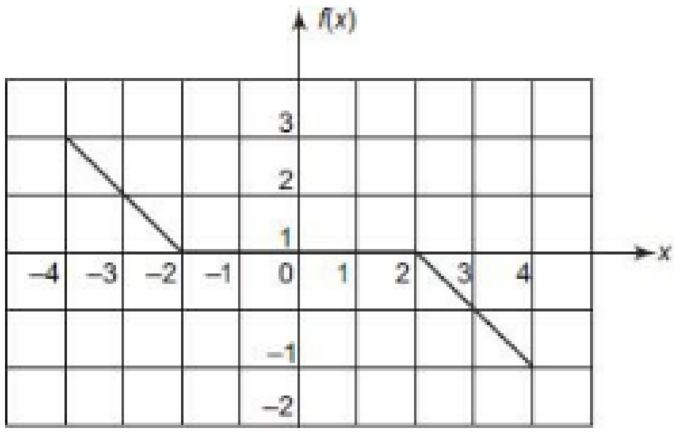
(b) if $f(x) = -f(-x)$

(c) if $f(x) = f(-x)$

(d) if $3f(x) = 6\,f(-x)$, for $x \geq 0$

15. 

16. 

17. 

Directions for Questions 18 and 19: Answer the questions based on the following information.

For a real number x, let **(CAT 2000)**

$$f(x) = \frac{1}{1+x} \quad \text{if } x \text{ is non-negative}$$

$$= 1 + x \quad \text{if } x \text{ is negative}$$

$$f^n(x) = f(f^{n-1}(x)),\ n = 2, 3, \ldots$$

18. What is the value of the product $f(2)\,f^2(2)\,f^3(2)\,f^4(2)\,f^5(2)$?

(a) 1/3

(b) 3

(c) 1/18

(d) None of these

19. r is an integer ≥ 2. Then what is the value of $f_{r-1}(-r)+f_r(-r)+f_{r+1}(-r)$?

(a) –1

(b) 0

(c) 1

(d) None of these

20. If the equation $x^3 - ax^2 + bx - a = 0$ has three real roots, then it must be the case that **(CAT 2000)**

(a) $b = 1$

(b) $b \neq 1$

(c) $a = 1$

(d) $a \neq 1$

21. The set of all positive integers is the union of two disjoint subsets: **(CAT 2000)**

$\{f(1), f(2), ..., f(n), ...\}$ and $\{g(1), g(2), ..., g(n), ...\}$,

where

$f(1) < f(2) < ... < f(n) ...$, and $g(1) < g(2) < ... < g(n) ...$, and

$g(n) = f(f(n)) + 1$ for all $n \geq 1$.

What is the value of $g(1)$?

(a) 0

(b) 2

(c) 1

(d) Cannot be determined

22. For all non-negative integers x and y, $f(x, y)$ is defined as below. **(CAT 2000)**

$f(0, y) = y + 1$

$f(x + 1, 0) = f(x, 1)$

$f(x + 1, y + 1) = f(x, f(x + 1, y))$

Then what is the value of $f(1, 2)$?

(a) 2

(b) 4

(c) 3

(d) Cannot be determined

23. x and y are real numbers satisfying the conditions $2 < x < 3$ and $-7 < y < -1$. Which of the following expressions will have the least value? **(CAT 2001)**

(a) $x2y$

(b) $xy2$

(c) $5xy$

(d) None of these

24. 'm' is the largest positive integer such that $n > m$. Also, it is known that $n3 - 7n2 + 11n - 5$ is positive. Then, the largest possible value for m is: **(CAT 2001)**

(a) 4

(b) 5

(c) 8

(d) None of these

25. Let x, y be two positive numbers such that $x + y = 1$. Then, the minimum value of $(x + 1/x)2 + (y + 1/y)2$ is... **(CAT 2001)**

(a) 12

(b) 20

(c) 12.5

(d) 13.3

26. The number of real roots of the equation $\frac{A^2}{x} + \frac{B^2}{x^{-1}} = 1$ where A and B are real numbers not equal to zero simultaneously, is **(CAT 2002)**

(a) 1

(b) 2

(c) 1 or 2

(d) None of these

27. Suppose for any real number x, $[x]$ denotes the greatest integer less than or equal to x. Let $L(x, y) = [x] + [y] + [x + y]$ and $R(x, y) = [2x] + [2y]$. Then it is impossible to find any two positive real numbers x and y for which **(CAT 2002)**

(a) $L(x, y) = R(x, y)$

(b) $L(x, y) \neq R(x, y)$

(c) $L(x, y) < R(x, y)$

(d) $L(x, y) > R(x, y)$

28. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f(x) + f(y)$ is: **(CAT 2002)**

(a) $f(x + y)$

(b) $f\{(x + y)/(1 + xy)\}$

(c) $(x + y)f\{1/(1 + xy)\}$

(d) $f(x) + f(y)/(1 + xy)$

29. If both a and b belong to the set {1, 2, 3, 4}, then the number of equations of the form $ax2 + bx + 1 = 0$ having real roots is **(CAT 2002)**

(a) 10

(b) 7

(c) 6

(d) 12

30. What is the sum of 'n' terms in the series

$\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3}\right) + \cdots?$ **(CAT 2002)**

(a) $\log\left[\frac{n^{n-1}}{m^{n+1}}\right]^{n/2}$

(b) $\log\left[\frac{m^m}{n^n}\right]^{n/2}$

(c) $\log\left[\frac{m^{1-n}}{n^{1-m}}\right]^{n/2}$

(d) $\log\left[\frac{m^{n+1}}{n^{n-1}}\right]^{\frac{n}{2}}$

31. The number of roots common between the two equations $x^3 + 3x^2 + 4x + 5 = 0$ and $x^3 + 2x^2 + 7x + 3 = 0$ is **(CAT 2003)**

(a) 0

(b) 1

(c) 2

(d) 3

32. A real number x satisfying $1 - \frac{1}{n} < x \leq 3 + \frac{1}{n}$, for every positive integer n, is best described by **(CAT 2003)**

(a) $1 < x < 4$

(b) $1 < x \leq 3$

(c) $0 < x \leq 4$

(d) $1 \leq x \leq 3$

Directions for Questions 33 and 35: Answer the questions on the basis of the tables given below.

Two binary operations ≈ and * are defined over the set $\{a, e, f, g, h\}$ as per the following tables:

⊕	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

*	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	f	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first table $f \oplus g = a$, while according to the second table $g*h = f$, and so on. Also, let $f^2 = f*f$, $g^3 = g*g*g$, and so on. **(CAT 2003)**

33. What is the smallest positive integer n such that $gn = e$?

(a) 4

(b) 5

(c) 2

(d) 3

34. Upon simplification, $f \oplus [f * \{f \oplus (f * f)\}]$ equals

(a) e

(b) f

(c) g

(d) h

35. Upon simplification, $\{a_{10} * (f_{10} \oplus g_9)\} \oplus e8$ equals

(a) e

(b) f

(c) g

(d) h

36. If n is such that $36 \le n \le 72$, then $x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4}$ satisfies **(CAT 2003)**

(a) $20 < x < 54$

(b) $23 < x < 58$

(c) $25 < x < 64$

(d) $28 < x < 60$

37. If $|b| \geq 1$ and $x = -|a|b$, then which one of the following is necessarily true? **(CAT 2003)**

(a) $a - xb < 0$

(b) $a - xb \geq 0$

(c) $a - xb > 0$

(d) $a - xb \leq 0$

38. If $\frac{1}{3}\log 3\ \text{M} + 3\log 3\ \text{N} = 1 + \log 0.008\ 5$, then: **(CAT 2003)**

(a) $M9 = 9/N$

(b) $N9 = 9/M$

(c) $M3 = 3/N$

(d) $N9 = 3/M$

39. Let $f(x) = ax2 - b\,|x|$, where a and b are constants. Then at $x \neq 0$, $f(x)$ is **(CAT 2003)**

(a) maximized whenever $a > 0, b > 0$

(b) maximized whenever $a > 0, b < 0$

(c) minimized whenever $a > 0, b > 0$

(d) minimized whenever $a > 0, b < 0$

40. The number of non-negative real roots of $2x - x - 1 = 0$ equals **(CAT 2003 L)**

(a) 0

(b) 1

(c) 2

(d) 3

41. When the curves $y = \log_{10} x$ and $y = x - 1$ are drawn in the $x - y$ plane, how many times do they intersect for values $x \geq 1$? **(CAT 2003)**

(a) Never

(b) Once

(c) Twice

(d) More than twice.

42. Let $g(x) = \max(5 - x, x + 2)$. The smallest possible value of $g(x)$ is: **(CAT 2003)**

(a) 4.0

(b) 4.5

(c) 1.5

(d) None of these

43. The function $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is a real number, attains a minimum at: **(CAT 2003)**

(a) $x = 2.3$

(b) $x = 2.5$

(c) $x = 2.7$

(d) None of these

44. Let p and q be the roots of the quadratic equation $x^2 - (a - 2)x - (a + 1) = 0$. What is the minimum possible value of $p^2 + q^2$? **(CAT 2003)**

(a) 0

(b) 3

(c) 4

(d) 5

45. If $\log_3 2$, $\log_3(2x - 5)$, $\log_3 (2x - 7/2)$ are in arithmetic progression, then the value of x is equal to:

(a) 5

(b) 4

(c) 2

(d) 3 **(CAT 2003)**

46. Consider the following two curves in the $x - y$ plane; $y = x^3 + x^2 + 5$ and $y = x^2 + x + 5$ which of the following statements is true for $-2 \le x \le 2$? **(CAT 2003)**

(a) The two curves intersect once.

(b) The two curves intersect twice.

(c) The two curves do not intersect.

(d) The two curves intersect thrice.

47. If $f(x) = x^3 - 4x + p$, and $f(0)$ and $f(1)$ are of opposite signs, then which of the following is necessarily true **(CAT 2004)**

(a) $-1 < p < 2$

(b) $0 < p < 3$

(c) $-2 < p < 1$

(d) $p > 3$ or $p < 0$

Directions for Questions 48 and 49: Answer the questions on the basis of the information given below:

$f1(x) = x \; 0 \le x \le 1$

$= 1 \; x \ge 1$

$= 0$ Otherwise

$f2(x) = f1(-x)$ for all x

$f3(x) = -f2(x)$ for all x

$f4(x) = f3(-x)$ for all x **(CAT 2004)**

48. How many of the following products are necessarily zero for every x.

$f1(x)f2(x), f2(x)f3(x), f2(x)f4(x)$?

(a) 0

(b) 1

(c) 2

(d) 3

49. Which of the following is necessarily true?

(a) $f4(x) = f1(x)$ for all x

(b) $f1(x) = -f3(-x)$ for all x

(c) $f2(-x) = f4(x)$ for all x

(d) $f1(x) = f3(x) = 0$ for all x

50. Let $u = (\log 2x)2 - 6\log 2x + 12$ where x is a real number. Then the equation $xu = 256$, has. **(CAT 2004)**

(a) no solution for x

(b) exactly one solution for x

(c) exactly two distinct solutions for x

(d) exactly three distinct solutions for x

51. For which value of k does the following pair of equations yield a unique solution of x such that the solution is positive? **(CAT 2005)**

$x2 - y2 = 0\ (x - k)2 + y2 = 1$

(a) 2

(b) 0

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

52. Let $g(x)$ be a function such that $g(x + 1) + g(x - 1) = g(x)$ for every real x. Then for what value of p is the relation $g(x + p) = g(x)$ necessarily true for every real x? **(CAT 2005)**

(a) 5

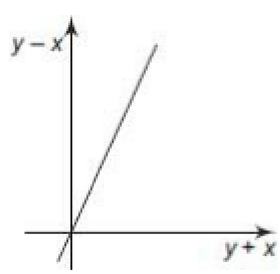
(b) 3

(c) 2

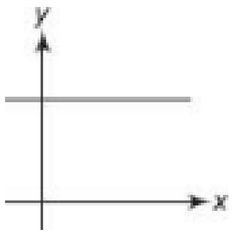
(d) 6

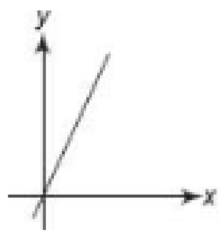
53. The graph of $y - x$ against $y + x$ is as shown below. (All graphs in this question are drawn to scale and the same scale has been used on each axis.)

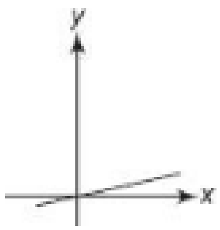
(CAT 2006)

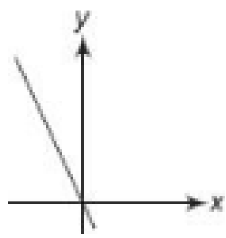


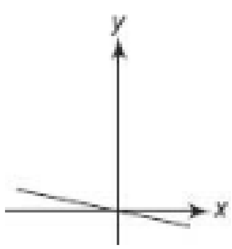
Which of the following shows the graph of *y* against *x*?

(a) 

(b) 

(c) 

(d) 

(e) 

54. What values of *x* satisfy $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 \le 0$ ('*x*' is a real number)? **(CAT 2006)**

(a) $-8 \le x \le 1$

(b) $-1 \le x \le 8$

(c) $1 < x < 8$

(d) $1 < x \le 8$

(e) $-8 \le x \le 8$

55. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number. Then the minimum possible value of $f(x)$ is: **(CAT 2006)**

(a) 1/3

(b) 1/2

(c) 2/3

(d) 4/3

(e) 5/3

56. If $\log Y\, x = (a \cdot \log z\, y) = (b \cdot \log x\, z) = ab$, then which of the following pairs of values for (a, b) is not possible? **(CAT 2006)**

(a) (–2, 1/2)

(b) (1, 1)

(c) (0.4, 2.5)

(d) (a, 1/a)

(e) (2,2)

57. A function $f(x)$ satisfies $f(1) = 3600$ and $f(1) + f(2) + \ldots + f(n) = n2\, f(n)$, for all positive integers $n > 1$. What is the value of $f(9)$? **(CAT 2007)**

(a) 80

(b) 240

(c) 200

(d) 100

58. A quadratic function $f(x)$ attains a maximum of 3 at $x = 1$. The value of the function at $x = 0$ is 1. What is the value $f(x)$ at $x = 10$? **(CAT 2007)**

(a) –119

(b) –159

(c) –110

(d) –180

(e) –105

Directions for Questions 59 and 60: Let $f(x) = ax^2 + bx + c$, where a, b and c are certain constants and $a \neq 0$. It is known that $f(5) = -3f(2)$, and that 3 is a root of $f(x) = 0$. **(CAT 2008)**

59. What is the other root of $f(x) = 0$?

(a) –7

(b) –4

(c) 2

(d) 6

(e) cannot be determined

60. What is the value of $a + b + c$?

(a) 9

(b) 14

(c) 13

(d) 37

(e) cannot be determined

61. Let $f(x)$ be a function satisfying $f(x)f(y) = f(xy)$ for all real x, y. If $f(2) = 4$, then what is the value of $f(1/2)$? **(CAT 2008)**

(a) 0

(b) 1/4

(c) 1/2

(d) 1

(e) cannot be determined

62. If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b? **(CAT 2008)**

(a) $-1/\sqrt{3}$

(b) -1

(c) 0

(d) 1

(e) $1/\sqrt{3}$

63. If x is real number such that $\log_3 5 = \log_5(2 + x)$, then which of the following is true? **(CAT 2017)**

(a) $0 < x < 3$

(b) $23 < x < 30$

(c) $x > 30$

(d) $3 < x < 23$

64. Let $f(x) = x^2$ and $g(x) = 2x$, for all real x. Then the value of $f(f(g(x)) + g(f(x)))$ at $x = 1$ is **(CAT 2017)**

(a) 16

(b) 18

(c) 36

(d) 40

65. The minimum possible value of the sum of the squares of the roots of the equation **(CAT 2017)**

$x^2 + (a + 3)x - (a + 5) = 0$ is

(a) 1

(b) 2

(c) 3

(d) 4

66. If $9^{x-1/2} - 2^{2x-2} = 4x - 32x-3$, then x is **(CAT 2017)**

(a) 3/2

(b) 2/5

(c) 3/4

(d) 4/9

67. If log($2a * 3b * 5c$) is the arithmetic mean of log($2^2 * 3^3 * {}^5$), log($2^6 * 3 * 57$), and log ($2 * 32 * 54$), then a equals **(CAT 2017)**

68. If $f(ab) = f(a)f(b)$ for all positive integers a and b, then the largest possible value of $f(1)$ is **(CAT 2017)**

69. Let $f(x) = 2x - 5$ and $g(x) = 7 - 2x$. Then $|f(x) + g(x)| = |f(x)| + |g(x)|$ if and only if

(a) $5/2 < x < 7/2$

(b) $x \leq 5/2$ or $x \geq 7/2$

(c) $x < 5/2$ or $x \geq 7/2$

(d) $5/2 \leq x \leq 7/2$ **(CAT 2017)**

70. The area of the closed region bounded by the equation $|x| + |y| = 2$ in the two-dimensional plane is

(a) 4p

(b) 4

(c) 8

(d) 2p **(CAT 2017)**

71. Suppose, $\log_3 x = \log_{12} y = a$, where x, y are positive numbers. If G is the geometric mean of x and y, then $\log_6 G$ is equal to **(CAT 2017)**

(a) $\sqrt{a}$

(b) $2a$

(c) $a/2$

(d) a

72. If $x + 1 = x2$ and $x > 0$, then $2x4$ is **(CAT 2017)**

(a) $6 + 4\sqrt{5}$

(b) $3 + 5\sqrt{5}$

(c) $5 + 3\sqrt{5}$

(d) $7 + 3\sqrt{5}$

73. The value of $\log 0.008\sqrt{5}$ + log÷381 – 7 is equal to **(CAT 2017)**

(a) 1/3

(b) 2/3

(c) 5/6

(d) 7/6

74. If $9_{2x-1} - 81_{x-1} = 1944$, then x is? **(CAT 2017)**

(a) 3

(b) 9/4

(c) 4/9

(d) 1/3

75. If $f_1(x) = x_2 + 11x + n$ and $f_2(x) = x$, then the largest positive integer n for which the equation $f1(x) = f2(x)$ has two distinct real roots, is **(CAT 2017)**

76. If $f(x) = (5x + 2)/(3x - 5)$ and $g(x) = x_2 - 2x - 1$, then the value of $g(f(f(3)))$ is **(CAT 2017)**

(a) 2

(b) 1/3

(c) 6

(d) 2/3

77. If $f(x + 2) = f(x) + f(x + 1)$ for all positive integers x, and $f(11) = 91$, $f(15) = 617$, then $f(10)$ equals **(CAT 2018)**

78. If $\log_2 (5 + \log 3\ a) = 3$ and $\log 5\ (4a + 12 + \log_2 b) = 3$, then $a + b$ is equal to **(CAT 2018)**

(a) 67

(b) 40

(c) 32

(d) 59

79. If x is a positive quantity such that $2^x = 3^{\log_5 2}$, then x is equal to **(CAT 2018)**

(a) $1 + \log_3 \frac{5}{3}$

(b) $\log_5 8$

(c) $\log_5 9$

(d) $1 + \log_5 \frac{3}{5}$

80. Let $f(x) = \min\{2x^2, 52 - 5x\}$, where x is any positive real number. Then the maximum possible value of $f(x)$ is **(CAT 2018)**

81. The smallest integer n such that $n^3 - 11n^2 + 32n - 28 > 0$ is **(CAT 2018)**

82. Let $f(x) = \max\{5x, 52 - 2x^2\}$, where x is any positive real number. Then the minimum possible value of $f(x)$ is **(CAT 2018)**

83. If a and b are integers such that $2x^2 - ax + 2 > 0$ and $x^2 - bx + 8 \geq 0$ for all real numbers x, then the largest possible value of $2a - 6b$ is **(CAT 2018)**

84. If $p^3 = q^4 = r^5 = s^6$, then the value of $\log_s (pqr)$ is equal to **(CAT 2018)**

(a) 24/5

(b) 16/5

(c) 47/10

(d) 1

85. $\frac{1}{\log_2 100} - \frac{1}{\log_4 100} + \frac{1}{\log_5 100} - \frac{1}{\log_{10} 100} + \frac{1}{\log_{20} 100} - \frac{1}{\log_{25} 100} + \frac{1}{\log_{50} 100} = ?$ **(CAT 2018)**

86. The real root of the equation $2^{6x} + 2^{3x+2} - 21 = 0$ is **(CAT 2019)**

(a) (log23)/3

(b) log29

(c) (log27)/3

(d) log227

87. What is the largest positive integer such that $(n^2 + 7n + 12)/(n^2 - n - 12)$ is also a positive integer? **(CAT 2019)**

(a) 6

(b) 8

(c) 16

(d) 12

88. Let f be a function such that $f(m \times n) = f(m) \times f(n)$ for every positive integers m and n. If $f(1)$, $f(2)$ and $f(3)$ are positive integers, $f(1) < f(2)$, and $f(24) = 54$, then $f(18)$ equals **(CAT 2019)**

89. Let A be a real number. Then the roots of the equation $x^2 - 4x - \log_2 A = 0$ are real and distinct if and only if **(CAT 2019)**

(a) $A < \frac{1}{16}$

(b) $A > \frac{1}{8}$

(c) $A > \frac{1}{16}$

(d) $A < \frac{1}{8}$

90. The quadratic equation $x^2 + bx + c = 0$, has two roots $4a$ and $3a$, where 'a' is an integer. Which of the following is a possible value of $b^2 + c$? **(CAT 2019)**

(a) 3721

(b) 549

(c) 361

(d) 427

91. If $5^x - 3^y = 13438$ and $5^{x-1} + 3^{y+1} = 9686$, then $x + y$ equals [TITA] **(CAT 2019)**

92. For any positive integer n, let $f(n) = n \times (n + 1)$ if n is even, and $f(n) = n + 3$, if n is odd. If m is a positive integer such that $8 \times f(m + 1) - f(m) = 2$, then m equals **(CAT 2019)**

93. Let x and y be positive real number such that $\log_5(x + y) + \log_5(x - y) = 3$, and $\log_2 y - \log_2 x = 1 - \log_2 3$. Then xy equals **(CAT 2019)**

(a) 25

(b) 150

(c) 250

(d) 100

94. The number of solutions of the equation $|x|(6x^2 + 1) = 5x^2$ is **(CAT 2019)**

95. The product of the distinct roots of $|x^2 - x - 6| = x + 2$ is **(CAT 2019)**

(a) –4

(b) –16

(c) –8

(d) –24

96. Consider a funtion $f(x + y) = f(x) f(y)$ where x, y are positive integers, and $f(1) = 2$. if $f(a + 1) + f(a + 2) + ... + f(a + n) = 16(2n - 1)$ then a is equal to: **(CAT 2019)**

97. If **x** is a real number, then $\sqrt{\log_e \frac{4x - x^2}{3}}$ is a real number if and only if **(CAT 2019)**

(a) $-3 \le x \le 3$

(b) $1 \le x \le 2$

(c) $1 \le x \le 3$

(d) $-1 \le x \le 3$

98. If $(5.55)x = (0.555)y = 1000$, then the value of $\frac{1}{x} - \frac{1}{y}$ is **(CAT 2019)**

(a) 1

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) 3

ANSWER KEY

1. (d)
2. (b)
3. (b)

4. (c)
5. (d)
6. (c)
7. (c)
8. (a)
9. (d)
10. (d)
11. (a)
12. (d)
13. (a)
14. (b)
15. (c)
16. (d)
17. (b)
18. (c)
19. (b)
20. (b)
21. (b)
22. (b)
23. (c)
24. (b)
25. (c)
26. (d)
27. (d)
28. (b)
29. (b)
30. (d)

31. (a)
32. (c)
33. (a)
34. (d)
35. (a)
36. (d)
37. (b)
38. (b)
39. (b)
40. (c)
41. (b)
42. (d)
43. (b)
44. (d)
45. (d)
46. (d)
47. (b)
48. (c)
49. (b)
50. (b)
51. (c)
52. (d)
53. (d)
54. (a)
55. (e)
56. (e)
57. (a)

58. (b)

59. (b)

60. (e)

61. (b)

62. (b)

63. (d)

64. (c)

65. (c)

66. (a)

67. 3

68. 1

69. (d)

70. (c)

71. (d)

72. (d)

73. (c)

74. (b)

75. 24

76. (a)

77. 54

78. (d)

79. (d)

80. 32

81. 8

82. 20

83. 36

84. (c)

85. $\frac{1}{2}$

86. (a)

87. (d)

88. 12

89. (c)

90. (b)

91. 13

92. 10

93. (b)

94. 5

95. (b)

96. 3

97. (c)

98. 1/3

Solutions

1. If you look at the graph closely, you can observe that as you move along the X-axis, from 0 towards the left, both the graphs of $F(x)$ and $F1(x)$ are the same. Thus, we can conclude that: For $x < 0$, $F1(x) = F(x)$

   When $x > 0$, the value of $F1(x)$ is opposite in sign to the value of $F(x)$. Hence, for $x > 0$, we can conclude $F1(x) = -F(x)$. Hence, option (d) is correct.

2. **Theory tip: In a function $f(x)$ when we change the sign of the argument of the function from 'x' to '$-x$', the graph of the function transforms to it's mirror image along the Y-axis. In other words, when we change**

the sign of the argument of the function from $f(x)$ to $f(-x)$, the graph formed is the mirror image of the graph of the $f(x)$ function. In the current question's graphs we can see that graph of $F1(x)$ is just the mirror image of the graph of $F(x)$. Thus, we conclude that, $F1(x) = F(-x)$. Option (b) is correct.

3. We see the same picture in this question – where the two graphs are mirror images of each other around the Y-axis. Hence, $F1(x) = F(-x)$. Option (b) is correct.

4. **Theory tip: An odd function can be spotted when we see a graph of a function, that is symmetrical about the origin. Symmetry about the origin means that a point of the graph in the first quadrant, would have it's image in the third quadrant and a point of the graph in the fourth quadrant would have it's image in the second quadrant. In other words, suppose you move from the origin to the right by a certain distance along the X-axis. You observe the value of the graph at that point - if the graph is in the first quadrant, then on moving left from the origin by a equal distance on the X-axis, the graph should have a corresponding point in the third quadrant. Also, the absolute value of the Y coordinate of the graph in the first quadrant, should be equal to the value of the Y-coordinate of the graph in the third quadrant for the corresponding values of x. For example, if we consider the graph of $y = x3$. For $x = 2$, the function's value would be 8 and lie in the first quadrant. For $x = -2$, the function's value would be –8 and lie in the third quadrant.**

In the figures for $F(x)$ and $F1(x)$, we see that points in the first quadrant in $F(x)$ are mirrored in the third quadrant in $F1(x)$ and likewise points in the fourth quadrant in $F(x)$ are mirrored in the second quadrant of $F1(x)$. So, we can conclude that $F1(x) = -F(-x)$ and hence option (c) is correct.

Solutions for Questions 5 to 7:

Special Tip: In such questions, normally students try to solve by taking values for the given variables and then try to match the answers from the options. A better way to solve such questions is to first read the functions in logical language before you proceed. In this question, for instance, $f(x,y)$ is a function with two arguments, namely x and y and it returns a value of the modulus of the sum of x and y. This means that $f(x,y)$ would always be a non-negative function. Then $F(f(x,y))$ is a function that would always give us a non positive value - since it is simply given by $-f(x,y)$. In effect this means that $F(f(x,y))$ is a function that simply changes the sign of the value returned by the function $f(x,y)$ and hence $f(x,y)$ and $F(f(x,y))$ would have the same magnitude, but would be opposite in sign. Armed with this information, if you look at $G(f(x,y))$, you realise that this function is again a sign change function from $F(f(x,y)$. This means, that the value of $G(f(x,y))$ after two sign changes would become the same as the value of $f(x,y)$. With this analysis of the functions, you can move onto the questions.

5. **If you read the options in your mind as:** Option (a) $F(f) \times G(f) = -F(f) \times G(f)$, you realise that the two sides of this equation would not be the same, as they would have different signs. Hence, option (a) can be rejected. Option (b): $F(f) \times G(f) > -F(f) \times G(f)$. On analysing this option, we realise that the Left Hand side of the option's inequality would be a negative value, while the right hand side would always be necessarily positive. Hence, option (b) can actually be seen to be always false. Option (c): $F(f) \times G(f) \neq G(f) \times F(f)$, has no reason to be true, since the value of a product is

always same and doesn't depend on the order in which the multiplication is done. Moving onto option (d) we see that in option (d) $F(f(x, y)) \times G(f(x, y)) \le G(f(x, y)) \times f(-x, -y)$, the left hand side would always be negative since in essence it is just $F(f) \times G(f)$. For analysing the right hand side, we need to think as follows: On the right hand side, there is no effect of changing the arguments of 'f' from x,y to $-x, -y$, since the value the function $f(x, y)$ would return would always be the modulus of the sum of the arguments. Thus, for instance, if we try to calculate the values of $f(2, 3)$ and $f(-2, -3)$, we would see the function has the same value. Hence, you can simply read the RHS as: $G(f(x, y)) \times f(x, y)$. It is self evident, based on our discussion above, that the values of $G(f(x, y))$ and $f(x, y)$ would always have the same sign. Hence, the product $G(f(x, y)) \times f(-x, -y)$ would always be positive. Hence, the RHS of option (d) would always be greater than or equal to the value of the LHS. Hence, option (d) is correct.

6. $f(G(f(1, 0)), f(F(f(1, 2)), G(f(1, 2)))) = f(G(f(1, 0)), f(3, -3)) = f(G(f(1, 0)), 0) = f(1, 0) = 1$. Hence, option (c) is correct.

7. Check the options: Option (c): $-f(f(x, x)).G(f(x, x)) \prod \log_2 16 =$

   $-f(f(x, x) \times f(x, x)) \prod \log224 = (2x \times 2x) \prod \log216 = \frac{4x^2}{\log_2 2^4} = x2$

8. When you check the first option, you see that: $@ (A, B) = (A + B)/2$. Then:

   $/ \{@ (A, B), 2\} = / \left\{\frac{(A+B)}{2}, 2\right\} = \frac{A+B}{2} \times 2 = a + B.$

9. By putting some values of A, B, C and checking the options, we get option (d) is correct. **Special Tip: In these situations, while taking the values of *A, B, C* assume values that are outside the first set of values that come to your mind. This is necessary because the question setter also knows that test takers would solve these questions using values.**

Hence, he would often try to set traps for test takers by creating confusing options to the question. In this process, the way the question setter thinks is that he tries to guess the set of values that test takers would assume - and then set incorrect options that would mislead them by giving the required numerical answer. Thus, for questions such as these, if you take values of *A*, *B* and *C* as 1, 2, 3 or 2, 2, 2 or 0, 1, 2 and such values, you expose yourself to the danger of falling into the question setter's trap options. Thus my advise is to avoid the first set of values that come to your mind - because the question setter can also guess those as the values that most test takers would take. To avoid, these traps, it is then best to take values that are outside the normal range that most people would take. Thus, something like 4, 8 and 9 would be a good set of values to take. (I would avoid taking something like 6, 8 and 10 - think why? Because the answer of the average of *A*, *B* and *C* would coincide with the value of *B* in such a case.)

The solution would go as follows, with *A* = 4, *B* = 8 and *C* = 9, we are looking for the option that gives us a value of 7 as its' answer.

Option (a) @ (/ (@ (/ (*B*, *A*), 2), *C*), 3) would be: @ (/ (@ (/ (8, 4), 2), 9), 3) =

@ (/ (@ (32, 2), 9), 3) = @ (/ (17, 9), 3) =@ (153, 3) = 78 ≠ 7. Hence, we reject this option.

Checking for Option (d) we see that: / (× (@ (/ (@ (*B*, *A*) 2), *C*), 3), 2) =

/ (× (@ (/ (@ (8, 4) 2), 9), 3), 2) = / (× (@ (/ (6, 2), 9), 3), 2) = / (× (@ (12, 9), 3), 2) =

/ (× (10.5, 3), 2) = / (3.5, 2) = 7. Thus, option (d) becomes the correct answer. (**Note**: in this approach, since we have not solved the question through algebraic solving - but just by taking values, we need to make sure that when we select an option as the answer it should be the only option satisfying the requirement of the question. Hence, in this question, we select option (d) not just because it is giving us the required value of 7, but because it is the only option giving us that value).

Solutions for Questions 10 and 11: **When we read the functions logically, in this case we see the following:**

$f(x, y)$ is always non-negative – since it returns the value of the positive square root of $(x + y)$ is $x + y \geq 0$ and if $x + y < 0$, then it would return the square of the value of $(x + y)$. Likewise, $g(x, y)$ would also always be positive – since if $x + y$ is negative, then we take the value of $g(x, y)$ as $-(x + y)$ → which would always give a positive value.

10. With this analysis, if we look at the options, it is clear that the value in option (d) $f(x, y) + g(x, y)$ would always be positive. Hence, we can directly mark option (d) as the correct answer. If you do not spot that in this question, you would have to solve the question by checking the options by putting different values of x, y. You would be able to get that options (a), (b) and (c) could be both negative as well as positive, depending on the values of x and y. But option (d) would always be positive. However, needless to say, this approach would end up taking much longer time than the approach that directly spots the answer.
11. Let's check the options one by one.

Option (a): Both x and y are less than –1.

Then, $f(x, y) = (x + y)2$ and $g(x, y) = -(x + y)$.

Substituting any value of $x, y < -1$, we get $f(x, y)$ always greater than $g(x, y)$.

We do not need to check the further options once we get that. (**Note:** Logically also you can see that $f(x, y)$ being the square of $(x + y)$ would increase the value of $(x + y)$ by squaring it – when x, y are both less than -1, while $g(x, y)$ would just give you a positive value by changing the sign of $(x + y)$ from negative to positive – without raising the value of the number. Hence, for $x, y < -1$, $f(x, y)$ would always be greater than $g(x, y)$.

Solutions for Questions 12 to 14:

A closer reading of the given functions tells us that the function:

$f(x, y, z)$ = Min(Max(x, y), Max(y, z), Max(z, x)) gives us the second highest value of the three numbers x, y, z;

$g(x, y, z)$ = Max(Min(x, y), Min(y, z), Min(z, x)) gives us the second highest value of the three numbers x, y, z;

$h(x, y, z)$ = Max(Max(x, y), Max(y, z), Max(z, x)) gives us the highest value of the three numbers x, y, z;

$j(x, y, z)$ = Min(Min(x, y), Min(y, z), Min(z, x)) gives us the minimum value of the three numbers x, y, z;

m(x, y, z) = Max(x, y, z)gives us the highest value of the three numbers x, y, z;

n(x, y, z) = Min(x, y, z) gives us the minimum value of the three numbers x, y, z;

Thus, we have: $f(x, y, z) = g(x, y, z)$; $h(x, y, z) = m(x, y, z)$ and $j(x, y, z) = n(x, y, z)$.

With this starting understanding if we move into the questions, the solving of the questions would be much easier than by blindly substituting values in the options and checking to see which answer is correct.

12. Since we are looking at a value 'necessarily greater than 1' it means we are looking for the numerator to be greater than the denominator. When we look at the first option, we see that: $(h - f)/j$ = (highest - second highest)/ lowest → Can be more than or less than one depending on the values we choose for x, y and z;

Option (b) j/h = minimum/maximum would necessarily be less than 1;

Option (c) f/g = (second highest)/(second highest) = 1;

Option (d) $[f + h - g]/j = h/j$ (since f and g are same) = maximum/minimum → necessarily greater than 1.

13. Option (a) is [second highest - highest]/[second highest - highest] = 1. Hence, option (a) is correct.

14. For an indeterminate expression, we need the denominator to be zero. This would occur only when the components of the denominator cancel out. Checking the options, the first option has a denominator of $g - h$ = second highest - highest ≠ 0. Hence, option (a) is not indeterminate. In option (b), we see that the denominator is $j + h - m - n$ = minimum + maximum – maximum – minimum = 0. Hence, the denominator is 0 and hence option (b) is indeterminate.

15. $f(x) = f(-x)$, so option (c) is correct.

16. $3f(x) = 6f(-x)$, option (d) is correct.

17. $f(x) = -f(-x)$, option (b) is correct

18. $f(2)=\frac{1}{1+2}=\frac{1}{3}, f^2(2)=f\left(\frac{1}{3}\right)=\frac{1}{1+\frac{1}{3}}=\frac{3}{4},$

$$f^3(2)=f\left(\frac{3}{4}\right)=\frac{4}{7}, f^4(2)=f\left(\frac{4}{7}\right)=\frac{7}{11},$$

$$f^5(2)=f\left(\frac{7}{11}\right)=\frac{11}{18}$$

$$f(2)f^2(2)f^3(2)f^4(2)f^5(2)=\frac{1}{3}\times\frac{3}{4}\times\frac{4}{7}\times\frac{7}{11}\times\frac{11}{18}=\frac{1}{18}$$

19. Let $r = 2$

$f(-2) = 1 - 2 = -1$

$f2(-2) = f(-1) = 1 - 1 = 0$

$f3(-2) = 1/1 + 0 = 1$

$fr - 1(-r) + fr(-r) + fr + 1(-r) = -1 + 0 + 1 = 0$

20. If $b = 1$, then the factors are $(x - a)(x^2 + 1)$. This yields 2 imaginary roots. So, option (b) ($b \neq 1$) is the required answer.

21. In the question, $f(n)$ represents the series of all odd numbers 1, 3, 5, 7, 9… and $g(n)$ represents the series of all even numbers 2, 4, 6, 8, 10. Hence, $g(1) = 2$.

22. $f(0, 1) = 1 + 1 = 2$

$f(x + 1, 0) = f(x, 1)$

put $x = 0$, $f(0 + 1, 0) = f(1, 0) = f(0, 1) = 2$

$f(x + 1, y + 1) = f(x, f(x + 1, y))$

put $x = 0$, $y = 0$ in the above equation we get:

$f(1, 1) = f(0, f(1, 0)) = f(0, 2) = 2 + 1 = 3.$

Now put $x = 0$, $y = 1$ in $f(x + 1, y + 1) = f(x, f(x + 1, y))$, we get:

$f(1, 2) = f(0, f(1, 1)) = f(0, 3) = 3 + 1 = 4.$

23. x^2y and $5xy$ both have negative value. For $2 < x < 3$ and $-7 < y < -1$, $5xy$ will be the smaller than x^2y. Hence, option (c) is correct.

24. $n^3 - 7n^2 + 11n - 5 = (n - 1)^2(n - 5) > 0$ for $n > 5$. Hence, the largest possible value of m is 5.

25. The required minimum value would occur when both x and y are taken as 1/2. Then the required sum = $\left(\frac{1}{2^2} + 2^2\right) + \left(\frac{1}{2^2} + 2^2\right) = 6.25 + 6.25 = 12.5$

26. The given would have only one real root if either A or B is equal to 0 (as both can never be zero simultaneously).

When both A and B are non-zero then, it would be a quadratic equation so in this case there would be two real roots.

So, option (d) is correct.

27. We can solve this problem just by inserting values for x and y in the given expressions. Put $x = 2.2$ and $y = 1.7$

$L(x, y) = 6$ & $R(x, y) = 7$ so we can eliminate option (b), (c).

Now put $x = y = 1$ then $L(x, y) = R(x, y)$. So we can eliminate option (a).

So, option (d) is the correct option.

28. The best and the quickest way to solve this problem is to put $x = y$ and check the options.

$$f(x) + f(y) = f(x) + f(x) = 2f(x) = 2\log\left(\frac{1+x}{1-x}\right)$$

Now check the options. Option (b) gives us:

$$F\{(x+y)/(1+xy)\} = f\{(x+x)/(1+x.x)\}$$

$$= f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+2x+x^2}{1+x^2-2x}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2 = 2\log\left(\frac{1+x}{1-x}\right)$$

Hence, this is the correct answer.

Alternately, you could also solve this question by taking $x = y = 0.5$. (**Note:** Since, there are no constraints on the values of x and y, you are free to take any value that is convenient). At $x = y = 0.5$: we get: $f(x) + f(y) = 2 \log 3 = \log 9$. Only, option (b) gives us log 9 as its' outcome if we insert $x = y = 0.5$ in it. Hence, this option would be correct.

29. $ax^2 + bx + 1 = 0$

For real roots

$b^2 - 4ac \geq 0$

$\therefore b^2 - 4a(1) \geq 0$

$\therefore b^2 \geq 4a$

For $a = 1$, possible values of b are 2, 3, 4.

$a = 2$, $4a = 8$, possible values of b are 3, 4.

$a = 3$, $4a = 12$, possible value of b is 4.

$a = 4$, $4a = 16$, possible value of b is 4.

$\therefore$ Number of equations possible = 7.

30. $\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \cdots n$ terms

$$\log \frac{m \cdot m^2 \cdot m^3 \ldots m^n}{1 \cdot n \cdot n^2 \cdot n^3 \ldots n^{n-1}} = \log m^{1+2+\ldots n} / n^{1.2\ldots n-1}$$

$$= \log \frac{m^{\frac{n(n+1)}{2}}}{n^{\frac{n(n-1)}{2}}} = \log\left[\frac{m^{n+1}}{n^{n-1}}\right]^{\frac{n}{2}}$$

31. The intersection points between two functions, are got by equating, the two functions. Thus, we have $x^3 + 3x^2 + 4x + 5 = x^3 + 2x^2 + 7x + 3 \rightarrow x =$ 1,2. However, we can see that none of these two intersection points is the root of either of the equations: $x^3 + 3x^2 + 4x + 5 = 0$ or $x^3 + 2x^2 + 7x + 3 = 0$

32. $1 - \frac{1}{n} < x \le 3 + \frac{1}{n}$

By putting $n = 1$, we get

$\therefore 0 < x \le 4$. Hence option (c) is correct.

33. $g^2 = g * g = h$

$g^3 = g^2 * g = h * g = f$

$g^4 = g^3 * g = f * g = e$

$\therefore n = 4$

34. $f \oplus [f * \{f \oplus (f * f)\}]$

$= f \oplus [f * \{f \oplus h\}]]$

$= f \oplus [f * e\}]$

$= f \oplus [f]] = h$

35. $e^8 = e^2 * e^2 * e^2 * e^2$

$= e * e * e * e = e$

$\therefore a^{10} = (a^2)^5 = (a * a)^5 = a^5 = a * (a * a)^2 = a * a = a$

$g^4 = e$ or $g^8 = g^4 * g^4 = e * e = e$

$g^9 = g^8 * g = g * e = g$

Similarly we can prove that f10 = h

$\therefore \{a_{10} * (f10 \oplus g_9)\} \oplus e8 = \{a * (h \oplus g)\} \oplus e$

$= a^* f \oplus e = a \oplus e = e$

36. $x = \dfrac{n^2 + 2\sqrt{n}\,(n+4) + 16}{n + 4\sqrt{n} + 4}$

Put $x = 36$

$$\therefore \quad x = \frac{(36)^2 + 2 \times 6 \times 40 + 16}{36 + 24 + 4}$$

$$= \frac{(36)^2 + 2 \times 6 \times 40 + 16}{64}$$

$$= \frac{81 + 30 + 1}{4} = \frac{112}{4} = 28$$

The least value of $x = 28$, option (d) is correct.

(**Note:** To confirm the upper end of the inequality you can try to put x = 72.)

37. Put $a = 0$ and $b = 1$

We get: $x = -|a|\ b = 0$

$a - xb = 0 - 0 = 0$. By putting $a = 1$ and $b = 1$ we get $x = -1$ and $a - xb = 2$

Hence, $a - xb \geq 0$.

38. We can solve this problem by checking the options.

$$1 + \log_{0.008} 5 = 1 + \frac{\log_{10} 5}{\log_{10} 0.008}$$

$$= 1 + \frac{\log 10 - \log 2}{3(\log 2 - 1)} = 1 - \frac{1}{3} = \frac{2}{3}$$

Option (b): $N_9 = 9/M$

$$\frac{1}{3}\log_3 M + 3\log_3 N$$

$$= \frac{1}{3}\log_3\left(\frac{9}{N^9}\right) + 3\log_3 N = \frac{2}{3}\log_3 3 = \frac{2}{3}$$

So, Option (b) is correct.

39. x_2 and $|x|$ are always positive for any value of x. For $a > 0$ and $b < 0$ the given expression will always be positive and attain its maximum value.

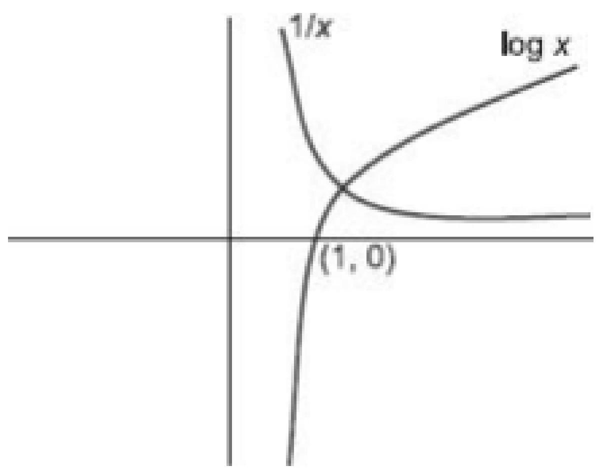
40. $2_x - x - 1 = 0$

$2x = x + 1$

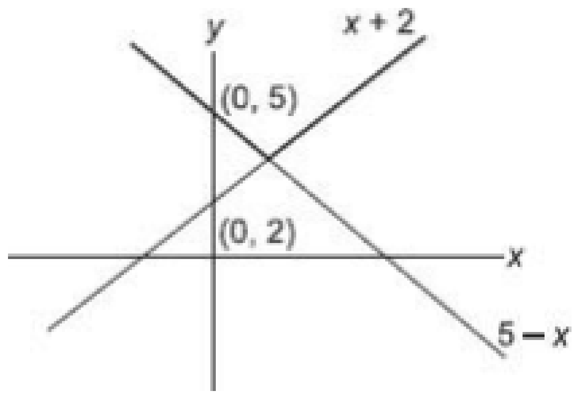
For $x = 0$ and 1, the LHS and the RHS of the above equation are equal. So, $x = 0$ is a solution of the above equation.

For $x = 2, 3, 4, \ldots$ $2_x > x + 1$. So the given equation has 2 solutions. Option (c) is correct.

41. This question can be conveniently handled if you know the graphs of the two functions for $x > 0$. The following picture shows quite clearly that they would intersect only once. Hence, option (b) is correct.



42. The bold portion of the curve given below is max(5 – x, x + 2).



$g(x)$ attains its minimum value at a point where $5 - x = 2 + x$

$5 - x = x + 2$

$x = 1.5$

At $x = 1.5$, $g(x) = 5 - 1.5 = 3.5$. Option (d) is correct.

43. **Theory tip: Any function of the type $|x - 3| + |5 - x|$ would attain its' least value when x is between 3 to 5. Also that least value would be 5 – 3 = 2. Thus, $|x - 3| + |5 - x|$ at $x = 3 = 0 + 2$; at $x = 3.1 = 0.1 + 1.9$; at $x = 3.5 = 0.5 + 1.5$ at $x = 5 = 2 + 0$. It can be seen to be equal to 2 for all values of x between 3 to 5. After 5 and before 3 on the x axis, the value of the function would start increasing from it's minimum value. Task for student: See how $|x - 2| + |3.6 - x|$ behaves as you alter the value of x between 2 to 3.6. Also, see the rate at which the function changes its' value when you go beyond 3.6 and when you go below 2 for the value of x.**

When you look at this function, you can easily realise that the value of the function $|x - 2| + |3.6 - x|$ would be equal to 1.6 whenever we take the value of x between 2 to 3.6. Naturally, then the function would attain its' minimum value when $|2.5 - x|$ is minimised. This would happen at $x = 2.5$.

So at $x = 2.5$, $f(x)$ attains its minimum value.

44. $(p^2 + q^2) = (p + q)^2 - 2pq = (a - 2)^2 + 2(a + 1) = a^2 - 2a + 6 = a^2 - 2a + 1 + 5 = (a - 1)^2 + 5$. This expression can be minimised at $a - 1 = 0$ and its minimum value is 5.

45. Put the values of x from the options to check which option works to solve this question. We get for $x = 3$, the three values are: $\log_3 2$, $\log_3 3$, and $\log_3 4.5$, which is equal to the series: $\log_3 2$; $\log_3 2 + \log_3 1.5$; $\log_3 2 + \log_3 1.5 + \log_3 1.5$ - three terms of an AP. Hence, option (d) is correct.

46. In order to find the number of intersection points of the curves between $-2 \le x \le 2$, we need to equate the two equations and look for the values of x at which the two are equal. Thus, $x^3 + x^2 + 5 = x^2 + x + 5$

$\rightarrow x^3 = x$ (We know that x^3 and x would be equal at three values viz: 0, 1 and –1)

$\therefore x = 0, -1, 1$

So the two curves intersect thrice for $-2 \le x \le 2$.

47. **Theory tip:** The reaction to two values of a function being of opposite sign is that their product would be negative. Thus we have: $f(0) \times f(1) < 0$.

Also we realise that for $x = 0$, the value of the function would be p; while for $x = 1$, its' value would be $p - 3$. Hence,

$p \times (p - 3) < 0$

$p \times (3 - p) > 0$

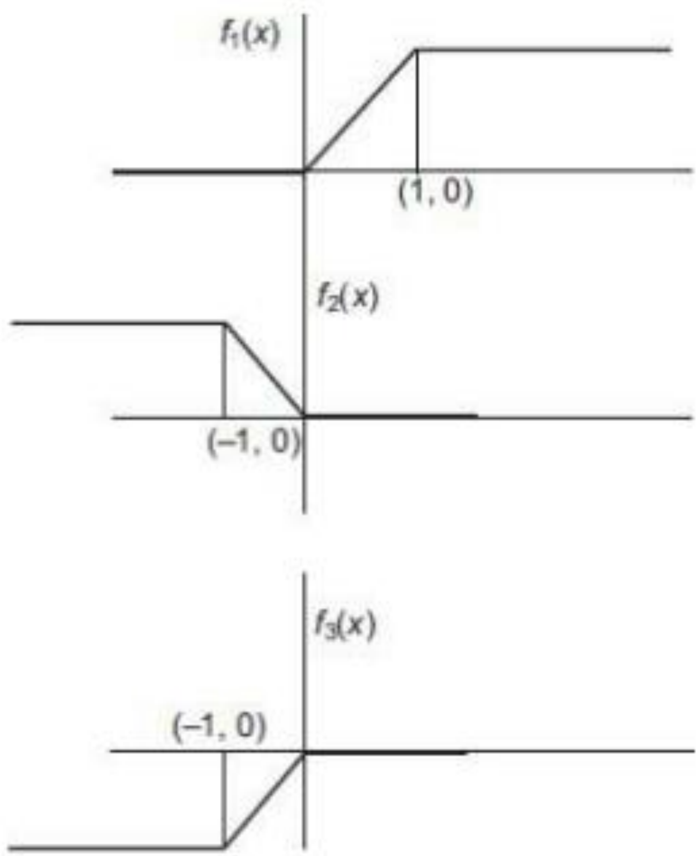
Only option (b) satisfies the above inequality.

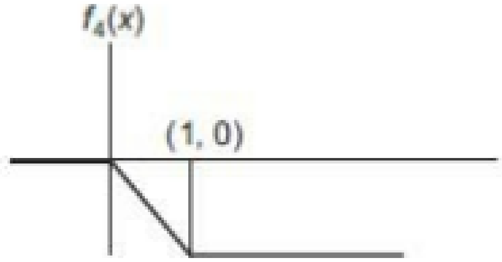
Solutions for Questions 48 and 49:

The best method to solve this problem is the graphical approach

$$\begin{aligned} f_1(x) &= x && 0 \le x \le 1 \\ &= 1 && x \ge 1 \\ &= 0 && \text{Otherwise} \end{aligned}$$

$$\begin{aligned} f_2(x) &= f_1(-x) && \text{for all } x \\ f_3(x) &= -f_2(x) && \text{for all } x \\ f_4(x) &= f_3(-x) && \text{for all } x \end{aligned}$$





48. From the above graphs it can be seen that $f1(x)f2(x) = 0$ and $f2(x)f4(x) = 0$ for all values of x. Hence, option (c) is the correct answer.

49. From the graphs it can be seen that option (b) is correct.

50. Since 28 = 256 = 44 = 162 = xu, the possible sets of values which would satisfy the given equation are 28, 44 or 162.

Possible values of (x, u) are (2, 8), (4, 4), (16, 2).

Now by putting these values one by one in the given equation we get only (4, 4) satisfies the given equation. Hence, option (b) is correct.

51. $(x - k)2 + y2 = 1$

$$x^2 + k^2 - 2kx + y^2 = 1$$

As $x^2 = y^2$, $x^2 + k^2 - 2kx + x^2 = 1$

$$2x^2 - 2kx + k^2 - 1 = 0$$

This is a quadratic equation and the question says that the solution is unique and positive so $D = 0$, which gives us: $4k2 - 8(k2 - 1) = 0$

Or $k = \sqrt{2}$

For $k = \sqrt{2}$ the sum and the product of the roots of the given equation are positive.

So $k = \sqrt{2}$ is the required answer.

52. We are given a starting relationship for the function $g(x)$, that says that:

$g(x + 1) + g(x - 1) = g(x)$

Now, in this expression, if we put $x = x + 1$ we get $g(x + 2) + g(x) = g(x + 1)$. From these two relationships, we see that:

$g(x) - g(x - 1) = g(x + 2) + g(x) \rightarrow g(x + 2) = -g(x - 1)$. Again by putting $x = x + 1$ we get: $g(x + 3) = -g(x)$.

By putting $x = x + 3$ we get $g(x + 6) = -g(x + 3) = g(x)$. Thus, we realise that $g(x) = g(x + 6)$. Hence, $p = 6$.

Note: This can be solved much simpler by taking a starting value of x and proceeding as follows: If $x = 1$ we get:

$g(1) = g(2) + g(0) \rightarrow g(0) = g(1) - g(2)$. Also, if $x = 2$ we get: $g(2) = g(1) + g(3) \rightarrow g(3) = g(2) - g(1)$. It can be seen that $g(0) = -g(3)$. By the same logic $g(6) = -g(3)$. This essentially means that $g(6) = g(0)$. Hence, if $p = 6$, we get $g(x + p) = g(x)$. Option (d) is the correct answer.

53. From the original graph it can be seen that when $y + x = 0$, $y - x = 0$. So, obviously if we take the value of x as 0, the value of y will also be 0 in the correct $X - Y$ graph. So, the correct graph has to pass through the origin. Hence, option (a) is rejected. Further looking at the original graph, we can see that y-x grows faster than $y + x$ when we move right on the $y + x$ axis. When $y + x = 1$, $y - x = 2$ (we can say this since the question says that all the graphs are drawn to scale). Thus, we need a point on the correct graph where, if $y = 1.5$, $x = -0.5$. Only the graph in option (d) satisfies this and hence option (d) is the correct answer.

54. Put $x = 8$ in the given inequality, we get:

$\rightarrow (8)2/3 + (8)1/3 - 2$

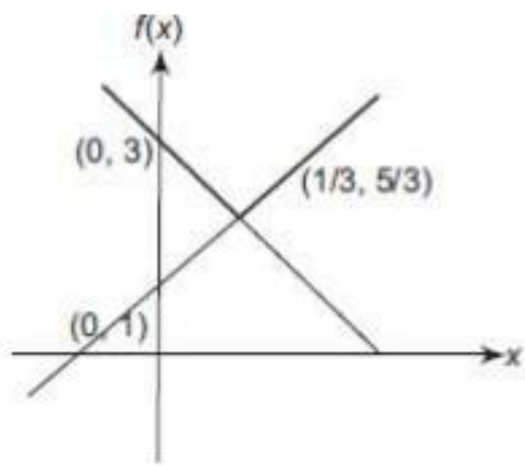
$\rightarrow 4 + 2 - 2 = 4$

So, $x = 8$ does not satisfy the given inequality.

Hence option (b), (c), (d) & (e) are wrong.

So, only option (a) is correct.

55. $f(x) = \max(2x + 1, 3 - 4x)$ → The following graph would depict the correct picture for the function ($f(x)$).



The intersection point of the two straight lines would be got by equating the two lines:

Thus, we have: $2x + 1 = 3 - 4x$

→ $6x = 2$

Hence, $x = 1/3$

$f(x)$at $x = 1/3 = 2.(1/3) + 1 = 5/3$. This is the minimum value of the function.

From the above curve it is clear that $f(x)$ (which is shown by the dark line) attains its minimum value at $x = 1/3$ and it's value is $5/3$ at $x = 1/3$. Hence, the correct option is option (e).

56. Logz $y = b$ or $y = z^b$

Logx $z = a$ or $z = x^a$

And $x = y^{ab}$

Substituting options in these expressions:

For option (a): $x = y^{-1}$ – (a), $y = z^{1/2}$ – (b) and $z = x$–2 – (c). Putting (c) in (b), we get that $y = x^{-1}$ which confirms the original expression. Hence, option (a) is possible and hence cannot be the answer.

For option (b): $x = y$, $y = z$ and $z = x$. Again all relationships are maintained. Hence, option (b) is possible and hence cannot be the answer.

For option (c): $x = y$, $y = z^{2.5}$ and $z = x^{0.4}$. Substituting the value of z from the third expression into the second, we again get $y = x$, which is consistent with the first expression viz: $x = y$. Hence, option (c) is possible and hence cannot be the answer.

For option (d): we get: $x = y$, $y = z^{1/a}$ and $z = xa$. Substituting the value of z from the third expression into the second, we again get $y = x$, which is consistent with the first expression viz: $x = y$. Hence, option (d) is possible and hence cannot be the answer. For option (e) we get: $x = y4$, $y = z^2$ and $z = x^2$. Substituting the value of z from the third expression into the second, we get $y = x4$, which is not consistent with the first expression viz: $x = y4$. Hence, option (e) is not a possible set of values for (a, b) and hence is the correct answer.

57. $f(1) + f(2) = 4f(2) \rightarrow f(1) = 3f(2) \rightarrow f(2) = 1200$

$f(1) + f(2) + f(3) = 9\,f(3) \rightarrow f(3) = 4800/8 = 600.$

$f(1) + f(2) + f(3) = 15f(4) \rightarrow f(4) = 5400/15 = 360$

$f(1) + f(2) + f(3) + f(4) = 24f(5) \rightarrow 5760/24 = f(5)$

= 240

At this stage, if you look at the chain of values you've created, you would see that the numbers are:

3600, 1200, 600, 360 and 240. Notice that $3600 \times 1/3 = 1200$; $1200 \times 2/4 = 600$; $600 \times 3/5 = 360$; $360 \times 4/6 = 240$. Thus, the next numbers in the series would be:

$f(6) = 240 \times 5/7 = 1200/7$

$f(7) = 1200/7 \times 6/8 = 900/7$

$f(8) = 900/7 \times 7/9 = 100$ and

$f(9) = 100 \times 8/10 = 80$.

Option (a) is correct.

58. There are three things given in the question. Since $f(x)$ is a quadratic function, we can start with let $f(x) = ax^2 + bx + c$;

At $x = 0$, $f(x) = 1$ can be interpreted as: $f(0) = 1$. But $f(0)$ for $ax^2 + bx + c$ would be equal to c. Hence, $c = 1$. Thus, the function becomes $f(x) = ax^2 + bx + 1$

Next, we are given that $f(1) = 3$. Thus, $f(1) = 3 = a + b + c$

or $a + b + 1 = 3$ or $a + b = 2$ (1)

Also, according to the question $f(x)$ attains its maximum value at $x = 1$

So at $x = 1$, $f'(x) = 0$. Also, we know that $f'(x) = 2ax + b$. At $x = 1$, we get $2a + b = 0$ or

$b = -2a$ (2)

Using (1) and (2) we get: $a = -2$ and $b = 4$. Thus, the quadratic function is:

$f(x) = -2x^2 + 4x + 1$

At $x = 10$ we get:

$f(10) = -200 + 40 + 1 = -159$

Option (b) is correct.

59. $f(x) = ax^2 + bx + c$ where a is not equal to 0 (means that this is a strictly quadratic function).

As 3 is a root of the equation, $f(x) = 0$ we have $f(3) = 0$. This gives us:

$9a + 3b + c = 0 \quad (1)$

Next, we are given that: $f(5) = -3f(2)$. This gives us:

$$25a + 5b + c = -3[4a + 2b + c]$$
$$25a + 5b + c = -12a - 6b - 3c$$
$$37a + 11b + 4c = 0 \qquad (2)$$

→ equation (2) –4 × equation (1) gives us

→ $a - b = 0$ or $a = b$

Sum of roots of $f(x) = -b/a = -b/b = -1$

Since, one root is given as 3, the other root must be –4. (Since, 3 + other root = –1)

60. This question is asking us to find the value of $a + b + c$. This is indeterminate – as there are infinite quadratic equations having roots as 3 and –4. This can be thought of mathematically as follows:

$$f(x) = a(x - 3)(x + 4)$$
$$= a(x^2 + x - 12) = ax^2 + ax - 12a$$

$b = a, c = -12a$

$a + b + c = a + a - 12a = -10a$

As we can see, the value of $a + b + c$ depends on the value of 'a' which is not known. So the value of '$a + b + c$' cannot be determined.

Option (e) is correct.

61. $f(x) \cdot f(y) = f(xy)$

Put $x = 2, y = 1$

$f(2) \cdot f(1) = f(2)$

$f(1) = 1$

Now put $x = 2, y = 1/2$

we get $f(2) \cdot f(1/2) = f(2 \cdot 1/2) = f\ (1)$

$f(1/2) = f(1)/f(2) = 1/4$

Option (b) is correct.

62. Let the roots of the equations are a – 1, a, a + 1. For a cubic equation of the form:

$x3 - ax2 + bx - c = 0$, the sum of the pair wise product of the roots would be given by b.

Thus, $b = \alpha(\alpha - 1) + (\alpha - 1)(a + 1) + a(a + 1)$

$b = \alpha2 + \alpha2 - 1 + \alpha2 = 3\alpha2 - 1$

$b = 3\alpha2 - 1$

$3\alpha2 > 0$. To minimize the value of b we need to minimize the value of $3\alpha2$ and minimum possible value of $3\alpha2 = 0$

b_{min} = –1. Hence, option (b) is correct.

Alternate Solution:

Since the question represents a cubic expression, and we want the smallest possible value of b, keeping the constraint of their roots being three consecutive values—a little bit of guess estimation would lead you to think of –1, 0 and 1 as the three roots for minimising the value of b.

Thus, the expression would be $(x + 1)(x)(x - 1) = (x^2 + x)(x - 1) = x^3 - x$. This gives us the value of b as –1.

It can be seen that changing the values of the roots from –1, 0 and 1 would result in increasing the coefficient of x—which is not what we want. Hence, the correct answer should be –1, option (b). (**Note:** For trial purposes if you were to take the values of the three roots as 0, 1 and 2, the expressions would become $x(x - 1)(x - 2) = (x^2 - x)(x - 2)$ which would lead to the coefficient of x being 2. This would obviously increase the value of the coefficient of x above –1.)

You could also go for changing the three consecutive integral roots in the other direction to –2, –1 and 0. In such a case the expression would become: $x(x + 2)(x + 1) = (x^2 + 2x)(x + 1)$ which would again give us the coefficient of x as +2.]

The total solving time for this question would be 30 seconds if you were to hit on the right logic for taking the roots as –1, 0 and 1. In case, you had to check for the value of b in different situations by altering the values of the roots (as explained above), the time would still be under 2 minutes.

63. No need to solve this question mathematically, we can solve it just by estimating the value ranges of $\log_3 5$ and then matching them with the

value range of $\log_5(2 + x)$ by picking up values of x from the options.

The first thing we realise is that: $\log_3 9 > \log_3 5 > \log_3 3$.

$2 > \log_3 5 > 1$. Hence, we want $\log_5(2 + x)$ to lie between 1 to 2. The only option that gives us the value of $\log_5(2 + x)$ between 1 to 2 would be the fourth one. Going through the other options (a to c) we can reject them as follows:

⇒ **Option (a):** When $0 < x < 3$ then $\log_5(2 + x)$ will always less than 1 because $2 + x = 2 + 3 = 5$ and $\log_5(2 + 3) = \log_5 5 = 1$. This option is not correct.

⇒ **Option (b):** When $23 < x < 29$, then minimum value of $\log_5(2 + x) = \log_5(2 + 23) = \log_5 25 = 2$. So this option is also not correct. Similarly option (c) is also not correct.

Hence, option (d) is correct.

64. $f(x) = x^2$, $g(x) = 2^x$

$$f(f(g(x)) + g(f(x)) = (2^{2x} + 2^{x^2})^2$$

At $x = 1$, $f(f(g(x)) + g(f(x)) = (2^{2x} + 2^{x^2})^2 = (2^{2 \times 1} + 2^{1^2})^2 = (4 + 2)^2 = 36$.
Hence, option (c) is correct.

65. Let the values of roots of the equation be p and q.

$p^2 + q^2 = (p + q)^2 - 2pq = (a + 3)^2 + 2(a + 5) = a^2 + 9 + 6a + 2a + 10$

$= a^2 + 8a + 16 + 3 = (a + 4)^2 + 3$

Minimum possible value of $(a + 4)^2 + 3$ is 3 (when we take $a = -4$). Hence, option (c) is correct.

66. By checking the options we can see that for $x = 3/2$ the LHS and RHS of the above equation are equal to each other. So, option (a) is correct.

67. Arithmetic mean = [log $(2^2 \times 3^3 \times 5)$ + log$(2^6 \times 3 \times 5^7)$

$+ \log (2 \times 3^2 \times 5^4)]/3 = \log (2^9 \times 3^6 \times 5^{12})^{\frac{1}{3}} = \log(2^3 \times 3^2 \times 5^4) = \log(2^a \times 3^b \times 5^c)$

By comparing we get $a = 3$.

68. $f(ab) = f(a)f(b)$

Put $a = 1, b = 0$

$f(0) = f(1)f(0)$

$\rightarrow f(1)=1$.

69. $|f(x) + g(x)| = |f(x)| + |g(x)|$

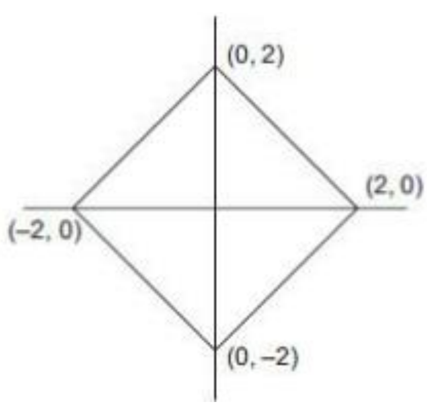
$f(x) = 2x - 5, g(x) = 7 - 2x$

$|2x - 5 + 7 - 2x| = |2x - 5| + |7 - 2x|$

$2 = |2x - 5| + |7 - 2x|$

By checking the options, we get that the above equation satisfies for any value of x which lies in between 5/2 and 7/2 (including both the values). So, option (d) is correct.

70. Curve of $|x| + |y| = 2$ is as shown below:



There are four triangles with base 2 and height 2 in this figure. The area of each triangle is 2. Hence, the area of the enclosed region is 2 × 4 = 8

71. $\log_3 x = \log_{12} y = a$

Or $x = 3a, y = 12a$

$$G = \sqrt{xy} = \sqrt{3^a \cdot 12^a} = \sqrt{6^{2a}} = 6^a$$

$$\log_6 G = \log_6 6^a = a$$

72. $x + 1 = x2$

$$x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

According to the question, $x > 0$, hence, $x = \frac{1+\sqrt{5}}{2}$

$$2x^4 = 2\left[\frac{1 \pm \sqrt{5}}{2}\right]^4 = \frac{2}{16}[1 + 5 + 2\sqrt{5}]^2 = \frac{1}{8}[6 + 2\sqrt{5}]^2$$

$$= \frac{1}{8}[36 + 20 + 24\sqrt{5}] = \frac{1}{8}[56 + 24\sqrt{5}] = 7 + 3\sqrt{5}$$

Option (d) is correct.

Alternately, from the point where we got the value of $x = \frac{1+\sqrt{5}}{2}$, we can use the value of the square root of 5 as 2.23 and check with the options to get the correct answer.

73. $\log_{0.008}\sqrt{5} = \frac{1}{2}\log_{0.008}5 = 1/6 \log_{0.2}5 = 1/(6\log_5 0.2) = 1/(6\log_5 1/5) = -1/6$

$$\log_{\sqrt{3}}81 = 2\log_3 3^4 = 8.$$

$$\text{Required answer} = -\frac{1}{6} + 8 - 7 = \frac{5}{6}$$

74. $9^{2x-1} - 81^{x-1} = 9^{2x-1} - 9^{2x-2} = 9^{2x-2}(9 - 1) = 1944$

$92^{x-2} = 243$

$34^{x-4} = 35$ or $4x - 4 = 5$ or $x = 9/4$

75. $f1(x) = x^2 + 11x + n$ and $f2(x) = x$

$x^2 + 11x + n = x$

$x^2 + 10x + n = 0$

If the above equation has two distinct real roots then $b^2 - 4ac > 0$

$102 - 4n > 0$

$100 - 4n > 0$ or $n < 25$

So the largest possible value of n is 24.

76. $f(3) = \frac{5 \times 3 + 2}{3 \times 2 - 5} = \frac{17}{4}$

Similarly $f(17/4) = 3$.

$g(3) = 32 - 3 \times 2 - 1 = 2$

77. It is obvious that $f(10) + f(11) = f(12)$ would have to be used in this question. However, the hurdle involved in getting $f(10)$ is to find the value of $f(12)$. Also, since we are given the value of $f(15)$, we can see that we would need to move from $f(11)$ towards $f(15)$. The following thought-process would give us the solution.

Given, $f(x + 2) = f(x) + f(x + 1)$

$f(11) = 91, f(15) = 617$

We get, $91 + f(12) = f(13)$

Let, $f(12) = X$

$X + 91 + X = f(14)$

So, $f(14) = 2X + 91$. Also, $f(13) + f(14) = f(15) = 617$

So, $91 + X + 2X + 91 = 617$

$3X + 182 = 617$

$X = 145$

Substituting the value of X and $f(11)$, in $f(12) = f(10) + f(11)$, we get

$f(10) + 91 = 145$

$f(10) = 54$

78. $\log_2 (5 + \log_3 a) = 3 \Rightarrow 5 + \log_3 a = 2^3 \Rightarrow \log_3 a = 3$ or $a = 27$

$\log_5(4a + 12 + \log_2 b) = 3$

$4a + 12 + \log_2 b = 5^3 = 125$ $(4 \times 27) + 12 + \log_2 b = 125$

$120 + \log_2 b = 125$

$\log_2 b = 5$

$b = 2^5 = 32$

$a + b = 27 + 32 = 59$

Hence, option (d) is correct.

79. $2^x = 3^{\log_5 2}$

$\log_3 2^x = \log_3 3^{\log_5 2}$

$x \log_3 2 = \log_5 2$

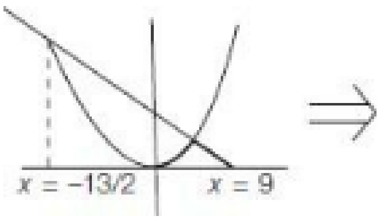
$x\log_2 5 = \log_2 3$

$\log_2 5^x = \log_2 3$

$5^x = 3$

$x = \log_5 3 = 1 + \log_5\left(\frac{3}{5}\right)$

Hence, option (d) is correct.

80. 

$2x^2 = 52 - 5x$

$2x^2 + 5x - 52 = 0$

$2x^2 + 13x - 8x - 52 = 0$

$x(2x + 13) - 4(2x + 13) = 0$

$x = 4$ or $-13/2$

for positive real value we should ignore the left portion of the above curve.

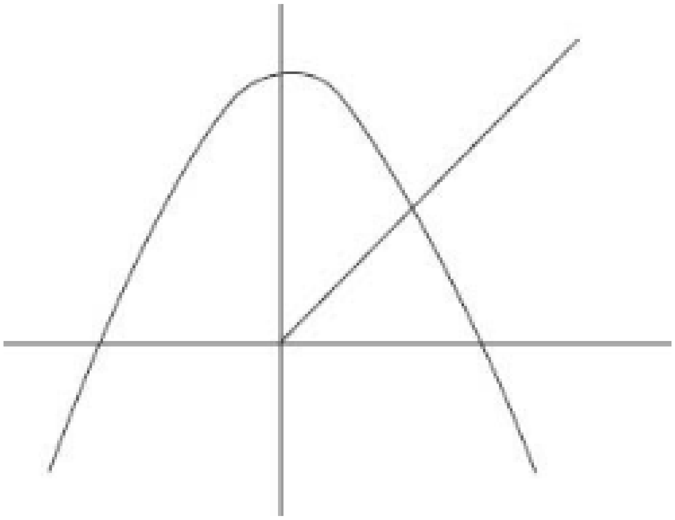
From the above curve it is clear that a $f(x)$ is maximum at $x = 4$.

$f(x)/x = 4 = 2(4)^2 = 32$

81. $n^3 - 11n^2 + 32n - 28 > 0$ $(n-7)(n-2)^2 > 0$ $(n-2)^2$ is always greater than 0.

Hence, $n - 7 > 0$ or $n > 7$

So the smallest possible integer must be 8.

82. 

For $x > 0$, $5x$ is an increasing function and $52 - 2x^2$ is a decreasing function. So, the minimum value of $f(x)$ is the value of $f(x)$ at the intersection point of $5x$ and $52 - 2x^2$.

$5x = 52 - 2x^2$

Or $x = -13/2, 4$

At $x = 4$, value of $f(x) = 20$.

83. $2x^2 - ax + 2 > 0$, means that the roots of the equation $2x^2 - ax + 2 = 0$ must be imaginary. For that to occur: $a^2 - 16 < 0$. So, possible values of 'a' are: $-4 < a < 4$. In other words, possible values for x are: –3, –2, –1, 0, 1, 2 or 3.

Likewise, for $x^2 - bx + 8 \geq 0$, $b^2 - 32 <= 0 \rightarrow$ values of 'b' should be –5, –4, …0,1,2,3,4,5.

For $2a - 6b$ to be maximum, we have: $2 \times 3 - 6 \times (-5) = 6 + 30 = 36$.

84. Let, $p^3 = q^4 = r^5 = s^6 = k^{60}$. Thus, $p = k^{20}$, $q = k^{15}$, $r = k^{12}$ and $s = k^{10}$.

Then, $\log_s (pqr) = 47/10$. Hence, option (c) is correct.

85. The given expression can be written as:

$\log_{100}2 - \log_{100}4 + \log_{100}5 - \log_{100}10 + \log_{100}20 - \log_{100}25 + \log_{100}50$

$$=\log_{100} \frac{50\times 20\times 5\times 2}{25\times 10\times 4} = \log_{100}10 = \frac{1}{2}$$

86. Check the options:

$2^{6x} + 2^{3x+2} - 21 = 0$

$2^{6x} + 2^2 \cdot 2^{3x} - 21 = 0$

Option (a): Put $x = \frac{\log_2 3}{3}$

$= 2^{\frac{6}{3}\times \log_2 3} + 4 \bullet 2^{3\times \frac{\log_2 3}{3}} - 21$

$= 2^{2\log_2 3} + 4 \bullet 2^{\log_2 3} - 21$

$= 2^{\log_2 3^2} + 4 \bullet 2^{\log_2 3} - 21$

$3^2 + 4 \cdot 3 - 21$

$9 + 12 - 21 = 0$

Hence, option (a) is correct.

87. The expression can be written as:

$$\frac{(n+3)(n+4)}{(n-4)(n+3)} = \frac{(n+4)}{(n-4)}$$

Checking the options, for $n = 16$, we get the value as $\frac{20}{12}$, which is not an integer.

For $n = 12$, we get $\frac{15\times 16}{8\times 15}$, which is an integer. Hence, the largest value of 'n' at which the given expression becomes an integer is $n = 12$.

88. $f(24) = f(2 \times 2 \times 2 \times 3) = [f(2)]^3 \times f(3) = 3^3 \times 2$. Since $f(1), f(2)$ and $f(3)$ are integers, we get on comparing: $f(2) = 3, f(3) = 2$. The value of $f(18)$ can be obtained using $f(18) = f(3 \times 3 \times 2) = [f(3)]^2 \times f(2) = 2^2 \times 3 = 12$.

89. The roots of $x^2 - 4x - \log_2 A = 0$, will be real and distinct if an only if $16 + 4\log_2 A > 0$.

Hence, $\log_2 A > -4 \rightarrow A > 2^{-4}$. Thus, $A > 1/16$ is the correct range of A. Hence, option (c) is correct.

90. The sum of the roots would be: $4a + 3a = 7a$. This would be equal to '$-b$'. Thus, we get that $b = -7a$.

Also, the value of 'c' can be calculated using the logic of the product of the roots →

$c = 4a \times 3a = 12a^2$

$b^2 + c = 49a^2 + 12a^2 = 61a^2$

As 'a' is an integer so a^2 must be an integer and perfect square which is possible for option (b) only.

91. $5^x - 3^y = 13438$

$5^{x-1} + 3^{y+1} = 9686$ or $0.2 \times 5^x + 3 \times 3^y = 9686$

On solving the above two equations, we get:

$$5^x = 15625 \rightarrow x = 6$$

$$3^y = 2187 \rightarrow y = 7$$

Hence, $x + y = 6 + 7 = 13$.

92. Assuming 'm' is even:

$8 \times f(m + 1) - f(m) = 2$

$8 \times (m + 1 + 3) - m \times (m + 1) = 2$

$8(m + 4) - m \times (m + 1) - 2 = 0$

$8m + 32 - m^2 - m - 2 = 0 \rightarrow$

$m^2 - 7m - 30 = 0 \rightarrow m = 10, -3$

Rejecting the negative value, we get $m = 10$.

If 'm' is odd on the other hand, we do not get integral values of m, and hence this case can be ignored.

93. $\log_5(x + y) + \log_5(x - y) = 3$

$\log_5(x + y)(x - y) = 3$ $(x^2 - y^2) = 5^3 = 125$ (1)

$\log_2 y - \log_2 x = 1 - \log_2 3$

$$\log_2\left[\frac{3y}{2x}\right] = 0$$

$$\frac{3y}{2x} = 1$$

$3y = 2x$ (2)

On solving equation (1) and equation (2), we get:

$x = 15, y = 10$

$xy = 150$

94. For $x \geq 0$, $x \times (6x^2 + 1) = 5x^2$

$x \times (6x^2 + 1 - 5x) = 0.$

For this situation, we have either $x = 0$ or $6x^2 - 5x + 1 = 0$. The roots of this quadratic equation are 0.33 and 0.5. Thus, for $x \geq 0$, we have three solutions for the given equation.

For $x < 0$, $-x \times (6x^2 + 1) = 5x^2$

$x \times (6x^2 + 1 + 5x) = 0$. For this to be true $6x^2 + 5x + 1 = 0$ should be satisfied (and that too only for x negative). The roots of the equation are: –0.5 and –0.33. Thus, there are five solutions to the given equation.

95. $|x^2 - x - 6| = x + 2$

For $x^2 - x - 6$ > greater than or equal to 0.

$$x^2 - x - 6 = x + 2$$

$$x^2 - 2x - 8 = 0$$

Roots = 4, –2

For $x^2 - x - 6 < 0$

$$-x^2 + x + 6 = x + 2$$

$$x^2 = 4$$

Roots = 2, –2. But, we will take only 2 as the root that satisifes the starting condition.

Product of roots = $4 \times 2 \times -2 = -16$

Hence, option (b) is correct.

96. $f(2) = f(1 + 1) = f(1) \cdot f(1) = 2^2$

$f(3) = f(2 + 1) = f(2) \cdot f(1) = 2^2 \cdot 2 = 2^3$

Similarly $f(n) = 2^n$

$f(a + 1) + f(a + 2) + f(a + 3) + \ldots\ldots + f(a + n) = 2^{a+1} + 2^{a+2} + \ldots\ldots\ldots + 2^{a+n} =$ $2^a(2 + 2^2 + 2^3 + \ldots\ldots + 2^n)$

$= 2a[2 \cdot (2n - 1)] = 16(2n - 1)$. Comparing LHS and RHS, we get:

$2^{a+1} = 16 = 2^4 \rightarrow a = 3$

97. $\sqrt{\log_e \frac{4x - x^2}{3}}$ is a real number if :

$$\log_e \frac{4x - x^2}{3} \geq 0$$

$$\frac{4x - x^2}{3} \geq e^0$$

$$\frac{4x - x^2}{3} \geq 1$$

$$4x - x^2 \geq 3$$
$$x^2 - 4x + 3 \leq 0$$
$$x^2 - 3x - x + 3 \leq 0$$
$$x(x - 3) - 1(x - 3) \leq 0$$
$$(x - 3)(x - 1) \leq 0$$

or $1 \leq x \leq 3$

Hence, option (c) is correct.

98. We have, $(5.55)^x = (0.555)^y = 1000$

Taking log in base 10 on both sides:

$x \log_{10} 5.55 = y \log_{10} 0.555 = \log_{10} 1000 = 3$

$x [\log_{10} 555 - 2] = 3$ (1)

$y [\log_{10} 555 - 3] = 3$ (2)

From equation (1) and equation (2)

$$\log_{10} 555 = \frac{3}{x} + 2 = \frac{3}{y} + 3$$

$$3\left[\frac{1}{x} - \frac{1}{y}\right] = 1$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{3}$$

Hence, option (b) is correct.

BLOCK VI
COUNTING

Chapter 17 – **Permutations and Combinations**

Chapter 18 – **Probability**

Chapter 19 – **Set Theory**

BACK TO SCHOOL...

In my experience, students can be divided into two broad categories on the basis of their ability in solving chapters of this block.

Category 1: Students who are comfortable in solving questions of this block, since they understand the underlying concepts well.

Category 2: Students who are not able to tackle questions of this block, since they are not conversant with the counting tools and methods in this block.

If you belong to the second category of students, the main thing you would need to do is to familiarise yourself with the counting methods and techniques of Permutations and Combinations. Once you are through with the same, you would find yourself relatively comfortable at both Permutations and Combinations (P & C) and Probability—the chapters in this block which create the maximum problems for students. Set Theory being a relatively easier chapter, the Back to School section of this block concentrates mostly on *P* & C and Probability. However, before we start looking at these counting methods, right at the outset, I would want you to remove any negative experiences you might have had while trying to study (P & C) and Probability. So if you belong to the second category of students, you are advised to read on:

Look at the following table:

Suppose, I was to ask you to count the number of cells in the table above, how would you do it??

$$5 \times 5 = 25!!$$

I guess, all of you realise the fact that the number of cells in this table is given by the product of the number of rows and columns. However, if you ask a five-year old child to count the same, he would be counting the number of cells physically. In fact, when you were five-years old, you would also have required doing a physical count of the number of cells in the table. However, there must have come a point where you must have understood that in all such situations (number of cells in a table, number of students in a class, etc.) the total count is obtained by simply multiplying the number of rows and the number of columns. What I am interested in pointing out to you, is that the discovery of this process for this specific counting situation surely made your counting easier!! What used to take you much longer started taking you shorter times. Not only that, in situations where the count was too large (e.g. 100 rows × 48 columns) an infeasible count became extremely easy. So why am I telling you this??

Simply because just as the rows into columns tool for counting had the effect of making your count easier, so also the tools for counting which P&C describe will also have the effect of making counting easier for you in the specific situations of counting that you will encounter. My experience of training the students shows that the only reason they have problems in this block is because it is not been explained properly to them in their +2 classes.

One must try to understand these chapters with the approach that they are not meant to complicate your life but to simplify it; you might end up finding out that there is nothing much to fear in this chapter.

TOOL BOX APPROACH TO P & C

While studying *P* & C, your primary objective should be to familiarise yourself with each and every counting situation. You also need to realise that there are a limited number of counting situations which you need to tackle. You can look at this as the process of the creation of what can be described as a counting tool box.

Once this tool box is created, you would be able to understand the basic situations for counting. Then solving tough questions becomes a matter of simplifying the language of the question into the language of the answer—a matter I would come back to later.

Let us now proceed to list out the specific counting situations which you need to get a hold on. You need to know and understand the following twelve situations of counting and the tools that are used in these situations. Please take a note that these tools are explained in detail in various parts of the text on the Permutations and Combinations chapter. You are required to keep these in mind along with the specific situations in which these apply. Look at these as a kind of comprehensive list of situations in which you should know how to count using mathematical tools for your convenience.

Twelve Counting Situations and Their Tools

Tool No. 1: **The $_nC_r$ tool**—This tool is used for the specific situation of counting the number of ways of selecting *r* things out of *n* **distinct** things.

Example: Selecting a team of eleven cricketers out of a team of sixteen distinct players will be $_{16}C_{11}$.

Tool No. 2: The tool for counting the number of selections of *r* things out of *n* **identical** things. (Always 1)

Example: The number of ways of selecting three letters out of five *A*'s. Note that in such cases there will only be one way of selecting them.

Tool No. 3: The 2^n tool—The tool for selecting any number (including 0) of things from *n* **distinct** things ($_nC_0 + {_nC_1} + {_nC_2} \ldots\ldots + {_nC_n} = 2^n$).

Example: The number of ways you can or cannot eat sweets at a party, if there are ten sweets and you have the option of eating one piece of as many sweets as you like or even of not eating any sweet. (Will be given by 2^{10}).

Corollary: The $2^n - 1$ tool—Used when zero selections are not allowed. For instance, in the above situation, if you are asked to eat at least one sweet and rejecting all sweets is not an option.

Tool No. 4: The $n + 1$ tool—The tool for selecting any number (including 0) of things from *n* **identical** things. For instance, if you have to select any number of letters from A, A, A, A and A, then you can do it in six ways.

Tool No. 5: **The $_{n+r-1}C_{r-1}$ tool**—The tool for distributing *n* identical things amongst *r* people/groups such that any person/group might get any number (including 0). For instance, if you have to distribute seven identical gifts amongst five children in a party then it can be done in $_{(7+5-1)}C_{(5-1)}$ ways, i.e. $_{11}C_4$ ways.

Tool No. 6: **The $_{n-1}C_{r-1}$ tool**—The tool for distributing *n* identical things amongst *r* people/groups such that any person/group might get any number (except 0).

Suppose in the above case you have to give at least one gift to each child; it can be done in: $_6C_4$ ways.

Tool No. 7: **The mnp rule tool**—The tool for counting the number of ways of doing three things when each of them has to be done and there are *m* ways of doing the first thing, *n* ways of doing the second thing and *p* ways of doing the last thing. Suppose you have to go from Lucknow to Varanasi to Patna to Ranchi to Jamshedpur and there are five trains from Lucknow to Varanasi and there are four trains from Varanasi to Patna, six from Patna to Ranchi and three from Ranchi to Jamshedpur, then the number of ways of going from Lucknow to Jamshedpur through Varanasi, Patna and Ranchi is $5 \times 4 \times 6 \times 3 = 360$.

Note that this tool is extremely crucial and is used to form numbers and words.

For example, how many four-digit numbers can you form using the digits 0, 1, 2, 3, 4, 5 and 6 only without repeating any digits? In this situation, the work outline is to choose a digit for the first place (=6, any one out of 1, 2, 3, 4, 5 and 6 as zero cannot be used there), then choosing a digit for the second place (=6, select any one out of 5 remaining digits plus 0), then a digit for the third place (five ways) and a digit for the fourth place (four ways).

Tool No. 8: **The *r*! tool for arrangement**—This tool is used for counting the number of ways in which you can arrange *r* distinct things in *r* places. Notice that this can be derived out of the mnp rule tool.

Tool No. 9: **The AND rule tool**—Whenever you describe the counting situation and connect two different parts of the count by using the conjunction 'AND', you will always replace the 'AND' with a multiplication sign between the two parts of the count. This is also used for solving probability questions. For instance, suppose you have to choose a vowel 'AND', a consonant from the letters of the word PERMIT, you can do it in ${}_4C_1 \times {}_2C_1$ ways.

Tool No. 10: **The OR rule tool**—Whenever you describe the counting situation and connect two different parts of the count by using the conjunction '**OR**, you will always replace the '**OR**' with an addition sign between the two parts of the count. This is also used for solving Probability questions. For instance, suppose you have to select either two vowels or a consonant from the letters of the word PERMIT, you can do it in ${}_4C_2 + {}_2C_1$ ways.

Tool No. 11: **The Circular Permutation Tool [(n – 1)! Tool]**—This gives the number of ways in which n distinct things can be placed around a circle. The need to reduce the value of the factorial by 1 is due to the fact that in a circle, there is no defined starting point.

Tool No. 12: **The Circular Permutation Tool when there is no distinction between clockwise and anti- clockwise arrangements [(n – 1)!/2 tool]**—This gives the number of ways in which n distinct things can be placed around a circle when there is no distinction between clockwise and anti-clockwise arrangements. In such a case the number of circular permutations is just divided by 2.

Two More Issues

(1) What about the $_nP_r$ tool? The ${}_nP_r$ tool is used to arrange r things amongst n. However, my experience shows that the introduction of this tool just adds to the confusion for most students. A little bit of smart thinking gets rid of all the problems that the ${}_nP_r$ tool creates in your mind. For this let us consider the difference between the $\mathbf{{}_nP_r}$ and the $\mathbf{{}_nC_r}$ tools.

$$\mathbf{{}_nC_r} = n!/[r! \times (n-r)!] \text{ while } \mathbf{{}_nP_r} = n!/[(n-r)!]$$

A closer look would reveal that the relationship between the ${}_nP_r$ and the ${}_nC_r$ tools can be summarised as:

$$ {}_nP_r = {}_nC_r \times r! $$

Read this as: If you have to arrange *r* things amongst *n* things, then simply select *r* things amongst *n* things and use the r! tool for the arrangement of the selected *r* items amongst themselves. This will result in the same answer as the ${}_nP_r$ tool.

In fact, for all questions involving arrangements, always solve the question in two parts—first finish the selection within the question and then arrange the selected items amongst themselves. This will remove a lot of confusions in questions of this chapter.

(2) Treat both permutations and combinations and probability as English language chapters rather than Maths chapters: One of the key discoveries in getting stronger at this block is that after knowing the basic twelve counting situations and their tools (as explained above), you need to stop treating the questions in this chapter as questions in Mathematics and rather treat them as questions in English. Thus, while solving a question in these chapters, your main concentration and effort should be on converting the question language into the answer language.

What do I mean by this?

Consider this:

Question Language: What is the number of ways of forming four-letter words from five vowels and six consonants, such that the word contains one vowel and three consonants?

To solve this question, first create the answer language in your mind.

Answer Language: Select one vowel out of five **AND** select three consonants out of six **AND** arrange the four selected letters amongst themselves.

Once you have this language, the remaining part of the answer is just a matter of using the correct tools.

Thus, the answer is ${}_5C_1 \times {}_6C_3 \times 4!$

As you can see, the main issue in getting the solution to this question was working out the language by connecting the various parts of the solution using the conjunction AND (In this case, the conjunction OR was not required.). Once, you had the language all you had to do was use the correct tools to count.

The most difficult of questions are solved this way. This is why, I advise you to treat this chapter as a language chapter and not as a chapter in Mathematics!!

Pre-assessment Test

1. How many words of eleven letters could be formed with all the vowels present in even places, using all the letters of the alphabet? (without repetitions)

   (a) ${}_{21}P_6.5!$

   (b) 21!

   (c) ${}_{21}P_5(5!)$

   (d) ${}_{26}P_8$

2. A candidate is required to answer six out of ten questions, which are divided into two groups each containing five questions, and he is not permitted to attempt more than four from each group. In how many ways can he make up his choice?

   (a) 300

   (b) 200

   (c) 400

(d) 100

3. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then find the number of ways in which they can be seated.

(a) $m! \times {}_{m+1}P_n$

(b) $m! \times {}_{\mathrm{m+1}}C_n$

(c) $m! \times {}_mP_n$

(d) $m! \times {}_mC_n$

4. How many different four-digit numbers can be made from the first four whole numbers, using each digit only once?

(a) 20

(b) 18

(c) 24

(d) 20

5. How many different four-digit numbers can be made from the first four natural numbers, if repetition is allowed?

(a) 128

(b) 512

(c) 256

(d) None of these

6. How many six-lettered words starting with the letter T can be made from all the letters of the word TRAVEL?

(a) 5

(b) 5!

(c) 124

(d) None of these

7. In the question above, how many words can be made with *T* and *L* at the end positions?

(a) 24

(b) 5!

(c) 6!

(d) None of these

8. If the letters of the word ATTEMPT are written down at random, find the probability that all the *T*'s are together.

(a) 2/7

(b) 1/7

(c) 4/7

(d) 3/7

9. A bag contains three white and two red balls. One by one, two balls are drawn without replacing them. Find the probability that the second ball is red.

(a) 2/5

(b) 1/10

(c) 2/10

(d) 3/5

Directions for questions 11 and 12: Read the passage below and solve the questions based on it.

India plays two matches each with New Zealand and South Africa. In any match, the probability of different outcomes for India is given below:

Outcome	*Win*	*Loss*	*Draw*
Probability	0.5	0.45	0.05
Points	2	0	1

Outcome of all the matches are independent of each other.

11. What is the probability of India getting at least seven points in the contest? Assume South Africa and New Zealand play two matches.

    (a) 0.025

    (b) 0.0875

    (c) 0.0625

    (d) 0.975

12. What is the probability of South Africa getting at least four points? Assume South Africa and New Zealand play two matches.

    (a) 0.2025

    (b) 0.0625

    (c) 0.0425

    (d) Cannot be determined

13. If ${}_nC_4 = 126$, then ${}_np_4 = ?$

    (a) 126

    (b) 126 * 2!

(c) 126 * 4!

(d) None of these

14. The number of ways, in which a student can choose five courses out of nine courses, if two courses are compulsory, is

(a) 53

(b) 35

(c) 34

(d) 32

15. What is the value of $_{18}C_{16}$?

(a) 5!

(b) 6!

(c) 153

(d) Cannot be determined

16. How many triangles can be drawn from *N* given points on a circle?

(a) N!

(b) 3!

(c) N! / 3!

(d) (N – 1) N(N – 2)/6

17. Following are the alphabets for which the alphabet and its mirror image are same: *A*, *H*, *I*, *M*, *O*, *T*, *U*, *V*, *W*, *Y*, *X*. These alphabets are called as symmetrical. Other are called as unsymmetrical. A password containing four alphabets (without repetitions) is to be formed using symmetrical alpha-

bets. How many such passwords can be formed?

(a) 720

(b) 330

(c) 7920

(d) Cannot be determined

18. How many five letter words can be formed out of ten consonants and four vowels, such that each contains three consonants and two vowels?

(a) ${}_{10}P_3 \times {}_4P_2 \times 5!$

(b) ${}_{10}C_3 \times {}_4C_2 \times 5!$

(c) ${}_{10}P_3 \times {}_4C_2 \times 5!$

(d) ${}_{10}C_2 \times {}_4P_2 \times 5!$

19. A question paper consists of two sections A and B having respectively three and four questions. Four questions are to be solved to qualify in that paper. It is compulsory to solve at least one question from Section A and two questions from Section B. In how many ways can a candidate select the questions to qualify in that paper?

(a) 30

(b) 18

(c) 48

(d) 60

20. A student is allowed to select at most n books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one book is 63, find the value of n.

(a) 4

(b) 3

(c) 5

(d) 6

21. A man and a woman appear in an interview for two vacancies for the same post. The probability of a man's selection is 1/4 and that of a woman's selection is 1/3. What is the probability that both of them will be selected?

(a) 1/12

(b) 1/3

(c) 2/5

(d) 3/7

22. In Question 21, what will be the probability that only one of them will be selected?

(a) 11/12

(b) 1/2

(c) 7/12

(d) 5/12

23. In Question 21, what will be the probability that none of them will be selected?

(a) 1/2

(b) 1/3

(c) 2/5

(d) 3/7

24. How many different signals can be made by hoisting five different coloured flags one above the other, when any number of them may be hoisted a time?

(a) 25

(b) ${}_5P_5$

(c) 325

(d) none of these

25. Find the number of ways in which the letters of the word MACHINE can be arranged so that the vowels may occupy only odd positions.

(a) $4! \times 4!$

(b) ${}_7P_3 \times 4!$

(c) ${}_7P_4 \times 3!$

(d) none of these

ANSWER KEY

1. (a)
2. (b)
3. (a)
4. (b)
5. (c)
6. (b)
7. (d)
8. (b)

9. (a)

10. (b)

11. (b)

12. (d)

13. (c)

14. (b)

15. (c)

16. (d)

17. (c)

18. (b)

19. (a)

20. (b)

21. (a)

22. (d)

23. (a)

24. (c)

25. (a)