**CBSE Test Paper 05**

**Chapter 8 Gravitation**

1. Deimos, a moon of Mars, is about 12 km in diameter with mass $2.0 \times 10^{15}$ kg. Suppose you are stranded alone on Deimos and want to play a one-person game of baseball. You would be the pitcher, and you would be the batter! You have to throw a baseball so that it would go into a circular orbit just above the surface and return to you so you could hit it. How long (in hours) after throwing the ball should you be ready to hit it? **1**
    a. 2.03 hr
    b. 2.11 hr
    c. 2.35 hr
    d. 2.20 hr
2. A uniform, solid, 1000.0-kg sphere has a radius of 5.00 m. Find the gravitational force this sphere exerts on a 2.00-kg point mass placed at a distance of 5.01 m from the center of the sphere **1**
    a. $5.31 \times 10^{-9}$ N
    b. $5.41 \times 10^{-9}$ N
    c. $5.63 \times 10^{-9}$ N
    d. $5.52 \times 10^{-9}$ N
3. On October 15, 2001, a planet was discovered orbiting around the star HD 68988. Its orbital distance was measured to be 10.5 million kilometers from the center of the star, and its orbital period was estimated at 6.3 days. What is the mass of HD 68988? **1**
    a. $2.1 \times 10^{30}$ kg
    b. $2.4 \times 10^{30}$ kg
    c. $2.3 \times 10^{30}$ kg
    d. $2.2 \times 10^{30}$ kg
4. A rocket is fired vertically with a speed of 5 km $sec^{-1}$ from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth= 6.0

$\times\ 10^{24}$ kg; mean radius of the earth = 6.4 $\times\ 10^{6}$ m; G = 6.67 $\times\ 10^{-11}$ N $m^2$ $kg^{-2}$. **1**

a. $8.0 \times 10^6$ m from the earth's center
b. $9.0 \times 10^6$ m from the earth's centre
c. $8.5 \times 10^6$ m from the earth's centre
d. $7.5 \times 10^6$ m from the earth's center

5. Calculate the escape speed from the Earth for a 5 000-kg spacecraft. mass of the earth = 6.0 $\times\ 10^{24}$ kg; radius of the earth = 6.4 $\times\ 10^{6}$ m; G = 6.67 $\times\ 10^{-11}$ N m2 $kg^{-2}$. **1**

   a. $1.52 \times 10^4$ m/s
   b. $1.32 \times 10^4$ m/s
   c. $1.12 \times 10^4$ m/s
   d. $1.72 \times 10^4$ m/s

6. A satellite of small mass burns during its desent and not during ascent. Why? **1**

7. Why is the atmosphere much rarer on the moon than on the earth? **1**

8. How is the gravitational force between two point masses affected when they are dipped in water keeping the separation between them the same? **1**

9. In kepler's law of period, $T^2 = kr^3$, the constant, k = $10^{-13} s^2 m^{-3}$. Express the constant, k in days and kilometres. The moon is at a distance of 3.84 $\times 10^5$ km from the earth. Obtain its time period of revolution in days. **2**

10. The mean orbital radius of the earth around the sun is 1.5 $\times\ 10^8$ km. Calculate mass of the sun if G = 6.67 $\times\ 10^{11}$ $Nm^2/kg^{-2}$? **2**

11. What will be the value of g at the bottom of sea 7 km deep? The diameter of the earth is 12800 km and g on the surface of the earth is 9.8 $ms^{-2}$. **2**

12. A comet orbits the sun in highly elliptical orbit. Does the comet has a constant **3**

    i. linear speed,
    ii. angular speed,
    iii. angular momentum,

iv. kinetic energy,
v. potential energy and
vi. total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the sun.

13. A star 2.5 times the mass of the sun and collapsed to a size of 12 km and rotates with a speed of 1.2 revolutions per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects, called pulsars, belong to this category.) Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = $2 \times 10^{30}$ kg) **3**

14. A simple pendulum has a time period exactly 2 s when used in a laboratory at north pole. What will be the time period if the same pendulum is used in a laboratory at the equator? Account for the earth's rotation only. Take g = $\frac{GM}{R^2}$ 9.8 m/s$^2$ and radius of earth = 6400 km. **3**

15. Let us assume that our galaxy consists of $2.5 \times 10^{11}$ stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be $10^5$ ly. **5**

## Answer

1. d. 2.20 hr

**Explanation:** $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$Here, G = 6.67 \times 10^{-11} N\ m^2\ kg^{-2}$

$M = 2 \times 10^{15} kg$

$r = \frac{12000}{2} = 6000m$

$\Rightarrow T = 2 \times \frac{22}{7} \times \sqrt{\frac{(6000)^3}{6.67 \times 10^{-11} \times 2 \times 10^{15}}}$

$\Rightarrow T = 2 \times \frac{22}{7} \times \sqrt{\frac{216 \times 10^9}{6.67 \times 10^{-11} \times 2 \times 10^{15}}}$

$\Rightarrow T = 2 \times \frac{22}{7} \times \sqrt{\frac{216 \times 10^9}{13.34 \times 10^4}}$

$\Rightarrow T = 2 \times \frac{22}{7} \times \sqrt{\frac{216 \times 10^9}{13.34 \times 10^4}}$

$\Rightarrow T = 2 \times \frac{22}{7} \times \sqrt{16.2 \times 10^5}$

$\Rightarrow T = 2 \times \frac{22}{7} \times \sqrt{1.6 \times 10^6}$

$\Rightarrow T = 2 \times \frac{22}{7} \times 1.26 \times 10^3$

$\Rightarrow T = 7920 \text{ sec} \Rightarrow T = 2.2 \text{ hr}$

2. a. $5.31 \times 10^{-9}$ N

**Explanation:** We know gravitational force

$F = \frac{GMm}{r^2}$

$Here\ G = 6.67 \times 10^{-11} Nm^2kg^{-2}$

M = 1000 kg

m = 2.0 kg

r = 5.01 m

$\Rightarrow F = \frac{6.67 \times 10^{-11} \times 1000 \times 2}{(5.01)^2}$

$\Rightarrow F = \frac{13.34 \times 10^{-8}}{25.10}$

$\Rightarrow F = 0.531 \times 10^{-8} = 5.31 \times 10^{-9} N$

3. c. $2.3 \times 10^{30}$ kg

**Explanation:** Following is the equation for the period T:

$$T = \frac{2\pi r^{3/2}}{\sqrt{Gm}}\ Or\ T^2 = \frac{4\pi^2 r^3}{Gm}$$

Solving this equation for the mass m, we get mass of the star HD68988 as follows:

$$m = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2 (10.5 \times 10^9 m)^3}{(5.443 \times 10^5 s)^2 (6.673 \times 10^{-11} N.m^2/kg^2)} = 2.3 \times 10^{30} kg$$

4. a. $8.0 \times 10^6$ m from the earth's center

**Explanation:** Work done in bringing a mass m against a mass M at adistance 'x' through a distance dx

$$= \text{Gravitational force} \times \text{Distance} = \frac{\text{GMm}}{x^2} dx$$

$$\text{Workdone in bringing it from } \infty \text{ to r} = \text{GMm}\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$= \text{- GMm}\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

Therefore Gravitational Potential Energy, W = $\frac{\text{- GMm}}{R}$

Initial Total Enrgy = Kinetic Energy + Potential Energy $= \frac{1}{2}\text{mv}^2$ - $\frac{\text{GMm}}{R}$ Final Total Energy = 0 - $\frac{\text{GMm}}{(R+h)}$

Applying law of conservation of energy: $\frac{1}{2}\text{mv}^2$ - $\frac{\text{GMm}}{R} = 0$ - $\frac{\text{GMm}}{(R+h)}$

$$\frac{\text{GMm}}{(R+h)} = \frac{\text{GMm}}{R} - \frac{1}{2}\text{mv}^2$$

$$\frac{GM}{(R+h)} = \frac{1}{R}\left(GM - \frac{1}{2}\text{Rv}^2\right)$$

$$\frac{(R+h)}{R} = \frac{GM}{(GM - \frac{1}{2}\text{Rv}^2)}$$

$$(R+h) = \frac{GMR}{(GM - \frac{1}{2}\text{Rv}^2)}$$

$$h = \frac{GMR}{(GM - \frac{1}{2}\text{Rv}^2)} - R$$

$$h = \frac{\frac{1}{2}\text{R}^2\text{v}^2}{(GM - \frac{1}{2}\text{Rv}^2)}$$

$$h = \frac{[\frac{1}{2} \times (6.4 \times 10^6)^2 \times (5 \times 10^3)^2]}{[(6.67 \times 10^{-11} \times 6 \times 10^{24}) - (\frac{1}{2} \times 6.4 \times 10^6 \times (5 \times 10^3)^2]} m$$

$$= \frac{(512 \times 10^{18})}{40 \times 10^{13} - 80 \times 10^{12}}$$

$$= 1.6 \times 10^6 m$$

Height achieved by the rocket with respect to the centre of the Earth = $R_e$ + h

$= 6.4 \times 10^6 + 1.6 \times 10^6 = 8 \times 10^6$ m.

5. c. $1.12 \times 10^4$ m/s

   **Explanation:** We know, $V_{esc} = \sqrt{\frac{2GM}{R}}$

   $Here\ G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

   $M = 6 \times 10^{24}\ kg$

   $R = 6.4 \times 10^6\ m$

   $\Rightarrow V_{esc} = \sqrt{\frac{2GM}{R}}$

   $= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}} = \sqrt{\frac{12 \times 6.67 \times 10^7}{6.4}} = 1.12 \times 10^4\ m/\sec$

6. The speed of satellite during descent is much larger than during ascent, and so heat produced is large. As the air resistance is directly proportional to velocity, therefore heat produced is large during descent and satellite burns up.

7. The value of escape velocity on the moon is small as compared to the value of the earth only 2.5 $kms^{-1}$. So, the molecules of air escape easily from the surface of the moon, hence there is no atmosphere.

8. The Newton's Universal law of gravitational force of attraction (F) between two bodies of masses $m_1, m_2$ separated by distance r is $F = \frac{Gm_1 m_2}{r^2}$
   G does not depend upon the medium. So force of attraction does not change if the masses are kept in water or any other medium.

9. Given,$k = 10^{-13} s^2/m^3$
   As, 1 s= $\frac{1}{24 \times 60 \times 60}$ day and 1m = $\frac{1}{1000}$ m
   $\therefore$ k = $10^{-12} \times \frac{1}{(24 \times 60 \times 60)^2}$ $(day)^2$ $\frac{1}{(1/1000)^3}$ $km^{-3}$
   = $1.33 \times 10^{-14}$ $(day)^2$ $km^{-3}$
   For the moon, $r = 3.84 \times 10^5 km$
   $\therefore T^2 = kr^3 = 1.33 \times 10^{-14} (3.84 \times 10^5)^3 = 753.087$
   $T = 27.3\ days$

10. Given that :

    R = $1.5 \times 10^8$ Km = $1.5 \times 10^{11}$ m

Time period of earth to complete one revolution around the sun is(T) = 365 days = 365 $\times$ 24 $\times$ 3600 s

Centripetal force = gravitational force

$M_s = \frac{4\pi^2 R^3}{GT^2} = \frac{4\times 9.87\times\left(1.5\times 10^{11}\right)^3}{6.64\times 10^{-11}\times(365\times 24\times 3600)^2}$

$= \frac{9.87\times\left(1.5\times 10^{11}\right)^3}{6.64\times 10^{-11}\times(31536000)^2}$

$M_s = 2.01 \times 10^{30}$ kg

11. Depth of sea, $d = 7\ km,\ g = 9.8ms^{-2}$
    Radius of the earth, $R = \frac{D}{2} = \frac{12800}{2}\ km = 6400km$
    Value of g at bottom of sea
    $g_d = g\left(1 - \frac{d}{R}\right) = 9.8\left(1 - \frac{7}{6400}\right)$ $ms^{-2}$
    $g_d = \frac{9.8\times 6393}{6400} ms^{-2} = 9.789\ ms^{-2}$

12. i. The speed of the comet does not remain constant because according to law of conservation of angular momentum, $L = mvr = constant$, therefore the comet moves faster when it is close to the sun and moves slower when it is farther away from the sun.
    ii. The angular speed of the comet does not remain constant because the linear speed varies, the angular speed also varies.
    iii. As no external torque is acting on the comet, therefore, according to law of conservation of angular momentum, the angular momentum of the comet remain constant.
    iv. The kinetic energy of the comet = $\frac{1}{2}mv^2$
    Its KE does not remain constant because the linear speed of the comet changes, its kinetic energy also changes.
    v. The potential energy of the comet changes as its kinetic energy changes.
    vi. Only angular momentum and total energy of a comet remain constant throughout its orbit.

13. We are given mass of neutron star,
    i.e. $M = 2.5 \times 2 \times 10^{30} kg$
    $= 5 \times 10^{30} kg$
    Radius of star, $R = 12km = 1.2 \times 10^4 m$

Frequency of rotation $\nu = 1.5$

If g is the acceleration due to gravity on the surface of the star

g = $\frac{GM}{R^2}$ = $\frac{6.67\times10^{-11}\times5\times10^{20}}{(1.2\times10^4)^2}$ m/s$^2$

$= 2.3 \times 10^{12} m/s^2$

Centrifugal acceleration (ac) produced in the object at the equator,

i.e. $a_e = R\omega^2$

= R × $(2\,\pi v)^2$ = $4\pi^2 v^2 R$

or $a_c = 4 \times 9.87 \times (1.5)^2 \times 1.2 \times 10^4 m/s^2$

$a_c = 1.1 \times 10^6 m/s^2$

Since g > $a_c$, the object will remain stuck to its surface due to gravity.

14. Consider the pendulum in its mean position at the north pole. As the pole is on the axis of rotation, the bob is in equilibrium. Hence in the mean position, the tension T is balanced by the earth's attraction. Thus,

$T = \frac{GMm}{R^2} = mg.$

The time period t is

$t = 2\pi\sqrt{\frac{l}{T/m}} = 2\pi\sqrt{\frac{l}{g}}$ ..(i)

At the equator, the lab and the pendulum rotate with the earth at angular velocity $\omega = \frac{2\pi \text{ radian}}{24 \text{ hour}}$ in a circle of radius equal to 6400 km.

**Using Newton's second law,**

$\frac{GMm}{R^2} - T = m\omega^2 R \; or, \; T' = m(g - \omega^2 R)$

where T' is the tension in the string.

The time period will be

$t' = 2\pi\sqrt{\frac{l}{(T'/m)}} = 2\pi\sqrt{\frac{l}{g-\omega^2 R}}$ ...(ii)

$By\ (i)\ and\ (ii),$

$\frac{t'}{t} = \sqrt{\frac{g}{g-\omega^2 R}} = \left(1 - \frac{\omega^2 R}{g}\right)^{-1/2}$

$\Rightarrow t' = t\left(1 + \frac{\omega^2 R}{2g}\right)$

Putting the values,

t' = 2.004 seconds.

15. Number of stars in our Galaxy (N) = 2.5 × $10^{11}$

Mass of each stars = $2 \times 10^{30}$kg

So, mass of the stars of the galaxy(M) = $2.5\times 10^{11}\times 2\times 10^{30}$

$= 5 \times 10^{41}$ Kg

Radius of orbit of a star (r) = 50000 light-years

We know,

1 light years = $9.46 \times 10^{15}$ m

So, r = $50000\times9.46\times10^{15}$ m

$= 5 \times 9.46 \times 10^{19}$ m

Centripital force = Gravitational force

$mv^2/r = GMm/r^2$

$v^2 = GM/r$

$(2\pi r/T)^2 = GM/r$ [$v = 2\pi r/T$]

$4\pi^2r^2/T^2 = GM/r$

$T^2 = 4\pi^2r^3/GM$

Put the values of r , G and M

$T = \sqrt\{ 4\times(3.14)^2\times (5\times9.46\times10^{19})^3/6.67\times10^{-11}\times5\times10^{41}\}$

$= 111.93 \times 10^{14}$ sec

$= 111.93 \times 10^{14}/(365\times24\times3600)$ yr

$= 3.55 \times 10^8$ yr