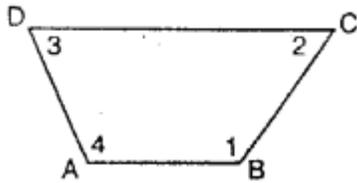


Quadrilateral

IMPORTANT POINTS

4. Quadrilateral: A quadrilateral is a plane figure enclosed by four sides. It has four sides, four interior angles and four vertices.



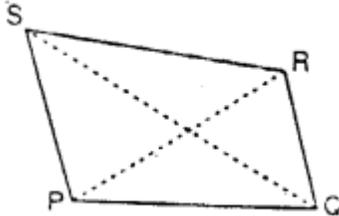
In quadrilateral ABCD, shown alongside:

(i) four sides are : AB, BC, CD and DA.

(ii) four angles are : $\angle ABC, \angle BCD, \angle CDA$ and $\angle DAB$; which are numbered $\angle 1, \angle 2, \angle 3$ and $\angle 4$ respectively.

(iii) four vertices are : A, B, C and D.

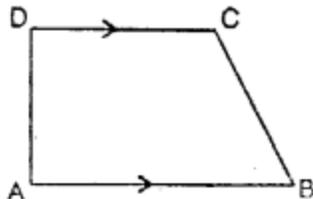
5. Diagonals of a Quadrilateral : The line segments joining the opposite vertices of a quadrilateral are called its diagonals.



The given figure shows a quadrilateral PQRS with diagonals PR and QS.

6. Types of Quadrilaterals :

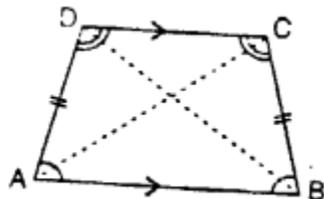
1. Trapezium: A trapezium is a quadrilateral in which one pair of opposite sides are parallel.



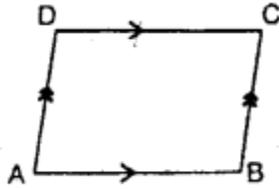
The figure, given alongside, shows a trapezium as its sides AB and DC are parallel i.e. $AB \parallel DC$.

When the non-parallel sides of the trapezium are equal in length, it is called an isosceles trapezium.

The given figure shows a trapezium ABCD whose non-parallel sides AD and BC are equal in length i.e. $AD = BC$; therefore it is an isosceles trapezium.



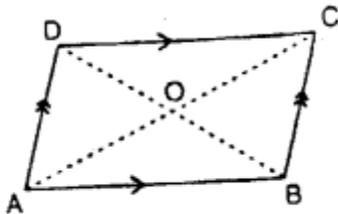
Also, in an isosceles trapezium :



- (i) base angles are equal:
i.e. $\angle A = \angle B$ and $\angle D = \angle C$
- (ii) diagonals are equal
i.e. $AC = BD$.

2.Parallelogram : A parallelogram is a quadrilateral, in which both the pairs of opposite sides are parallel.

The quadrilateral ABCD, drawn alongside, is a parallelogram; since, AB is parallel to DC and AD is parallel to BC i.e.



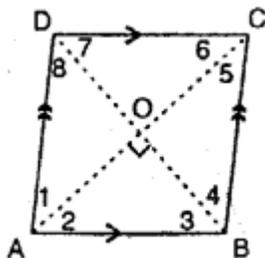
$AB \parallel DC$ and $AD \parallel BC$.

Also, in a parallelogram ABCD:

- (i) opposite sides are equal:
i.e. $AB = DC$ and $AD = BC$.
- (ii) opposite angles are equal:
i.e. $\angle ABC = \angle ADC$ and $\angle BCD = \angle BAD$
- (iii) diagonals bisect each other :
i.e. $OA = OC = \frac{1}{2} AC$ and $OB = OD = \frac{1}{2} BD$.

7. Some special types of Parallelograms

(a) Rhombus : A rhombus is a parallelogram in which all its sides are equal.



\therefore In a rhombus ABCD :

- (i) opposite sides are parallel:
i.e. $AB \parallel DC$ and $AD \parallel BC$.
- (ii) all the sides are equal:
i.e. $AB = BC = CD = DA$.
- (iii) opposite angles are equal:

i.e. $\angle A = \angle C$ and $\angle B = \angle D$.

(iv) diagonals bisect each other at right angle :

i.e. $OA = OC = \frac{1}{2} AC$; $OB = OD = \frac{1}{2} BD$.

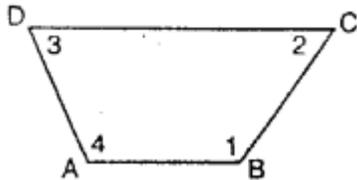
and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$

(v) diagonals bisect the angles at the vertices :

i.e. $\angle 1 = \angle 2$; $\angle 3 = \angle 4$; $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$.

IMPORTANT POINTS

4. Quadrilateral: A quadrilateral is a plane figure enclosed by four sides. It has four sides, four interior angles and four vertices.



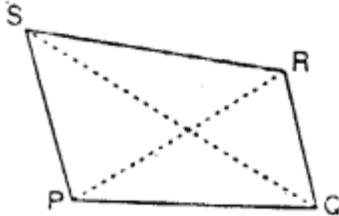
In quadrilateral ABCD, shown alongside:

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(ii) four angles are : $\angle ABC, \angle BCD, \angle CDA$ and $\angle DAB$; which are numbered $\angle 1, \angle 2, \angle 3$ and $\angle 4$ respectively.

(iii) four vertices are : A, B, C and D.

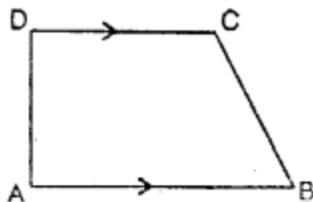
5. Diagonals of a Quadrilateral : The line segments joining the opposite vertices of a quadrilateral are called its diagonals.



The given figure shows a quadrilateral PQRS with diagonals PR and QS.

6. Types of Quadrilaterals :

1. Trapezium: A trapezium is a quadrilateral in which one pair of opposite sides are parallel.

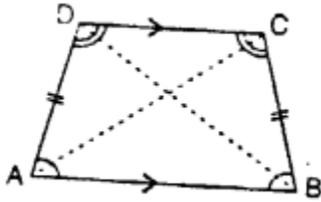


The figure, given alongside, shows a trapezium as its sides AB and DC are parallel i.e. $AB \parallel DC$.

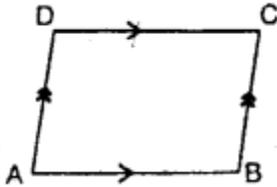
When the non-parallel sides of the trapezium are equal in length, it is called an isosceles trapezium.

The given figure shows a trapezium ABCD whose non-parallel sides AD and BC are

equal in length i.e. $AD = BC$; therefore it is an isosceles trapezium.



Also, in an isosceles trapezium :



(i) base angles are equal:

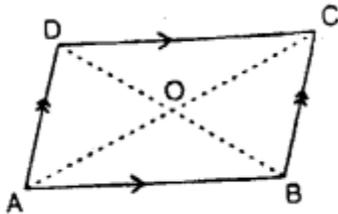
i.e. $\angle A = \angle B$ and $\angle D = \angle C$

(ii) diagonals are equal

i.e. $AC = BD$.

2.Parallelogram : A parallelogram is a quadrilateral, in which both the pairs of opposite sides are parallel.

The quadrilateral ABCD, drawn alongside, is a parallelogram; since, AB is parallel to DC and AD is parallel to BC i.e.



$AB \parallel DC$ and $AD \parallel BC$.

Also, in a parallelogram ABCD:

(i) opposite sides are equal:

i.e. $AB = DC$ and $AD = BC$.

(ii) opposite angles are equal:

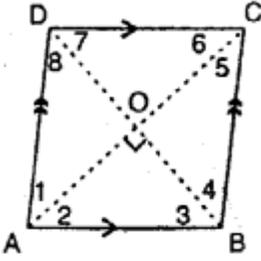
i.e. $\angle ABC = \angle ADC$ and $\angle BCD = \angle BAD$

(iii) diagonals bisect each other :

i.e. $OA = OC = \frac{1}{2} AC$ and $OB = OD = \frac{1}{2} BD$.

7. Some special types of Parallelograms

(a) Rhombus : A rhombus is a parallelogram in which all its sides are equal.



∴ In a rhombus ABCD :

(i) opposite sides are parallel:

i.e. $AB \parallel DC$ and $AD \parallel BC$.

(ii) all the sides are equal:

i.e. $AB = BC = CD = DA$.

(iii) opposite angles are equal:

i.e. $\angle A = \angle C$ and $\angle B = \angle D$.

(iv) diagonals bisect each other at right angle :

i.e. $OA = OC = \frac{1}{2} AC$; $OB = OD = \frac{1}{2} BD$.

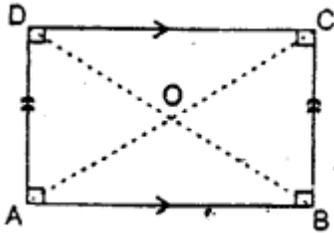
and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$

(v) diagonals bisect the angles at the vertices :

i.e. $\angle 1 = \angle 2$; $\angle 3 = \angle 4$; $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$.

(b) Rectangle : A rectangle is a parallelogram whose any angle is 90° .

A rectangle is also defined as a quadrilateral whose each angle is 90° .



Note : If any angle of a parallelogram is 90° ; automatically its each angle is 90° ; the reason being that the opposite angles of a parallelogram are equal.

Also, in a rectangle:

(i) opposite sides are parallel.

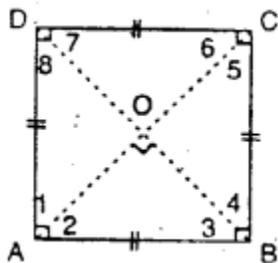
(ii) opposite sides are equal.

(iii) each angle is 90° .

(iv) diagonals are equal.

(v) diagonals bisect each other.

(c) Square : A square is a parallelogram, whose all side are equal and each angle is 90° .



A square can also be defined as :

- (i) a rhombus whose any angle is 90° .
- (ii) a rectangle whose all sides are equal.
- (iii) a quadrilateral whose all sides are equal and each angle is 90° .

\therefore If ABCD is a square :

(i) all its sides are equal, i.e. $AB = BC = CD = DA$

(ii) each angle of it is 90° .

i.e. $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

Also,

(iii) diagonals are equal.

i.e. $AC = BD$.

(iv) diagonals bisect each other at 90° .

i.e. $OA = OC = \frac{1}{2} AC$; $OB = OD = \frac{1}{2} BD$

and $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$.

Since, diagonals AC and BD are equal; therefore ; $OA = OC = OB = OD$.

(v) diagonals bisect the angles at the vertices

i.e. $\angle 1 = \angle 2 = 45^\circ$ [$\because \angle 1 + \angle 2 = 90^\circ$]

Similarly; $\angle 3 = \angle 4 = 45^\circ$;

$\angle 5 = \angle 6 = 45^\circ$ and $\angle 7 = \angle 8 = 45^\circ$.

EXERCISE 27 (A)

Question 1.

Two angles of a quadrilateral are 89° and 113° . If the other two angles are equal; find the equal angles.

Solution:

Let the other angle = x°

According to given,

$$89^\circ + 113^\circ + x^\circ + x^\circ = 360^\circ$$

$$2x^\circ = 360^\circ - 202^\circ$$

$$2x^\circ = 158^\circ$$

$$x^\circ = \frac{158}{2} = 79^\circ$$

\therefore other two angles = 79° each

Question 2.

Two angles of a quadrilateral are 68° and 76° . If the other two angles are in the ratio 5 : 7; find the measure of each of them.

Solution:

Two angles are 68° and 76°

Let other two angles be $5x$ and $7x$

$$\therefore 68^\circ + 76^\circ + 5x + 7x = 360^\circ$$

$$12x + 144^\circ = 360^\circ$$

$$12x = 360^\circ - 144^\circ$$

$$12x = 216^\circ$$

$$x = 18^\circ$$

angles are $5x$ and $7x$

i.e. $5 \times 18^\circ$ and $7 \times 18^\circ$ i.e. 90° and 126°

Question 3.

Angles of a quadrilateral are $(4x)^\circ$, $5(x+2)^\circ$, $(7x-20)^\circ$ and $6(x+3)^\circ$. Find

(i) the value of x .

(ii) each angle of the quadrilateral.

Solution:

Angles of quadrilateral are,

$(4x)^\circ$, $5(x+2)^\circ$, $(7x-20)^\circ$ and $6(x+3)^\circ$.

$$4x + 5(x+2) + (7x-20) + 6(x+3) = 360^\circ$$

$$4x + 5x + 10 + 7x - 20 + 6x + 18 = 360^\circ \quad 22x + 8 = 360^\circ$$

$$22x = 360^\circ - 8^\circ$$

$$22x = 352^\circ$$

$$x = 16^\circ$$

Hence angles are,

$$(4x)^\circ = (4 \times 16)^\circ = 64^\circ,$$

$$5(x+2)^\circ = 5(16+2)^\circ = 90^\circ,$$

$$(7x-20)^\circ = (7 \times 16 - 20)^\circ = 92^\circ$$

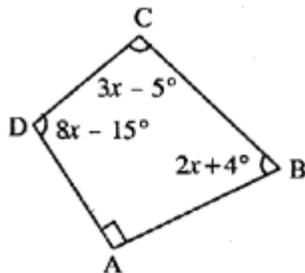
$$6(x+3)^\circ = 6(16+3) = 114^\circ$$

Question 4.

Use the information given in the following figure to find :

(i) x

(ii) $\angle B$ and $\angle C$

**Solution:**

$$\therefore \angle A = 90^\circ \text{ (Given)}$$

$$\angle B = (2x + 4^\circ)$$

$$\angle C = (3x - 5^\circ)$$

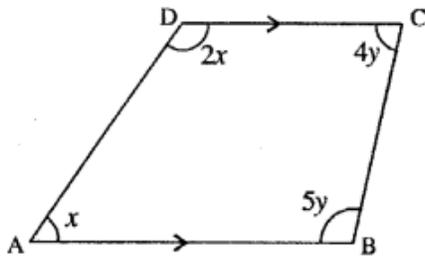
$$\begin{aligned} \angle D &= (8x - 15^\circ) \\ \angle A + \angle B + \angle C + \angle D &= 360^\circ \\ 90^\circ + (2x + 4^\circ) + (3x - 5^\circ) + (8x - 15^\circ) &= 360^\circ \\ 90^\circ + 2x + 4^\circ + 3x - 5^\circ + 8x - 15^\circ &= 360^\circ \\ \Rightarrow 74^\circ + 13x &= 360^\circ \\ \Rightarrow 13x &= 360^\circ - 74^\circ \\ \Rightarrow 13x &= 286^\circ \\ \Rightarrow x &= 22^\circ \\ \therefore \angle B &= 2x + 4 = 2 \cdot 22^\circ + 4 = 48^\circ \\ \angle C &= 3x - 5 = 3 \times 22^\circ - 5 = 61^\circ \\ \text{Hence (i) } 22^\circ \text{ (ii) } \angle B &= 48^\circ, \angle C = 61^\circ \end{aligned}$$

Question 5.

In quadrilateral ABCD, side AB is parallel to side DC. If $\angle A : \angle D = 1 : 2$ and $\angle C : \angle B = 4 : 5$

- Calculate each angle of the quadrilateral.
- Assign a special name to quadrilateral ABCD.

Solution:



$$\begin{aligned} \therefore \angle A : \angle D &= 1 : 2 \\ \text{Let } \angle A &= x \text{ and } \angle B = 2x \\ \therefore \angle C : \angle B &= 4 : 5 \text{ Let } \angle C = 4y \text{ and } \angle B = 5y \\ \therefore AB &\parallel DC \\ \angle A + \angle D &= 180^\circ \quad x + 2x = 180^\circ \\ 3x &= 180^\circ \quad x = 60^\circ \\ \therefore \angle A &= 60^\circ \\ \angle D &= 2x = 2 \times 60 = 120^\circ \text{ Again } \angle B + \angle C = 180^\circ \\ 5y + 4y &= 180^\circ \\ 9y &= 180^\circ \\ y &= 20^\circ \\ \therefore \angle B &= 5y = 5 \times 20 = 100^\circ \\ \angle C &= 4y = 4 \times 20 = 80^\circ \\ \text{Hence } \angle A &= 60^\circ ; \angle B = 100^\circ ; \angle C = 80^\circ \text{ and } \angle D = 120^\circ \end{aligned}$$

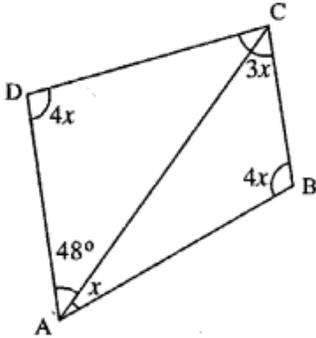
Question 6.

From the following figure find ;

- x,
- $\angle ABC$,
- $\angle ACD$.

Solution:

(i) In Quadrilateral ABCD,



$$x + 4x + 3x + 4x + 48^\circ = 360^\circ$$

$$12x = 360^\circ - 48^\circ$$

$$12x = 312$$

$$(ii) \angle ABC = 4x$$

$$4 \times 26 = 104^\circ$$

$$(iii) \angle ACD = 180^\circ - 4x - 48^\circ$$

$$= 180^\circ - 4 \times 26^\circ - 48^\circ$$

$$= 180^\circ - 104^\circ - 48^\circ$$

$$= 180^\circ - 152^\circ = 28^\circ$$

Question 7.

Given : In quadrilateral ABCD ; $\angle C = 64^\circ$, $\angle D = \angle C - 8^\circ$;

$\angle A = 5(a+2)^\circ$ and $\angle B = 2(2a+7)^\circ$.

Calculate $\angle A$.

Solution:

$$\because \angle C = 64^\circ \text{ (Given)}$$

$$\therefore \angle D = \angle C - 8^\circ$$

$$= 64^\circ - 8^\circ$$

$$= 56^\circ$$

$$\angle A = 5(a + 2)^\circ$$

$$\angle B = 2(2a + 7)^\circ$$

$$\text{Now } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$5(a + 2)^\circ + 2(2a + 7)^\circ + 64^\circ + 56^\circ = 360^\circ$$

$$5a + 10 + 4a + 14 + 64^\circ + 56^\circ = 360^\circ$$

$$9a + 144^\circ = 360^\circ$$

$$9a = 360^\circ - 144^\circ$$

$$9a = 216^\circ$$

$$a = 24^\circ$$

$$\therefore \angle A = 5(a + 2)$$

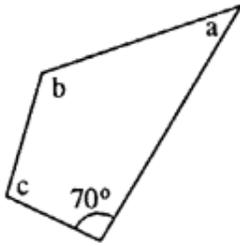
$$= 5(24 + 2)$$

$$= 130^\circ$$

Question 8.

In the given figure :

$\angle b = 2a + 15$
and $\angle c = 3a + 5$; find the values of b and c .



Solution:

$$\angle b = 2a + 15$$

$$\& \angle c = 3a + 5$$

\therefore Sum of angles of quadrilateral = 360°

$$70^\circ + a + 2a + 15 + 3a + 5 = 360^\circ$$

$$6a + 90^\circ = 360^\circ$$

$$6a = 270^\circ$$

$$a = 45^\circ$$

$$\therefore b = 2a + 15 = 2 \times 45 + 15 = 105^\circ$$

$$c = 3a + 5 = 3 \times 45 + 5 = 140^\circ$$

$$105^\circ \text{ and } 140^\circ$$

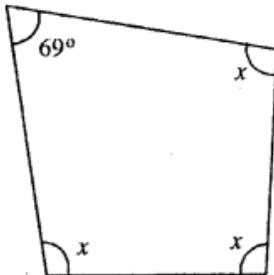
Question 9.

Three angles of a quadrilateral are equal. If the fourth angle is 69° ; find the measure of equal angles.

Solution:

Let each equal angle be

$$x^\circ \quad x + x + x + 69^\circ = 360^\circ$$



$$3x = 360^\circ - 69 \quad 3x = 291 \quad x = 97^\circ$$

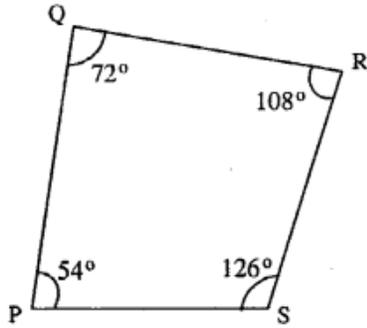
$$\text{Each equal angle} = 97^\circ$$

Question 10.

In quadrilateral PQRS, $\angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$.

Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other. Is PS also parallel to QR ?

Solution:



$$\therefore \angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$$

$$\text{Let } \angle P = 3x$$

$$\angle Q = 4x$$

$$\angle R = 6x \text{ \& } \angle S = 7x$$

$$\therefore \angle P + \angle Q + \angle R + \angle S = 360^\circ$$

$$3x + 4x + 6x + 7x = 360^\circ$$

$$20x = 360^\circ$$

$$x = 18^\circ$$

$$\therefore \angle P = 3x = 3 \times 18 = 54^\circ$$

$$\angle Q = 4x = 4 \times 18 = 72^\circ$$

$$\angle R = 6x = 6 \times 18 = 108^\circ$$

$$\angle S = 7x = 7 \times 18 = 126^\circ$$

$$\angle Q + \angle R = 72^\circ + 108^\circ = 180^\circ \text{ or } \angle P + \angle S = 54^\circ + 126^\circ = 180^\circ$$

Hence $PQ \parallel RS$

$$\text{As } \angle P + \angle Q = 72^\circ + 54^\circ = 126^\circ$$

Which is $\neq 180^\circ$.

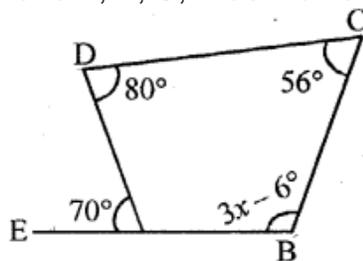
$\therefore PS$ and QR are not parallel.

Question 11.

Use the information given in the following figure to find the value of x .

Solution:

Take A, B, C, D as the vertices of quadrilateral and BA is produced to E (say).



$$\text{Since } \angle EAD = 70^\circ$$

$$\therefore \angle DAB = 180^\circ - 70^\circ = 110^\circ \text{ [}\because \text{EAB is a straight line and AD stands on it]}$$

$$\therefore \angle EAD + \angle DAB = 180^\circ$$

$$\therefore 110^\circ + 80^\circ + 56^\circ + 3x = 360^\circ$$

[\because sum of interior angles of a quadrilateral = 360°]

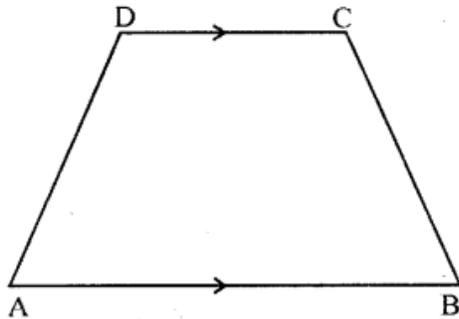
$$\therefore 3x = 360^\circ - 110^\circ - 80^\circ - 56^\circ + 6^\circ$$

$$3x = 360^\circ - 240^\circ = 120^\circ$$

$$\therefore x = 40^\circ$$

Question 12.

The following figure shows a quadrilateral in which sides AB and DC are parallel. If $\angle A : \angle D = 4 : 5$, $\angle B = (3x - 15)^\circ$ and $\angle C = (4x + 20)^\circ$, find each angle of the quadrilateral ABCD.



Solution:

$$\text{Let } \angle A = 4x$$

$$\angle D = 5x$$

$$\text{Since } \angle A + \angle D = 180^\circ \text{ [}\because \text{AB} \parallel \text{DC}]$$

$$\therefore 4x + 5x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ \Rightarrow x = 20^\circ$$

$$\therefore \angle A = 4(20) = 80^\circ, \angle D = 5(20) = 100^\circ \text{ Again } \angle B + \angle C = 180^\circ \text{ [}\because \text{AB} \parallel \text{DC}]$$

$$\therefore 3x - 15^\circ + 4x + 20^\circ = 180^\circ$$

$$7x = 180^\circ - 5^\circ$$

$$\Rightarrow 7x = 175^\circ \Rightarrow x = 25^\circ$$

$$\therefore \angle B = 75^\circ - 15^\circ = 60^\circ \text{ and } \angle C = 4(25) + 20 = 100^\circ + 20^\circ = 120^\circ$$

EXERCISE 27 (B)

Question 1.

In a trapezium ABCD, side AB is parallel to side DC. If $\angle A = 78^\circ$ and $\angle C = 120^\circ$, find angles B and D.

Solution:

\because AB \parallel DC and BC is transversal

\therefore $\angle B$ and $\angle C$, $\angle A$ and $\angle D$ are Cointerior angles with their sum = 180°

$$\text{i.e. } \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + 120^\circ = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - 120^\circ$$

$$\Rightarrow \angle B = 60^\circ$$

Also $\angle A + \angle D = 180^\circ$

$$\Rightarrow 78^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 78^\circ$$

$$\angle D = 102^\circ$$

Question 2.

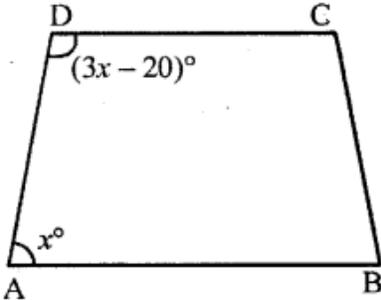
In a trapezium ABCD, side AB is parallel to side DC. If $\angle A = x^\circ$ and $\angle D = (3x - 20)^\circ$; find the value of x.

Solution:

$\therefore AB \parallel DC$ and BC is transversal

$\therefore \angle A$ and $\angle D$ are Co-interior angles with their sum = 180°

i.e. $\angle A + \angle D = 180^\circ$



$$\Rightarrow x^\circ + (3x - 20)^\circ = 180^\circ$$

$$\Rightarrow x^\circ + 3x^\circ - 20^\circ = 180^\circ$$

$$\Rightarrow 4x^\circ = 180^\circ + 20^\circ$$

$$x^\circ = \frac{200}{4} = 50^\circ$$

\therefore Value of x = 50°

Question 3.

The angles A, B, C and D of a trapezium ABCD are in the ratio 3 : 4 : 5 : 6.

Le. $\angle A : \angle B : \angle C : \angle D = 3:4: 5 : 6$. Find all the angles of the trapezium. Also, name the two sides of this trapezium which are parallel to each other. Give reason for your answer

Solution:

As the trapezium ABCD is a quadrilateral,

$$\therefore \text{Sum of its interior angles} = 360^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 3x + 4x + 5x + 6x = 360^\circ$$

$$\Rightarrow 18x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{18} = 20^\circ$$

$$\therefore \angle A = 3x = 3 \times 20^\circ = 60^\circ$$

$$\text{and } \angle B = 4x = 4 \times 20^\circ = 80^\circ$$

$$\text{and } \angle C = 5x = 5 \times 20^\circ = 100^\circ$$

$$\text{and } \angle D = 6x = 6 \times 20^\circ = 120^\circ$$

AB is parallel to DC.

$$\therefore \angle A + \angle D = 180^\circ,$$

$\angle A$ and $\angle D$ are co-interior angles
whose sum = 180°

Question 4.

In an isosceles trapezium one pair of opposite sides are to each other and the other pair of opposite sides are to each other.

Solution:

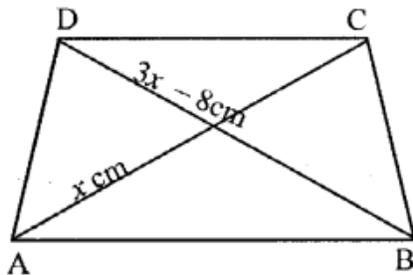
In an isosceles trapezium one pair of opposite sides are **parallel** to each other and the other pair of opposite sides are **equal** to each other.

Question 5.

Two diagonals of an isosceles trapezium are x cm and $(3x - 8)$ cm. Find the value of x .

Solution:

\therefore The diagonals of an isosceles trapezium are of equal length



$$\therefore 3x - 8 = x$$

$$\Rightarrow 3x - x = 8 \text{ cm}$$

$$\Rightarrow 2x = 8 \text{ cm}$$

$$\Rightarrow x = 4 \text{ cm}$$

\therefore The value of x is 4 cm

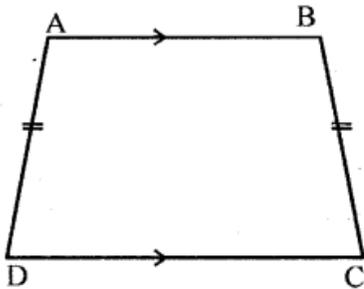
Question 6.

Angle A of an isosceles trapezium is 115° ; find the angles B, C and D.

Solution:

Since, the base angles of an isosceles trapezium are equal,

$$\therefore \angle A = \angle B = 115^\circ$$



Also, $\angle A$ and $\angle D$ are co-interior angles and their sum = 180°

$$\therefore \angle A + \angle D = 180^\circ$$

$$\Rightarrow 115^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 115^\circ$$

$$\Rightarrow \angle D = 65^\circ$$

Also, $\angle D = \angle C = 65^\circ$

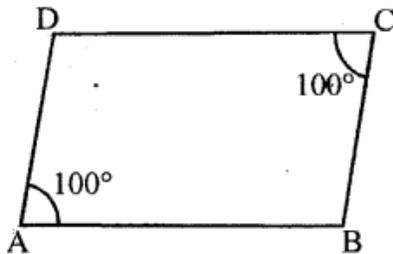
$$\therefore \angle B = 115^\circ, \angle C = 65^\circ \text{ and } \angle D = 65^\circ$$

Question 7.

Two opposite angles of a parallelogram are 100° each. Find each of the other two opposite angles.

Solution:

Given : Two opposite angles of a parallelogram are 100° each



\therefore Adjacent angles of a parallelogram are supplementary,

$$\therefore \angle A + \angle B = 180^\circ$$

$$\Rightarrow 100^\circ + \angle B = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - 100^\circ$$

$$\Rightarrow \angle B = 80^\circ$$

Also, opposite angles of a parallelogram are equal

$$\therefore \angle D = \angle B = 80^\circ$$

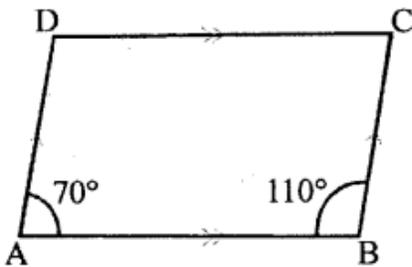
$$\therefore \angle B = \angle D = 80^\circ$$

Question 8.

Two adjacent angles of a parallelogram are 70° and 110° respectively. Find the other two angles of it.

Solution:

Given two adjacent angles of a parallelogram are 70° and 110° respectively.



Since, we know that opposite angles of a parallelogram are equal

$$\therefore \angle C = \angle A = 70^\circ \text{ and } \angle D = \angle B = 110^\circ$$

Question 9.

The angles A, B, C and D of a quadrilateral are in the ratio 2:3: 2 : 3. Show this quadrilateral is a parallelogram.

Solution:

Given, Angles of a quadrilateral are in the ratio 2 : 3 : 2 : 3

i.e. A : B : C : D are in the ratio 2 : 3 : 2 : 3

To prove - Quadrilateral ABCD is a parallelogram

Proof - Let us take $\angle A = 2x$, $\angle B = 3x$, $\angle C = 2x$ and $\angle D = 3x$

We know, that the sum of interior angles of a quadrilateral = 360°

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 2x + 3x + 2x + 3x = 360^\circ$$

$$\Rightarrow 10x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{10} = 36^\circ$$

$$\therefore \angle A = \angle C = 2x = 2 \times 36^\circ = 72^\circ$$

$$\angle B = \angle D = 3x = 3 \times 36^\circ = 108^\circ$$

Now, A quadrilateral ABCD is considered as a parallelogram.

(i) When opposite angles are equal,

i.e. $\angle A = \angle C = 72^\circ$ and $\angle B = \angle D = 108^\circ$

(ii) When adjacent angles are supplementary

i.e. $\angle A + \angle B = 180^\circ$

and $\angle C + \angle D = 180^\circ$

$$\Rightarrow 72^\circ + 108^\circ \text{ and } 72^\circ + 108^\circ = 180^\circ$$

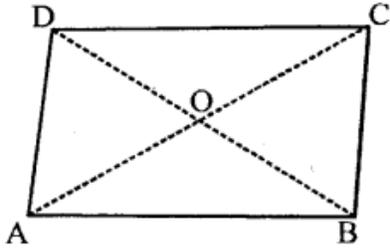
$$\Rightarrow 180^\circ = 180^\circ \text{ and } 180^\circ = 180^\circ$$

Since, quadrilateral ABCD fulfils the conditions

\therefore Quadrilateral ABCD is a parallelogram.

Question 10.

In a parallelogram ABCD, its diagonals AC and BD intersect each other at point O.



If $AC = 12$ cm and $BD = 9$ cm ; find; lengths of OA and OD .

Solution:

\therefore When diagonal AC and BD intersect each other at point O ,

$$\text{then } OA = OC = \frac{1}{2} AC$$

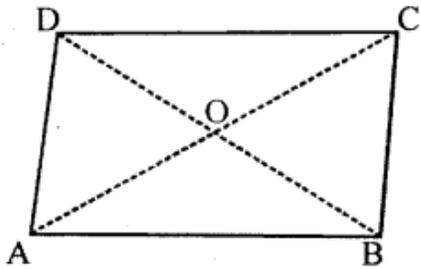
$$\text{and } OB = OD = \frac{1}{2} BD$$

$$\therefore OA = \frac{1}{2} \times AC = \frac{1}{2} \times 12 = 6 \text{ cm}$$

$$\text{and } OB = \frac{1}{2} \times BD = \frac{1}{2} \times 9 = 4.5 \text{ cm}$$

Question 11.

In parallelogram $ABCD$, its diagonals intersect at point O . If $OA = 6$ cm and $OB = 7.5$ cm, find the length of AC and BD .



Solution:

∴ When diagonal AC and BD intersect each other at point O,

$$\text{then } OA = OC = \frac{1}{2} AC$$

$$\text{and } OB = OD = \frac{1}{2} BD$$

$$\therefore OA = \frac{1}{2} \times AC \Rightarrow AC = 2 \times OA$$

$$\Rightarrow AC = 2 \times 6 \text{ cm} = 12 \text{ cm,}$$

$$\text{and } OB = \frac{1}{2} \times BD \Rightarrow BD = 2 \times OB$$

$$\Rightarrow BD = 2 \times 7.5 \text{ cm} \Rightarrow BD = 15 \text{ cm}$$

Question 12.

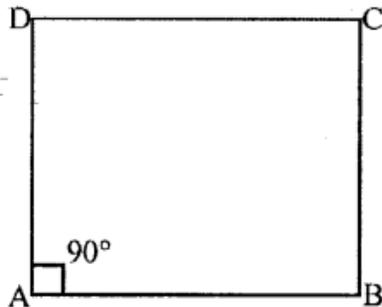
In parallelogram ABCD, $\angle A = 90^\circ$

(i) What is the measure of angle B.

(ii) Write the special name of the parallelogram.

Solution:

In parallelogram ABCD, $\angle A = 90^\circ$



(i) ∴ In a parallelogram, adjacent angles are supplementary

$$\therefore \angle A + \angle B = 180^\circ$$

$$\Rightarrow 90^\circ + \angle B = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - 90^\circ$$

$$\Rightarrow \angle B = 90^\circ$$

(ii) The name of the given parallelogram is a rectangle.

Question 13.

One diagonal of a rectangle is 18 cm. What is the length of its other diagonal?

Solution:

∴ In a rectangle, diagonals are equal

$$\Rightarrow AC = BD$$

Given, one diagonal of a rectangle = 18cm

∴ Other diagonal of a rectangle will be = 18cm

i.e. $AC = BD = 18\text{cm}$.

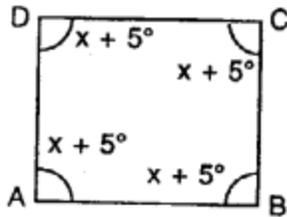
Question 14.

Each angle of a quadrilateral is $x + 5^\circ$. Find:

(i) the value of x

(ii) each angle of the quadrilateral.

Give the special name of the quadrilateral taken.



Solution:

(i) We have,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

∴ We know that the sum of interior angles of a quadrilateral is 360°

$$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow x + 5^\circ + x + 5^\circ + x + 5^\circ + x + 5^\circ = 360^\circ$$

$$\Rightarrow 4x + 20^\circ = 360^\circ$$

$$\Rightarrow 4x = 360^\circ - 20^\circ$$

$$\Rightarrow x = \frac{340^\circ}{4} = 85^\circ$$

(ii) Each angle of the quadrilateral

$$ABCD = x + 5^\circ$$

$$= 85^\circ + 5^\circ$$

$$= 90^\circ$$

The name of the given quadrilateral is a rectangle.

Question 15.

If three angles of a quadrilateral are 90° each, show that the given quadrilateral is a rectangle.

Solution:

The given quadrilateral ABCD will be a rectangle, if its each angle is 90°

Since, the sum of interior angles of a quadrilateral is 360° .

$$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow 90^\circ + 90^\circ + 90^\circ + \angle D = 360^\circ$$

$$\Rightarrow 270^\circ + \angle D = 360^\circ$$

$$\Rightarrow \angle D = 360^\circ - 270^\circ$$

$$\Rightarrow \angle D = 90^\circ$$

Since, each angle of the quadrilateral is 90° .

\therefore The given quadrilateral is a rectangle.

Question 16.

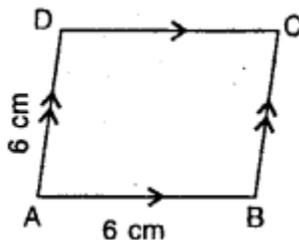
The diagonals of a rhombus are 6 cm and 8 cm. State the angle at which these diagonals intersect.

Solution:

The diagonals of a Rhombus always intersect at 90° .

Question 17.

Write, giving reason, the name of the figure drawn alongside. Under what condition will this figure be a square.

**Solution:**

Since, all the sides of the given figure are equal.

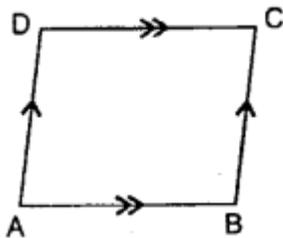
i.e. $AB = BC = CD = DA = 6 \text{ cm}$

\therefore The given figure is a rhombus.

This figure shall be considered as a square, if any angle is 90° .

Question 18.

Write two conditions that will make the adjoining figure a square.

**Solution:**

The conditions that will make the adjoining figure a square are :

(i) All the sides must be equal.

(ii) Any angle is 90° .