GOVERNMENT OF TAMIL NADU

STANDARD EIGHT

TERM - I

VOLUME - 2

MATHEMATICS

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Untouchability is Inhuman and a Crime

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Mathematics is a unique symbolic language in which the whole world works and acts accordingly. This text book is an attempt to make learning of Mathematics easy for the students community.

Mathematics is not about numbers, equations, computations or algorithms; it is about understanding

— William Paul Thurston

The main goal of Mathematics in School Education is to mathematise the child's thought process. It will be useful to know how to mathematise than to know a lot of Mathematics.

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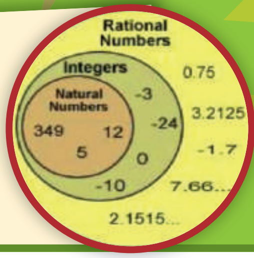
1 RATIONAL NUMBERS



Learning Outcomes

- To understand the necessity for extending fractions to rational numbers.
- To represent rational numbers on the number line.
- To understand that between any two given rational numbers, there lies many rational numbers.
- To learn and perform the four basic operations on rational numbers.
- To solve the word problems on all the operations.
- To understand the properties, the additive identity and inverse and the multiplicative identity and inverse of rational numbers.
- To know how to simplify expressions with atmost three brackets.

B7XMD8

1.1 Introduction

Think about the situation

Observe the following conversation:

Pari : My dear friend Sethu, I have a doubt about fractions on the number line. Can you please clear that doubt?

Sethu : Tell me Pari, I will be happy to help you.

Pari : We know about fractions, right? Fractions like $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ are obviously clear, that $\frac{1}{4}$ represents 1 out of 4 parts, $\frac{1}{2}$ is 1 out of 2 parts and so on. But, where are they on the number line?

Sethu : Pari, it is easy to identify where they are on the number line. The fractions you have given here are proper fractions. Aren't they? As we know, proper fractons are greater than zero but definitely less than one.

Pari : Yes, I do agree to that, Sethu.

Sethu : Now, let me tell you where they lie on the number line. See the line that I have drawn for you.

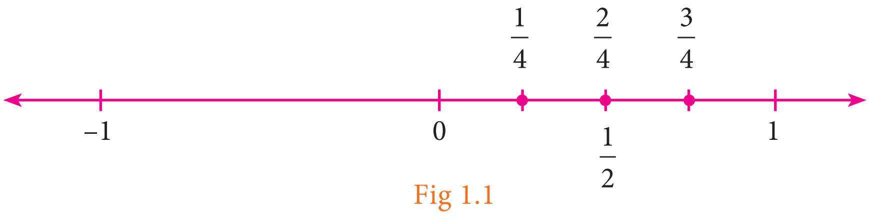


Fig 1.1

As you see here Pari, $\frac{1}{2}$ is exactly at the middle of 0 and 1 whereas $\frac{1}{4}$ is exactly at the middle of 0 and $\frac{1}{2}$. Also $\frac{3}{4}$ is exactly at the middle of $\frac{1}{2}$ and 1. Also, when we divide the distance between 0 and 1 roughly into 3 equal parts, the second part of it, represents $\frac{2}{3}$. Is it clear now?

Pari : Fine, it is very clear now. I think that these fractions $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and $\frac{2}{3}$ also correspond to the decimals 0.25, 0.50, 0.75 and 0.66. Am I correct, Sethu?

Sethu : Yes, you are correct.

Pari : By the way, I think that the improper fractions $\frac{13}{5}$, $\frac{10}{3}, \frac{31}{7}$ etc., should be converted into mixed fractions as $2\frac{3}{5}$, $3\frac{1}{3}$, $4\frac{3}{7}$ respectively, so as to locate them easily on the number line as given below.

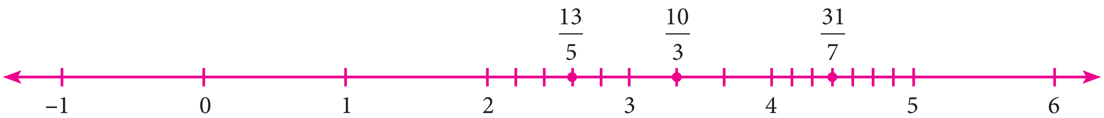


Fig 1.2

Sethu : Yes Pari, you are right. Here, it is clear that $\frac{13}{5}$ lies between 2 and 3, $\frac{10}{3}$ lies between 3 and 4 whereas $\frac{31}{7}$ lies between 4 and 5.

Pari : Sethu, let me ask you another question. If 50 students in a class contribute equally to a total of ₹ 35 for a cause, how much does each one contribute?

Sethu : It is simple. Each one's contribution is $\frac{35}{50}$, simplified to $\frac{7}{10}$ of a rupee, which is 70 paise (or) ₹ 0.70. Why do you ask this question here, Pari?

Pari : Wait Sethu. Tell me, what if they (50 students) have a debt of ₹ 35? Shall I denote it by a negative sign as $\frac{-7}{10}$?

Sethu : I also think so! As we have seen the extension of whole numbers to integers, these negative fractions need to be accommodated somewhere on the number line.

Teacher : Pari and Sethu, I have been listening to your conversation for a while now. You have almost got everything correct! Now, we know that 0 acts as the mirror to the natural numbers (right of 0) to reflect negative integers (left of 0). By the same way, we can indicate the negative fractions on to the left of 0.

Sethu : Thank you, Teacher. We have now understood what you said and know how to mark negative fractions on the number line as under.

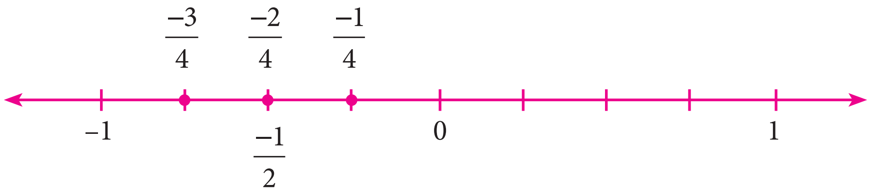


Fig 1.3

Observing the above conversation, one can see the need of negative fractions coming into the system of numbers that we have already know about.

Recap

Now let us recall about Fractions

1. Write the following fractions in the appropriate boxes.

$$\frac{-4}{5}, \frac{6}{7}, \frac{8}{3}, 4\frac{2}{3}, \frac{10}{7}, \frac{9}{12}, \frac{-12}{17}, 1\frac{4}{5}$$

Proper fraction	Improper fraction	Mixed fraction	Negative Fraction

2. Which of the following is not an equivalent fraction of $\frac{8}{12}$?

(A) $\frac{2}{3}$ (B) $\frac{16}{24}$ (C) $\frac{32}{60}$ (D) $\frac{24}{36}$

3. The simplest form of $\frac{125}{200}$ is __________.

4. Which is bigger $\frac{4}{5}$ or $\frac{8}{9}$?

5. Add the fractions : $\frac{3}{5}+\frac{5}{8}+\frac{7}{10}$

6. Simplify : $\frac{1}{8} - \left(\frac{1}{6}-\frac{1}{4}\right)$

7. Multiply $2\frac{3}{5}$ and $1\frac{4}{7}$.

8. Divide $\frac{7}{36}$ by $\frac{35}{81}$.

9. Fill in the boxes : $\frac{\square}{66} = \frac{70}{\square} = \frac{28}{44} = \frac{\square}{121} = \frac{7}{\square}$

10. In a city, $\frac{7}{20}$ of the population are women and $\frac{1}{4}$ are children. Find the fraction of the population of men.

1.1.1 Necessity for extending fractions to rational numbers

For the easy understanding and mathematical clarity, we shall introduce the rational numbers abstractly by focusing on two properties, namely every number has an opposite and every non-zero number has a reciprocal.

(i) Firstly, take the integers and form all possible 'fractions' where the numerators are integers and the denominators are non-zero integers. In this method, a rational number is defined as a 'ratio' of integers. The collection of rational numbers defined in this way will include the opposites of the fractions.

(ii) Secondly, we could take all the fractions together with their opposites. This would give us a new collection of numbers, called the fractions and numbers such as $\frac{-3}{4}, \frac{5}{-9}, \frac{-13}{2}$ etc.,

We know that, the fraction $\frac{4}{5}$ satisfies the equation $5x = 4$ since $5 \times \frac{4}{5} = 4$ and -2 satisfies the equation $x + 2 = 0$, since $-2 + 2 = 0$. However, there is neither a fraction nor an integer that satisfies the equation $5x + 2 = 0$.

We have studied about integers. We you add, subtract or multiply two or more integers, you will get only an integer. If we divide two integers, we will not always get an integer. For example, $\frac{-3}{5}$ and $\frac{-4}{-8}$ are not integers. These situations can be handled by extending the numbers to another collection of numbers called as rational numbers.

The following figure shows how rational numbers are an extension of the fractions and the integers.

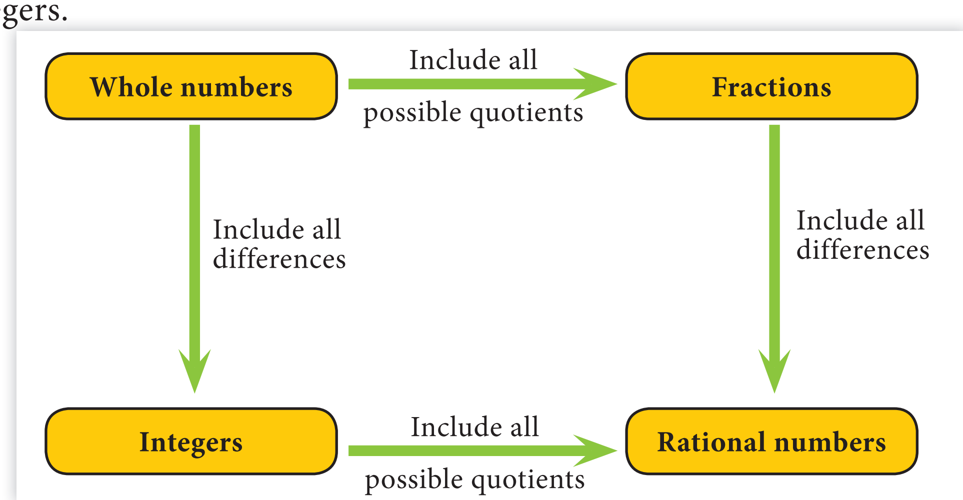
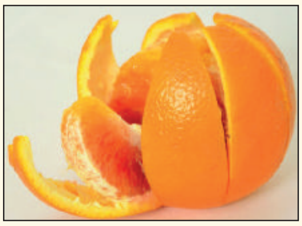


Fig 1.4

MATHEMATICS ALIVE	RATIONAL NUMBERS IN REAL LIFE
 If an orange is peeled off and 8 carpels are found, then one part of it is represents the rational number $\frac{1}{8}$.	 An LPG domestic cylinder showing the weight of the tare and gas in decimal form.

1.2 Rational numbers - Definition

The collection of all numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is called rational numbers which is denoted by the letter Q. Here, the top number a is called the numerator and the bottom number b is called the denominator.

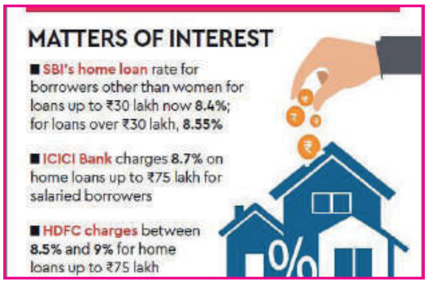
Examples:

$\frac{1}{3}, \frac{6}{11}, \frac{-3}{5}$ and $\frac{-11}{-25}$ are some examples of rational numbers. Also, integers like 7, –4 and 0 are rational numbers as they can be written in the form $\frac{7}{1}$, $\frac{-4}{1}$ and $\frac{0}{1}$. Mixed numbers such as $-4\frac{2}{5} = \frac{-22}{5}$, $-5\frac{1}{3} = \frac{-16}{3}$, $3\frac{1}{2} = \frac{7}{2}$ etc., are also rational numbers. So, all integers as well as fractions are rational numbers. The decimal numbers too, like 0.75, 1.3, 0.888 etc., are also rational numbers since they can be written in fractions form as :

$$0.75 = \frac{75}{100} = \frac{3}{4}$$

$$1.3 = \frac{13}{10} \quad \text{and}$$

$$0.888 = \frac{888}{1000} = \frac{222}{250} = \frac{111}{125}$$



In banks, home loans are given for a pre-determined interest rate as given above in decimal percentages which can be converted into rational numbers.

Note

The word 'ratio' in math refers to comparison of the sizes of two different quantities of any kind. For example, if there is one teacher for every 20 students in a class, then the ratio of teachers to students is 1:20. Ratios are often written as fractions and so $1:20 = \frac{1}{20}$. For this reason, numbers in the fractions form are called rational numbers.

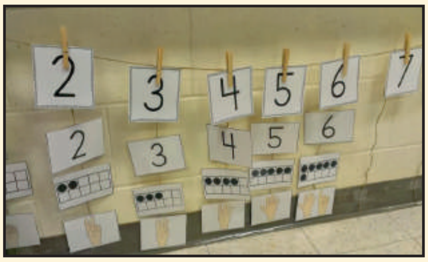
Try these

(i) Is the number $\frac{7}{-5}$ a fraction or a rational number ? Why?

(ii) Write any 6 rational numbers of your choice.

Activity-1

Use a string as a number line and fix it on the wall, for the length of the class room. Just mark the integers spaciously and ask the students to pick the rational number cards from a box and fix it at the right place on the number line string. This can be played between teams and the team which fixes more number of cards correctly (by marking) will be the winner.



1.2.1 Rational numbers on a number line

We know how the integers are represented on a number line. The same way, rational numbers can also be represented on a number line.

Now, let us represent the number $\frac{-3}{4}$ on the number line. Being negative, $\frac{-3}{4}$ would be marked to the left of 0 and it is between 0 and –1.

We know that in integers, 1 and –1 are equidistant from 0 and so are the pairs 2 and –2, 3 and -3 from 0. This remains the same for rational numbers too. Now, as we mark $\frac{3}{4}$ to the right of zero, at 3 parts out of 4 between 0 and 1, the same way, we mark $\frac{-3}{4}$ to the left of zero, at 3 parts out of 4 between 0 and –1 as shown below.

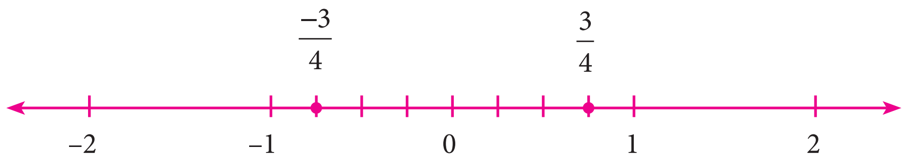


Fig. 1.5

Similarly, it is easy to say that $\frac{-5}{2}$ lies between –2 and -3 as $\frac{-5}{2} = -2\frac{1}{2}$.

Remember that all proper rational numbers lie between 0 and 1 (or) 0 and –1 just like the fractions.

Now, where do these rational numbers $\frac{18}{5}$ and $-\frac{32}{7}$ lie on a number line?

Here, $\frac{18}{5} = 3\frac{3}{5}$ and $-\frac{32}{7} = -4\frac{4}{7}$

Now, $\frac{18}{5}$ lies between 3 and 4 on the number line. The unit part between 3 and 4 is divided into 5 equal parts and the third part is marked as $\frac{3}{5}$. Thus, the arrow mark indicates $3\frac{3}{5}=\frac{18}{5}$. Also, it is clear that the rational number $-\frac{32}{7}$ which is $-4\frac{4}{7}$ lies between -4 and -5 on the number line. Here, the unit part between -4 and -5 is divided to 7 equal parts and fourth part is marked as $\frac{4}{7}$. Thus the arrow mark indicates $-4\frac{4}{7}=-\frac{32}{7}$. These rational numbers are shown on the number line as shown below.

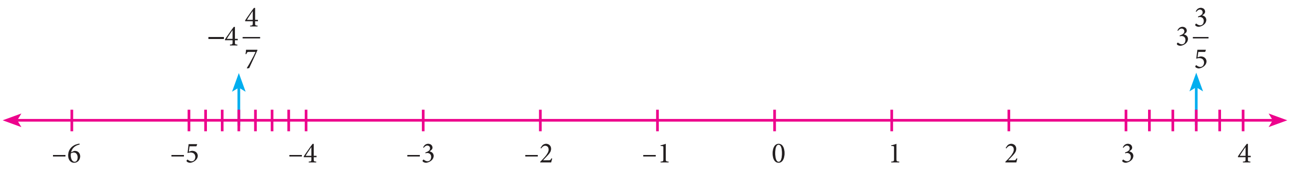


Fig. 1.6

1.2.2 Writing a decimal as a rational number

Every terminating or repeating decimal can be written in the rational form as shown in the following example.

Example 1.1

Write the following decimal numbers as rationals.

(i) 3.0 (ii) 0.25 (iii) 0.666… (iv) −5.8 (v) 1.15

Solution:

(i) $3.0=\frac{30}{10}=\frac{3}{1}$ (ii) $0.25=\frac{25}{100}=\frac{1}{4}$

(iii) $0.666...=\frac{2}{3}$ (check… you will know how in IX std)

(iv) $-5.8=\frac{-58}{10}=\frac{-29}{5}=-5\frac{4}{5}$

(v) $1.15=\frac{115}{100}=\frac{23}{20}=1\frac{3}{20}$

Note

There are decimal numbers which are non-terminating and non-repeating, such as $\pi=3.1415926535...$, 3.01002000400005...etc., They are not rational numbers because they cannot be written in $\frac{a}{b}$ form.

Try this

Explain why the following statements are true?

(i) $0.8=\frac{4}{5}$ (ii) $1.4>\frac{1}{4}$ (iii) $0.74<\frac{3}{4}$

(iv) $0.4>0.386$ (v) $0.096<0.24$ (vi) $0.128=0.1280$

1.2.3 Equivalent rational numbers

If the numerator and denominator of a rational number (say $\frac{a}{b}$) is multiplied by a non-zero integer (say c), we obtain another rational number which is equivalent to the given rational number. This is exactly the same way of getting equivalent fractions.

For example, take $\frac{a}{b} = \frac{-4}{7}$ and $c = 5$

Now, $\frac{a}{b} \times \frac{c}{c} = \frac{a \times c}{b \times c} = \frac{-4 \times 5}{7 \times 5} = \frac{-20}{35}$ is an equivalent rational number to $\frac{-4}{7}$ and if c is taken as 2,3,-4 etc., the corresponding rational numbers are $\frac{-8}{14}, \frac{-12}{21}, \frac{16}{-28}$ respectively.

1.2.4 Rational numbers in standard form

If in a rational number $\frac{a}{b}$, the only common factor of a and b is 1 and b is positive, then the rational number is said to be in standard form.

The rational numbers $\frac{4}{5}, \frac{-3}{7}, \frac{1}{6}, \frac{-4}{13}, \frac{-50}{51}$ etc., are all said to be in standard form.

If a rational number is not in the standard form, then it can be simplified to get the standard form.

Note

The quotient of two numbers with different signs is a negative number.

That is, $\frac{-4}{5} = -\frac{4}{5}$ and $\frac{4}{-5} = -\frac{4}{5}$ and so $\frac{-4}{5} = \frac{4}{-5} = -\frac{4}{5}$

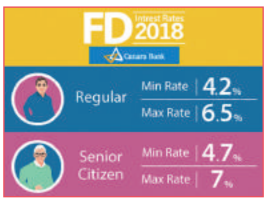
Also, if the two numbers are of the same sign, then the quotient is a positive number.

That is, $\frac{+3}{+4} = \frac{3}{4}$ and $\frac{-3}{-4} = \frac{3}{4}$.

0 is neither a positive nor a negative rational number.

DO YOU KNOW?

The collection of rational numbers is denoted by the letter Q because it is formed by considering all quotients, except those involving division by 0. Decimal numbers can be put in quotient form and hence they are also rational numbers.



Example 1.2

Reduce to the standard form (i) $\frac{48}{-84}$ (ii) $\frac{-18}{-42}$

Solution:

(i) **Method 1:**

$$\frac{48}{-84}=\frac{48\div(-2)}{-84\div(-2)}=\frac{-24\div 2}{42\div 2}=\frac{-12\div 3}{21\div 3}=\frac{-4}{7}$$ (dividing by –2,2 and 3 successively)

Method 2:

The HCF of 48 and 84 is 12 (Find it). Thus, we can get its standard form by dividing it by –12.

$$=\frac{48\div(-12)}{-84\div(-12)}=\frac{-4}{7}$$

(ii) **Method 1:**

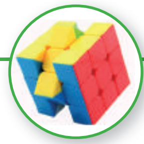
$$\frac{-18}{-42}=\frac{-18\div(-2)}{-42\div(-2)}=\frac{9\div 3}{21\div 3}=\frac{3}{7}$$ (dividing by –2 and 3 successively)

Method 2:

The HCF of 18 and 42 is 6 (Find it). Thus, we can get its standard form by dividing it by 6.

$$\frac{-18}{-42}=\frac{-18\times(-1)}{-42\times(-1)}=\frac{18}{42}=\frac{18\div 6}{42\div 6}=\frac{3}{7}$$

Try these



1. Which of the following pairs represents equivalent rational numbers?

   (i) $\frac{-6}{4}, \frac{18}{-12}$ (ii) $\frac{-4}{-20}, \frac{1}{-5}$ (iii) $\frac{-12}{-17}, \frac{60}{85}$,

2. Find the standard form of

   (i) $\frac{36}{-96}$ (ii) $\frac{-56}{-72}$ (iii) $\frac{27}{18}$

3. Mark the following rational numbers on a number line.

   (i) $\frac{-2}{3}$ (ii) $\frac{-8}{-5}$ (iii) $\frac{5}{-4}$

1.2.5 Comparison of rational numbers

You know how to compare integers and fractions taking two at a time and say which is smaller or greater. Now you will learn how to compare a pair of rational numbers.

- Two positive rational numbers, say $\frac{3}{5}$ and $\frac{5}{6}$ can be compared as studied earlier in comparison of two fractions.

- Two negative rational numbers, say $\frac{-1}{2}$ and $\frac{-4}{5}$ can be compared as follows.

Find the LCM of the denominators 2 and 5. Mark these rational numbers on a number line by finding their equivalent rational numbers having common denominator. Now, the equivalent rational numbers having the LCM 10 as common denominator are found as,

$$\frac{-1}{2}\times\frac{5}{5}=\frac{-5}{10} \text{ and } \frac{-4}{5}\times\frac{2}{2}=\frac{-8}{10}$$

We know that $4 < 8$ and so $-4 > -8$

$$\therefore \frac{-8}{10}<\frac{-5}{10}. \text{ Thus, } \frac{-4}{5}<\frac{-1}{2}.$$

- If one of the two rational numbers is negative, say $\frac{3}{8}$ and $\frac{-2}{3}$, we can easily say that $\frac{3}{8}>\frac{-2}{3}$ (or) $\frac{-2}{3}<\frac{3}{8}$ because we know that a positive number is always greater than a negative number.

Example 1.3

Find which rational number is greater?

(i) $\frac{5}{-4},\frac{-11}{-7}$ (ii) $\frac{-10}{3},\frac{14}{-5}$

Solution:

(i) Now, $\frac{5}{-4}=\frac{5\times(-1)}{-4\times(-1)}=\frac{-5}{4}$

Also, $\frac{-11}{-7}=\frac{-11\times(-1)}{-7\times(-1)}=\frac{11}{7}$

Here, $\frac{11}{7}$ is positive and $\frac{-5}{4}$ is a negative rational number.

$\therefore \frac{11}{7}>\frac{-5}{4}$, that is $\frac{-11}{-7}>\frac{5}{-4}$

(ii) First, make the denominator of the rational number $\frac{14}{-5}$ to be positive as $\frac{-14}{5}$.

Then, make the denominators the same by finding the LCM of the denominators.

So, $\frac{-10}{3}=\frac{-10}{3}\times\frac{5}{5}=\frac{-50}{15}$ and $\frac{-14}{5}=\frac{-14}{5}\times\frac{3}{3}=\frac{-42}{50}$

As $50>42,$ we have $-50<-42$.

Hence, $\frac{-50}{15}<\frac{-42}{15}$ and so $\frac{-42}{15}>\frac{-50}{15}$. Thus, $\frac{14}{-5}>\frac{-10}{3}$.

Activity-2

Compare many rational numbers using the class room situations like the number of boys or girls to the total number of students, their test marks etc., to say which rational number is bigger or smaller?

Example 1.4

Write the following rational numbers in descending and ascending order.

$$\frac{-3}{5}, \frac{7}{-10}, \frac{-15}{20}, \frac{14}{-30}, \frac{-8}{15}$$

Solution:

First make the denominators positive and write the numbers in standard form as $\frac{-3}{5}, \frac{-7}{10}, \frac{-15}{20}, \frac{-14}{30}, \frac{-8}{15}$. Now, the LCM of 5,10,15,20 and 30 is 60 (How?). Change the given rational numbers to their equivalent form with common denominator 60.

$$\frac{-3}{5} = \frac{-3}{5} \times \frac{12}{12} = \frac{-36}{60}$$

$$\frac{7}{-10} = \frac{-7}{10} \times \frac{6}{6} = \frac{-42}{60}$$

$$\frac{-15}{20} = \frac{-15}{20} \times \frac{3}{3} = \frac{-45}{60}$$

$$\frac{-14}{30} = \frac{-14}{30} \times \frac{2}{2} = \frac{-28}{60}$$

$$\frac{-8}{15} = \frac{-8}{15} \times \frac{4}{4} = \frac{-32}{60}$$

Now, comparing the numerators $-36, -42, -45, -28$ and -32 we see that

$$-28 > -32 > -36 > -42 > -45$$

That is, $\frac{-28}{60} > \frac{-32}{60} > \frac{-36}{60} > \frac{-42}{60} > \frac{-45}{60}$

and so, $\frac{14}{-30} > \frac{-8}{15} > \frac{-3}{5} > \frac{7}{-10} > \frac{-15}{20}$

Hence, the descending order of the given rational numbers is $\frac{14}{-30}, \frac{-8}{15}, \frac{-3}{5}, \frac{7}{-10}$ and $\frac{-15}{20}$ and its reverse order gives the ascending order. Hence the ascending order of the given rational numbers is $\frac{-15}{20}, \frac{7}{-10}, \frac{-3}{5}, \frac{-8}{15}$ and $\frac{14}{-30}$.

1.2.6 Rational numbers between any two given rational numbers

Think about the situation:

Seyyon wanted to know the number of integers between −10 and 20. He found that there are 9 negative integers, zero and 19 positive integers, a total of 29 integers between −10 and 20 (excluding −10 and 20). He also finds that there is no other integer between any two consecutive integers.

Is this true for rational numbers too?

Seyyon took two rational numbers $\frac{-3}{4}$ and $\frac{-2}{5}$. He converted them to rational numbers having the same denominators (find the LCM of the denominators).

So, $\frac{-3}{4}=\frac{-3}{4}\times\frac{5}{5}=\frac{-15}{20}$

and $\frac{-2}{5}=\frac{-2}{5}\times\frac{4}{4}=\frac{-8}{20}$

He could find rational numbers $\frac{-9}{20},\frac{-10}{20},\frac{-11}{20},\frac{-12}{20},\frac{-13}{20}$ and $\frac{-14}{20}$ between $\frac{-8}{20}$ and $\frac{-15}{20}$ but Seyyon doubts… Are these the only rational numbers between $\frac{-15}{20}$ and $\frac{-8}{20}$?

No, if we get the multiples of the denominator, then we can insert as many rational numbers as we want between any two given rational numbers.

For example,

$$\frac{-3}{4}=\frac{-15}{20}=\frac{-30}{40} \text{ and } \frac{-2}{5}=\frac{-8}{20}=\frac{-16}{40}$$

Now, between $\frac{-30}{40}$ and $\frac{-16}{40}$, we could easily find as many as 13 rational numbers like $\frac{-29}{40},\frac{-28}{40}\ldots\ldots\frac{-17}{40}$.

Let us find more rational numbers say between $\frac{3}{7}$ and $\frac{4}{7}$. This is clearly understood in the following visual explanation on the number line. We shall write $\frac{3}{7}$ as $\frac{30}{70}$ and $\frac{4}{7}$ as $\frac{40}{70}$. Now, we see that there are 9 rational numbers between $\frac{3}{7}$ and $\frac{4}{7}$ as given in the number line below .

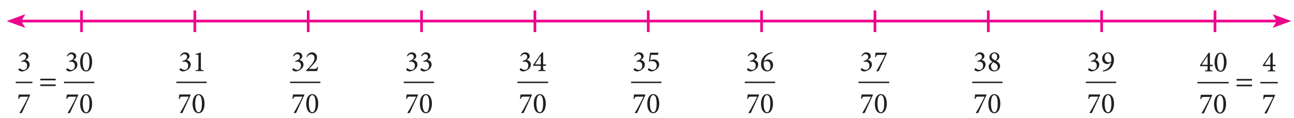


Fig. 1.7

Now, if we want more rational numbers between say $\frac{37}{70}$ and $\frac{38}{70}$ we can write $\frac{37}{70}$ as $\frac{370}{700}$ and $\frac{38}{70}$ as $\frac{380}{700}$. Then again, we will get nine rational numbers between $\frac{37}{70}$ and $\frac{38}{70}$ as $\frac{371}{700}, \frac{372}{700}, \frac{373}{700}, \frac{374}{700}, \frac{375}{700}, \frac{376}{700}, \frac{377}{700}, \frac{378}{700}$ and $\frac{379}{700}$.

The following diagram helps us to understand this nicely with a magnifying lens used between 0 and 1 and further zoomed into the fractional parts also.

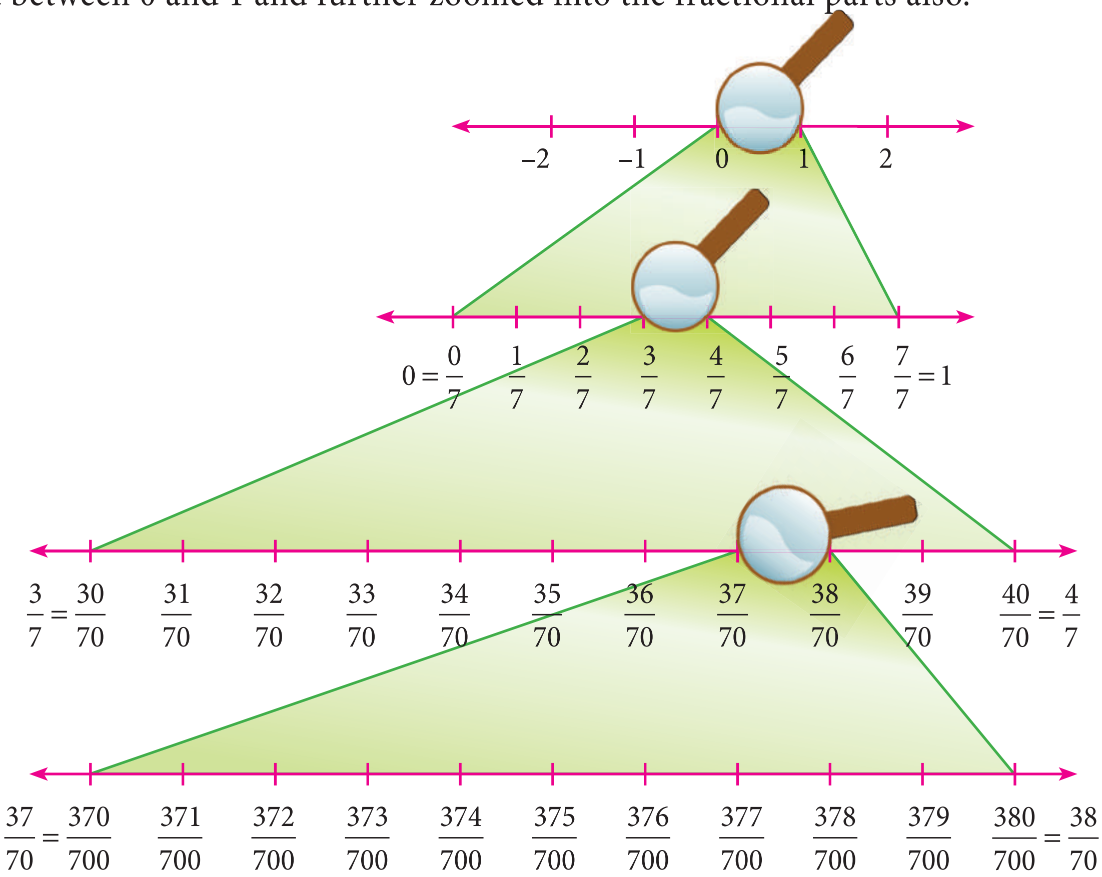


Fig. 1.8

Now, Seyyon understands that he can find an unlimited bunch of rational numbers between any two given rational numbers.

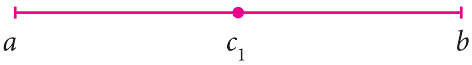
1.2.7 Alternative method for finding rational numbers between any two rational numbers by average concept.

In this method, we shall use the average concept.

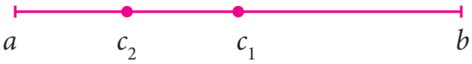
The average of two numbers a and b is $\frac{1}{2}(a+b)$.

Let a and b any two given rational numbers. By using the average, we can find many rational numbers between a and b as c_1, c_2, c_3, c_4, c_5 etc., as explained in the following.

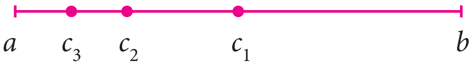
$c_1 = \frac{1}{2}(a+b)$



$c_2 = \frac{1}{2}(a+c_1)$

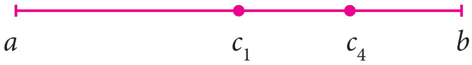


$c_3 = \frac{1}{2}(a+c_2)$

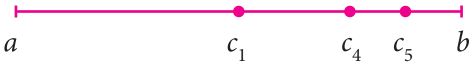


The numbers c_2, c_3 are to the left c_1. Similarly, we have c_4, c_5 on the right of c_1 as given below.

Here, $c_4 = \frac{1}{2}(c_1+b)$



$c_5 = \frac{1}{2}(c_4+b)$



This clearly shows that average of two numbers, in general, always lie between them.

For example, the average of $\frac{1}{4}$ and $\frac{3}{8}$ $= \frac{1}{2}\left(\frac{1}{4}+\frac{3}{8}\right)$

$$= \frac{1}{2}\left(\frac{5}{8}\right) = \frac{5}{16}$$

Here, $\frac{1}{4} = \frac{1}{4}\times\frac{4}{4} = \frac{4}{16}$ and $\frac{3}{8} = \frac{3}{8}\times\frac{2}{2} = \frac{6}{16}$

Thus, we find that the average $\frac{5}{16}$ lies between $\frac{4}{16}$ and $\frac{6}{16}$.

That is, $\frac{1}{4} < \frac{5}{16} < \frac{3}{8}$

Hence, the average of two numbers lies between the two numbers.

Example 1.5

Find 6 rational numbers between $\frac{-7}{11}$ and $\frac{5}{-9}$.

Solution:

Method 1:

LCM of 11 and 9 = 11 × 9 =99

$$\frac{-7}{11} = \frac{-7}{11}\times\frac{9}{9} = \frac{-63}{99}$$

$$\frac{5}{-9} = \frac{5\times(-1)}{-9\times(-1)} = \frac{-5}{9}\times\frac{11}{11} = \frac{-55}{99}$$

Therefore, 6 rational numbers between $\frac{-7}{11}\left(=\frac{-63}{99}\right)$ and $\frac{5}{-9}\left(=\frac{-55}{99}\right)$ are

$$\frac{-63}{99}\frac{-56}{99},\frac{-57}{99},\frac{-59}{99},\frac{60}{99},\frac{-61}{99},\frac{-62}{99},\frac{-55}{99}.$$

Method 2:

> **Think**
>
> 
>
> Are there any rational numbers between $\frac{-7}{11}$ and $\frac{6}{-11}$?

The average of a and b is $\frac{1}{2}(a+b)$

The average of $\frac{-7}{11}$ and $\frac{5}{-9}$ is

$$c_1 = \frac{1}{2}\left(\frac{-7}{11}+\frac{-5}{9}\right)$$

$$= \frac{1}{2}\left(\frac{-63-55}{99}\right) = \frac{1}{2}\left(\frac{-118}{99}\right)$$

$$c_1 = \frac{-59}{99}$$

$$\therefore \quad \frac{-7}{11} < \frac{-59}{99} < \frac{5}{-9} \qquad \text{...(1)}$$

The average of $\frac{-7}{11}$ and $\frac{-59}{99}$ is

$$c_2 = \frac{1}{2}\left(\frac{-7}{11}+\frac{-59}{99}\right)$$

$$= \frac{1}{2}\left(\frac{-63-59}{99}\right)$$

$$c_2 = \frac{1}{2}\times\frac{-122}{99} = \frac{-61}{99}$$

$$\therefore \quad \frac{-7}{11} < \frac{-61}{99} < \frac{-59}{99} \qquad \text{...(2)}$$

The average of $\frac{-59}{99}$ and $\frac{-5}{9}$ is

$$c_3 = \frac{1}{2}\left(\frac{-5}{9}+\frac{-59}{99}\right)$$

$$= \frac{1}{2}\left(\frac{-55-59}{99}\right)$$

$$c_3 = \frac{1}{2}\left(\frac{-114}{99}\right) = \frac{-57}{99}$$

$$\therefore \quad \frac{-59}{99} < \frac{-57}{99} < \frac{5}{-9} \qquad \text{...(3)}$$

Combining (1), (2) and (3), we get $\frac{-7}{11} < \frac{-61}{99} < \frac{-59}{99} < \frac{-57}{99} < \frac{5}{-9}$. Thus, we have found 3 rational numbers between $\frac{-7}{11}$ and $\frac{5}{-9}$. Similarly, try to find 3 more rational numbers in between $\frac{-7}{11}$ and $\frac{5}{-9}$ in the same way.

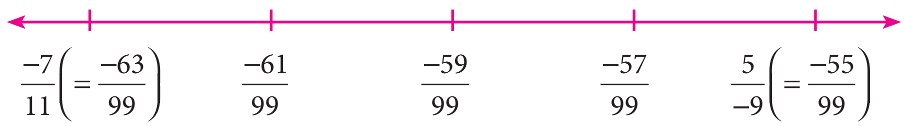


Fig. 1.9

That is, $\frac{-7}{11} < \frac{-61}{99} < \frac{-59}{99} < \frac{-57}{99} < \frac{5}{-9}$

Note

We can find many rational numbers between $\frac{-7}{11}$ and $\frac{5}{-9}$ quickly as given below:

The range of rational numbers can be got by the cross multiplication of denominators with the numerators after writing the given fractions in standard form. The cross multiplication here $\frac{-7}{11} \quad \frac{-5}{9}$ gives the range of rational numbers from −63 to −55 with denominator 99.

1.3 Four basic operations on rational numbers

1.3.1 Addition

(i) Addition of rational numbers with the same denominators

Add only the numerators of the two or more rational numbers and write the same denominator.

Example 1.6

Add: $\frac{-6}{11}, \frac{8}{11}, \frac{-12}{11}$

Solution:

Write the given rational numbers in the standard form and add them.

$$\frac{-6}{11} + \frac{8}{11} + \frac{-12}{11} = \frac{-6+8-12}{11} = \frac{-10}{11}$$

(ii) Addition of rational numbers with different denominators:

After writing the given rational numbers in the standard form, take the LCM of the denominators of the given rational numbers and convert them to equivalent rational numbers with common denominators (LCM) and then, add the numerators.

Example 1.7

Add : $\frac{-5}{9}, \frac{-4}{3}, \frac{7}{12}$

Solution:

LCM of $9, 3, 12 = 36$

$$\frac{-5}{9} + \frac{-4}{3} + \frac{6}{12} = \frac{-5}{9} \times \frac{4}{4} + \frac{-4}{3} \times \frac{12}{12} + \frac{7}{12} \times \frac{3}{3}$$

$$= \frac{-20}{36} + \frac{-48}{36} + \frac{21}{36} = \frac{-20-48+21}{36}$$

$$= \frac{-47}{36}$$

1.3.2 Additive Inverse

What is $\frac{-8}{11}+\frac{8}{11}$?

Now, $\frac{-8}{11}+\frac{8}{11}=\frac{-8+8}{11}=\frac{0}{11}=0$

Also, $\frac{8}{11}+\left(\frac{-8}{11}\right)=\frac{8-8}{11}=\frac{0}{11}=0$

In the case of integers, we say −5 as the additive inverse of 5 and 5 as the additive inverse of −5. Here, for rational numbers, $\frac{-8}{11}$ is the additive inverse of $\frac{8}{11}$ and $\frac{8}{11}$ is the additive inverse of $\frac{-8}{11}$.

1.3.3 Subtraction

Subtracting two rational numbers, is the same as adding the additive inverse of the second rational number to the first rational number.

(i) Subtraction of rational numbers with the same denominators

Subtract only the numerators of the two or more rational numbers and write the same denominator.

(ii) Subtraction of rational numbers with different denominators:

After writing the given rational numbers in the standard form, take the LCM of the denominators of the two given rational numbers and convert them to equivalent rational numbers with common denominators (LCM) and then, subtract the numerators.

Example 1.8

Subtract $\frac{9}{17}$ from $\frac{-12}{17}$

Solution:

Now, $\frac{-12}{17}-\frac{9}{17}=\frac{-12-9}{17}=\frac{-21}{17}$

Example 1.9

Subtract $\frac{-180}{225}$ from $\frac{200}{225}$

Solution:

Now, $\frac{200}{225}-\left(\frac{-180}{225}\right)=\frac{200+180}{225}=\frac{300}{225}=\frac{4}{3}$

Example 1.10

Subtract $\left(-2\frac{6}{11}\right)$ from $\left(-4\frac{5}{22}\right)$

Solution:

$$\text{Now, } \left(-4\frac{5}{22}\right)-\left(-2\frac{6}{11}\right)$$

$$=\frac{-93}{22}-\left(\frac{-28}{11}\right)$$

$$=\frac{-93}{22}+\frac{28}{11}$$

$$=\frac{-93+28\times 2}{22}$$

$$=\frac{-93+56}{22}=\frac{-37}{22}=-1\frac{15}{22}$$

1.3.4 Multiplication

Product of two or more rational numbers can be found by multiplying the corresponding numerators and denominators of the numbers and then write them in the standard form.

Example 1.11

evaluate (i) $\frac{-5}{8}\times 7$ (ii) $\frac{-6}{-11}\times(-4)$

Solution:

(i) $\frac{-5}{8}\times 7=\frac{-5}{8}\times\frac{7}{1}=\frac{-5\times 7}{8\times 1}=\frac{-35}{8}$

(ii) $\frac{-6}{-11}\times(-4)=\frac{6}{11}\times\frac{(-4)}{1}=\frac{6\times(-4)}{11\times 1}=\frac{-24}{11}$

1.3.5 Product of reciprocals and the Multiplicative Inverse

If the product of two rational numbers is 1, then one rational number is said to be the reciprocal or the multiplicative inverse of the other.

For the rational number *a*, its reciprocal is $\frac{1}{a}$ as $a\times\frac{1}{a}=\frac{a}{a}\times 1=1$.

For the rational number $\frac{a}{b}$, its multiplicative inverse is $\frac{b}{a}$ as $\frac{a}{b}\times\frac{b}{a}=\frac{b}{a}\times\frac{a}{b}=1$.

1.3.6 Division

We have seen about the reciprocals of fractions in the earlier classes. The same idea of reciprocals is extended to rational numbers also.

To divide a rational number by another rational number, we have to multiply the rational number by the reciprocal of another rational number.

Example 1.12

Divide $\frac{7}{-8}$ by $\frac{-3}{4}$

Solution:

$$\frac{7}{-8} \div \frac{-3}{4} = \frac{-7}{8} \times \frac{-4}{3} = \frac{7}{6}$$

1.4 Word problems on these operations

Example 1.13

The sum of two rational numbers is $\frac{4}{5}$. If one number is $\frac{2}{15}$, find the other.

Solution:

Let the other number be x

Given, $\frac{2}{15} + x = \frac{4}{5}$

$$\Rightarrow x = \frac{4}{5} - \frac{2}{15} = \frac{12-2}{15} = \frac{10}{15}$$

$$\Rightarrow x = \frac{2}{3}$$

Try this

Divide :

(i) 5 by $\frac{-7}{3}$

(ii) $\frac{-7}{3}$ by 5

Example 1.14

The product of two rational numbers is $\frac{-2}{3}$. If one number is $\frac{3}{7}$, find the other.

Solution:

Let the other number be x

Given, $\frac{3}{7}x = \frac{-2}{3}$

Multiplying by the reciprocal of $\frac{3}{7}$, that is $\frac{7}{3}$,

$$\Rightarrow \frac{7}{3} \times \frac{3}{7} \times x = \frac{7}{3} \times \frac{-2}{3}$$

$$\Rightarrow x = \frac{-14}{9}$$

Aliter

$$\frac{3}{7}x = \frac{-2}{3}$$

$$\Rightarrow x = \frac{-2}{3} \times \frac{7}{3} = \frac{-14}{9}$$

Example 1.15

One roll of ribbon is $18\frac{3}{4}$ m long. Sankari has four full rolls and one – third of a roll. How many metres of ribbon does Sankari have in total?

Solution:

Number of metres of ribbon Sankari has

$$= 18\frac{3}{4} \times 4\frac{1}{3}$$

$$= \frac{75}{4} \times \frac{13}{3} = \frac{325}{4} = 81\frac{1}{4} \ m$$

Fig. 1.10

Special Examples:

Example 1.16

Find the rational numbers that should be added and subtracted that will make the sum $3\frac{1}{2}+1\frac{3}{4}+2\frac{3}{8}$ to the nearest whole number.

Solution:

Now, $3\frac{1}{2}+1\frac{3}{4}+2\frac{3}{8}$

$$=\frac{7}{2}+\frac{7}{4}+\frac{19}{8}$$

$$=\frac{7\times4+7\times2+19\times1}{8}=\frac{28+14+19}{8}$$

$=\frac{61}{8}\ =7\frac{5}{8}$, which lies between the whole numbers 7 and 8.

Now, $\frac{64}{8}=8$ and $\frac{56}{8}=7$.

Therefore, the rational number to be added to $\frac{61}{8}$ to get $\frac{64}{8}$ is $\frac{64}{8}-\frac{61}{8}=\frac{3}{8}$ and the rational number to be subtracted from $\frac{61}{8}$ to get $\frac{56}{8}$ is $\frac{61}{8}-\frac{56}{8}=\frac{5}{8}$.

Example 1.17

A student instead of multipling a number by $\frac{8}{9}$, divided it by $\frac{8}{9}$ by mistake. If the difference between the answers got by him is 34, find the number.

Solution:

Let x be the number.

The student had to find $\frac{8x}{9}$ but,

he had found $\frac{x}{\left(\frac{8}{9}\right)}$, that is $\frac{9x}{8}$

Now, $\frac{9x}{8}-\frac{8x}{9}\ =\ 34$

$$\frac{81x-64x}{72}\ =34\Rightarrow\frac{17x}{72}=34$$

$$x\ =\ \frac{34\times72}{17}=144$$

Note

For any non-zero b, c, and d, we have

(i) $\left(\frac{a}{b}\right)\div c=\frac{a}{bc}$

(ii) $a\div\left(\frac{b}{c}\right)=\frac{ac}{b}$

(iii) $\left(\frac{a}{b}\right)\div\left(\frac{c}{d}\right)=\frac{ad}{bc}$

Example 1.18

Evaluate: $\left(\frac{4}{3}-\left(\frac{-3}{2}\right)\right)+\left(\frac{-5}{3}\div\frac{30}{12}\right)+\left(\frac{-12}{9}\times\frac{-27}{16}\right)$

Solution:

$$\left(\frac{4}{3}-\left(\frac{-3}{2}\right)\right)+\left(\frac{-5}{3}\div\frac{30}{12}\right)+\left(\frac{-12}{9}\times\frac{-27}{16}\right)=\left(\frac{4}{3}+\frac{3}{2}\right)+\left(\frac{-5}{3}\times\frac{12}{30}\right)+\left(\frac{-12}{9}\times\frac{-27}{16}\right)$$

$$=\left(\frac{8}{6}+\frac{9}{6}\right)+\left(\frac{-1}{1}\times\frac{4}{6}\right)+\left(\frac{-3}{1}\times\frac{-3}{4}\right)$$

$$=\left(\frac{17}{6}\right)+\left(\frac{-4}{6}\right)+\left(\frac{9}{4}\right)$$

$$=\left(\frac{17-4}{6}\right)+\frac{9}{4}=\frac{13}{6}+\frac{9}{4}$$

$$=\frac{26+27}{12}=\frac{53}{12}$$

Example 1.19

In the NEET exam, out of a total of 180 questions, Jeyanth answered $\frac{19}{30}$ of the questions correctly and $\frac{5}{18}$ of the questions incorrectly. How many questions did Jeyanth not attend at all?

Solution:

Questions answered correctly by Jeyanth $=\frac{19}{30}\times 180=19\times 6=114$

Questions answered incorrectly by Jeyanth $=\frac{5}{18}\times 180=50$

$\therefore$ Number of questions not attended by Jeyanth $=180-(114+50)$

$=180-164=16$

Exercise 1.1

1. Fill in the blanks:

   (i) $\frac{-19}{5}$ lies between the integers ___________ and ___________.

   (ii) The rational number that is represented by 0.44 is ___________.

   (iii) The standard form of $\frac{+58}{-78}$ is ___________.

   (iv) The value of $\frac{-5}{12}+\frac{7}{15}=$ ___________.

   (v) The value of $\left(\frac{-15}{23}\right)\div\left(\frac{+30}{-46}\right)$ is ___________.

2. Say True or False:

   (i) 0 is the smallest rational number.

   (ii) There are an unlimited number of rational numbers between 0 and 1.

   (iii) The rational number which does not have a reciprocal is 0.

(iv) The only rational number which is its own reciprocal is –1.

(v) The rational numbers that are equal to their additive inverses are 0 and–1.

3. List five rational numbers between

(i) –2 and 0 (ii) $\frac{-1}{2}$ and $\frac{3}{5}$ (iii) 0.25 and 0.35 (iv) –1.2 and -2.3

4. Write four rational numbers equivalent to

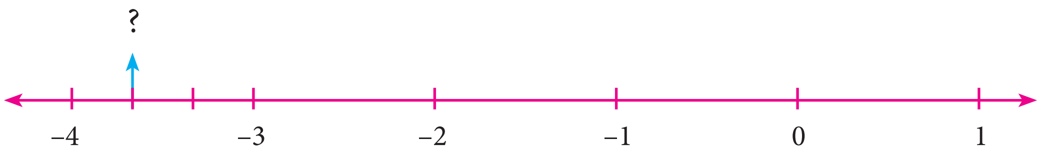
(i) $\frac{-3}{5}$ (ii) $\frac{7}{-6}$ (iii) $\frac{8}{9}$

5. Draw the number line and represent the following rational numbers on it.

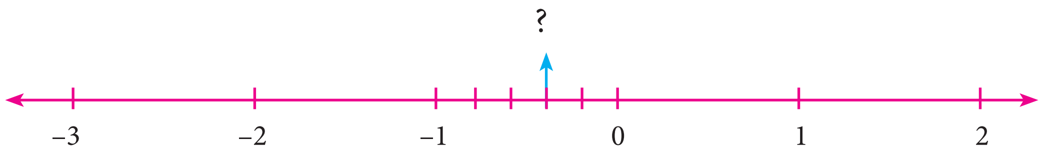
(i) $\frac{9}{4}$ (ii) $\frac{-8}{3}$ (iii) $\frac{-17}{-5}$ (iv) $\frac{15}{-4}$

6. Find the rational numbers for the question marks marked on the number line.

(i)



(ii)



(iii)

7. Using average, write 3 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$.

8. Verify that $-(-x)$ is the same x for: (i) $x = \frac{11}{15}$ (ii) $x = \frac{-31}{45}$

9. Re-arrange suitably and add : $\frac{-3}{7} + \frac{5}{6} + \frac{4}{7} + \frac{1}{3} + \frac{13}{-6}$

10. What should be added to $\frac{-8}{9}$ to get $\frac{2}{5}$?

11. Subtract $\frac{-8}{44}$ from $\frac{-17}{11}$.

12. Evaluate: (i) $\frac{9}{2} \times \frac{-11}{3}$ (ii) $\frac{-7}{27} \times \frac{24}{-35}$

13. Divide i) $\frac{-21}{5}$ by $\frac{-7}{-10}$ (ii) $\frac{-3}{13}$ by -3 (iii) –2 by $\frac{-6}{15}$

14. Simplify $\left(\frac{2}{5} + \frac{3}{2}\right) \div \frac{3}{10}$ as a rational number and show that it is between 6 and 7.

15. Write five rational numbers which are less than –2.

16. Compare the following pairs of rational numbers.

(i) $\frac{-11}{5}, \frac{-21}{8}$ (ii) $\frac{3}{-4}, \frac{-1}{2}$ (iii) $\frac{2}{3}, \frac{4}{5}$.

17. Arrange the following rational numbers in ascending and descending order.

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$ (ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

Objective Type Questions

18. The number which is subtracted from $\frac{-6}{11}$ to get $\frac{8}{9}$ is

(A) $\frac{34}{99}$ (B) $\frac{-142}{99}$ (C) $\frac{142}{99}$ (D) $\frac{-34}{99}$

19. Which of the following rational numbers is the greatest?

(A) $\frac{-17}{24}$ (B) $\frac{-13}{16}$ (C) $\frac{7}{-8}$ (D) $\frac{-31}{32}$

20. $\frac{-5}{4}$ is a rational number which lies between

(A) 0 and $\frac{-5}{4}$ (B) -1 and 0 (C) –1 and –2 (D) –4 and –5

21. The standard form of $\frac{3}{4}+\frac{5}{6}+\left(\frac{-7}{12}\right)$ is

(A) $\frac{1}{22}$ (B) $\frac{-1}{2}$ (C) $\frac{1}{12}$ (D) 1

22. The sum of the digits of the denominator in the simplest form of $\frac{112}{528}$

(A) 4 (B) 5 (C) 6 (D) 7

23. The rational number (numbers) which has (have) additive inverse is (are)

(A) 7 (B) $\frac{-5}{7}$ (C) 0 (D) all of these

24. Which of the following pairs is equivalent?

(A) $\frac{-20}{12}, \frac{5}{3}$ (B) $\frac{16}{-30}, \frac{-8}{15}$ (C) $\frac{-18}{36}, \frac{-20}{44}$ (D) $\frac{7}{-5}, \frac{-5}{7}$

25. $\frac{3}{4} \div \left(\frac{5}{8}+\frac{1}{2}\right) =$

(A) $\frac{13}{10}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{8}$

1.5 Properties for rational numbers

We know that a rational number can always be represented as the quotient of two integers $\frac{a}{b}$. The properties that we have seen for integers are recalled here and we will see the same for the rational numbers too.

Operations	Recalling the properties of Integers. (If a, b, c are integers then, $-a, -b, -c$ are also integers)					
	Closure	Commutative	Associative	Identity	Inverse	Distributive
Addition	$a+b$ belongs to Z . Eg. $5+(-3)=2$ $\Rightarrow 2$ is an integer	$a+b=b+a$ Eg. $5+(-3)=(-3)+5$ $\Rightarrow 2=2$	$(a+b)+c$ $=a+(b+c)$ Eg. $(2+3)+(-4)=1$ $2+\{3+(-4)\}=1$	$a+0$ $=0+a=a$ Eg.$(-4)+0$ $=0+(-4)$ $=-4$	$a+(-a)$ $=(-a)+a=0$ Eg. $5+(-5)$ $=(-5)+5=0$	$a\times(b+c)$ $=(a\times b)+(a\times c)$ Eg. $2\times[3+(-5)]=-4$ $(2\times3)+[2\times(-5)]$ $=-4$
Multiplication	ab ibelongs to Z. Eg.________	$a\times b=b\times a$ Eg.________	$(a\times b)\times c$ $=a\times(b\times c)$ Eg. $(2\times3)\times(-6)=-36$ $2\times[3\times(-6)]=-36$	$a\times1$ $=1\times a=a$ Eg.________	Does not exist	Not Applicable
Subtraction	$a-b$ ibelongs to Z. Eg.________	$a-b\neq b-a$ Eg.________	$(a-b)-c$ $\neq a-(b-a)$ Eg.________	$a-0\neq0-a$ Eg. $5-0=5$ $0-5=-5$ $5\neq-5$	$a-(-a)$ $\neq(-a)-a$ Eg. $2-(-2)=4$ $(-2)-2=-4$ $4\neq-4$	$a\times(b-c)$ $=(a\times b)-(a\times c)$ Eg.________
Division	$a\div b$ does not belong to Z. Eg. $3\div5=\frac{3}{5}$ does not belong to Z.	Fails	Fails	Fails	Fails	Not applicable

1.5.1 Closure property

The collection of rational numbers (Q) is closed under addition and multiplication. This means for any two rational numbers a and b, $a+b$ and $a \times b$ are unique rational numbers.

Illustration

Take $a = \frac{3}{4}$ and $b = \frac{-1}{2}$

Now, $a+b = \frac{3}{4} + \frac{-1}{2} = \frac{3}{4} + \frac{-2}{4} = \frac{3-2}{4} = \frac{1}{4}$ is in Q

Also, $a \times b = \frac{3}{4} \times \frac{-1}{2} = \frac{-3}{8}$ is in Q

Try this

The closure property on integers holds for subtraction and not for division. What about rational numbers? Verify.

1.5.2 Commutative property

Addition and multiplication are commutative for rational numbers.

That is, for any two rational numbers a and b,

(i) $a+b = b+a$ and

(ii) $a \times b = b \times a$

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Illustration

Take $a = \frac{-7}{8}$ and $b = \frac{3}{5}$

Now, $a+b = \frac{-7}{8} + \frac{3}{5} = \frac{-7 \times 5 + 3 \times 8}{40} = \frac{-35+24}{40} = \frac{-11}{40}$

Also, $b+c = \frac{3}{5} + \frac{-7}{8} = \frac{3 \times 8 + -7 \times 5}{40} = \frac{24-35}{40} = \frac{-11}{40}$

We find $a+b = b+a$ and hence addition is commutative.

Further,

$$a \times b = \frac{-7}{8} \times \frac{3}{5} = \frac{-7 \times 3}{8 \times 5} = \frac{-21}{40}$$

Also, $b \times a = \frac{3}{5} \times \frac{-7}{8} = \frac{3 \times -7}{5 \times 8} = \frac{-21}{40}$

Thus, $a \times b = b \times a$ and hence multiplication is commutative.

Try these

(i) Check whether $\frac{3}{5} - \frac{7}{8} = \frac{7}{8} - \frac{3}{5}$.

(ii) Is $\frac{3}{5} \div \frac{7}{8} = \frac{7}{8} \div \frac{5}{3}$? So, what do you conclude?

1.5.3 Associative property

Addition and multiplication are associative for rational numbers.

That is, for any three rational numbers a, b and c,

(i) $(a+b)+c = a+(b+c)$ and

(ii) $(a \times b) \times c = a \times (b \times c)$

Illustration

Take rational numbers a, b, c as $a=\frac{-1}{2}$, $b=\frac{3}{5}$ and $c=\frac{-7}{10}$

Now, $a+b=\frac{-1}{2}+\frac{3}{5}=\frac{-5}{10}+\frac{6}{10}$ (equivalent rationals with common denominators)

$$a+b = \frac{-5+6}{10}=\frac{1}{10}$$

$$(a+b)+c = \frac{1}{10}+\left(\frac{-7}{10}\right)=\frac{1-7}{10}=\frac{-6}{10}=\frac{-3}{5} \quad \text{...(1)}$$

Also, $$b+c = \frac{3}{5}+\frac{-7}{10}=\frac{6}{10}+\frac{-7}{10}=\frac{6-7}{10}=\frac{-1}{10}$$

$$a+(b+c) = \frac{-1}{2}+\frac{-1}{10}=\frac{-5}{10}+\frac{-1}{10}=\frac{-5-1}{10}=\frac{-6}{10}=\frac{-3}{5} \quad \text{...(2)}$$

(1) and (2) shows that $(a+b)+c=a+(b+c)$ is true for rational numbers.

Now, $$a\times b = \frac{-1}{2}\times\frac{3}{5}=\frac{-1\times3}{2\times5}=\frac{-3}{10}$$

$$(a\times b)\times c = \frac{-3}{10}\times\frac{-7}{10}=\frac{-3\times-7}{10\times10}=\frac{21}{100} \quad \text{...(3)}$$

Also, $$b\times c = \frac{3}{5}\times\frac{-7}{10}=\frac{3\times-7}{5\times10}=\frac{-21}{50}$$

$$a\times(b\times c) = \frac{-1}{2}\times\frac{-21}{50}=\frac{-1\times-21}{2\times50}=\frac{21}{100} \quad \text{...(4)}$$

(3) and (4) shows that

$(a\times b)\times c=a\times(b\times c)$ is true for rational numbers. Thus, the associative property is true for addition and multiplication of rational numbers.

Try this

Check whether associative property holds for subtraction and division.

Think

Observe that, $\frac{1}{1.2}+\frac{1}{2.3}=\frac{2}{3}$

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}=\frac{3}{4}$$

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}=\frac{4}{5}$$

Use your reasoning skills, to find the sum of the first 7 numbers in the pattern given above.

1.5.4 Additive and Multiplicative Identity property

The identity for addition is 0 and the identity for multiplication is 1.

For any rational number a there exists unique identity elements 0 and 1 such that

(i) $0+a=a$ and

(ii) $1\times a=a$

Illustration

Take $a=\dfrac{3}{-7}$ that is, $a=\dfrac{-3}{7}$. Now $\dfrac{-3}{7}+0=\dfrac{-3}{7}=0+\dfrac{-3}{7}$ (Why?)

Hence, 0 is the additive identity for $\dfrac{-3}{7}$

Also, $\dfrac{-3}{7}\times 1=\dfrac{-3}{7}=1\times\dfrac{-3}{7}$ (Why?)

Hence, 1 is the multiplicative identity for $\dfrac{-3}{7}$.

1.5.5 Additive and Multiplicative Inverse property

For any rational number a there exists a unique rational number $-a$ such that $a+(-a)=0=(-a)+a$ (Additive Inverse property).

For any non-zero rational number b there exists a unique rational number $\dfrac{1}{b}$ such that $b\times\dfrac{1}{b}=1=\dfrac{1}{b}\times b$ (Multiplicative Inverse property).

Illustration

Take $a=\dfrac{-11}{23}$ Now, $-a=-\left(\dfrac{-11}{23}\right)=\dfrac{11}{23}$

Now, $a+(-a)=\dfrac{-11}{23}+\dfrac{11}{23}=\dfrac{-11+11}{23}=\dfrac{0}{23}=0$

Also, $(-a)+a=\dfrac{11}{23}+\dfrac{-11}{23}=\dfrac{11-11}{23}=\dfrac{0}{23}=0$

$\therefore$ $a+(-a)=(-a)+a=0$ is true.

Also, take $b=\dfrac{-17}{29}$ Now, $\dfrac{1}{b}=\dfrac{29}{-17}=\dfrac{-29}{17}$

$b\times\dfrac{1}{b}=\dfrac{-17}{29}\times\dfrac{-29}{17}=1$

Also, $\dfrac{1}{b}\times b=\dfrac{-29}{17}\times\dfrac{-17}{29}=1$

$\therefore$ $b\times\dfrac{1}{b}=\dfrac{1}{b}\times b=1$ is true.

DO YOU KNOW?

We know that different operations with the same rational numbers usually give different answers. But the following calculations are some interesting exceptions in rational numbers.

(i) $\dfrac{13}{4}+\dfrac{13}{9}=\dfrac{13}{4}\times\dfrac{13}{9}$ (ii) $\dfrac{169}{30}+\dfrac{13}{15}=\dfrac{169}{30}\div\dfrac{13}{15}$

Amazing …! Isn't it? Try a few more like these, if possible.

1.5.6 Distributive property

Multiplication is distributive over addition for the collection of rational numbers.

For any three rational numbers a, b and c,

$$a\times(b+c)=(a\times b)+(a\times c)$$

Illustration

Take rational numbers a,b,c as $a=\frac{-7}{9}$, $b=\frac{11}{18}$ and $c=\frac{-14}{27}$

Now, $b+c \;=\; b+c=\frac{11}{18}+\frac{-14}{27}=\frac{33}{54}+\frac{-28}{54}=\frac{33-28}{54}=\frac{5}{54}$

(Equivalent rational numbers with common denominators)

$$\therefore\ a\times(b+c) = \frac{-7}{9}\times\frac{5}{54}=\frac{-7\times5}{9\times54}=\frac{-35}{486} \qquad \text{---(1)}$$

Also, $a\times b = \frac{-7}{9}\times\frac{11}{18}=\frac{-7\times11}{9\times18}=\frac{-77}{9\times9\times2}$

$a\times c = \frac{-7}{9}\times\frac{-14}{27}=\frac{7\times14}{9\times9\times3}=\frac{98}{9\times9\times3}$

$$\therefore\ (a\times b)+(a\times c) = \frac{-77}{9\times9\times2}+\frac{98}{9\times9\times3}$$

$$=\frac{-77\times3+98\times2}{9\times9\times2\times3}$$

$$=\frac{-231+196}{486}=\frac{-35}{486} \qquad \text{---(2)}$$

(1) and (2) shows that $a\times(b+c)=(a\times b)+(a\times c)$.

Hence, multiplication is distributive over addition for rational numbers Q.

Exercise 1.2

1. Fill in the blanks:

(i) The multiplicative inverse of $2\frac{3}{5}$ is ________.

(ii) If $-3\times\frac{6}{-11}=\frac{6}{-11}\times x$, then x is ________.

(iii) If distributive property is true for $\left(\frac{3}{5}\times\frac{-4}{9}\right)+\left(x\times\frac{15}{17}\right)=\frac{3}{5}\times(y+z)$, then x,y,z are ________, ________ and ________.

(iv) If $x\times\frac{-55}{63}=\frac{-55}{63}\times x=1$, then x is called the ________________ of $\frac{55}{63}$.

(v) The multiplicative inverse of -1 is ________.

2. Say True or False:

(i) $\frac{-7}{8}\times\frac{-23}{27}=\frac{-23}{27}\times\frac{-7}{8}$ illustrates the closure property of rational numbers.

(ii) Associative property is not true for subtraction of rational numbers.

(iii) The additive inverse of $\frac{-11}{-17}$ is $\frac{11}{17}$.

(iv) The product of two negative rational numbers is a positive rational number.

(v) The multiplicative inverse exists for all rational numbers.

3. Verify the closure property for addition and multiplication of the rational numbers $\frac{-5}{7}$ and $\frac{8}{9}$.

4. Verify the associative property for addition and multiplication of the rational numbers $\frac{-10}{11},\frac{5}{6},\frac{-4}{3}$.

5. Check the commutative property for addition and multiplication of the rational numbers $\frac{-10}{11}$ and $\frac{-8}{33}$.

6. Verify the distributive property $a\times(b+c)=(a\times b)+(a+c)$ for the rational numbers $a=\frac{-1}{2}$, $b=\frac{2}{3}$ and $c=\frac{-5}{6}$.

7. Evaluate: $\left(\frac{13}{18}\times\frac{-12}{39}\right)-\left(\frac{8}{9}\times\frac{-3}{4}\right)+\left(\frac{-7}{-9}\div\frac{63}{-36}\right)$

8. Evaluate using appropriate properties.

(i) $\left\{\frac{2}{3}\times\frac{-5}{12}\right\}+\left\{\frac{-4}{6}\times\frac{-8}{12}\right\}+\left\{\frac{-1}{4}\times\frac{2}{3}\right\}$ (ii) $\left\{\frac{1}{2}\times\frac{-3}{4}\right\}-\left\{\frac{3}{8}\times\frac{-1}{4}\right\}+\left\{\frac{-3}{5}\times\frac{-1}{4}\right\}$

9. Use commutative and distributive properties to simplify $\frac{4}{5}\times\frac{-3}{8}-\frac{3}{8}\times\frac{1}{4}+\frac{19}{20}$.

Objective Type Questions

10. Multiplicative inverse of 0 (is)

(A) 0 (B) 1 (C) -1 (D) does not exist

11. Which of the following illustrates the inverse property for addition?

(A) $\frac{1}{8}-\frac{1}{8}=0$ (B) $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ (C) $\frac{1}{8}+0=\frac{1}{8}$ (D) $\frac{1}{8}-0=\frac{1}{8}$

12. Closure property is not true for division of rational numbers because of the number

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$

13. $\frac{1}{2}-\left(\frac{3}{4}-\frac{5}{6}\right)\neq\left(\frac{1}{2}-\frac{3}{4}\right)-\frac{5}{6}$ illustrates that subtraction does not satisfy the ________ property for rational numbers.

(A) commutative (B) closure (C) distributive (D) associative

14. $\left(1-\frac{1}{2}\right)\times\left(\frac{1}{2}-\frac{1}{4}\right)\div\left(\frac{3}{4}-\frac{1}{2}\right)=$

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{-1}{2}$

Note

Why $\frac{a}{b}+\frac{c}{d}\neq\frac{a+c}{b+d}$?

A mathematical statement is true only if it is 100% true without any exception. If we do the addition, say $\frac{3}{4}+\frac{1}{4}$ as $\frac{3+1}{4+4}=\frac{4}{8}=\frac{1}{2}$ this is wrong, since $\frac{3}{4}+\frac{1}{4}=\frac{3+1}{4}=\frac{4}{4}=1$.

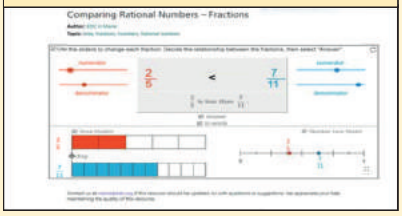
ICT CORNER

Step-1 Open the Browser and type www.Geogebra.com (or)scan the QR CODE given below.

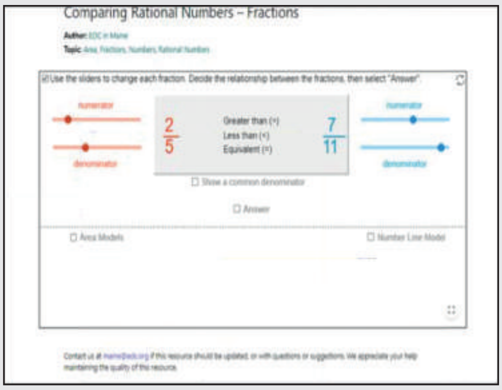
Step-2 Type Rational Numbers on the search column

Step-3 Move the numerator and denominator slide to show the different rational numbers and click the number line models to show the number line.

Through this activity you will know about the rational numbers, operations on them and study their properties as well.



Step 1



Step 2

Web URL rational numbers (rational number links)

Web URL https://www.geogebra.org/m/n92AKzBF#material/ca5D7VbZ

*Pictures are indicatives only.

*If browser requires, allow Flash Player or Java Script to load the page.

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Exercise 1.3

Miscellaneous Practice Problems

N4JH34

1. Match the following appropriately.

A	B
(i) $\left(\frac{5}{6}+-1\right)+\frac{-3}{4}=\frac{5}{6}+\left(-1+\frac{-3}{4}\right)$	a rational number
(ii) $\frac{-1}{3}\left(\frac{1}{4}+\frac{2}{3}\right)=\left(\frac{-1}{3}\times\frac{1}{4}\right)+\left(\frac{-1}{3}\times\frac{2}{3}\right)$	multiplicative inverse property
(iii) $\frac{-4}{9}\times\frac{9}{-4}=\frac{9}{-4}\times\frac{-4}{9}=1$	does not exist
(iv) $\frac{1}{0}$	distributive property of multiplication over addition
(v) $\frac{22}{7}$	associative property of addition

2. Which of the following properties hold for subtraction of rational numbers? Why?
   (a) closure (b) commutative (c) associative
   (d) identity (e) inverse

3. Subbu spends $\frac{1}{3}$ of his monthly earnings on rent, $\frac{2}{5}$ on food and $\frac{1}{10}$ on monthly usuals. What fractional part of his earnings is left with him for other expenses?

4. In a constituency, $\frac{19}{25}$ of the voters had voted for candidate A whereas $\frac{7}{50}$ had voted for candidate B. Find the rational number of the voters who had voted for others.

5. If $\frac{3}{4}$ of a box of apples weighs 3 *kg* and 225 *gm*, how much does a full box of apples weigh?

6. Mangalam buys a water jug of capacity $3\frac{4}{5}$ *litres*. If she buys another jug which is $2\frac{2}{3}$ times as large as the smaller jug, how many litres can the larger one hold?

7. In a recipe making, every $1\frac{1}{2}$ cup of rice requires $2\frac{3}{4}$ cups of water. Express this, in the ratio of rice to water.

8. Ravi multiplied $\frac{25}{8}$ and $\frac{16}{15}$ to obtain $\frac{400}{120}$. He says that the simplest form of this product is $\frac{10}{3}$ and Chandru says the answer in the simplest form is $3\frac{1}{3}$. Who is correct? (or) Are they both correct? Explain.

9. A piece of wire is $\frac{4}{5}m$ long. If it is cut into 8 pieces of equal length, how long will each piece be?

10. Find the length of a room whose area is $\frac{153}{10}$ *sq.m* and whose breadth is $2\frac{11}{20}m$.

Challenging problems

1. Show that $\left(\dfrac{\frac{7}{9}-5}{\frac{4}{3}}\right)\div\frac{3}{2}+\frac{4}{9}-\frac{1}{3}=-2$.

2. If A walks $\frac{7}{4}$ *km* and then jogs $\frac{3}{5}$ *km*, find the total distance covered by A. How much did A walk rather than jog?

3. In a map, if 1 inch refers to 120 *km*, then find the distance between two cities B and C which are $4\frac{1}{6}$ *inches* and $3\frac{1}{3}$ *inches* from the city A which is in between the cities B and C.

4. Give an example and verify each of the following statements.

   (i) The collection of all non-zero rational numbers is closed under division.

   (ii) Subtraction is not commutative for rational numbers.

   (iii) Division is not associative for rational numbers.

   (iv) Distributive property of multiplication over subtraction is true for rational numbers. That is, $a(b-c)=ab-ac$.

   (v) The mean of two rational numbers is rational and lies between them.

5. If $\frac{1}{4}$ of a *ragi adai* weighs 120 *grams*, what will be the weight of $\frac{2}{3}$ of the same *ragi adai*?

6. Find the difference between the greatest and the smallest of the following rational numbers.

   $$\frac{-7}{12},\frac{2}{-9},\frac{-11}{36},\frac{-5}{-6}$$

7. If $p+2q=18$ and $pq=40$, find $\frac{2}{p}+\frac{1}{q}$.

8. Find 'x' if $5\frac{x}{5}\times3\frac{3}{4}=21$.

9. The difference between a number and its two third is 30 more than one -fifth of the number. Find the number.

10. By how much does $\dfrac{1}{\left(\frac{10}{11}\right)}$ exceed $\dfrac{\left(\frac{1}{10}\right)}{11}$?

Summary

- A number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ is called a rational number.
- All natural numbers, whole numbers, integers and fractions are rational numbers.
- All terminating and repeating decimals can be written as rational numbers.
- All non-terminating and non-recurring decimals are not rational numbers.
- Every rational number can be represented on a number line.
- If the numerator and denominator of a rational number is multiplied or divided by the same non-zero integer, we get an equivalent rational number.
- 0 is neither a positive nor a negative rational number.
- A rational number $\frac{a}{b}$ is said to be in the standard form if its denominator b is a positive integer and HCF $(a,b)=1$
- There are unlimited numbers of rational numbers between two rational numbers.
- Two rational numbers with different denominators can be added after converting them to equivalent rational numbers having the same denominators (LCM).
- Subtracting two rational numbers is the same as adding the additive inverse of the second number to the first rational number.
- Multiplying two rational numbers is the same as multiplying their numerators and denominators separately and then writing the product in the standard form.
- Dividing a rational number by another rational number is the same as multiplying the first rational number by the reciprocal of the second rational number.
- The following table is about the properties of rational numbers.

Q	Closure	Commutative	Associative	Multiplication is distributive over +/-
+	✓	✓	✓	✓
-	✓	×	×	✓
×	✓	✓	✓	-
÷	×	×	×	-

- 0 and 1 are respectively the additive and the multiplicative identities of rational numbers.
- The additive inverse for $\frac{a}{b}$ is $\frac{-a}{b}$ and vice – versa.
- The reciprocal or the multiplicative inverse of a rational number $\frac{a}{b}$ is $\frac{b}{a}$ and $\frac{a}{b} \times \frac{b}{a} = 1$.

2 MEASUREMENTS

Learning Outcomes

- To know the parts of a circle.
- To calculate the length of arc, area and circumference of a sector.
- To calculate the area and the perimeter of combined plane figures.
- To understand representation of 3-D shapes in 2-D.
- To understand representation of 3-D objects with cubes.

U2UWF2

2.1 Introduction

An important aspect in everyone's day-to-day life is **'to measure'.** Measuring the length of a rope, the distance between two places, finding the perimeter, area of plots and lands, building the structures under specified measures etc., are a few of the many situations where the concept of 'measure' is used.

It is said that the biggest invention by man is the wheel. The wheel is otherwise called as the **'cradle of invention'.** What is the shape of a wheel? Circle, isn't it? In the things we use, apart from circles, we can also see different shapes like triangles, squares, rectangles etc.,

Measurements play a vital role in everyone's life. Not only the person who learns proper mathematics in schools, but also the layman uses his logical thinking to apply the concept of measurements when he needs. For example, a carpenter who carves out the wooden wheels of a temple car wants to protect the outer surface by an iron strap. With all his experience, he says that a 22 feet strap will be required for a 7 feet high (diameter) wooden wheel.

MATHEMATICS ALIVE — **AREA AND CIRCUMFERENCE IN REAL LIFE**

Sectors are used in manufacturing the dining table

Combined shapes are used in the construction of building

We already have learnt how to find the perimeter and the area of shapes like circles, triangles, squares, rectangles, trapeziums, parallelograms etc. In this chapter, we shall see the parts of a circle and how to find the perimeter and the area of a sector and some combined shapes.

Recap

The teacher asks the students to calculate the area of the circle of radius 7 *cm*. Many students have solved it as in method 1, but a few of them have done it as in method 2.

Method:1	**Method:2**
Area of the circle, A $= \pi r^2$ *sq.units* $= \frac{22}{7} \times 7 \times 7$ $= 154$ *sq.cm*	Area of the circle, A $= \pi r^2$ *sq.units* $= 3.14 \times 7 \times 7$ $= 153.86$ *sq.cm*

Then they had the following conversation:

Sudha : Which of these two answers is correct, Teacher ?

Teacher : Both answers are approximately correct.

Sudha : We get two answers for the same question. How is it possible, teacher?

Teacher : We can infact solve this exactly as $\pi \times 7 \times 7 = 49\pi$ *sq.cm*.

Meena : When we find the area of a square, a rectangle, etc., we all get unique answers, but why is the area of a circle not unique ?

Teacher : What is the value of π ?

Meena : The value of π is $\frac{22}{7}$ or 3.14

Teacher : Actually it is neither $\frac{22}{7}$ nor 3.14. They are approximate values.

Think

$\frac{22}{7}$ and 3.14 are rational numbers. Is 'π' a rational number? Why?

Meena : Then, what is the exact value of π?

Teacher : Though π is the constant, it is a non-terminating and non-recurring decimal number. So, we are not able to use its exact value and to make the calculations easy, $\frac{22}{7}$ or 3.14 is used as the approximate value of π.

Meena : Is the area of a circle always approximate, teacher?

Teacher : No, it is an exact answer unless we substitute the value of π as $\frac{22}{7}$ or 3.14.

Meena : Then, for the above problem, 49π *sq.cm* is an exact answer whereas 154 *sq.cm* and 153.86 *sq.cm.* are approximately correct answers. Am I right, teacher?

Teacher : Yes, you are right Meena.

Activity-1

Cut a circle of diameter 7 units from a card board and mark a number line from 0 to 30 units on a sheet of paper. Now, rotate the circle, from 0 to complete one full round (The length covered in a full round is equal to its circumference).

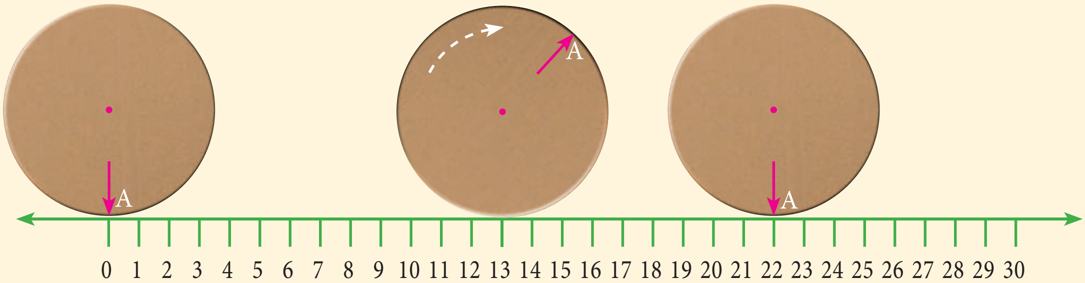


Fig. 2.1

(i) What is the circumference of a circle, whose diameter is 7 *units*?

(ii) $\pi = \dfrac{circumference}{diameter} = ?$

2) Divide the circumference of your bicycle's wheel by its diameter and find the value of π. Do the same for coins, circle shaped buttons and some circular objects. Discuss the answer with your friends.

3) Count and write the number of letters in each word of the sentence "May I have a large container of coffee, cream and sugar". (First 3 words are counted and written for you)

3	.	1	4								

This number 3.1415926535 represents the value of 'π' correct to 10 decimals

4) When is the 'π' day celebrated? Why?

DO YOU KNOW?

The circumference of a circle is $2\pi r$ *units*, which can also be written as πd *units*. As π =3.14 (approximately) and it is slightly more than 3, for some quick guess, we shall say that a circle whose diameter is *d units* shall have its circumference slightly more than three times its diameter.

For example,

(i) If a round table of diameter of 3 *feet* is to be decorated by flower strings around it, it will require a little more than 9 *feet* of flower strings.

(ii) To make a circular kite of diameter 10 *cm*, we need a wire which is more than slightly 30 *cm* to make it. Here, slightly means 0.14 (or) $\frac{1}{7}$ of a part.

2.2 Parts of a Circle

A circle is the path traced by a moving point so that its distance from a fixed point is e always a constant. The fixed point of the circle is called its **'centre'** and the constant distance is called its **'radius'.**

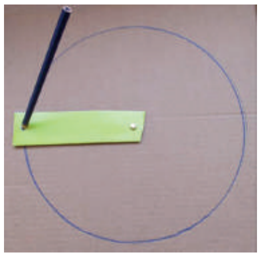


Fig. 2.2

Further, if any two points on a circle are joined by a line segment, then the line segment is called a **'chord'.** A chord divides a circle into two parts. A chord which passes through the centre of a circle is called as a **'diameter'.** A diameter of a circle divides it into two equal parts. It is also the 'longest chord' of a circle.

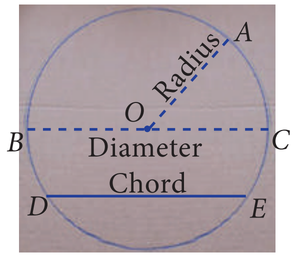


Fig. 2.3

Activity-2

1. Using a bangle, draw a circle on the paper and cut it. Then mark any two points A and B on it and fold the circle so that the fold has A and B on it. Now, this line segment represents a chord.
2. By paper folding, find two diameters and hence the centre of a circle.
3. Check whether the diameter of a circle is twice its radius.

2.2.1 Circular Arc and Circular Sector

Look at the glass bangle and the pizza given in Fig. 2.4 carefully.

Though both are of circular shapes, what we understand is that, the bangle indicates the boundary of the circle, whereas the pizza indicates the plane enclosed within the boundary of the circle. Thus, it is clear that the bangle indicates the circumference of the circle whereas the pizza indicates the area of the circle.

Fig. 2.4

From them, cut a part as shown in Fig. 2.5.

Each of the glass bangle pieces represents the circular arcs. Here, arc AB $\left(\overset{\frown}{AB}\right)$ is the smaller one and arc BA ($\overset{\frown}{BA}$) is the larger one. Each of the parts of the pizza represents the circular sectors. Here A_1 is the minor sector and A_2 is the major sector.

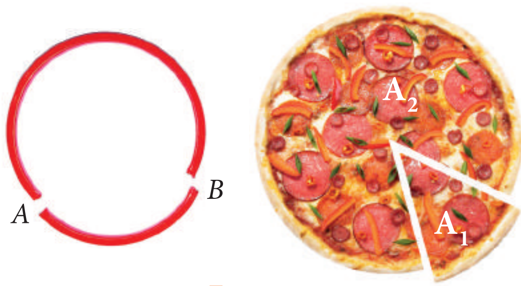


Fig. 2.5

- A part of the circumference of a circle is called a circular arc .
- The plane surface that is enclosed between two radii and the circular arc of a circle is called a sector.
- Each part of a circle which is divided by a chord is called a segment.

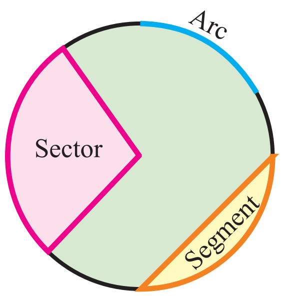
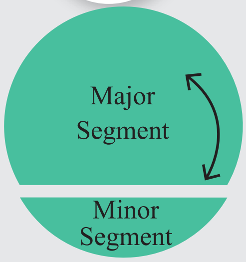


Fig. 2.6

Note

The part which has a smaller arc is called as the **'minor segment'** and the part which has a larger arc is called as the **'major segment'**.



Think


Fig. 2.7

The given circular figure is divided into six equal parts. Can we call the equal parts as sectors? Why?

Central Angle

The angle formed by a sector of a circle at its centre is called the central angle. The vertex of the central angle of the sector is the centre of the circle. The two arms of its are the radii. In the Fig.2.8, the shaded sector has the central angle, $\underline{|AOB} = \theta^\circ$ (read as theta) and its two arms OA and OB are the radii of the circle.

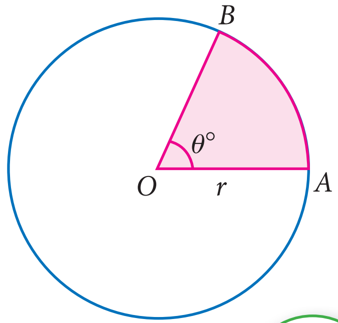


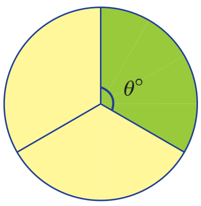
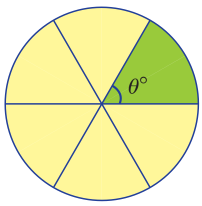
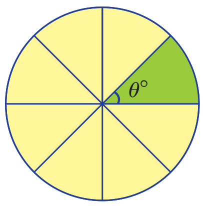
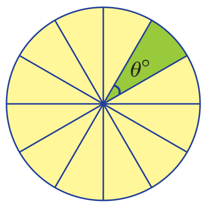
Fig. 2.8

The central angle of a circle is 360°. If a circle is divided into 'n' equal sectors, the central angle of each of the sectors is $\theta^\circ = \dfrac{360^\circ}{n}$.

For example, the central angle of a semicircle $= \dfrac{360^\circ}{2} = 180^\circ$ and the central angle of a quadrant of a circle $= \dfrac{360^\circ}{4} = 90^\circ$.

Try these

Fill the central angle of the shaded sector (each circle is divided into equal sectors)

Sectors				
Central angle $\theta^\circ = \left(\dfrac{360^\circ}{n}\right)$	$\theta^\circ = 120^\circ$			

2.2.2 Length of the arc and Area of the sector

We already have learnt that, a circle of radius 'r' *units* has

Central angle = 360°

Circumference of the circle = $2\pi r$ *units* and

Area of the circle = πr^2 *Sq.units.*

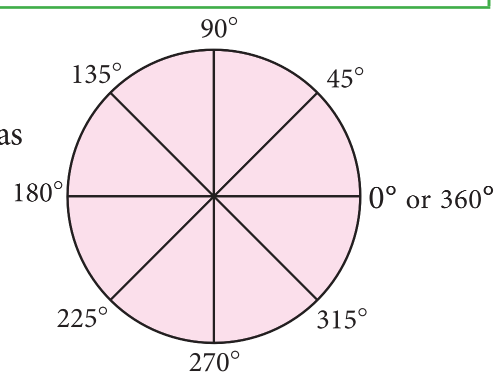


Fig. 2.9

First, let us take a semicircle . If a circle is divided into 2 equal sectors we will get 2 semi-circles. The length of a semicircular arc is half of the circumference of the circle.

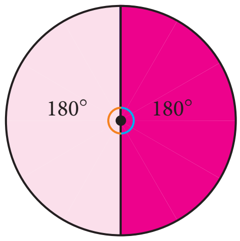


Fig. 2.10

$\therefore$ Length of the semicircular arc $\left(\overset{\frown}{AB}\right) = \frac{1}{2}\times 2\pi r$ *units* and the area of the semicircular sector is half of the area of circle,

that is, area of the semicircle $= \frac{1}{2}\times \pi r^2$ sq.*units*

Here, the semicircular sector makes an angle $180^\circ \left(=\frac{360^\circ}{2}\right)$ at the centre of the circle.

The ratio of the central angle (180°) of a semicircular sector and the central angle (360°) of a circle is $\frac{1}{2}\left(=\frac{180^\circ}{360^\circ}\right)$. Hence, to find the length of a semicircular arc and its area, instead of multiplying by $\frac{1}{2}$, we shall multiply by $\frac{180^\circ}{360^\circ}$.

$\therefore$ Length of the semicircular arc $= \frac{1}{2}\times 2\pi r = \frac{180^\circ}{360^\circ}\times 2\pi r$ *units.*

The area of the semicircle $= \frac{1}{2}\times \pi r^2 = \frac{180^\circ}{360^\circ}\times \pi r^2$ *sq.units.*

Now, if a circle is divided into 3 equal sectors, each of their central angles is $120^\circ \left(=\frac{360^\circ}{3}\right)$. The ratio of 120° to 360° is $\frac{1}{3}\left(=\frac{120^\circ}{360^\circ}\right)$.

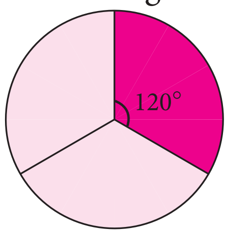


Fig. 2.11

Now, the length of the arc of this sector $= \frac{1}{3}\times 2\pi r = \frac{120^\circ}{360^\circ}\times 2\pi r$ *units*

and the area of the sector $= \frac{1}{3}\times \pi r^2 = \frac{120^\circ}{360^\circ}\times \pi r^2$ *sq. units.*

In the same way, if a circle is divided into 4 equal sectors, what would you call them as? Quadrant of a circle. What is the central angle of a quadrant? Yes, $90^\circ\left(=\frac{360^\circ}{4}\right)$.

The ratio 90° to 360° is $\frac{1}{4}\left(=\frac{90^\circ}{360^\circ}\right)$

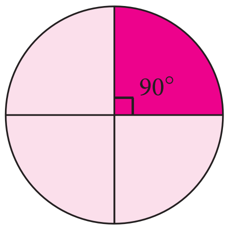


Fig. 2.12

$\therefore$ Length of the arc of the circular quadrant $= \frac{1}{4}\times 2\pi r = \frac{90^\circ}{360^\circ}\times 2\pi r$ *units.*

and area of the quadrant $= \frac{1}{4}\times \pi r^2 = \frac{90^\circ}{360^\circ}\times \pi r^2$ *sq.units*

What do we know from this?

If the ratio of the central angle of a sector to the central angle of a circle is multiplied with the circumference and the area of the circle, we can find the length of the arc of that sector and its area respectively.

That is, if we assume that the central angle of a sector of radius '*r*' *units* as θ°, then, the ratio of the central angle $\theta°$ to 360° is $\dfrac{\theta°}{360°}$.

$\therefore$ **Length of the arc,** $l = \dfrac{\theta°}{360°} \times 2\pi r$ units and

Area of the sector, $A = \dfrac{\theta°}{360°} \times \pi r^2$ sq.units.

Note

1. If a circle of radius r units divided into n equal sectors, then the

Length of the arc of each of the sectors $= \dfrac{1}{n} \times 2\pi r$ *units* and

Area of each of the sectors $= \dfrac{1}{n} \times \pi r^2$ *sq.units*

2. Also, the area of the sector is derived as

$$= \frac{\theta°}{360°} \times \pi r^2$$
$$= \frac{1}{2}\left(\frac{\theta°}{360°} \times 2\pi r\right) \times r$$
$$= \left(\frac{1}{2} \times l\right) \times r = \frac{lr}{2} \text{ sq.units}$$

2.2.3 Perimeter of a sector

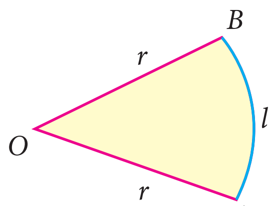


Fig. 2.13

We already know that, the total length of the boundary of a closed part is its perimeter, Isn't it? What is the boundary of a sector? **Two radii** (OA and OB) and **an arc** $\left(\overset{\frown}{AB}\right)$.

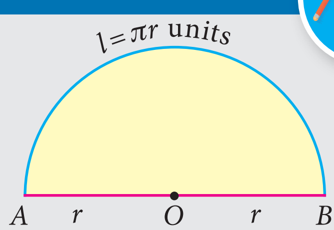
So, the perimeter of a sector = length of the arc + length of two radii

$$P = l + 2r \text{ units}$$

$\therefore$ **Perimeter of a sector,** $P = l + 2r$ *units*

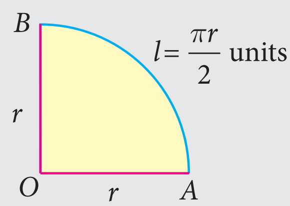
Note

1. The perimeter of a semi-circle



$$P = l + 2r \text{ units}$$
$$= \pi r + 2r$$
$$= (\pi + 2)r \text{ units}$$

2. The perimeter of a circular quadrant



$$P = l + 2r = \frac{\pi r}{2} + 2r = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$

Tamil people always have a prominent role in the history of Mathematics. They had recorded in the form of a song on how to find the area of a circle, a few thousand years before itself, in the book titled 'Kanakkathikaram'. (கணக்கதிகாரம்). The song is as follows

"வட்டத்தரை கொண்டு விட்டத்தரை தாக்கச்
சட்டெனத் தோன்றும் குழி "

Meaning:

'**வட்டத்தரை**' represents half of the circumference and '**விட்டத்தரை**' represents half of the diameter which is the radius and '**குழி**' represents the area. In this song, the Tamil people had recorded that, if half the circumference is multiplied by half the diameter, the area of the circle can be calculated.

Area of the circle = **வட்டத்தரை** × **விட்டத்தரை**

$$= \frac{1}{2} \text{ of circumference} \times \frac{1}{2} \text{ of diameter}$$

$$= \left(\frac{1}{2} \times 2\pi r\right) \times r$$

$\therefore$ Area of the circle, $A = \pi r^2$ *sq.units*

Example 2.1

The radius of a sector is $21cm$ and its central angle is $120°$. Find (i) the length of the arc (ii) area of the sector and (iii) perimeter of sector.

Solution:

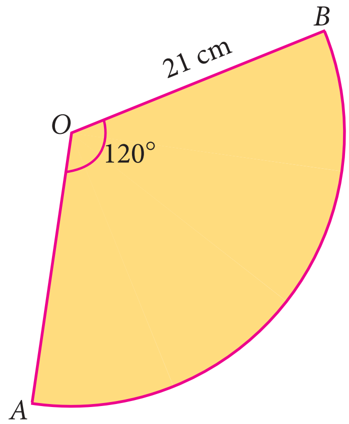


Fig. 2.14

(i) length of the arc , $l = \frac{\theta°}{360°} \times 2\pi r$ units

$$= \frac{120°}{360°} \times 2 \times \pi \times 21$$

$$= \frac{1}{3} \times 2 \times \pi \times 21$$

$$l = 14\pi \ cm \ \text{(or)}$$

$$= 14 \times \frac{22}{7}$$

$$l = 44 \text{ cm (approximately)}$$

(ii) Area of the sector, $A = \frac{\theta°}{360°} \times \pi r^2$

$$= \frac{120^\circ}{360^\circ} \times \pi \times 21 \times 21$$

$$= \frac{1}{3} \times \pi \times 21 \times 21$$

$$A = 147\pi \ sq.cm \text{ (or)}$$

$$= 147 \times \frac{22}{7}$$

A = 462 *sq.cm* (approximately)

Aliter:

Area of the sector

$$A = \frac{lr}{2} sq.units$$

$$= \frac{44 \times 21}{2}$$

A = 462 cm^2 (approximately)

(iii) Perimeter of the sector, $P = l + 2r$ *units*

= 44+2×21

= 44+42

P = 86 *cm* (approximately)

Example 2.2

Find the central angle and area of a palm leaf fan (sector) of radius 10.5 *cm* and whose perimeter is 43 *cm*. $\left(\pi = \frac{22}{7}\right)$

Solution:

Perimeter of the palm leaf fan= 43 *cm*

That is, $l + 2r = 43$

$$l + 2 \times (10.5) = 43$$

l = 43 – 21

∴ the length of the arc l = 22 *cm*.

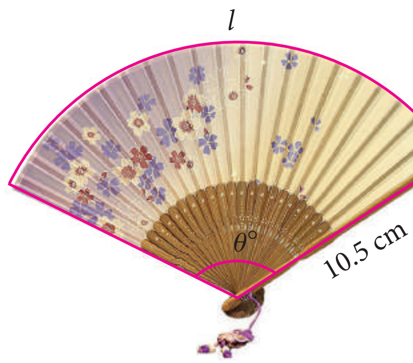


Fig. 2.15

Length of the arc $l = \frac{\theta^\circ}{360^\circ} \times 2\pi r$ *units*

$$22 = \frac{\theta^\circ}{360^\circ} \times 2 \times \frac{22}{\cancel{7}} \times \overset{1.5}{\cancel{10.5}}$$

$$\theta^\circ = \frac{360^\circ}{3} = 120^\circ$$

Also, area of the palm leaf fan

$$A = \frac{lr}{2} sq.units$$

$$= \frac{22 \times 10.5}{2}$$

A =115.5 cm^2 (approximately)

Aliter:

Area of the sector

$$A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 10.5 \times 10.5$$

A =115.5 cm^2 (approximately)

Example 2.3

A circular shaped gymnasium ring of radius 35*cm* is divided into 5 equal arcs shaded with different colours. Find the length of each of the arcs.

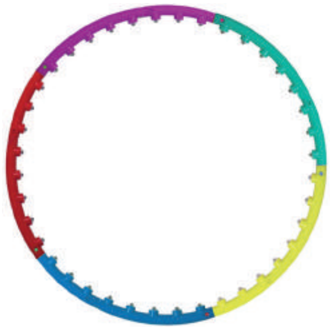


Fig. 2.16

Solution:

Length of each of the arcs, $l = \frac{1}{n} \times 2\pi r$ *units*

$$= \frac{1}{5} \times 2 \times \pi \times 35$$

$$l = 14\pi \text{ cm.}$$

Example 2.4

A spinner of radius 7.5 *cm* is divided into 6 equal sectors. Find the area of each of the sectors.

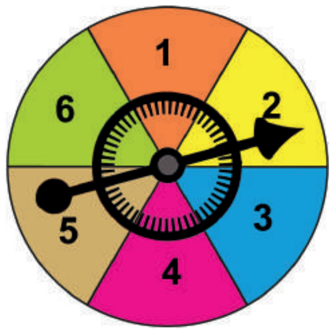


Fig. 2.17

Solution:

Area of each of the sectors, $A = \frac{1}{n} \times \pi r^2$

$$= \frac{1}{6} \times \pi \times 7.5 \times 7.5$$

$$= 9.375\pi \text{ sq. cm} \quad \text{(or)}$$

$$= 9.375 \times \frac{22}{7}$$

$$A = 29.46 \text{ sq. cm.} \text{(approximately)}$$

Example 2.5

From the three vertices of an equilateral triangle of side 12 *cm*, Nishanth cuts sectors of 5 *cm* radius each and forms the following shape. Find the area of that shape. $(\pi = 3.14)$

Solution:

Since, the sectors are cut from an equilateral triangle, the central angle of each of them is $60°$.

$\therefore$ Area of the shape formed, $A = 3 \times \left(\frac{\theta°}{360°} \times \pi r^2\right)$

$$= 3 \times \frac{60°}{360°} \times \pi \times 5 \times 5$$

$$= 3 \times \frac{1}{6} \times \pi \times 5 \times 5$$

$$= 12.5\pi \text{ sq.cm}$$

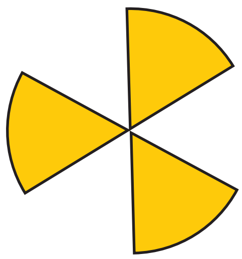


Fig. 2.18

Example 2.6

Pradeep wants to make a semicircular arch design at the entrance of his house with three equal sectors, as shown in the Fig.2.19 to be fitted in the iron frame. Find the length of the iron frame required and also the area of each of the sectors for which the mirrors to be fixed.

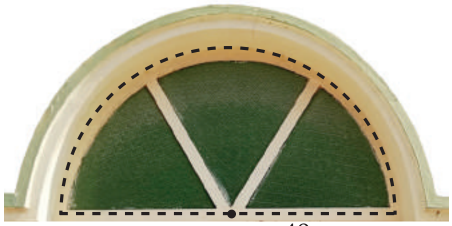


Fig. 2.19

Solution:

(i) The length of the iron frame required

$$= \text{length of the arc} + 4r$$
$$= \pi r + 4r$$
$$= \left(\frac{22}{7} \times 49\right) + (4 \times 49)$$
$$= 154 + 196$$
$$= 350\ cm \quad \text{(approximately)}$$

(ii) Area of each of the mirror sectors

$$= \frac{\theta^\circ}{360^\circ} \times \pi r^2$$
$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 49 \times 49$$
$$= 1257.67\ sq.cm \quad \text{(approximately)}$$

Example 2.7

Kamalesh has a dining table, circular in shape of radius 70 *cm*, whereas Tharun has a circular quadrant dining table of radius 140 *cm*. Whose dining table has a greater area?

Solution:


Fig. 2.20

Area of the dining table of Kamalesh $= \pi r^2\ sq.\ units$

$$= \frac{22}{7} \times 70 \times 70$$
$$A = 15400\ sq.cm \text{ (approximately.)}$$

Area of the circular quadrant dining table of Tharun

$$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 140 \times 140$$
$$A = 15400\ sq.cm \text{ (approximately.)}$$


Fig. 2.21

We find that, the area of the dining tables of both of them have the same area.

Think

If the radius of a circle is doubled, what will happen to the area of the new so formed?

Example 2.8

Four identical medals, each of diameter 7*cm* are placed as shown in Fig. 2.20. Find the area of the shaded region between the coins.

Solution:

Area of the shaded region = Area of the square – 4 × Area of the circular quadrant

$$= a^2 - 4 \times \frac{1}{4}\pi r^2$$

$$= (7 \times 7) - \left(4 \times \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$$

$= 49 - 38.5 = 10.5$ *sq.cm.* (approx.)

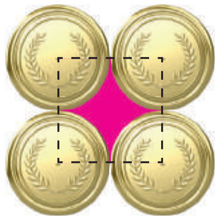


Fig. 2.22

Exercise 2.1

1. Fill in the blanks:

    (i) The ratio between the circumference and diameter of any circle is ________.

    (ii) A line segment which joins any two points on a circle is a ____________.

    (iii) The longest chord of a circle is ___________.

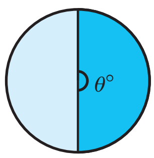
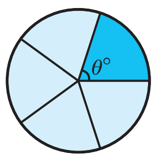
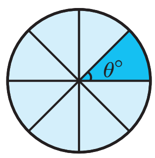
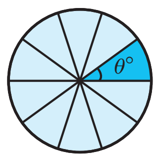
    (iv) The radius of a circle of diameter 24 *cm* is ________.

    (v) A part of circumference of a circle is called as ________.

2. Match the following:

(i) Area of a circle	-(a) $\frac{1}{4}\pi r^2$
(ii) Circumference of a circle	-(b) $(\pi + 2)r$
(iii) Area of the sector of a circle	-(c) πr^2
(iv) Circumference of a semicircle	-(d) $2\pi r$
(v) Area of a quadrant of a circle	-(e) $\frac{\theta°}{360°} \times \pi r^2$

3. Find the central angle of the shaded sectors (each circle is divided into equal sectors).

Sectors				
Central angle of each sector ($\theta°$)				

4. For the sectors with given measures, find the length of the arc, area and perimeter. (π=3.14)

    (i) central angle 45°, r = 16 *cm* (ii) central angle 120°, d =12.6 *cm*

    (iii) central angle 60°, r =36 *cm* (iv) central angle 72°, d =10 *cm*

5. From the measures given below, find the area of the sectors.

S.No	length of the arc (l)	radius (r)
(i)	48 m	10 m
(ii)	12.5 cm	6 cm
(iii)	50 cm	13.5 cm

6. Find the central angle of each of the sectors whose measures are given below. $\left(\pi = \frac{22}{7}\right)$

S.No	area (A)	length of the arc (l)	radius (r)
(i)	462 cm^2	-	21cm
(ii)	18.48 cm^2	-	8.4 cm
(iii)		44 m	35 m
(iv)	-	22 mm	105 mm

7. Answer the following questions:

   (i) A circle of radius 120 m is divided into 8 equal sectors. Find the length of the arc of each of the sectors.

   (ii) A circle of radius 70 cm is divided into 5 equal sectors. Find the area of each of the sectors.

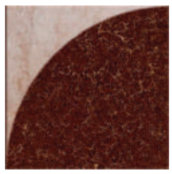
8. Find the area of a sector whose length of the arc is 50 mm and radius is 14 mm.

9. Find the area of a sector whose perimeter is 64 cm and length of the arc is 44 cm.

10. A sector of radius 4.2 cm has an area 9.24 cm^2. Find its perimeter.

11. Infront of a house, flower plants are grown in a circular quardant shaped pot whose radius is 2 feet. Find the area of the pot in which the plants grow. $(\pi = 3.14)$

12. Dhamu fixes a square tile of 30 cm on the floor. The tile has a sector design on it as shown in the figure. Find the area of the sector. $(\pi = 3.14)$.



13. A circle is formed with 8 equal granite stones as shown in the figure each of radius 56 cm and whose central angle is 45°. Find the area of each of the granite. $\left(\pi = \frac{22}{7}\right)$

2.3 Combined shapes

In our day-to-day life, we use infinitely many things of shapes like circle, triangle, square, rectangle, rhombus etc., don't we? We use them separately as well as with two or three or more shapes combined together. For example,

			
Glass window	Model house	Invitation card	Locker

What do we observe from the above figures?

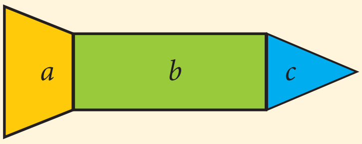
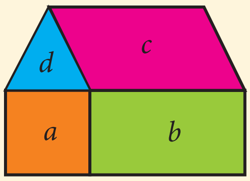
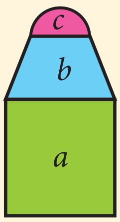
The shape of a glass window is like a semi-circle placed over a rectangle, whereas the front – facing wall of a model house looks like a triangle over a square.

List the shapes used to make an invitation card and a locker.

Thus, two or more plane figures joined with the sides of same measure give rise to a new shape called combined shapes.

Activity-3

If the following figures are combined shapes, find out by which simple shapes they are formed.

S.No	Combined shapes	Simple shapes attached
1		a. Trapezium b. Rectangle c. Triangle
2		a. b. c. d.
3		a. b. c.

2.3.1 Perimeter of combined shapes

The perimeter of a combined shape is the sum of all the lengths of the sides that form a closed boundary.

For example, observe the Fig. 2.23 which has a square of side *a units* and an equilateral triangle of side *a units* combined together.

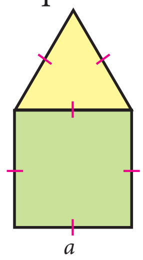


Fig. 2.23

Though a square has 4 sides and an equilateral triangle has 3 sides, when they are combined together, we will have a total of only 5 sides for the boundary of the combined shape and not 7 sides. Isn't it? So, the perimeter of the combined shape, here is 5*a units*.

2.3.2 Area of combined shapes

To find the area of a combined shape, split the combined shape into known simple shapes, find their area separately and then add them up. That is, the area of combined shapes is nothing but the sum of all the areas of the simple shapes in it.

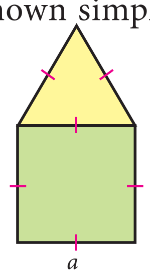


Fig. 2.24

To find the area of the Fig.2.24, find the area of the square and the area of the equilateral triangle separately and then add them up.

The combined shapes that we come across mostly in our day-to-day life are irregular polygons. To find their areas, we must identify the simple shapes in them, find their areas separately and then add them up.

For example, an irregular polygonal field can be split into known simpler shapes as given below and its area can be found.

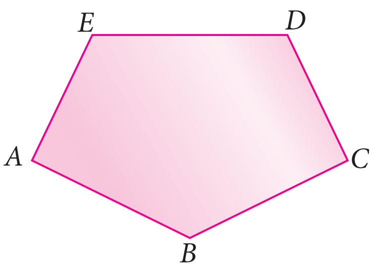


Fig. 2.25

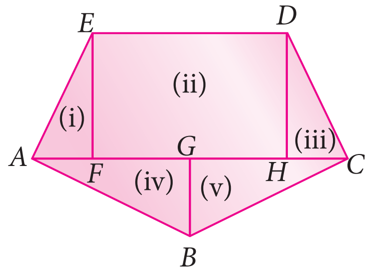


Fig. 2.26

DO YOU KNOW?

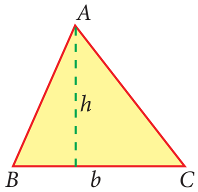
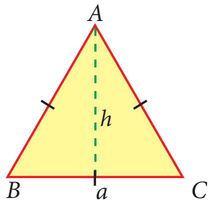
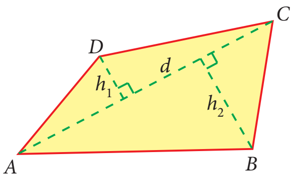
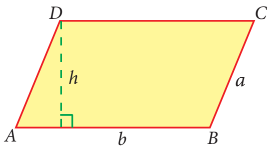
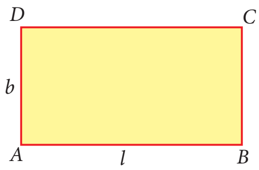
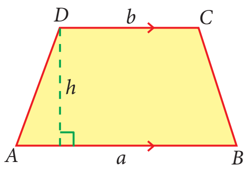
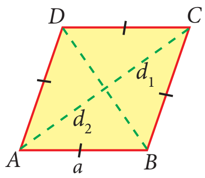
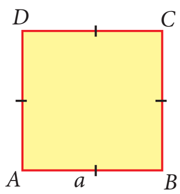
A closed plane figure formed by three or more sides is called a 'polygon' .Based on the sides, some of the polygons are tabulated as given below.

Number of sides	3	4	5	6	7	8	9	10
Name of the Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon

If all sides and all angles of a polygon are equal, then it is called as a regular polygon. Examples: equilateral triangle, square etc., Other polygons are irregular polygons. Examples: scalene triangle, rectangle etc.,

All the sides of a rhombus are equal. Is it a regular polygon?

The formulae to find the area and the perimeter of some plane figures learnt in the earlier class are tabulated below will be helpful to find the area and the perimeter of combined figures.

S.No	Shape	Name	Area (*sq.units*)	Perimeter (*units*)
1		Triangle	$\frac{1}{2}\times b\times h$	Sum of all three sides
2		Equilateral triangle	$\frac{\sqrt{3}}{4}a^2\left(h=\frac{\sqrt{3}}{2}a\right)$	$3a$
3		Quadrilateral	$\frac{1}{2}\times d\times(h_1+h_2)$	Sum of all the four sides
4		Parallelogram	$b\times h$	$2(a+b)$
5		Rectangle	$l\times b$	$2(l+b)$
6		Trapezium	$\frac{1}{2}\times h\times(a+b)$	Sum of all the four sides
7		Rhombus	$\frac{1}{2}\times d_1\times d_2$	4a
8		Square	a^2	4a

Example 2.9

Find the perimeter and area of the given Fig.2.27. $\left(\pi = \frac{22}{7}\right)$

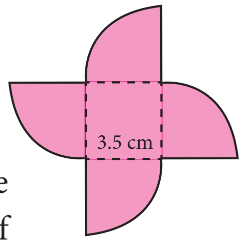


Fig. 2.27

Solution:

The given figure is formed by the joining of 4 quadrants of a circle with each side of a square. The boundary of the given figure consists of 4 arcs and 4 radii.

(i) Perimeter of the given combined shape

$$= 4 \times \text{ length of the arcs of the quadrant of a circle} + 4 \times \text{radius}$$

$$= \left(4 \times \frac{1}{4} \times 2\pi r\right) + 4r$$

$$= \left(4 \times \frac{1}{4} \times 2 \times \frac{22}{7} \times 3.5\right) + (4 \times 3.5)$$

$$= 22 + 14 = 36 \ cm \quad \text{(approximately)}$$

(ii) Area of the given combined shape

$$= \text{area of the square} + 4 \times \text{area of the quadrants of the circle}$$

$$= a^2 + \left(4 \times \frac{1}{4} \pi r^2\right)$$

$$= (3.5 \times 3.5) + \left(\frac{22}{7} \times 3.5 \times 3.5\right)$$

$$A = 12.25 + 38.5 = 50.75 \ cm^2 \quad \text{(approximately)}$$

Example 2.10

Find the area of the blue shaded and the grey shaded part of the given Fig.2.28. $(\pi = 3.14)$

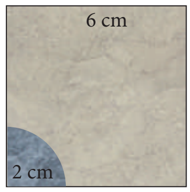


Fig. 2.28

Solution:

(i) Area of the blue shaded part = Area of the quadrant of a circle

$$= \frac{1}{4} \times \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times 2 \times 2$$

$$= 3.14 \ cm^2 \text{ (approximately)}$$

(ii) Area of the grey shaded part = Area of the square – Area of the blue shaded part

$$= a^2 - \frac{1}{4} \pi r^2$$

$$= 6 \times 6 - 3.14$$

$$= 36 - 3.14$$

$$= 32.86 \ cm^2 \text{ (approximately)}$$

Example 2.11

Thiyagu has fixed a door for the entrance of his house which is in the shape of a semicircle over a rectangle. The total height and width of the door is 9 *feet* and 3.5 *feet* respectively. Find the area of the door. $\left(\pi = \frac{22}{7}\right)$

Fig. 2.29

Solution:

The door is made of semicircle and rectangular shapes.

$$\text{Length of the rectangle} = 9 - 3.5 = 5.5 \textit{ feet}$$

$$\text{Its breadth} = 3.5 \textit{ feet}$$

$$\text{Diameter of the semicircle} = 3.5 \textit{ feet}$$

$$\therefore \quad \text{Radius} = \frac{3.5}{2} = 1.75 \textit{ feet}$$

$\therefore$ Area of the door = Area of the rectangle + Area of the semicircle

$$= (l \times b) + \left(\frac{1}{2}\pi r^2\right)$$

$$= (5.5 \times 3.5) + \left(\frac{1}{2} \times \frac{22}{7} \times 1.75 \times 1.75\right)$$

$$= 19.25 + 4.81 = 24.06 \textit{ sq.feet} \text{ (approx.)}$$

Example 2.12

A key-chain is in the form of an equilateral triangle and a semicircle attached to a square of side 5 *cm* as shown in the Fig.2.30. Find its area. $\left(\pi = 3.14, \sqrt{3} = 1.732\right)$

Solution:

Side of the square = 5 *cm*

Diameter of the semi-circle = 5 *cm*

$\therefore$ Radius = 2.5 *cm*

Side of the equilateral triangle = 5 *cm*

Fig. 2.30

$\therefore$ Area of the keychain = area of the semi circle + area of the square + area of the equilateral triangle

$$= \frac{1}{2}\pi r^2 + a^2 + \frac{\sqrt{3}}{4}a^2$$

$$= \left(\frac{1}{2} \times 3.14 \times 2.5 \times 2.5\right) + (5 \times 5) + \left(\frac{\sqrt{3}}{4} \times 5 \times 5\right)$$

$$= 9.81 + 25 + 10.83$$

$$= 45.64 \textit{ cm}^2 \text{ (approx.)}$$

Example 2.13

Find the area of the door mat whose measures are as given in the Fig.2.31. $(\pi = 3.14)$

Fig. 2.31

Solution:

The door mat has two semi-circles attached to a rectangle.

Length of the rectangle $= 90-40 = 50$ cm

Its breadth $= 40\ cm$

Diameter of the semi-circle $= 40\ cm$

Its radius $= 20\ cm$

$\therefore$ Area of the door mat = area of the rectangle + 2 × area of the semicircle

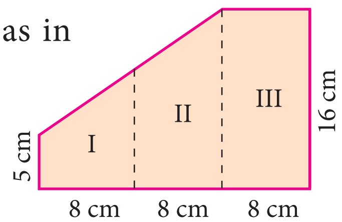
$$= (l \times b) + \left(2 \times \frac{1}{2}\pi r^2\right)$$

$$= (50 \times 40) + (3.14 \times 20 \times 20)$$

$= 2000+1256 = 3256\ sq.cm$ (approximately)

Example 2.14

A 3-fold invitation card is given with measures as in the Fig.2.32. Find its area.



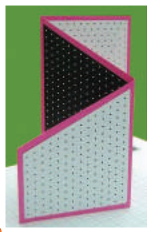


Fig. 2.32

Solution:

Figures I and II separately as well as combinedly are trapeziums.

The parallel sides of the combined trapezium (I and II) are 5 *cm* and 16 *cm*

Its height, $h = 8 + 8 = 16\,cm$

Length of the rectangle $= 16\ cm$

Breadth of the rectangle $= 8\ cm$

$\therefore$ Area of the combined invitation card

= area of the combined trapezium + area of the rectangle

$$= \left(\frac{1}{2} \times h \times (a+b)\right) + (l \times b)$$

$$= \left(\frac{1}{2} \times 16 \times (5+16)\right) + (16 \times 8)$$

$$= 168 + 128 = 296\ cm^2$$

Aliter:

Area of the invitation card = area of the outer rectangle – area of the right angled triangle

$$= (l\times b) - \left(\frac{1}{2}\times b\times h\right)$$

$$= (24\times 16) - \left(\frac{1}{2}\times 11\times 16\right)$$

$$= 384 - 88 = 296\ cm^2$$

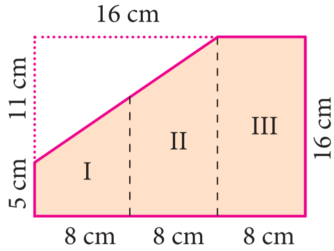


Fig. 2.33

Example 2.15

Seenu wants to buy a floor mat for the kitchen as given in the figure. If the cost of the mat is ₹ 20 per square foot, what will be the cost of the entire mat?

Fig. 2.34

Solution:

The mat given in the figure can be split into two rectangles as follows

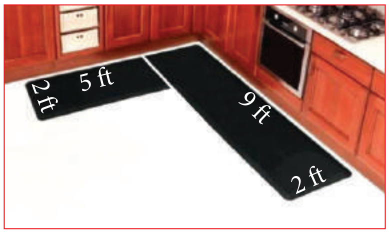


Fig. 2.35

∴ Area of the entire mat

= area of the I rectangle + area of the II rectangle

$$= (l_1\times b_1) + (l_2\times b_2)$$

$$= (5\times 2) + (9\times 2) = 10 + 18 = 28\ sq.feet$$

Cost per sq. foot = ₹ 20

∴ The total cost of the entire mat = 28 × ₹ 20 = ₹ 560.

Try this

In the above example split the given mat as into two trapeziums and verify your answer.

Example 2.16

Find the area of the shaded region in the square of side 10cm as given in the fig.2.36.

Solution:

Mark the unshaded parts of the given figure as I, II, III and IV

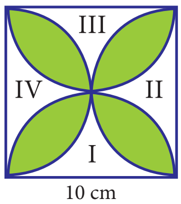


Fig. 2.36

Area of the I and III parts = Area of the square – Area of 2 semicircles

$$= a^2 - \left(2\times\frac{1}{2}\pi r^2\right)$$

$$= (10\times 10) - \left(\frac{22}{7}\times 5\times 5\right) = 100 - 78.57 = 21.43\ cm^2 \text{ (approx.)}$$

Similarly, the area of the II and IV parts = 21.43 cm^2 (approx.)

∴ Area of the unshaded parts (I, II, III and IV)

$= 21.43 \times 2 = 42.86\ cm^2$ (approx.)

∴ Area of the shaded part $=$ area of the square – area of the unshaded parts

$= 100 - 42.86 = 57.14\ cm^2$ (approx.)

DO YOU KNOW?

If the biggest circle is cut from a square of side 'a' *units*, then the remaining area in the square is $\frac{3}{14}a^2$ *sq.units.* $\left(\pi = \frac{22}{7}\right)$

Proof: Area of the remaining part = Area of the square – area of the circle

$$= a^2 - \pi r^2$$

$$= a^2 - \pi\left(\frac{a}{2}\right)^2$$

$$= a^2 - \frac{22}{7} \times \frac{a^2}{4}$$

$$= \frac{14a^2 - 11a^2}{14}$$

$$= \frac{3}{14}a^2 \text{ sq. units (approx.)}$$

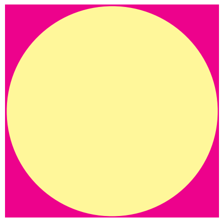
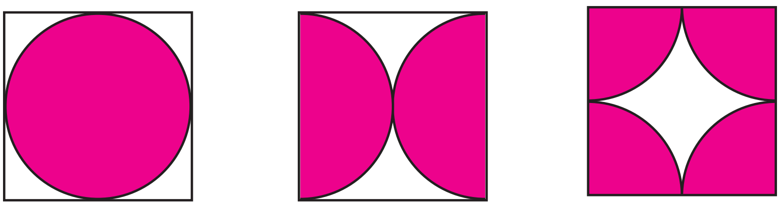


Fig. 2.37

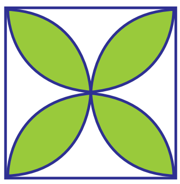
Note : The area of the biggest circle cut out from the square of 'a' *units* $= \frac{11}{14}a^2$ *sq. units* (approx.)

Try these

1. Show that the area of the unshaded regions in each of the squares of side a units are the same in all the cases given below.



2. If $\pi = \frac{22}{7}$, show that the area of the unshaded part of a square of side a units is approximately $\frac{3}{7}a^2$ *sq.units* and that of the shaded part is approximately $\frac{4}{7}a^2$ *sq.units* for the given figure.



Example 2.17

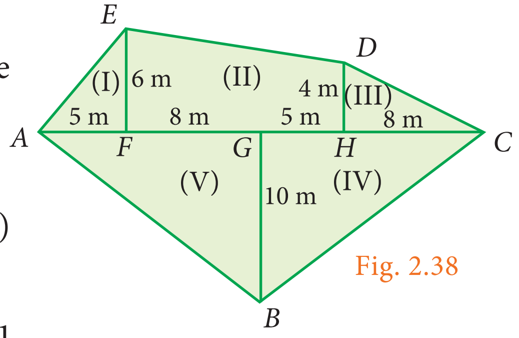


Fig. 2.38

Find the area of the irregular polygon field whose measures are as given in the figure.

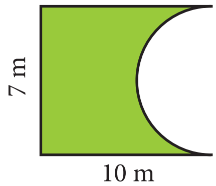
Solution:

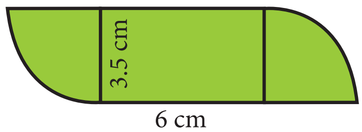
The given field has four triangles (I, III, IV & V) and a trapezium (II).

Area of the triangle (I)	$=\frac{1}{2}\times b\times h=\frac{1}{2}\times 5\times 6=15\ m^2$
Area of the trapezium (II)	$=\frac{1}{2}h(a+b)=\frac{1}{2}\times 13\times(6+4)=65\,m^2$
Area of the triangle (III)	$=\frac{1}{2}\times b\times h=\frac{1}{2}\times 8\times 4=\frac{32}{2}=16\ m^2$
Area of the triangle (IV)	$=\frac{1}{2}\times b\times h=\frac{1}{2}\times 13\times 10=65\ m^2$
Area of the triangle (V)	$=\frac{1}{2}\times b\times h=\frac{1}{2}\times 13\times 10=65\ m^2$
$\therefore$ The total area of the field	$=15+65+16+65+65\ =226\ m^2$

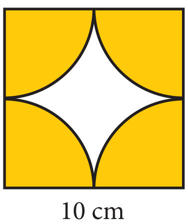
Exercise 2.2

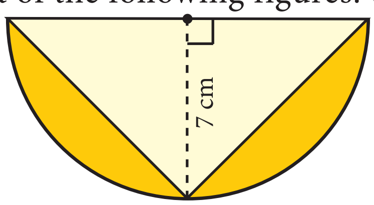
1. Find the perimeter and area of the combined figures given below. $\left(\pi=\frac{22}{7}\right)$

(i) 

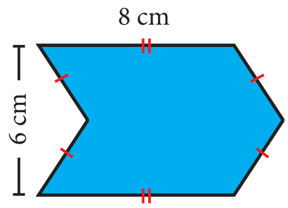
(ii) 

2. Find the area of the shaded part of the following figures. ($\pi=3.14$)

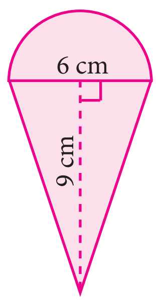
(i) 

(ii) 

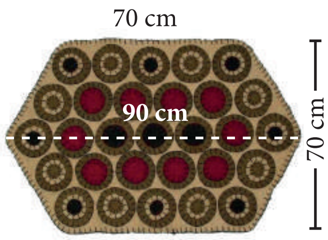
3. Find the area of the combined figure given which is got by joining of two parallelograms.



4. Find the area of the combined figure given, formed by joining a semicircle of diameter 6 *cm* with a triangle of base 6 cm and height 9 *cm*. ($\pi=3.14$)

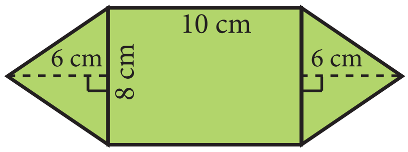


5. The door mat which is in a hexagonal shape has the following measures as given in the figure. Find its area.

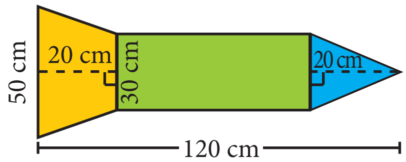


6. Find the area of an invitation card which has two semicircles attached to a rectangle as in the figure given. $\left(\pi = \frac{22}{7}\right)$

7. Find the area of the combined figure given, which has two triangles attached to a rectangle.



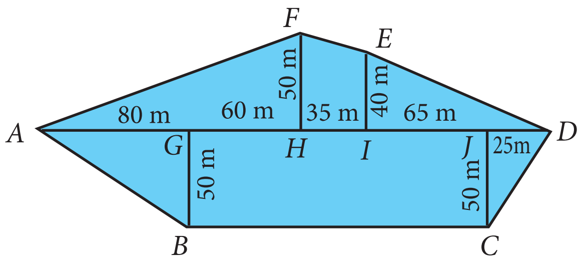
8. A rocket drawing has the measures as given in the figure. Find its area.



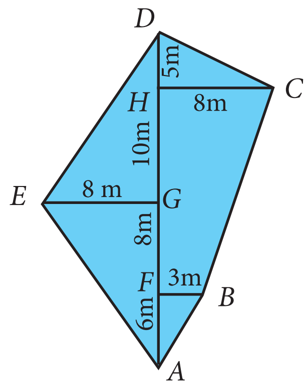
9. Find the area of the glass painting which has a triangle on a square as given in the figure.

10. Find the area of the irregular polygon shaped fields given below.

(i)



(ii)



2.4 Three dimensional (3-D) shapes

Trace the outline of a ₹ 2 coin, ₹ 10 note and a square shaped biscuit on a paper.

Fig. 2.39

What shapes have you traced? A **circle**, a **rectangle** and a **square**. These shapes represents the plane figures. Also, these plane figures have two dimensions namely length and breadth. Now, you place some coins of rupees two, some notes of rupees ten and some square shaped biscuits respectively on the drawn shapes as shown in the figure.

Fig. 2.40

What shapes do you get now? A **cylinder, a cuboid and a cube.** These shapes do not lie completely on the plane and they occupy some space also. That is, they have the third dimension namely the height along with the dimensions length and breadth. Thus, **the shapes which have three dimensions namely length, breadth and height (depth) are called three dimensional shapes, simply called as 3-D shapes.** Some examples of 3-D shapes are

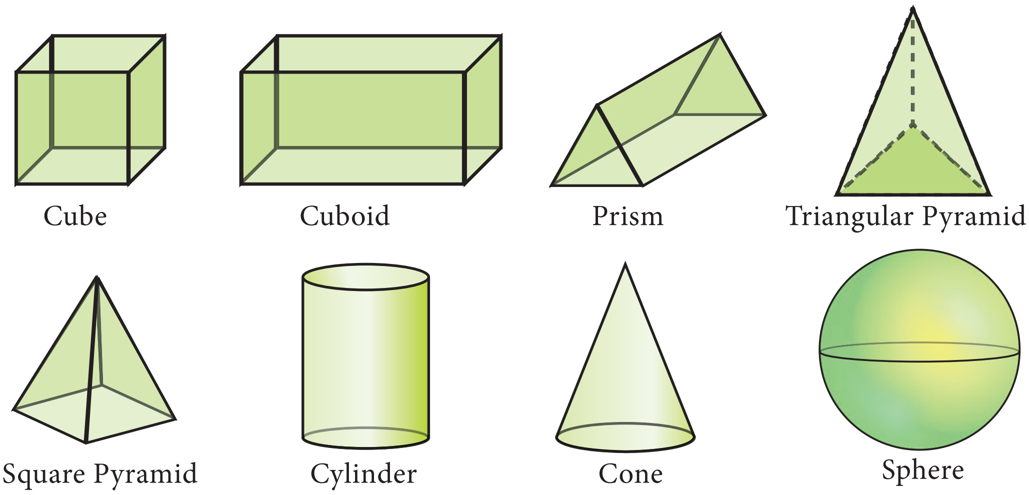
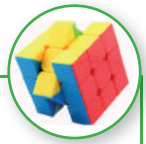


Fig. 2.41

Try this



List out atleast three objects in each category which are in the shape of cube, cuboid, cylinder, cone and sphere.

2.4.1 Faces, Edges and Vertices

Observe the following shape. What is its name? A cube. A cube is made of six, square shaped planes. These 6, square shaped planes of the cube are known as its **faces.**

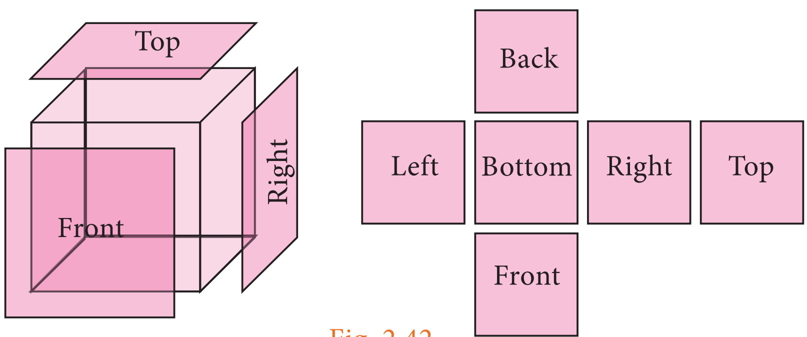
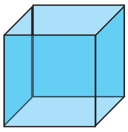
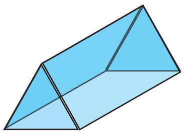
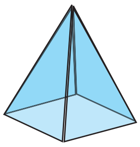
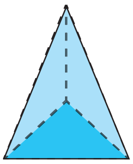


Fig. 2.42

A line segment which connects any two faces of a cubes called as **Edge** and each corner where three edges meet is called as **Vertex**. So, **a cube has 6 faces, 12 edges and 8 vertices.**

Try this

Tabulate the number of faces(F), vertices(V) and edges(E) for the following polyhedron. Also find F+V–E

Solid	Name	F	V	E	F+V–E
	Cuboid	6	8	12	
	Cube				
	Triangular Prism				
	Square Pyramid				
	Triangular Pyramid				

What do you observe from the above table? We observe that, F+V–E =2 in all the cases. This is true for any polyhedron and this relation F+V-E =2 is known as **Euler's formula**.

2.4.2 The nets for building three dimensional (3-D) shapes

When we buy sweets, a shop keeper picks a flat shaped card which has some flips and makes a rectangular shaped box (cuboid) by folding it as shown in the figure. Then, he arranges the sweets in the box and gives it to us.

The flat shaped card already designed for making the box excluding flaps (dotted lines) is known as net.

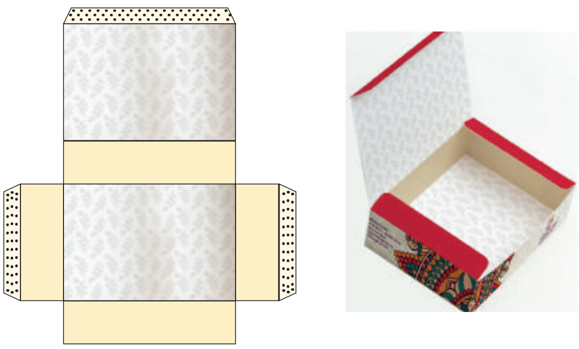
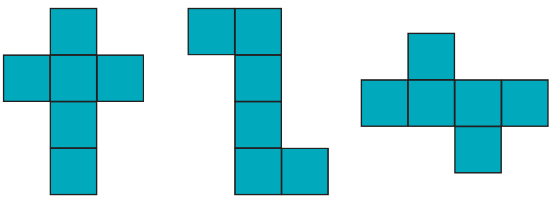
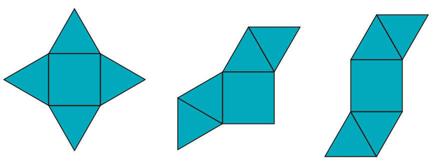


Fig. 2.43

For example, the following nets build cubes and square pyramids.



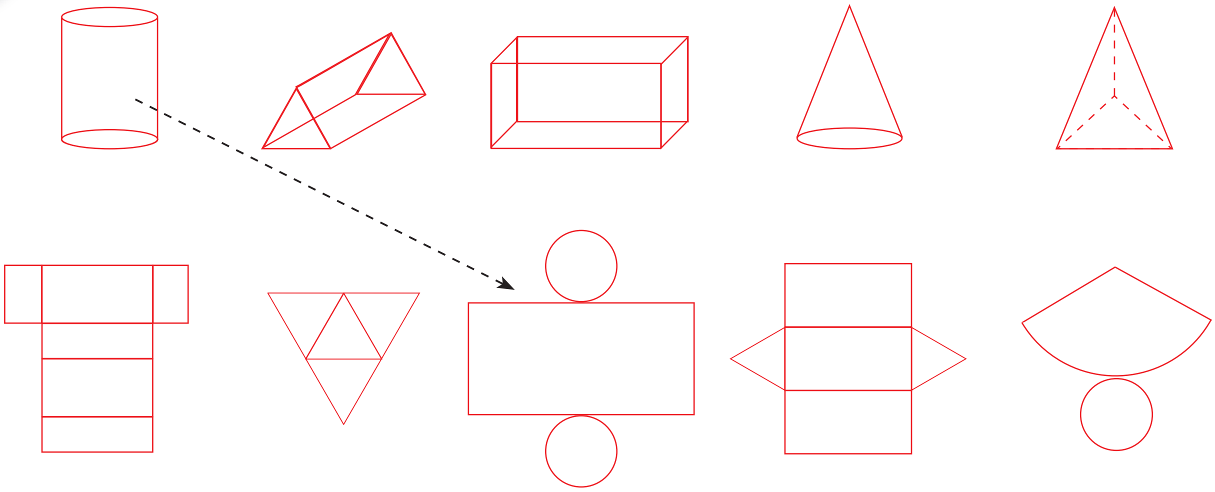
Nets for cubes



Nets for square pyramids

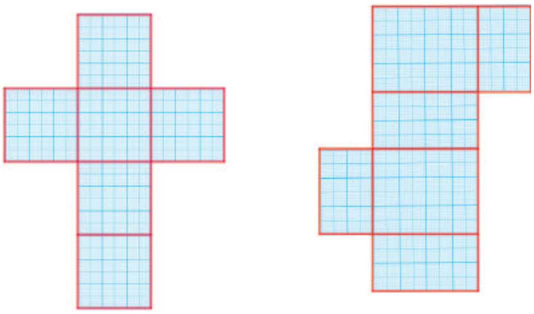
Activity-4

Draw a line to match the following shapes to their relevant nets.



Try this

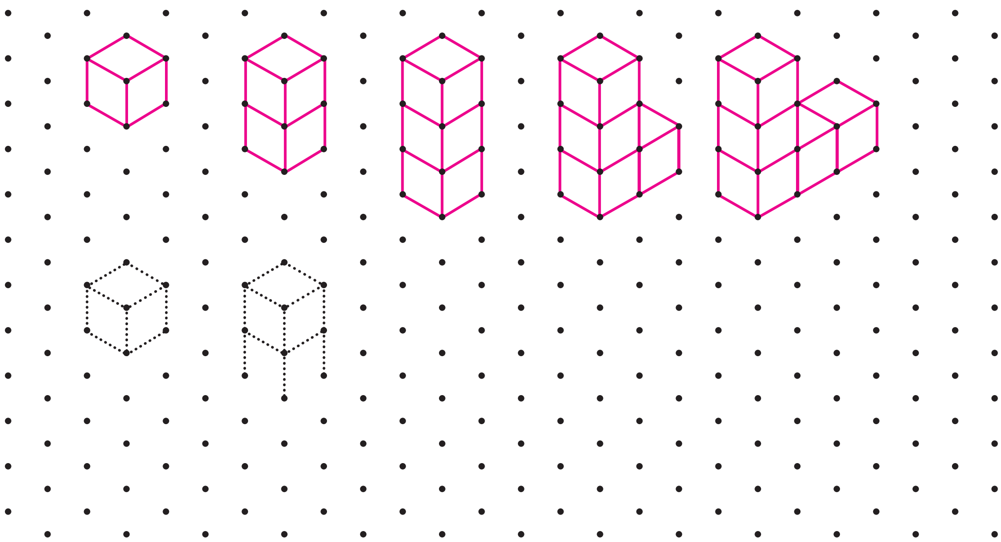
Find the area of the given nets



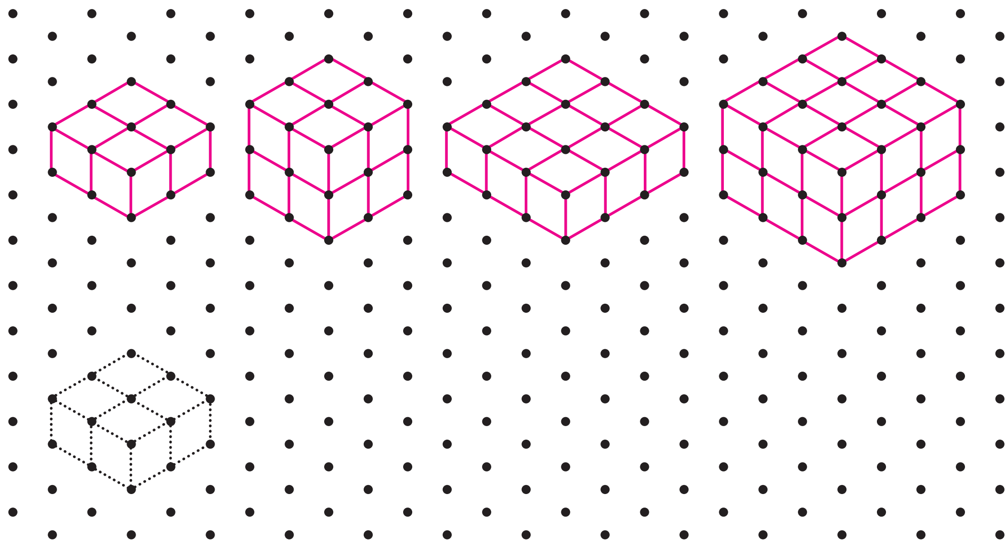
2.4.3 Drawing 3-D shapes using isometric dot sheets and grid sheets.

Activity-5

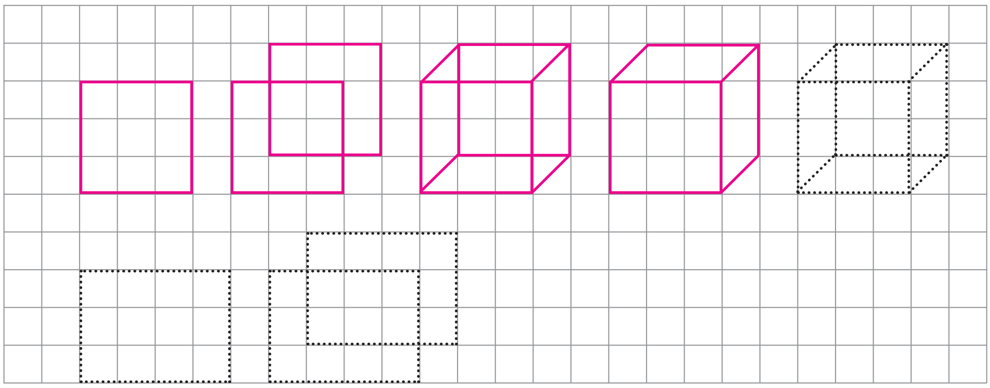
1. Draw each of the given solid figures on an isometric dot sheet



2. Draw each of the given solid figures on an isometric dot sheet



3. Draw each of the given solid figures on a grid paper



2.4.4 Cross section of solid shapes

When we cut the vegetables in cross section for cooking purpose, we see some plane figures in it. For example, the cross section of a carrot and a plantain stem is a circle.

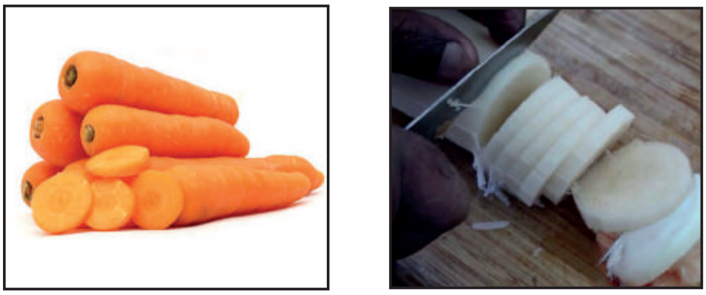


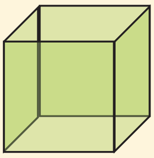
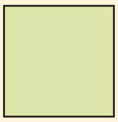
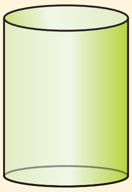
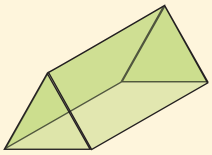
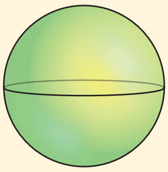
Fig. 2.44

In the same way, we can see rectangles and squares in the cross section of a bread loaf and bricks etc.,

Fig. 2.45

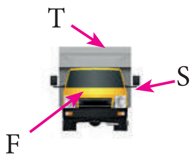
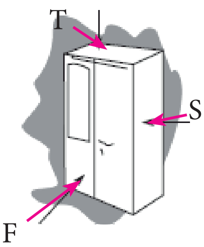
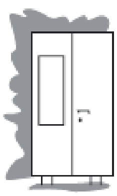
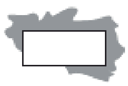
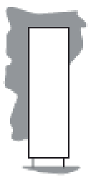
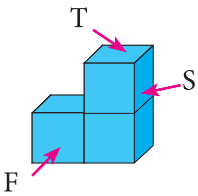
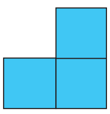
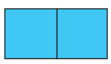
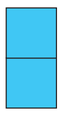
Activity-6

Draw and name the two dimensional shapes (2-D) which you get in the cross section of the following solid shapes.

Solids	2-D Shape	Name
		Square
		
		
		

2.4.5 3-D shapes in different views

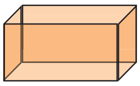
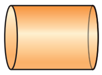
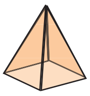
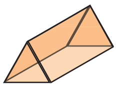
A 3-D object may look different from different positions. View of a 3D shape is what you see while observing the object from different positions. Some of the views are front view, top view and side view. The different views of some of the objects are as shown below.

Object	Front View	Top View	Side View
			
			
			

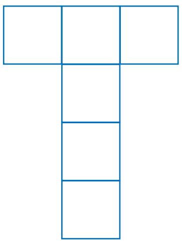
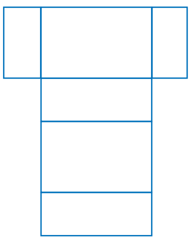
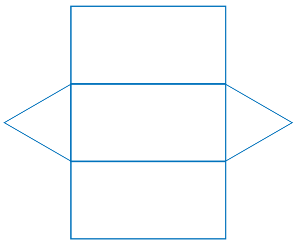
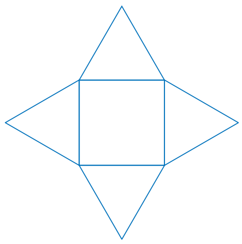
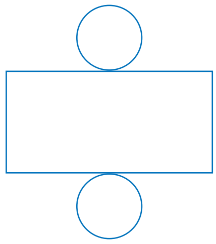
Exercise 2.3

1. Fill in the blanks:
   (i) The three dimensions of a cuboid are ______________.
   (ii) The meeting point of more than two edges is called as ___________.
   (iii) A cube has ___________ faces.
   (iv) The cross section of a solid cylinder is ___________.
   (v) If a net of a 3-D shape has six plane squares, then it is called _______.

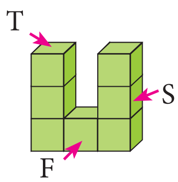
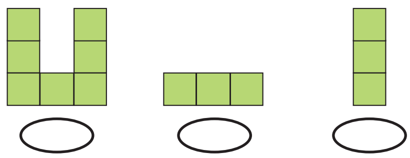
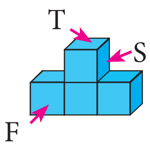
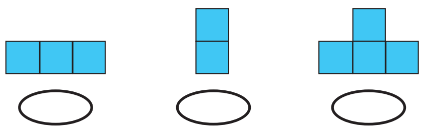
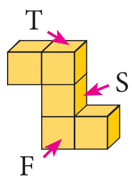
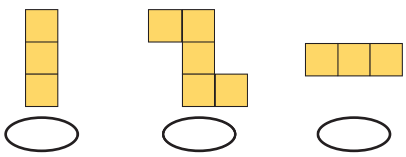
2. Match the following :

(i)		-	(a) Cylinder
(ii)		-	(b) Cuboid
(iii)		-	(c) Triangular Prism
(iv)		-	(d) Square Pyramid

3. Which 3-D shapes do the following nets represent? Draw them.

(i)

(ii)

(iii)

(iv)

(v)


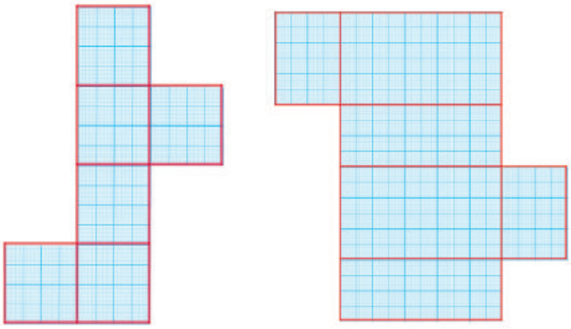
4. For each solid, three views are given. Identify for each solid, the corresponding top, front and side (T, F and S) views.

Solid	Three views
	
	
	

5. Verify Euler's formula for the table given below.

S.No.	Faces	Vertices	Edges
(i)	4	4	6
(ii)	10	6	12
(iii)	12	20	30
(iv)	20	13	30
(v)	32	60	90

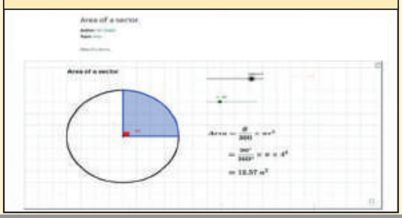
6. Find the area of the given nets.



7. Can a polyhedron have 12 faces, 22 edges and 17 vertices?

ICT CORNER

Through this activity you will know about how to find the perimeter and area of the sectors .

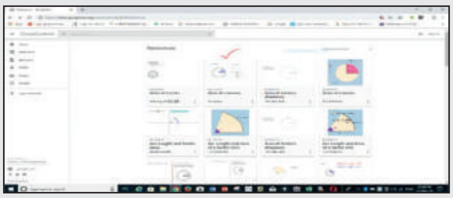


Step-1 Open the Browser and type www.Geogebra.com (or)scan the QR CODE given below.

Step-2 Type Area of sector on the search column

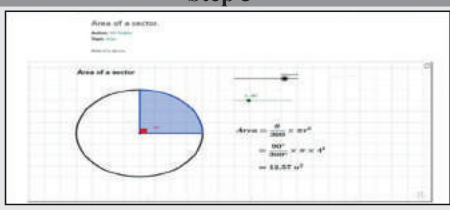
Step-3 Move the radius slide and degree slide. You will get various results.

Step 1



Step 2

Step 3



Web URL Measurement (measurements links)
https://www.geogebra.org/m/FSqNDNxN
*Pictures are indicatives only.
*If browser requires, allow Flash Player or Java Script to load the page.

B358_8_MATHS_EM

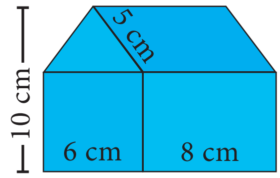
Exercise 2.4

Miscellaneous Practice Problems

1. Two gates are fitted at the entrance of a library. To open the gates easily, a wheel is fixed at 6 feet distance from the wall to which the gate is fixed. If one of the gates is opened to 90°, find the distance moved by the wheel $(\pi = 3.14)$.

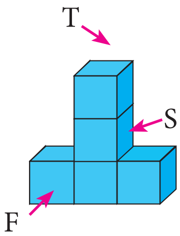
2. With his usual speed, if a person covers a circular track of radius 150 m in 9 *minutes*, find the distance that he covers in 3 *minutes* $(\pi = 3.14)$.

3. Find the area of the house drawing given in the figure.

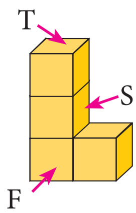


4. Draw the top, front and side view of the following solid shapes

(i)



(ii)



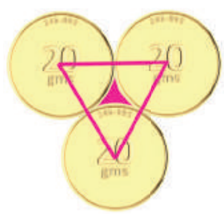
5. Draw the net for the cube of side 4 cm in a graph sheet.

Challenging problems

6. Guna has fixed a single door of 3 *feet* wide in his room where as Nathan has fixed a double door, each $1\frac{1}{2}$ *feet* wide in his room. From the closed state, if each of the single and double doors can open up to 120°, whose door requires a minimum area?

7. In a rectangular field which measures 15 m × 8m, cows are tied with a rope of length 3m at four corners of the field and also at the centre. Find the area of the field where none of the cow can graze. $(\pi = 3.14)$

8. Three identical coins, each of diameter 6 cm are placed as shown. Find the area of the shaded region between the coins. $(\pi = 3.14)$ $(\sqrt{3} = 1.732)$



9. Using graph sheet, draw the net for the cuboid whose length is 5 cm, breadth is 4 cm and height is 3 cm and also find its area.

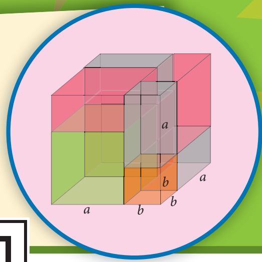
10. Using Euler's formula, find the unknowns.

S.No.	Faces	Vertices	Edges
(i)	?	6	14
(ii)	8	?	10
(iii)	20	10	?

Summary

- If any two points on a circle are joined by a line segment, then the line segment is called a 'chord'.
- A diameter of a circle divides it into two equal parts. It is the longest chord of a circle.
- A part of the circumference of a circle is called the circular arc .
- The plane surface that is enclosed between two radii and the circular arc of a circle is called a 'sector'.
- The angle made by the sector with the centre of the circle is called the 'central angle'.
- The perimeter of a combined shape is the sum of all the lengths of the sides that form a closed boundary.
- The area of combined shapes is nothing but the sum of all areas of the simple shapes in it.
- The shapes which have three dimensions namely length, breadth and height are called three dimensional shapes (or) simply called as 3-D shapes.
- A cube has 6 faces, 12 edges and 8 vertices.

3 ALGEBRA



2AGZUB

Learning Outcomes

- To recall addition and subtraction of expressions.
- To know how to multiply algebraic expressions with integer co-efficients
- To know how to divide algebraic expressions by monomials.
- To recall the identities $(a+b)^2, (a-b)^2, (a^2-b^2)$ and $(x+a)(x+b)$
- To understand the identities $(a+b)^3, (a-b)^3, (x+a)(x+b)(x+c)$ and apply them in problems.
- To understand the geometrical illustration of identities $(a+b)^3$, $(a-b)^3$ and $(x+a)\ (x+b)\ (x+c)$
- To recognize expressions that are factorizable of the type (a^3+b^3) and (a^3-b^3)

Recap

In our earlier classes, we have learnt about constants, variables, like terms, unlike terms, co-efficients, numerical and algebraic expressions. Later, we have done some basic operations like addition and subtraction on algebraic expressions. Now, we shall recollect them and extend the learning.

Further, we are going to learn about multiplication and division of algebraic expressions and algebraic identities.

Answer the following questions :

1. Write the number of terms in the following expressions

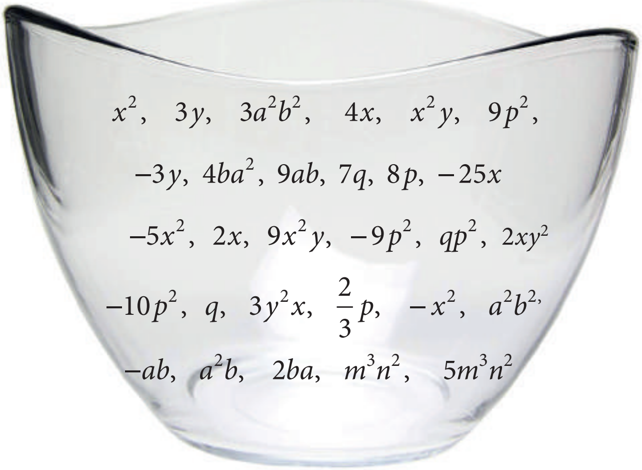
   (i) $x+y+z-xyz$ (ii) m^2n^2c

   (iii) $a^2b^2c-ab^2c^2+a^2bc^2+3abc$ (iv) $8x^2-4xy+7xy^2$

2. Identify the numerical co-efficient of each term in the following expressions.

   (i) $2x^2-5xy+6y^2+7x-10y+9$ (ii) $\frac{x}{3}+\frac{2y}{5}-xy+7$

3. Pick out the like terms from the following:



Like Terms

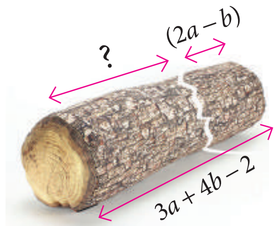
The variables of the terms along with their respective exponents must be same

Examples : $x^2, 4x^2$

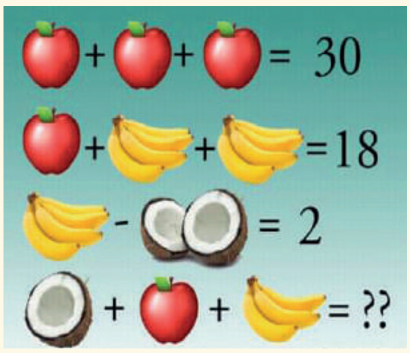
$a^2b^2, -5a^2b^2$

$2m, -7m$

4. Add : $2x, 6y, 9x-2y$
5. Simplify : $(5x^3y^3-3x^2y^2+xy+7)+(2xy+x^3y^3-5+2x^2y^2)$
6. The sides of a triangle are $2x-5y+9$, $3y+6x-7$ and $-4x+y+10$. Find the perimeter of the triangle .
7. Subtract $-2mn$ from $6mn$.
8. Subtract $6a^2-5ab+3b^2$ from $4a^2-3ab+b^2$.
9. The length of a log is $3a+4b-2$ and a piece $(2a-b)$ is removed from it. What is the length of the remaining log?



10. A tin had 'x' s of oil. Another tin had $(3x^2+6x-5)$ *litres* of oil. The shopkeeper added $(x+7)$ *litres* more to the second tin. Later, he sold (x^2+6) *litres* of oil from the second tin. How much oil was left in the second tin?

MATHEMATICS ALIVE	**ALGEBRA IN REAL LIFE**
 A vaccine box is in the shape of a cuboid.	 Expressions are useful to solve the unknown values.

DO YOU KNOW? The word *algebra* comes from the title of the Arabic book Ilm al-jabrwa'l-mukābala by the Persian mathematician and astronomer al-Khwarizmi. Algebra is the study of mathematical symbols and rules for calculating these symbols. In arithmetic, only numbers and their arithmetical operations (such as +, −, ×, ÷) occur. In algebra, numbers are often represented by symbols called variables.

3.1 Introduction

Let us consider the given situation, Ganesh planted saplings in his garden. He planted 10 rows each with 5 saplings. Can you say, how many saplings were planted?

Yes, we know that, the total number of saplings is the product of number of rows and number of saplings in each row.

Hence, the total number of saplings $=$ 10 rows $\times 5$ saplings in each row $= 10 \times 5 = 50$ saplings

Likewise, David planted some saplings. Not knowing the total number of rows and saplings in each row, how will you express the total number of saplings?

For the unknown quantities, we call them as 'x' and 'y'. Therefore, the total number of saplings $=$ 'x' rows $\times$ 'y' sapling in each row

$$= \text{'}x \times y\text{'} = xy \quad \text{saplings}$$

Let us extend this situation, Rahim planted saplings where the number of rows are $(2x^2 + 5x - 7)$ and each row contains $3y^2$ saplings. Now the above idea will help us to find the total number of saplings planted by Rahim.

The total number of saplings $=$ $(2x^2 + 5x - 7)$ rows $\times\, 3y^2$ saplings in each row.

$$= 3y^2 \times (2x^2 + 5x - 7)$$

How do we find the product of the above algebraic expression?

Now,we will learn to find the product of algebraic expressions, there are four ways by which multiplication of expressions can be done.

They are:

- Product of a Monomial with a Monomial
- Product of a Polynomial with a Monomial
- Product of a Binomial with a Binomial
- Product of a Polynomial with a Polynomial

Note

Polynomial	A special kind of algebraic expression is a polynomial. In a polynomial all variables are raised to only whole number powers. $a^2+2ab+b^2$ $\quad 4x^2+3x-7$ $x+y+z$ $\quad mn$
Monomial	An expression which contains only one term is called a monomial. Examples: $4x,\ 3x^2y,\ -2y^2$.
Binomial	An expression which contains only two terms is called a binomial. Examples: $2x+3,\ 5y^2+9y,\ a^2b^2+2b$.
Trinomial	An expression which contains only three terms is called a trinomial. Examples : $2a^2b-8ab+b^2,\ \ m^2-n^2+3$.
Polynomial	An expression which contains more than three terms is called a polynomial. Examples: $x^2+(a+b)x+ab+5,\quad a^3+b^3+c^3+\text{abc}$

We shall learn first three ways of multiplication here.

3.2 Multiplication of Algebraic Expressions

Before doing the product of algebraic expressions, we should follow the steps given below.

- Multiply the signs of the terms. That is, the product of two like signs are positive and the product of two unlike signs are negative.

Like signs	$(+)\times(+)=+$	$(-)\times(-)=+$
Unlike signs	$(+)\times(-)=-$	$(-)\times(+)=-$

- Multiply the corresponding co-efficients of the terms.
- Multiply the variable factors by using laws of exponents.

If 'x' is a variable and m, n are positive integers then,

$$x^m \times x^n = x^{m+n}$$

For example, $\quad x^3 \times x^4 = x^{3+4} \quad = x^7$

Think

Every algebraic expression is a polynomial. Is this statement true? Why?

DO YOU KNOW?

The difference between an algebraic expression and a polynomial.

Algebraic Expression

May contain whole numbers, fractions, negative powers on their variables.

Example: $4x^{3/2}-3x+9$

$2y^2+\frac{5}{y}-3,\ 3x^2-4x+1$

Polynomial

contains only whole numbers as the power of their variables.

Example: $4x^2-3x+9$

$2y^6+5y^3-3$

Note

Product of two terms is represented by the symbols ( ), dot (.) or $\times$.

For example,

multiplying $4x^2$ and xy can be written in any one of the following ways.

$(4x^2)(xy)$	$4x^2 \times xy$	$4x^2(xy)$	$(4x^2)\times xy$	$4x^2 \cdot xy$

3.2.1 Multiplication of two or more monomials

Consider that, Geetha buys 3 pens each @ ₹ 5, how much she has to pay to the shopkeeper?

Geetha has to pay to the shopkeeper $= 3 \times$ ₹ 5

$=$ ₹ 15

If there are 'x' pens and the cost of each pen is ₹ 'y', then the cost of $(3x^2)$ pens bought by Geetha @ ₹ $5y$

$$= (3x^2) \times 5y$$
$$= (3 \times 5)(x^2 \times y)$$
$$= ₹\ 15x^2y$$

Example 3.1

If the side of a square carpet is $3x^2$ metre, then find its area.

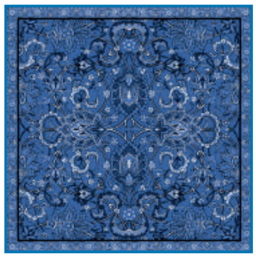
Solution:

The area of the square carpet, $A = (side \times side)$ *sq. units.*

$$= 3x^2 \times 3x^2$$
$$= 3 \times 3 \times x^2 \times x^2$$
$$A = 9x^4\ sq.m$$

$3x^2$

$3x^2$



Example 3.2

If the length and breadth of a rectangular painting are $4xy^3$ and $3x^2y$. Find its area.

Solution:

Area of the rectangular painting, $A = (l \times b)\ sq.units$

$$= (4xy^3) \times (3x^2y)$$
$$= (4 \times 3)(x \times x^2)(y^3 \times y)$$
$$A = 12x^3y^4\ sq.units$$

$3x^2y$

$4xy^3$

Example 3.3

Find the product of $3x^2$ and $(-4x)$

Solution:

We have, $(3x^2)\times(-4x)$

$$=(+)\times(-)(3\times4)(x^2\times x)$$
$$=-12x^3$$

Try these

Find the product of

(i) $3ab^2, -2a^2b^3$

(ii) $4xy, 5y^2x, (-x^2)$

(iii) $2m, -5n, -3p$

Example 3.4

Find the product of $2x^2y^2$, $3y^2z$ and $-z^2x^3$

Solution:

We have, $(2x^2y^2)\times(3y^2z)\times(-z^2x^3)$

$$=(+)\times(+)\times(-)(2\times3\times1)(x^2\times x^3)(y^2\times y^2)(z\times z^2)$$
$$=-6x^5y^4z^3$$

3.2.2 Multiplication of a polynomial by a monomial

In a school, there are 8 classrooms and each room has 12 sets of benches and desks, now the total number of benches and desks.

$=$ number of classrooms $\times$ number of benches and desks in each room

$=8\times12$

$=96$ sets of benches and desks.

If there are 'a' shops and each shop has 'x' apples in 8 baskets and 'y' oranges in 3 baskets and 'z' bananas in 5 baskets, then the total number of apples, oranges and bananas are

$=a\times(8x+3y+5z)$

$=a(8x)+a(3y)+a(5z)$ (using distributive law).

$=8ax+3ay+5az$

Note

Distributive law	If a is a constant, x and y are variables then $a(x+y)=ax+ay$ For example, $5(x+y)=5x+5y$

Example 3.5

Multiply $(3xy+7)$ by $(-4y)$

Solution:

Now, $-4y\times(3xy+7)$ $=-4y(3xy)+(-4y)(7)$

$$=(-4\times3)x\times y\times y+(-4\times7)y$$
$$=-12xy^2-28y$$

Think

$-5y^2+2y-6$

$=-(5y^2+2y-6)$

Is this correct?

If not, correct the mistake.

Example 3.6

Multiply $3x^2y$ and $(2x^3y^3 - 5x^2y + 9xy)$

Think

Which is corrcet?

$(3a)^2$ is equal to

(i) $3a^2$ (ii) 3^2a

(iii) $6a^2$ (iv) $9a^2$

Solution:

Now, $(3x^2y) \times (2x^3y^3 - 5x^2y + 9xy)$

$= 3x^2y(2x^3y^3) - 3x^2y(5x^2y) + 3x^2y(9xy)$

multiplying each term of the polynomial by the monomial

$= (3\times 2)(x^2 \times x^3)(y \times y^3) - (3\times 5)(x^2 \times x^2)(y \times y) + (3\times 9)(x^2 \times x)(y \times y)$

$= 6x^5y^4 - 15x^4y^2 + 27x^3y^2$

Example 3.7

If Guru wants to multiply the expressions $(2x + 3y + 50)$ and $3xy$, what is the resultant expression?

Solution:

Think

Why $3+(4x-7y) \neq 12x-21y$?

The resultant expression

$= 3xy \times (2x + 3y + 50)$

$= 3xy(2x) + 3xy(3y) + 3xy(50)$

$= 6x^2y + 9xy^2 + 150xy$

Example 3.8

Ram deposited 'x' number of ₹ 2000 notes, 'y' number of ₹ 500 notes, 'z' number of ₹ 100 notes in a bank and Velan deposited '$3xy$' times of amount of what Ram had deposited. How much amount did Velan deposit in the bank?

Solution:

Amount deposited by Ram

$= (x \times ₹2000 + y \times ₹500 + z \times ₹100)$

$= ₹(2000x + 500y + 100z)$

Amount deposited by Velan $= 3xy$ times $\times$ Amount deposited by Ram

$= 3xy \times (2000x + 500y + 100z)$

$= (3\times 2000)(x \times x \times y) + (3 \times 500)(x \times y \times y) + (3 \times 100)(x \times y \times z)$

$= ₹(6000x^2y + 1500xy^2 + 300xyz)$

Try these

Multiply

(i) $(5x^2 + 7x - 3)$ by $4x^2$ (ii) $(10x - 7y + 5z)$ by $6xyz$

(iii) $(ab + 3bc - 5ca)$ by $-3a^2bc$

3.2.3 Multiplication of two binomials

What is the area of the rectangular table top whose length is 4 *feet* and breadth is 3 *feet*?

Area of the rectangular table top $= l \times b$

$= 4 \times 3$

$= 12$ *sq.feet*

Consider that a rectangular flower bed whose length is decreased by 5 *units* from the original length and whose breadth is increased by 3 *units* to the original breadth. What is the area of the rectangular flower bed?

Area of the rectangular flower bed $= l \times b$

$$A = (l-5)\times(b+3) \text{ sq.units}$$

How do we multiply here?

Now, let us learn how to multiply two binomials.

If $(x+y)$ and $(p+q)$ are two binomials, we can find their product as given below,

$$(x+y)\ (p+q) = x(p+q) + y(p+q)$$

$$= xp + xq + yp + yq$$

Aliter

$\times$	x	y
p	xp	yp
q	xq	yq

$= xp + xq + yp + yq$

Or we can do it directly like, $(x+y)\ (p+q) = x(p) + x(q) + y(p) + y(q)$

We can extend this method for product of two polynomials also.

So, the above area of the rectangle $= (l-5)\times(b+3)$

$= l(b+3) - 5(b+3)$

$A = (lb + 3l - 5b - 15)$ *sq.u*

Aliter

$\times$	l	-5
b	lb	$-5b$
3	$3l$	-15

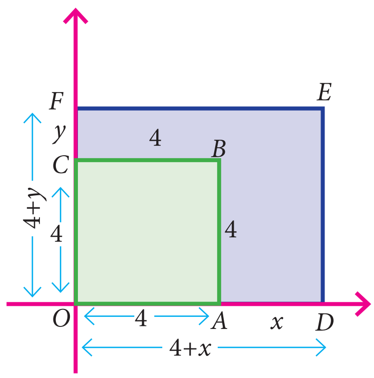
$= (lb + 3l - 5b - 15)$

Let us consider one more example. Consider the given figure , In the square *OABC*,

OA= 4 *units* ; *OC* =4 *units*

The area of the square *OABC* $= 4 \times 4$

$A = 16$ *sq.units*



If the sides of the square are increased by '*x*' *units* and '*y*' *units* respectively.

we get the rectangle *ODEF* whose sides are *OD*=(4+*x*) *units* and *OF*=(4+*y*) *units*.

Now, the area of the rectangle $ODEF$ $= (4+x)\ (4+y)$

$= 4\,(4+y) + x\,(4+y)$

$A = 16+ 4y + 4x + xy$ *sq. units.*

Example 3.9

Multiply $(2x+5y)$ and $(3x-4y)$

Solution:

$(2x+5y)(3x-4y) = 2x\,(3x-4y) + 5y\,(3x-4y)$

$= 6x^2 - 8xy + 15xy - 20y^2$

$= 6x^2 + 7xy - 20y^2$ (simplify the like terms)

Aliter

$\times$	$2x$	$5y$
$3x$	$6x^2$	$15xy$
$-4y$	$-8xy$	$-20y^2$

$= 6x^2 - 8xy + 15xy - 20y^2$

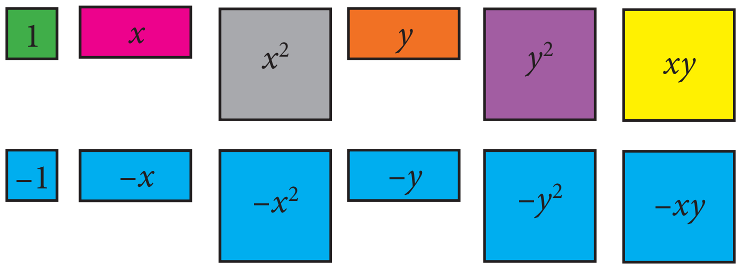
$= 6x^2 + 7xy - 20y^2$

DO YOU KNOW?

Algebra tiles are helpful to the student to understand the concept of algebra in a better way. Algebra tiles are made up of small square, rectangles and large squares.

Multiplication using Algebra tiles :

Keys



(i) $(2x+y-1)\times 3x$

$\times$	x	x	x
x	x^2	x^2	x^2
x	x^2	x^2	x^2
y	xy	xy	xy
-1	$-x$	$-x$	$-x$

$= 6x^2+3xy-3x$

(ii) $(y-1)(y-3)$

$\times$	y	-1	-1	-1
y	y^2	$-y$	$-y$	$-y$
-1	$-y$	1	1	1

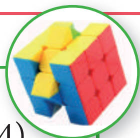
$= y^2-4y+3$

(iii) $(x+2)(x+3)$

$\times$	x	1	1	1
x	x^2	x	x	x
1	x	1	1	1
1	x	1	1	1

$= x^2+5x+6$

Try these



Multiply

(i) $(a-5)$ and $(a+4)$

(ii) $(a+b)$ and $(a-b)$

(iii) (m^4+n^4) and $(m-n)$

(iv) $(2x+3)(x-4)$

(v) $(x-5)(3x+7)$

(vi) $(x-2)(6x-3)$

Think

$3x^2\,(x^4-7x^3+2)$, what is the highest power in the expression.

Exercise 3.1

1. Multiply a monomial by a monomial.

   (i) $6x, 4$ (ii) $-3x, 7y$ (iii) $-2m^2, (-5m)^3$

   (iv) $a^3, -4a^2b$ (v) $2p^2q^3, -9pq^2$

2. Complete the table.

$\times$	$2x^2$	$-2xy$	x^4y^3	$2xyz$	$(\_\_)xz^2$
x^4					
$(\_\_\_)$			$4x^5y^4$		
$-x^2y$					
$2y^2z$					$-10xy^2z^3$
$-3xyz$					
$(\_\_\_)$				$-14xyz^2$	

3. Find the product of the terms.

   (i) $-2mn, (2m)^2, -3mn$ (ii) $3x^2y, -3xy^3, x^2y^2$

4. If $l = 4pq^2$, $b = -3p^2q, h = 2p^3q^3$ then, find the value of $l \times b \times h$.

5. Expand

   (i) $5x(2y-3)$ (ii) $-2p(5p^2-3p+7)$

   (iii) $3mn(m^3n^3-5m^2n+7mn^2)$ (iv) $x^2(x+y+z)+y^2(x+y+z)+z^2(x-y-z)$

6. Find the product of

   (i) $(2x+3)(2x-4)$ (ii) $(y^2-4)(2y^2+3y)$

   (iii) $(m^2-n)(5m^2n^2-n^2)$ (iv) $3(x-5)\times 2(x-1)$

7. Find the missing term.

   (i) $6xy \times \_\_\_\_\_\_\_\_ = -12x^3y$ (ii) $\_\_\_\_\_\_\_\_ \times (-15m^2n^3p) = 45m^3n^3p^2$

   (iii) $2y(5x^2y - \_\_\_ + 3\_\_\_) = 10x^2y^2 - 2xy + 6y^3$

8. Match the following.

   a) $4y^2 \times -3y$ (i) $20x^2y - 20x$

   b) $-2xy(5x^2-3)$ (ii) $5x^3 - 5xy^2 + 5x^2y$

   c) $5x(x^2-y^2+xy)$ (iii) $4x^2-9$

   d) $(2x+3)(2x-3)$ (iv) $-12y^3$

   e) $5x(4xy-4)$ (v) $-10x^3y+6xy$

   A) iv, v, ii, i, iii B) v, iv, iii, ii, i C) iv, v, ii, iii, i D) iv, v, iii, ii, i

9. A car moves at a uniform speed of $(x+30)$ km/hr. Find the distance covered by the car in $(y+2)$ hours. (Hint: distance = speed × time).

Objective type Questions:

10. The product of $7p^3$ and $(2p^2)^2$ is

(A) $14p^{12}$ (B) $28p^7$ (C) $9p^7$ (D) $11p^{12}$

11. The missing terms in the product $-3m^3n \times 9(\_\_) = \_\_\_\_\_\_\_\_\_ m^4n^3$ are

(A) $mn^2, 27$ (B) $m^2n, 27$ (C) $m^2n^2, -27$ (D) $mn^2, -27$

12. If the area of a square is $36x^4y^2$ then, its side is ____________

(A) $6x^4y^2$ (B) $8x^2y^2$ (C) $6x^2y$ (D) $-6x^2y$

13. If the area of a rectangle is $48m^2n^3$ and whose length is $8mn^2$ then, its breadth is__.

(A) 6 mn (B) $8m^2n$ (C) $7m^2n^2$ (D) $6m^2n^2$

14. If the area of a rectangular land is (a^2-b^2) $sq.units$ whose breadth is $(a-b)$ then, its length is__________

(A) $a-b$ (B) $a+b$ (C) a^2-b (D) $(a+b)^2$

3.3 Division of Algebraic Expressions

In the previous sessions, we have learnt to add, subtract and multiply algebraic expressions. Now we are going to learn about another basic operation **'division'** on algebraic expressions. We know that the division is the reverse operation of multiplication.

Now, the cost of 10 balls at the rate of ₹ 5 each $= 10 \times 5$

$= ₹\ 50$

whereas if we have ₹ 50 and we want to buy 10 balls then,

the cost of each ball is $= ₹\ \frac{50}{10}$

$= ₹\ 5$

What we have seen above is division on numbers. But how will you divide an algebraic expression by another algebraic expression?

Of course, the same procedure has to be followed for the algebraic expressions with the help of laws of exponents.

If 'x' is a variable and m, n are constants, then $x^m \div x^n = x^{m-n}$ where $m > n$.

There are four ways of division on algebraic expressions.

They are division of,

- Monomial by a Monomial
- Polynomial by a Monomial
- Polynomial by a Binomial
- Polynomial by a Polynomial

Now, we are going to learn the first two ways of division only. Later, you will learn to do the remaining ways of division on algebraic expressions.

3.3.1 Division of a monomial by another monomial

Dividing a monomial $10p^4$ by another monomial $2p^3$, we get

$$10p^4 \div 2p^3$$

$$\frac{10p^4}{2p^3} = \frac{\overset{5}{\cancel{10}} \times \cancel{p} \times \cancel{p} \times \cancel{p} \times p}{\cancel{2} \times \cancel{p} \times \cancel{p} \times \cancel{p}} \quad \text{(expansion of power)}$$

$$= 5p$$

Think

Are the following correct?

(i) $\dfrac{x^3}{x^8} = x^{8-3} = x^5$

(ii) $\dfrac{10m^4}{10m^4} = 0$

(iii) When a monomial is divided by itself, we will get 1?

However, we can also follow laws of exponents to divide as,

$$\frac{10p^4}{2p^3} = 5p^{4-3}$$

$$= 5p$$

$$\boxed{\frac{x^m}{x^n} = x^{m-n}}$$

Example 3.10

Velu pastes '$4xy$' pictures in one page of his scrap book. How many pages will he need to paste $100x^2y^3$ pictures? (x, y are positive integers)

Solution:

Total number of pictures $= 100x^2y^3$

Each page contains $= 4xy$ pictures

Total number of pages needed $= \dfrac{\text{Total number of pictures}}{\text{pictures in one page}}$

$$= \frac{\overset{25}{\cancel{100}}x^2y^3}{\cancel{4}xy} = 25x^{2-1}y^{3-1}$$

$= 25xy^2$ pages

Try these

Divide

(i) $12x^3y^2$ by x^2y (ii) $-20a^5b^2$ by $2a^3b^7$ (iii) $28a^4c^2$ by $21ca^2$

3.3.2 Division of an algebraic expression (polynomial) by a monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 3.11

Divide : $(5y^3 - 25y^2 + 8y)$ by $5y$

Solution:

We have, $(5y^3 - 25y^2 + 8y) \div 5y$

$$= \frac{5y^3 - 25y^2 + 8y}{5y}$$

$$= \frac{\cancel{5}y^3}{\cancel{5}y} - \frac{\overset{5}{\cancel{25}}y^2}{\cancel{5}y} + \frac{8y}{5y}$$

$$= y^{3-1} - 5y^{2-1} + \frac{8}{5}$$

$$= y^2 - 5y + \frac{8}{5}$$

Think

Are the following divisions correct?

(i) $\frac{4y+3}{4} = y + 3$

(ii) $\frac{5m^2+9}{9} = 5m^2$

(iii) $\frac{2x^2+8}{4} = 2x^2 + 2$

If not , correct it.

Example 3.12

Sethu travelled $(4x^2 + 3xy^2 + 5x)\,km$ in '$2x$' hrs. Find his speed of travel.

Solution:

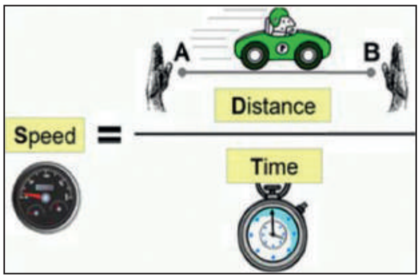
$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{4x^2 + 3xy^2 + 5x}{2x}$$

$$= \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}x} + \frac{3xy^2}{2x} + \frac{5x}{2x}$$

$$= 2x^{2-1} + \frac{3}{2}y^2 + \frac{5}{2}$$

$$\text{Speed} = \left(2x + \frac{3}{2}y^2 + \frac{5}{2}\right) km/hr$$



Example 3.13

Divide $(10m^2 - 5m)$ by $(2m - 1)$

Solution:

We have $\frac{10m^2 - 5m}{(2m-1)}$

$$= \frac{5m\cancel{(2m-1)}}{\cancel{(2m-1)}} \quad \text{(taking common factor from the numerator)}$$

$$= 5m$$

Try these

(i) $(16y^5 - 8y^2) \div 4y$

(ii) $(p^5q^2 + 24p^3q - 128q^3) \div 6q$

(iii) $(4m^2n + 9n^2m + 3mn) \div 4mn$

3.4 Avoid some common errors

	Error	Correct	Reason
1.	$2xx = 2x$	$2xx = 2\times x^1 \times x^1 = 2x^2$	Product of variables
2.	$7xxy = 7xy$	$7xxy = 7\times x^1 \times x^1 \times y^1$ $= 7x^2y$	Product of variables
3.	$-3x-4x=-1x$	$-3x-4x=-7x$	Same sign factors should be added and put the same sign.
4.	$4y+3y+y=7y$	$4y+3y+y=8y$	y is same as $1y$,co-efficient 1 of a term is usually not written.
5.	$5x+3x=8x^2$	$5x+3x=8x$	When we add or subtract like terms, add or subtract only the co-efficient of the like terms, keep the variable as it is.
6.	$9x+1=10x$	$9x+1=9x+1$	Unlike terms cannot be added or subtracted
7.	$3x+4y=7xy$	$3x+4y=3x+4y$	Unlike terms cannot be added
8.	$x+2=2x$	$x+2=x+2$	Unlike terms cannot be added
9.	$3(4x+9)$ $=12x+9$	$3(4x+9)=12x+27$	3 is common factor multiply both the terms.
10.	$5+(3y-4)$ $=15y-20$	$5+(3y-4)$ $=5+3y-4$	Addition symbol is in between the terms, not multiplication
11.	$-3x(2x^2y-5xy^2+9)$ $=-6x^3y-5xy^2+9)$	$-3x(2x^2y-5xy^2+9)$ $=-6x^3y+15x^2y^2-27x$	Multiply each term of the polynomial.
12.	$(-7x^2+2x+3)$ $=-(7x^2+2x+3)$	$-7x^2+2x+3$ $=-(7x^2-2x-3)$	Taking (-1) as common, we will have change in sign for all the terms.
13.	$(2x)^2=2x^2$	$(2x)^2=2^2\times x^2=4x^2$	Power is common for all the basic factors within the bracket
14.	$(2x-5)(3x-4)$ $=6x^2+20$	$(2x-5)(3x-4)$ $=2x(3x-4)-5(3x-4)$ $=6x^2-8x-15x+20$ $=6x^2-23x+20$	Distributive law should be followed

15.	$(x-9)^2 = x^2 - 9^2$	$(x-9)^2 = (x-9)(x-9)$ $= x^2 - 2(x)(9) + 9^2$ $= x^2 - 18x + 81$	Product of binomials $(a+b)^2$, $(a-b)^2$ use identities
16.	$\frac{p^2}{p^5} = p^{5-2} = p^3$	$\frac{p^2}{p^5} = \frac{1}{p^{5-2}} = \frac{1}{p^3}$	Law of exponents $x^m \div x^n = x^{m-n}$ when $m > n$
17.	$\frac{x^2+5}{5} = x^2$	$\frac{x^2+5}{5} = \frac{x^2}{5} + \frac{5}{5}$ $= \frac{x^2}{5} + 1$	Divide each term of the expressions by the denominator
18.	$\frac{5\cancel{m}^2}{5\cancel{m}^2} = 0$	$\frac{5m^2}{5m^2} = 1$	a term divided by itself gives 1

Exercise 3.2

1. Fill in the blanks

   (i) $\frac{18m^4(\_\_)}{2m^3n^3} = \_\_\_\_\_\_ mn^5$ (ii) $\frac{l^4m^5n^{(\_\_)}}{2lm^{(\_\_)}n^6} = \frac{l^3m^2n}{2}$ (iii) $\frac{42a^4b^5(\_\_)}{6a^4b^2} = (\_\_)b^3c^2$

2. Say True or False

   (i) $8x^3y \div 4x^2 = 2xy$ (ii) $7ab^3 \div 14ab = 2b^2$

3. Divide

   (i) $27y^3 \div 3y$ (ii) $x^3y^2 \div x^2y$

   (iii) $45x^3y^2z^4 \div (-15xyz)$ (iv) $(3xy)^2 \div 9xy$

4. Simplify

   (i) $\frac{3m^2}{m} + \frac{2m^4}{m^3}$ (ii) $\frac{14p^5q^3}{2p^2q} - \frac{12p^3q^4}{3q^3}$

5. Divide:

   (i) $(32y^2 - 8yz)$ by $2y$ (ii) $(4m^2n^3 + 16m^4n^2 - mn)$ by $2mn$

   (iii) $10(4x - 8y)$ by $5(x - 2y)$ (iv) $81(p^4q^2r^3 + 2p^3q^3r^2 - 5p^2q^2r^2)$ by $(3pqr)^2$

6. Find Adirai's percentage of marks who scored $25m^3n^2p$ out of $100m^2np$.

7. Identify the errors and correct them.

   (i) $7y^2 - y^2 + 3y^2 = 10y^2$ (ii) $6xy + 3xy = 9x^2y^2$

   (iii) $m(4m - 3) = 4m^2 - 3$ (iv) $(4n)^2 - 2n + 3 = 4n^2 - 2n + 3$

   (v) $(x-2)(x+3) = x^2 - 6$

3.5 Identities

We have studied in the previous class about standard algebraic identities. An identity is an equation satisfied by any value that replaces its variable(s). Now, we shall recollect four known identities, which are,

$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)^2 = a^2 - 2ab + b^2$
$(a^2 - b^2) = (a+b)(a-b)$	$(x+a)(x+b) = x^2 + (a+b)x + ab$

Try these

Expand the following

1. $(p+2)^2 =$ ....
2. $(3-a)^2 =$ ....
3. $(6^2 - x^2) =$ ....
4. $(a+b)^2 - (a-b)^2 =$ ....
5. $(a+b)^2 = (a+b)\times$ ....
6. $(m+n)(.....) = m^2 - n^2$
7. $(m+-)^2 = m^2 + 14m + 49$
8. $(k^2 - 36) = (k+...)(k-...)$
9. $m^2 - 6m + 9 =$ ....
10. $(m-10)(m+5) =$ ....

3.5.1 Application of Identities

The identities give an alternative method of solving problems on multiplication of algebraic expressions and also of numbers.

Note

$(c+d)^2 = c^2+d^2$ is wrong.
The correct expansion is
$(c+d)^2 = c^2+2cd+d^2$

Example 3.14

Find the value of $(3a+4c)^2$ by using $(a+b)^2$ identity.

Solution:

Comparing $(3a+4c)^2$ with $(a+b)^2$, we have $a = 3a, b = 4c$

$\boxed{\text{Now } (a+b)^2 = a^2 + 2ab + b^2}$

$\therefore \quad (3a+4c)^2 = (3a)^2 + 2(3a)(4c) + (4c)^2$ (replacing a and b values)

$$= 3^2a^2 + (2\times3\times4)(a\times c) + 4^2c^2$$

$$(3a+4c)^2 = 9a^2 + 24ac + 16c^2$$

Example 3.15

Find the value of 998^2 by using $(a-b)^2$ identity.

Solution:

We know, 998 can be expressed as $(1000-2)$

$$\therefore \quad (998)^2 = (1000-2)^2$$

This is in the form of $(a-b)^2$, we get $a = 1000, \quad b = 2$

$\boxed{\text{Now } (a-b)^2 = a^2 - 2ab + b^2}$

$$(1000-2)^2 = (1000)^2 - 2(1000)(2) + (2)^2$$
$$= 1000000 - 4000 + 4$$
$$(998)^2 = 996004$$

Example 3.16

Simplify $(3x+5y)(3x-5y)$ by using $(a+b)$ $(a-b)$ identity.

Solution:

We have $(3x+5y)(3x-5y)$

Comparing it with $(a+b)(a-b)$ we get $a = 3x \quad b = 5y$

$\boxed{\text{Now } (a+b)(a-b) = a^2 - b^2}$

$$(3x+5y)(3x-5y) = (3x)^2 - (5y)^2 \qquad \text{(replacing } a \text{ and } b \text{ values)}$$
$$= 3^2x^2 - 5^2y^2$$
$$(3x+5y)(3x-5y) = 9x^2 - 25y^2$$

Example 3.17

Expand $y^2 - 16$ by using a^2-b^2 identity

Solution:

$y^2 - 16$ can be written as $y^2 - 4^2$

Comparing it with $a^2 - b^2$, we get $a = y, b = 4$

$\boxed{\text{Now } a^2 - b^2 = (a+b)(a-b)}$

$$y^2 - 4^2 = (y+4)(y-4)$$
$$y^2 - 16 = (y+4)(y-4)$$

Example 3.18

Simplify $(5x+3)(5x+4)$ by using $(x+a)$ $(x+b)$ identity.

Solution:

We have $(5x+3)(5x+4)$

Comparing it with $(x+a)(x+b)$, we get $x = 5x$ and $a = 3, b = 4$

We know $(x+a)(x+b) = x^2 + (a+b)x + ab$ (replacing x, a and b values)

$$(5x+3)(5x+4) = (5x)^2 + (3+4)\,(5x) + (3)\,(4)$$
$$= 5^2x^2 + (7)(5x) + 12$$
$$(5x+3)(5x+4) = 25x^2 + 35x + 12$$

Try these

Expand using appropriate identities.

1) $(3p+2q)^2$ 2) $(105)^2$ 3) $(2x-5d)^2$

4) $(98)^2$ 5) $(y-5)(y+5)$ 6) $(3x)^2-5^2$

7) $(2m+n)(2m+p)$ 8) 203×197

9) Find the area of the square whose side is $(x-2)$

10) Find the area of the rectangle whose length and breadth are $(y+4)$ and $(y-3)$.

3.6 Cubic identites

I. $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

We shall prove it now,

LHS $= (a+b)^3$

$= [(a+b)(a+b)](a+b)$ (expanded form)

$= (a+b)^2(a+b)$

$= (a^2+2ab+b^2)(a+b)$ (using identity)

$= a(a^2+2ab+b^2)+b(a^2+2ab+b^2)$ (using distributive law)

$= a^3+2a^2b+ab^2+ba^2+2ab^2+b^3$

$= a^3+(2a^2b+ba^2)+(ab^2+2ab^2)+b^3$ (grouping 'like' terms)

$= a^3+3a^2b+3ab^2+b^3$

$= RHS$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

Hence, we proved the cubic identity by direct multiplication.

Aliter

$\times$	a^2	$2ab$	b^2
a	a^3	$2a^2b$	ab^2
b	a^2b	$2ab^2$	b^3

$= a^3+3a^2b+3ab^2+b^3$

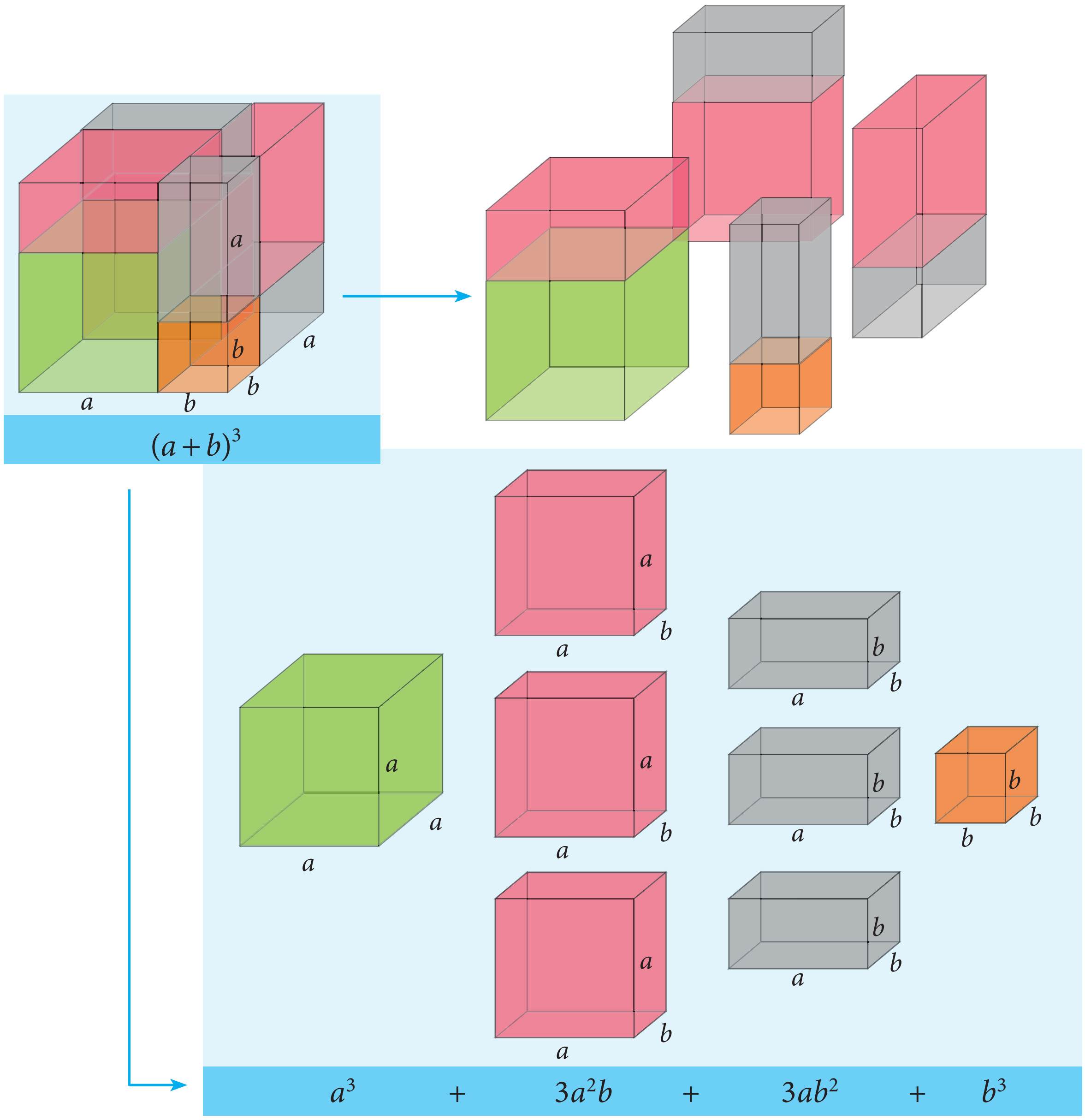
Geometrical Illustration:

We have $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

Here in the left hand side, there is a cube whose side is $(a+b)$ then its volume is equal to, sum of a cube whose side is 'a' *units*, 3 cuboids whose sides are (a, a and b) *units*, 3 more cuboids whose sides are (a, b, b) *units* and a cube whose is 'b' *units*.

We can visualize these 8 objects by doing an activity, and fit them in the cube of side $(a+b)$ *units*.

Therefore we conclude that the volume of the cube whose side $(a+b)$ has 2 cubes and 6 cuboids and a total of 8 objects in it.



In a plane the geometrical proof of $(a+b)^2$ is easy to prove, because it has only length and breadth whereas $(a+b)^3$ has a cube and cubiod of 3 dimensions.Geometrical proof of cannot be visualized in a plane, because a plane doesnot have the height.

II $\quad (a-b)^3 = a^3 - 3a^2b + 3a^2b - b^3$

We can prove this identity by direct multiplication

We have $(a-b)^3 = (a-b)(a-b)(a-b)$

$$= (a-b)^2 \times (a-b)$$

$$= (a^2 - 2ab + b^2)(a-b)$$

$$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$$

$$= a^3 - 2a^2b + ab^2 - ba^2 + 2ab^2 - b^3$$

Activity-1

You can visualize the proof of $(a+b)^3$ with the help of your teacher.

$$= a^3 - 2a^2b + ba^2 + ab^2 + 2ab^2 - b^3$$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

$$= RHS$$

$$(a-b)^3 = a^3 - 3a^2b + 3a^2b - b^3$$

Hence, we proved.

Aliter

$\times$	a^2	$-2ab$	b^2
a	a^3	$-2a^2b$	ab^2
$-b$	$-a^2b$	$2ab^2$	$-b^3$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

Geometrical illustration:

$$(a-b)^3 = a^3 - 3a^2b + 3a^2b - b^3$$

Let us take a cube whose side is 'a' *units*, make one more cube whose side is 'b' *units* $(b < a)$ at one of its corners. Complete the diagram with respect to sides to get cubes and cuboids in it.

Hence the volume of the cube of side $(a-b)$ is obtained by removing 4 elements namely 3 cuboids whose sides are (a, a and b) *units* and cube of side 'b' *units* from the cube whose side is 'a' *units* and finally 3 cuboids of sides (a, b and b)*units* are added to get the cube whose side is $(a-b)$.

III $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

We know the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$. Let us multiply this by a binomial $(x+c)$. Then we get.

$$(x+a)(x+b)(x+c) = \left[(x+a)(x+b)\right](x+c)$$

$$= (x^2 + (a+b)x + ab) \times (x+c)$$

$$= x\left[x^2 + (a+b)x + ab\right] + c\left[x^2 + (a+b)x + ab\right] \quad \text{(distributive law)}$$

$$= x^3 + (a+b)x^2 + abx + cx^2 + (a+b)xc + abc$$

$$= x^3 + ax^2 + bx^2 + abx + cx^2 + acx + bcx + abc$$

$$= x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc \quad \text{(Combine } x^2, x \text{ terms)}$$

$$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

Geometrical illustration:

The geometrical illustration of the above identity is the cuboid whose length,breadth,and height are $(x+a)$, $(x+b)$, $(x+c)$ and whose volume is $(l\times b\times h)$. That is $v= (x+a)\ (x+b)(x+c)$. This cuboid contains *a* cube of side 'x' *units* , 3 cuboids of sides (a,x,x) (b,x,x) and (c,x,x), 3 cuboids of sides (a,b,x) (b,c,x) and (c,a,x) and one cuboid of side (a,b,c)

Thus, we summarise the cubic identities as :

- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Deductions:

The above identities can also be deducted as

(i) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + 3ab(a+b) + b^3$

$$\boxed{(a+b)^3 = a^3 + b^3 + 3ab(a+b)}$$

From the above, $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ (taking out the common factor $(a+b)$]

$= (a+b)\left((a+b)^2 - 3ab\right)$

$= (a+b)\left(a^2 + b^2 + 2ab - 3ab\right)$ (using identity $(a+b)^2$)

$= (a+b)\left(a^2 + b^2 - ab\right)$

$$\boxed{a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)}$$

(ii) We have $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a-b)^3 = a^3 - 3ab(a-b) - b^3$

$$\boxed{(a-b)^3 = a^3 - b^3 - 3ab(a-b)}$$

Again from the above,

$a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

$= (a-b)\left((a-b)^2 + 3ab\right)$ (taking out the common factor $(a-b)$

$= (a-b)\left(a^2 + b^2 - 2ab + 3ab\right)$

$= (a-b)\left(a^2 + b^2 + ab\right)$

$$\boxed{a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)}$$

3.6.1 Application of Cubic Identities

I. Using the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Example 3.19

Expand $(x+4)^3$

Solution:

Comparing $(x+4)^3$ with $(a+b)^3$, we get $a=x\,,b=4$

We know $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$\boxed{\begin{array}{l}(4)^2 = 4\times 4 = 16\\ (4)^3 = 4\times 4\times 4 = 64\end{array}}$

$$(x+4)^3 = (x)^3+3(x)^2(4)+3(x)(4)^2+(4)^2 \quad \text{(replacing } a,\ b \text{ values)}$$

$$= (x)^3+3x^2(4)+3(x)(16)+16$$

$$(x+4)^3 = x^3+12x^2+48x+16$$

Example 3.20

Find the value of $(103)^3$

Solution:

Now, $(103)^3 = (100+3)^3$

Comparing this with $(a+b)^3$, we get $a=100\,,b=3$

$$(a+b)^3 \ = a^3+3a^2b+3ab^2+b^3 \quad \text{replacing} \quad a,\ b \text{ values,}$$

$$(100+3)^3 \ = (100)^3+3(100)^2(3)+3(100)(3)^2+(3)^3$$

$$= 1000000+3(10000)(3)+3(100)(9)+27$$

$$= 1000000+90000+2700+27$$

$$(103)^3 \ = 1092727$$

II. Using the identity $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example 3.21

Expand: $(y-5)^3$

Solution:

Comparing $(y-5)^3$ with $(a-b)^3$, we get $a=y\,,b=5$

$$(a-b)^3 \ = a^3-3a^2b+3ab^2-b^3$$

$$(y-5)^3 \ = (y)^3+3(y)^2(5)+3(y)(5)^2-(5)^3$$

$$= (y)^3+3y^2(5)+3(y)(25)-125$$

$$(y-5)^3 \ = y^3-15y^2+75y-125$$

Example 3.22

Find the value of $(98)^3$

Solution:

Now, $\quad (98)^3 = (100-2)^3$

Comparing this with $(a-b)^3$, we get $a=100\,,b=2$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(100-2)^3 = (100)^3 - 3(100)^2(2) + 3(100)(2)^2 + (2)^2$$

$$= 1000000 - 3(10000)(2) + 3(100)(4) - 8$$

$$= 1000000 - 60000 + 1200 - 8$$

$$= 941192$$

III. Using the identity $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Example 3.23

Expand: $(x+3)(x+5)(x+2)$

Solution:

Comparing this with $(x+a)(x+b)(x+c)$, we get $x = x$, $a = 3$, $b = 5$, $c = 2$

$$\boxed{(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc}$$

$$(x+3)(x+5)(x+2) = (x)^3 + (3+5+2)(x)^2 + (3\times5 + 5\times2 + 2\times3)x + (3)(5)(2)$$

$$= x^3 + 10x^2 + (15+10+6)x + 30$$

$$(x+3)(x+5)(x+2) = x^3 + 10x^2 + 31x + 30$$

Try these

Expand : (i) $(x+4)^3$ (ii) $(y-2)^3$ (iii) $(x+1)(x+3)(x+5)$

Exercise 3.3

1. Expand

   (i) $(3m+5)^2$ (ii) $(5p-1)^2$ (iii) $(2n-1)(2n+3)$ (iv) $4p^2 - 25q^2$

2. Expand

   (i) $(3+m)^3$ (ii) $(2a+5)^3$ (iii) $(3p+4q)^3$

   (iv) $(52)^3$ (v) $(104)^3$

3. Expand

   (i) $(5-x)^3$ (ii) $(2x-4y)^3$ (iii) $(ab-c)^3$

   (iv) $(48)^3$ (v) $(97xy)^3$

4. Simplify

   (i) $(5y+1)(5y+2)(5y+3)$ (ii) $(p-2)(p+1)(p-4)$

5. Find the volume of the cube whose side is $(x+1)$ cm

6. Find the volume of the cuboid whose dimensions are $(x+2), (x-1)$ and $(x-3)$

3.7 Factorisation

Teacher : Can you express the number 12 in as many ways as the product of two numbers?

Student : Yes sir, 12 can be written as $1 \times 12 = 12$, $2 \times 6 = 12$, $3 \times 4 = 12$ and again by the reverse product of numbers.

Teacher : What are the numbers 1, 2, 3, 4, 6 and 12 called as?

Student : They are called as factors of 12

Teacher : Yes, we can find factors of any number by using multiplication tables.

Expressing any number as the product of two or more numbers is called as **factorisation.** The same number 12 can be expressed as the product of prime factors also like $12 = 2 \times 2 \times 3$. This is called prime Factorisation.

Note

Prime numbers	A number which is divisible by 1 and itself (or) A number which has only 2 factors. Example: 2, 3, 5. 7. 11, ...
Composite numbers	A number which has more than 2 factors. Example: 4, 6, 8, 9, 10, 12, ...

1 is neither prime nor composite.
2 is the only one even prime number.
1 is a factor for all numbers.

Teacher : How will you factorise an algebraic expression? Yes. Expressing an algebraic expression as the product of two or more expressions is called the Factorisation.

For example, (i) $a^2 - b^2 = (a+b)(a-b)$ Here, $(a+b)$ and $(a-b)$ are the two factors of $a^2 - b^2$

(ii) $5y + 30 = 5(y+6)$, Here 5 and $(y+6)$ are the factors of $5y + 30$

Any expression can be factorized as (1) × (expression)

For example, $a^2 - b^2$ can also be factorised as

$$(1)\times\left(a^2 - b^2\right) \text{or } (-1)\times\left(b^2 - a^2\right)$$

because '1' is a factor for all numbers and expressions

So, when we factorise the expressions, follow the suitable type of factorisation given below to get two or more factors other than 1. Stop doing the factorisation process once you have taken out all the common factors from the expression and then list out the factors.

Type: 1 Factorisation by taking out the common factor from each term.

Example 3.24

Factorise: $2m^3 - 5m^2 + 9m$

Solution:

We have, $2m^3 - 5m^2 + 9m$ taking out the common factor 'm' from each term, we get

$$= m(2m^2 - 5m + 9)$$

Example 3.25

Factorise: $4x^2y + 8xy$

Solution:

We have, $4x^2y + 8xy$

This can be written as, $= (2 \times 2 \times x \times x \times y) + (2 \times 2 \times 2 \times x \times y)$

Taking out the common factor $2, 2, x, y$, we get

$$= 2 \times 2 \times x \times y(x + 2)$$

$$= 4xy(x + 2)$$

Type: 2 Factorisation by taking out the common binomial factor from each term

Example 3.26

Factorise: $(2x+5)(x-y)+(4y)(x-y)$

Solution:

We have $(2x+5)(x-y)+(4y)(x-y)$

Taking out the common binomial factor $(x-y)$

We get, $(x-y)(2x+5+4y)$

Type: 3 Factorisation by grouping

Sometimes, the terms of a given expression are grouped suitably in such a way that they have a common factor so that the factorisation is easy to take out common factor from those terms.

Example 3.27

Factorise : $x^2 + yz + xy + xz$

Solution:

We have, $x^2 + yz + xy + xz$

Group the terms suitably as, $= (x^2 + xy) + (yz + xz)$

$= x(x+y) + z(y+x)$

$= x(x+y) + z(x+y)$ (addition is commutative)

$= (x+y)[x+z]$ [taking out the common factor $(x+y)$]

Type: 4 Factorisation using Identities

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$

Example 3.28

Factorise : $x^2 + 8x + 16$

Solution:

Now, $x^2 + 8x + 16$

This can be written as $x^2 + 8x + 4^2$

Comparing this with $a^2 + 2ab + b^2 = (a+b)^2$ we get $a = x$; $b = 4$

$$(x^2) + 2(x)(4) + (4)^2 = (x+4)^2$$

$$x^2 + 8x + 16 = (x+4)^2$$

Example 3.29

Factorise $49x^2 - 84xy + 36y^2$

Solution:

$$\text{Now, } 49x^2 - 84xy + 36y^2 = 7^2x^2 - 84xy + 6^2y^2$$

$$= (7x)^2 - 2(7x)(6y) + (6y)^2$$

Comparing this with $a^2 - 2ab + b^2 = (a-b)^2$ we get $a = 7x$, $b = 6y$

$$(7x)^2 - 2(7x)(6y) + (6y)^2 = (7x-6y)^2$$

$$\therefore \quad 49x^2 - 84xy + 36y^2 = (7x-6y)^2$$

Example 3.30

Factorise : $49x^2 - 64y^2$

Solution:

Now, $49x^2 - 64y^2 = 7^2x^2 - 8^2y^2$

$$= (7x)^2 - (8y)^2$$

Comparing this with $a^2 - b^2 = (a+b)(a-b)$

we get $a = 7x$, $b = 8y$

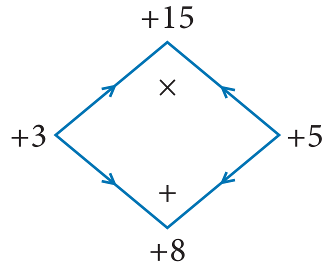
$$(7x)^2 - (8y)^2 = (7x+8y)(7x-8y)$$

Type: 5 Factorisation of the expression (ax^2+bx+c)

Example 3.31

Factorise $x^2+8x+15$

Product = 15	Sum = 8
1×15 = 15	1+15 = 16
3 × 5 = 15	**3+5 = 8**



Solution:

This is in the form of ax^2+bx+c

We get $a=1, b=8, c=15$

Now, the product $=a\times c$ and sum $=b$

$=1\times 15$ $\quad b=8$

$=x^2+8x+15$

$=(x^2+3x)+(5x+15)$ (the middle term $8x$ can be written as $3x+5x$)

$=x(x+3)+5(x+3)$ (taking out the common factor $x+3$)

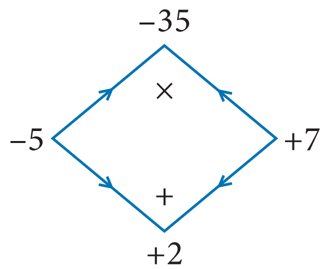
$=(x+3)(x+5)$

$x^2+8x+15=(x+5)(x+3)$

Example 3.32

Factorise $7c^2+2c-5$

Product = –35	Sum = 2
1×(–35) = –35	1–35 = –34
–1 × 35 = –35	–1+35 = 34
5 × (–7) = –35	5–7 = –2
–5 × 7 = –35	**–5+7 = 2**



Solution:

This is in the form of ax^2+bx+c

We get $a=7,\ b=2,\ c=-5$

Now, the product $=a\times c=7\times(-5)=-35$ and sum $b=2$

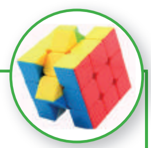
$=7c^2+2c-5$

$=(7c^2-5c)+(7c-5)$ (the middle term $2c$ can be written as $-5c+7c$)

$=c(7c-5)+1(7c-5)$ (taking out the common factor $7c-5$)

$=(7c-5)(c+1)$

Try these



Factorize the following :

1) $3y+6$ 2) $10x^2+15y^2$ 3) $7m(m-5)+1(5-m)$ 4) $64-x^2$

3.7.1 Factorisation using cubic identities

The cubic identities are

(i) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

(ii) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

The factorisable forms of the above identities are

(i) $a^3+b^3=(a+b)(a^2-ab+b^2)$

(ii) $a^3-b^3=(a-b)(a^2+ab+b^2)$

I. Factorise using the identity $a^3+b^3=(a+b)(a^2-ab+b^2)$

Note

$8a^3=2\times2\times2\times a^3$

$=2^3a^3 \;=(2a)^3$

Example 3.33

Factorise: x^3+125

Solution:

We have x^3+125, this can be written as x^3+5^3

Comparing x^3+5^3 with a^3+b^3 we get $a=x$, $b=5$

We know,

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$x^3+5^3=(x+5)(x^2-(x)(5)+5^2)$$

$$x^3+5^3=(x+5)(x^2-5x+25)$$

Now, 125 = 5×5×5

$=5^3$

Note

Perfect cube numbers

A number which can be written in the form of $x\times x\times x$ is called perfect cube number

Examples

$8=2\times2\times2=2^3$

$27=3\times3\times3=3^3$

$125=5\times5\times5=5^3$

Here 8, 7, 125 are some perfect cube numbers

Example 3.34

Factorize: $8p^3+q^3$

Solution:

We have $8p^3+q^3$

This can be written as

$$=2^3p^3+q^3$$

$$=(2p)^3+q^3$$

Comparing this with a^3+b^3, we get $a=2p, b=q$

We know $a^3+b^3=(a+b)(a^2-ab+b^2)$

$$(2p)^3+q^3=(2p+q)[(2p)^2-(2p)(q)+q^2]$$

$$8p^3+q^3=(2p+q)[4p^2-2pq+q^2]$$

II. Factorise using the identity $a^3-b^3=(a-b)(a^2+ab+b^2)$

Example 3.35

Factorise: a^3-8

Solution:

Here a^3-8 can be written as a^3-2^3

Comparing this with a^3-b^3, we get $a=a, b=2$

$$\therefore, \quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 - 2^3 = (a-2)(a^2 + a(2) + 2^2)$$

$$a^3 - 8 = (a-2)(a^2 + 2a + 4)$$

Example 3.36

Factorise: $81x^3 - 3y^3$

Solution:

We have $81x^3 - 3y^3 = 3(27x^3 - y^3)$

$$= 3(3^3x^3 - y^3)$$

$$= 3[(3x)^3 - y^3]$$

Comparing this $a^3 - b^3$, we get $a = 3x$, $b = y$

$$= 3\left\{(3x-y)\left[(3x)^2 + (3x)(y) + y^2\right]\right\}$$

Therefore, $81x^3 - 3y^3 = 3\left\{(3x-y)[9x^2 + 3xy + y^2]\right\}$

What we discussed so far are some factorisable forms of expressions only. Apart from this, there are more expressions that cannot be factorised by the ways explained above. For example, $x^2 + 3x + 13$, $2y^2 - 7x + 17$ are not factorisable. You will learn to factorise such expressions, in the higher classes.

ICT CORNER

Step-1 Open the Browser and type the URL given below.

Step-2 Click on any one of the link in the items to know about the basics in algebra, exponents, polynomial, quadric equation etc.

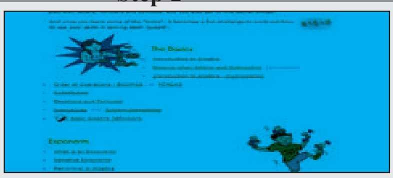
Step-3 For example, click on the "Balance While adding and subtracting", link under the Basic menu. A new tab will open in the browser where you can see the interactive game on adding and subtracting algebra.

Step-4 Likewise you can learn all the concepts in algebra.

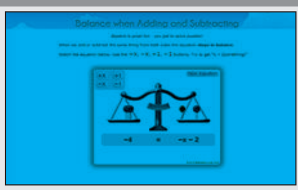
Through this activity you will know about the Algebra, operations on them and study their properties as well.

Step 1

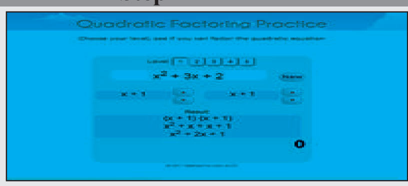
Step 2



Step 3



Step 4



B358_8_MATHS_EM

Web URL Algebra
https://www.mathsisfun.com/algebra/index.html
*Pictures are indicatives only.
*If browser requires, allow Flash Player or Java Script to load the page.

Exercise 3.4

1. Factorise the following by taking out the common factor

(i) $18xy-12yz$ (ii) $9x^5y^3+6x^3y^2-18x^2y$

(iii) $x(b-2c)+y(b-2c)$ (iv) $(ax+ay)+(bx+by)$

(v) $2x^2(4x-1)-4x+1$ (vi) $3y(x-2)^2-2(2-x)$

(vii) $6xy-4y^2+12xy-2yzx$ (viii) a^3-3a^2+a-3

(ix) $3y^3-48y$ (x) $ab^2-bc^2-ab+c^2$

2. Factorise the following expressions

(i) $x^2+14x+49$ (ii) $y^2-10y+25$

(iii) $c^2-4c-12$ (iv) m^2+m-72

(v) $4x^2-8x+3$

3. Factorise the following expressions using $a^3+b^3=(a+b)(a^2-ab+b^2)$ identity

(i) h^3+k^3 (ii) $2a^3+16$ (iii) x^3y^3+27

(iv) $64m^3+n^3$ (v) r^4+27p^3r

4. Factorise the following expressions using $a^3-b^3=(a-b)(a^2+ab+b^2)$ identity

(i) y^3-27 (ii) $3b^3-192c^3$ (iii) $-16y^3+2x^3$

(iv) $x^3y^3-7^3$ (v) $c^3-27b^3a^3$

Exercise 3.5

Miscellaneous Practice Problems

E73UJ7

1. Subtract: $-2(xy)^2(y^3+7x^2y+5)$ from $5y^2(x^2y^3-2x^4y+10x^2)$

2. Multiply $(4x^2+9)$ and $(3x-2)$

3. Find the simple interest on Rs. $5a^2b^2$ for $4ab$ years at $7b\%$ per annum.

4. The cost of a note book is Rs. $10ab$. If Babu has Rs. $(5a^2b+20ab^2+40ab)$. Then how many note books can he buy?

5. Factorise : $(7y^2-19y-6)$

Challenging problems

6. A contractor uses the expression $4x^2+11x+6$ to determine the amount of wire to order when wiring a house. If the expression comes from multiplying the number of rooms times the number of outlets and he knows the number of rooms to be $(x+2)$, find the number of outlets in terms of 'x'. [Hint : factorise $4x^2+11x+6$]

7. A mason uses the expression x^2+6x+8 to represent the area of the floor of a room. If the decides that the length of the room will be represented by $(x+4)$, what will the width of the room be in terms of x?

8. Find the missing term: $y^2+(-)x+56=(y+7)(y+-)$

9. Factorise : $16p^4-1$

10. Factorise : $(x^{15}-64y^3)$

Summary

- When the product of two algebraic expressions we follow,

  Multiply the signs of the terms,

  Multiply the corresponding co-efficients of the terms.

  Multiply the variable factors by using laws of exponents.

- For dividing a polynomial by a monomial, divide each term of the polynomial by a monomial.

- **Identity:** An identity is an equation is satisfied by any value that replaces its variables (s).

  $(a+b)^2=a^2+2ab+b^2$ $\qquad$ $(x+a)(x+b)=x^2+(a+b)x+ab$

  $(a^2-b^2)=a^2-2ab+b^2$ $\qquad$ $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

  $a^2-b^2=(a+b)(a-b)$ $\qquad$ $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

  $(x+b)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

- Identities give an alternative method of solving problems on multiplication of algebraic expressions and also of numbers.

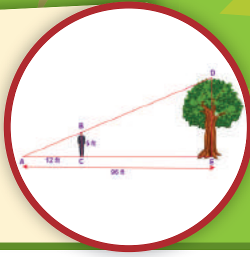
- **Factorisation:** Expressing an algebraic expression as the product of two or more expression is called Factorisation.

- Factorisation form of the cubic identities a^3+b^3 and a^3-b^3 are

  $a^3+b^3=(a+b)(a^2-ab+b^2)$

  $a^3-b^3=(a-b)(a^2+ab+b^2)$

4 GEOMETRY



Learning Outcomes

- To recall the similar and congruence and basic properties of triangles.
- To understand theorems based on these properties of triangles and apply them appropriately to problems.
- To construct quadrilaterals of various types.

K5E9W5

4.1 Introduction

Geometry, as we all know studies shapes by looking at the properties and relations of points, circles, triangles of two dimensions and solids.

In the earlier classes, we have seen a few properties of triangles. In this term, we are going to recall them and also see the similarity and congruence in triangles. Also, we will construct quadrilaterals based on the given sides and angles.

MATHEMATICS ALIVE	GEOMETRY IN REAL LIFE
	
Enlargements in proportion are similar. Small giraffe looks similar to her mother.	For better strength and stability, congruent triangles in the construction of buildings.

Recalling the Properties of Triangles

Answer the following questions:

1. The sum of the three angles of a triangle is ______________ .
2. The exterior angle of a triangle is equal to the sum of the _____________ angles to it.
3. In a triangle, the sum of any two sides is _____________ than the third side.

4. The difference between any two sides of a triangle is _______ than the third side.
5. Angles opposite to equal sides are _______ and vice – versa.
6. The angles of a triangle are in the ratio 4:5:6.
   (i) Is it an acute, right or obtuse triangle?
   (ii) Is it scalene, isosceles or equilateral?
7. What is $\angle A$ in the triangle ABC?

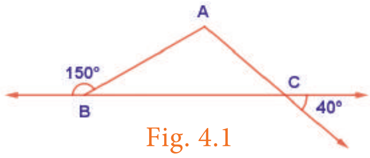


Fig. 4.1

8. Can a triangle have two supplementary angles? Why?
9. __________ figures have the same shapes but different sizes.
10. __________ figures are exactly the same in shape and size.

Think about the situation

In a Math class, the teacher is ready to teach a concept in geometry. The teacher asks the following questions and tries to elicit the answers from the students.

Teacher : Students, Tell me, what do you know about Similar or Congruent figures?

Terasa : Teacher, Similar figures are same in shape and different in size where as Congruent figures are same in shape and size too.

Teacher : Good answer Terasa. You are clear about them. Can you identify a pair of triangles as Similar or Congruent easily, if I give you any such pair?

Barani : No teacher. It is not that easy. We have to see some properties that match with the given pair of triangles and then say whether the pair is Similar or Congruent.

Teacher : Well said Barani. Can anyone of you list out all the properties that are possible by the elements (3 sides and 3 angles) to prove the Similarity (or) Congruence in triangles?

Saleem : Yes Teacher, I can state a few of them which I have already studied in VII Standard. To prove Similarity in triangles, there are SSS, SAS, AAA and RHS properties. To prove Congruence in triangles, there are properties such as SSS, SAS, ASA, RHS etc., Have I stated them all correctly, Teacher?

Teacher : Yes Saleem, you have almost stated them correctly. I will explain them a little further. If we shall write the properties using sides and angles, there are only 8 possibilities in order namely, SSS, SAS, ASA, AAS, SAA, SSA, ASS and AAA.

Terasa : Are all of them unique properties that prove the Congruence or Similarity in triangles, Teacher?

Teacher : No Terasa, some properties like AAS and SAA can be combined with ASA by finding the third angle.

Barani : Can AAA be used to prove Congruence teacher?

Teacher : No Barani, it can be used only for proving Similarity in triangles.

Saleem : Teacher, please tell about SSA and ASS properties.

Teacher : Saleem, these are tricky properties that do not convey clearly to say about Similarity or Congruence in triangles. We shall see the explanation later.

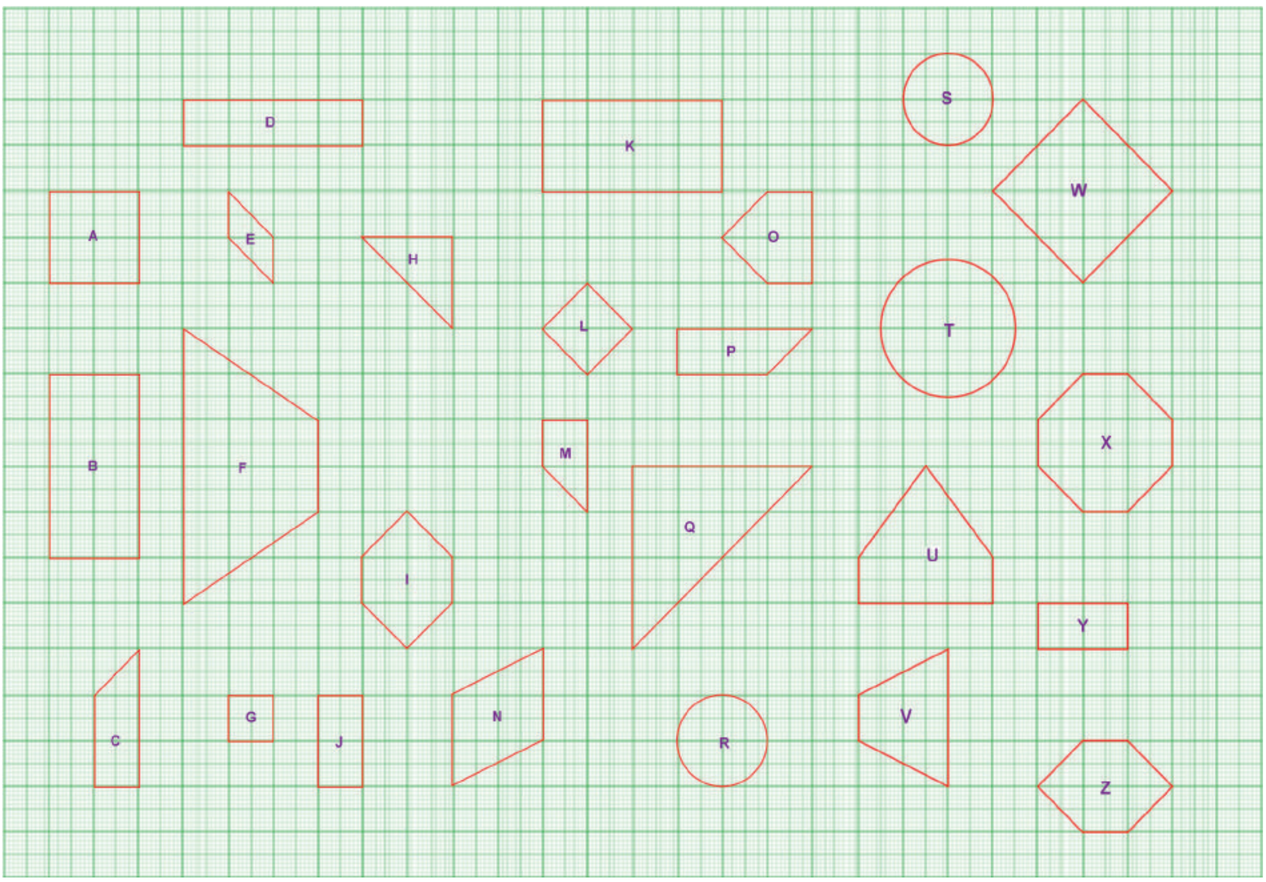
The above conversation leads us to know and use the properties in detail so as to prove a pair of triangles to be Similar or Congruent.

Activity-1

Draw a quadrilateral, a pentagon and a hexagon and find the sum of the interior angles in them using the triangle's angle sum property.

Try this

Identify the pairs of figures which are similar and csongruent and write the letter pairs.



4.2 Similar Triangles

Similar figures mathematically have the same shape but different sizes. Two geometrical figures are said to be similar ($\sim$) if the measures of one to the corresponding measures of the other are in a constant ratio. In other words, every part of a photographic enlargement is similar to the corresponding part of the original.

Some examples where similar triangles are seen and used in real life are :

(i) to determine the distances between light and the target in the light beams, the height of any building, objects, people etc., by analysing the shadows and using the scale modelling.

(ii) to analyse the stability of bridges.

(iii) in designing the work by the architects.

Similarity Properties:

1. Two triangles are similar if two angles of one triangle are equal respectively to two angles of the other triangle. In the Fig. 4.2, $\angle A = \angle P$, $\angle B = \angle Q$ Therefore, Δ ABC $\sim$ Δ PQR. This is, AA Similarity. This is also called as AAA Similarity.

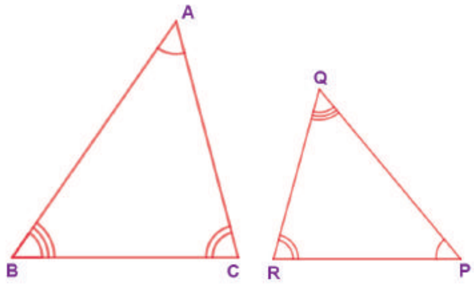


Fig. 4.2

2. Two triangles are similar if two sides of one triangle are proportional to two sides of the other triangle and the included angles are equal. In the Fig. 4.3, $\frac{AC}{PQ} = \frac{AB}{PR}$ and $\angle A = \angle P$ and hence Δ ACB $\sim$ Δ PQR. This is SAS Similarity.

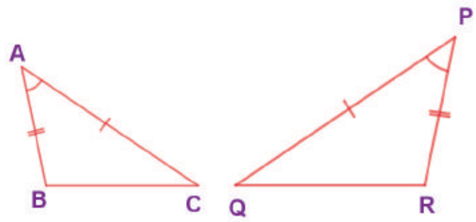


Fig. 4.3

3. Two triangles are similar if their corresponding sides are in the same ratio. That is, if $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$, then $\Delta\, ABC \sim \Delta\, PQR$. This is SSS Similarity.

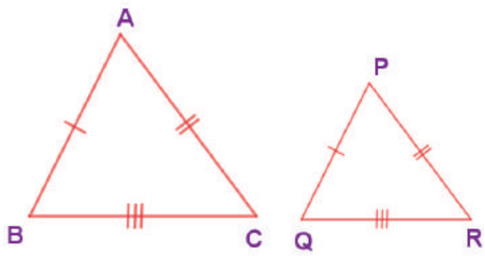


Fig. 4.4

4. Two right triangles are similar if the hypotenuse and a leg of one triangle are respectively proportional to the hypotenuse and a leg of the other triangle. This is RHS Similarity.

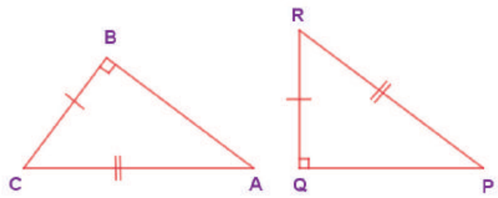


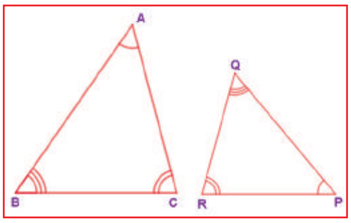
Fig. 4.5

Activity-2

Cut a square from a card board and mark its sides and diagonals and list out all pairs of similar triangles in it.

DO YOU KNOW?

If $\Delta\ ABC \sim \Delta\ PQR$, then the corresponding sides to AB, BC and AC of $\Delta\ ABC$ are PQ, QR and PR respectively and the corresponding angles to A, B and C are P, Q and R respectively. Naming a triangle has a significance. For example, if $\Delta\ ABC \sim \Delta\ PQR$ then, $\Delta\ BAC$ is not similar to $\Delta\ PQR$. If two triangles are Similar, then their angles are respectively equal and their corresponding sides are proportional.



Example 4.1 If $PQ \parallel RS$ and $\angle ONR = 30^\circ$ find $\angle MON$ and hence $\angle MOX$.

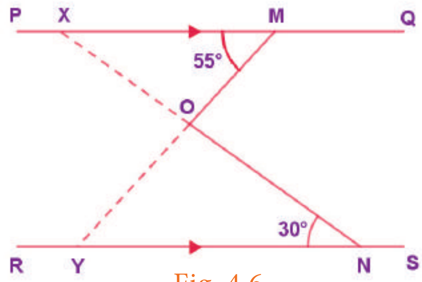


Fig. 4.6

Solution:

Extend MO and let it meet RS at Y. Extend NO and let it meet PQ at X.

As $PQ \parallel RS$,

$$\angle OYN = \angle OMX = 55^\circ \text{ (alternate angles are equal)}$$

$\therefore \quad \angle MON = \angle OYN + \angle ONY$ (exterior angle of $\Delta\ OYN$ = sum of interior opposite angles)

$$= 55^\circ + 30^\circ = 85^\circ$$

$$\Rightarrow \angle MOX = 180^\circ - 85^\circ = 95^\circ \text{ (} \angle MON, \angle MOX \text{ are linear pair)}$$

Example 4.2

In ΔPAT, the bisector of $\angle P$, meets AT at S.

If $\angle APT = 70^\circ$ and $\angle ASP = 100^\circ$, find $\angle A$ and $\angle T$.

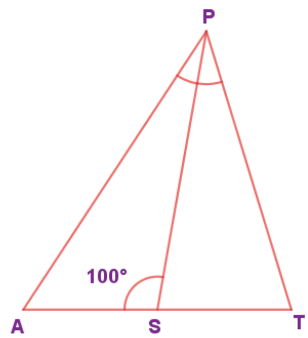


Fig. 4.7

Solution:

Now, PS is the bisector of $\angle P$

$\therefore \ \angle APS = \angle TPS = 35^\circ$

From ΔAPS,

$$\angle A + \angle APS + \angle ASP = 180^\circ \text{ (angle sum property in } \Delta \text{ APS)}$$

$$\angle A + 35^\circ + 100^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 135^\circ = 45^\circ$$

From ΔTPS,

$$\angle T + \angle TPS + \angle TSP = 180^\circ \text{ (angle sum property in } \Delta \text{ TPS)}$$

$$\Rightarrow \angle T + 35^\circ + 80^\circ = 180^\circ$$

$$\Rightarrow \angle T = 180^\circ - 115 = 65^\circ$$

Example 4.3

Find the unknowns in the following figures

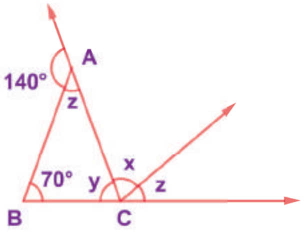
(i) 

Fig. 4.8(i)

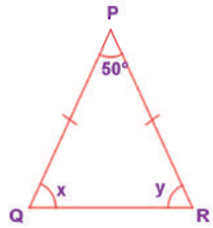
(ii) 

Fig. 4.8(ii)

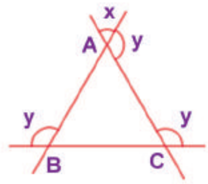
(iii) 

Fig. 4.8(iii)

Solution:

(i) Now, from Fig. 4.8(i), $\angle 140^\circ + \angle z = 180^\circ$ (linear pair)

$$\Rightarrow \angle z = 180^\circ - 140^\circ = 40^\circ$$

Also $\angle x + \angle z = \angle 70 + \angle z$ (exterior angle property)

$$\Rightarrow \angle x = 70^\circ$$

Also $\angle z + \angle y + 70^\circ = 180^\circ$ (angle sum property in Δ ABC)

$$\Rightarrow 40^\circ + \angle y + 70^\circ = 180^\circ$$

$$\Rightarrow \angle y = 180^\circ - 110^\circ = 70^\circ$$

(ii) Now, from Fig. 4.8(ii), $PQ = PR$

$\Rightarrow \angle Q = \angle R$ (angles opposite to equal sides are equal)

$$\Rightarrow \angle x = \angle y$$

$\Rightarrow \angle x + \angle y + 50^\circ = 180^\circ$ (angle sum property in $\Delta\, PQR$)

$$\Rightarrow 2\angle x = 130^\circ$$

$$\Rightarrow \angle x = 65^\circ$$

$$\Rightarrow \angle y = 65^\circ$$

(iii) Now, from Fig. 4.8(iii), in ΔABC $\angle A = x$ (vertically opposite angles)

Similarly $\angle B = \angle C = x$ (Why?)

$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$ (angle sum property in $\Delta\, ABC$)

$$\Rightarrow 3x = 180^\circ$$

$$\Rightarrow x = 60^\circ$$

$$\Rightarrow y = 180^\circ - 60^\circ = 120^\circ$$

Note

- All circles and squares are similar to each other.
- Not all rectangles need to be similar always.

Example 4.4 In the Fig. 4.8,

if $\Delta PQR \sim \Delta XYZ$, find a and b.

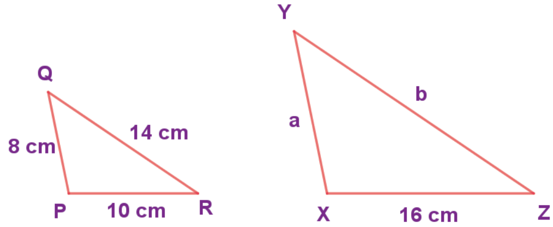


Fig. 4.9

Solution:

Given that $\Delta PQR \sim \Delta XYZ$

$\therefore$ Their corresponding sides are proportional.

$$\Rightarrow \frac{PQ}{XY} = \frac{QR}{YZ} = \frac{PR}{XZ}$$

$$\Rightarrow \frac{8}{a} = \frac{14}{b} = \frac{10}{16}$$

$$\Rightarrow \frac{8}{a} = \frac{10}{16}$$

$$\Rightarrow a = \frac{8\times16}{10} = \frac{128}{10}$$

$$a = 12.8\ cm$$

Also, $$\frac{14}{b} = \frac{10}{16}$$

$$\Rightarrow b = \frac{14\times16}{10} = \frac{224}{10}$$

$$\therefore\ b = 22.4\ cm$$

Example 4.5

In Fig. 4.10, if $\Delta PEN \sim \Delta PAD$, then find x and y.

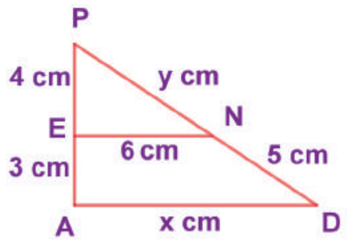


Fig. 4.10

Solution:

Given that $\Delta PEN \sim \Delta PAD$,

$$\therefore \quad \frac{PE}{PA} = \frac{EN}{AD} \Rightarrow \frac{4}{7} = \frac{6}{x} \Rightarrow x = \frac{42}{4} = 10.5\ cm$$

Also, $$\frac{PE}{PA} = \frac{PN}{PD} = \frac{y}{y+5}$$

i.e. $$\frac{4}{7} = \frac{y}{y+5} \Rightarrow 4y + 20 = 7y \Rightarrow 7y - 4y = 20$$

$$3y = 20 \Rightarrow y = \frac{20}{3} cm$$

Example 4.6 (Illustrating AA Similarity)

In the Fig. 4.11 , $\angle ABC \equiv \angle EDC$ and the perimeter of ΔCDE is 27 *units*, prove that $AB \equiv EC$.

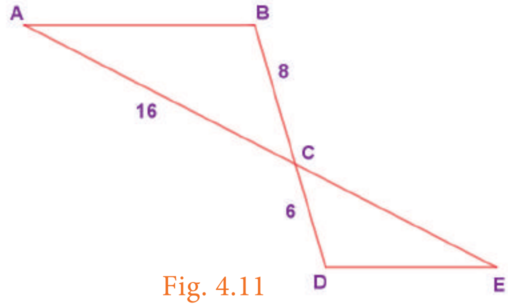


Fig. 4.11

Proof :

	Statements	Reasons
1	$\angle ABC \equiv \angle EDC$	given
2	$\angle BCA \equiv \angle DCE$	vertically opposite angles are equal
3	$\Delta ABC \sim \Delta EDC$	by AA property (1, 2)
4	$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$ $\Rightarrow \frac{8}{6} = \frac{16}{EC} \Rightarrow EC = 12$ *units*	corresponding sides are proportional by 3

Given, the perimeter of $\Delta CDE = 27$ *units*,

$\therefore$ $ED + DC + EC = 27$

$ED + 6 + 12 = 27$

$ED = 27 - 18 = 9$ *units*

$$\therefore \quad \frac{AB}{9} = \frac{8}{6} \Rightarrow AB = 12 \text{ units and hence } AB = EC.$$

Example 4.7 (Illustrating AA Similarity)

In the given Fig. 4.12, if $\angle 1 \equiv \angle 3$ and $\angle 2 \equiv \angle 4$ then, prove that $\Delta BIG \sim \Delta FAT$. Also find FA.

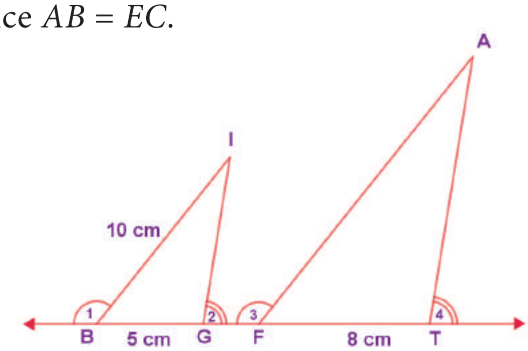


Fig. 4.12

Proof:

	Statements	Reasons
1	$\angle 1 \equiv \angle 3$	given
2	$\angle IBG \equiv \angle AFT$	supplements of congruent angles are congruent.
3	$\angle 2 \equiv \angle 4$	given
4	$\angle IGB \equiv \angle ATF$	supplement of congruent angles are congruent
5	$\Delta BIG \sim \Delta FAT$	by AA property (2, 4)

Also, their corresponding sides are proportional

$$\Rightarrow \frac{BI}{FA} = \frac{BG}{FT} \Rightarrow \frac{10}{FA} = \frac{5}{8}$$

$$\Rightarrow FA = \frac{10 \times 8}{5} = \frac{80}{5} = 16\,cm$$

Example 4.8 (Illustrating SAS Similarity)

If A is the midpoint of RU and T is the midpoint of RN, prove that $\Delta RAT \sim \Delta RUN$.

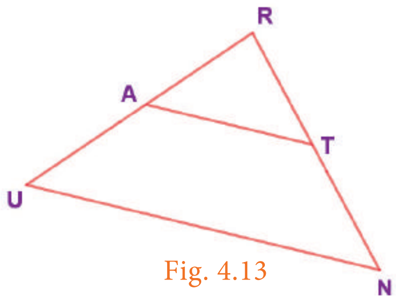


Fig. 4.13

Proof:

	Statements	Reasons
1	$\angle ART = \angle URN$	$\angle R$ is common in ΔRAT and ΔRUN
2	$RA = AU = \frac{1}{2}RU$	A is the midpoint of RU
3	$RT = TN = \frac{1}{2}RN$	T is the midpoint of RN
4	$\frac{RA}{RU} = \frac{RT}{TN} = \frac{1}{2}$	the sides are proportional from 3 and 4
5	$\Delta RAT \sim \Delta RUN$	by SAS (1, 2, 3)

Example 4.9 (Illustrating SSS similarity)

Prove that $\Delta PQR \sim \Delta PRS$ in the given Fig. 4.14.

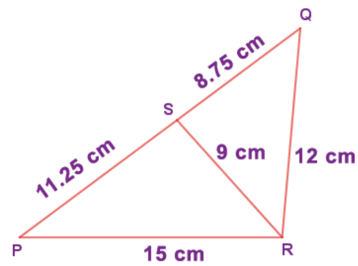


Fig. 4.14

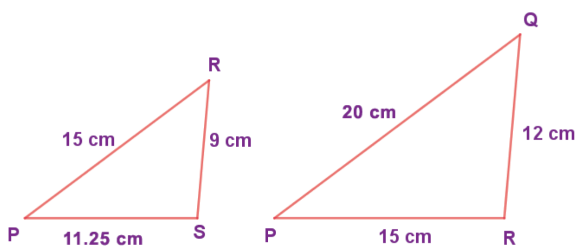
Solution:

Now, $\frac{PQ}{PR} = \frac{20}{15} = \frac{4}{3}$

$\frac{PR}{PS} = \frac{15}{11.25} = \frac{4}{3}$

Also, $\frac{QR}{RS} = \frac{12}{9} = \frac{4}{3}$

We find $\frac{PQ}{PR} = \frac{PR}{PS} = \frac{QR}{RS}$



That is, their corresponding sides are proportional.

$\therefore$ By SSS Similarity, $\Delta PQR \sim \Delta PRS$

Example 4.10 (Illustrating RHS similarity)

The height of a man and his shadow form a triangle similar to that formed by a nearby tree and its shadow. What is the height of the tree?

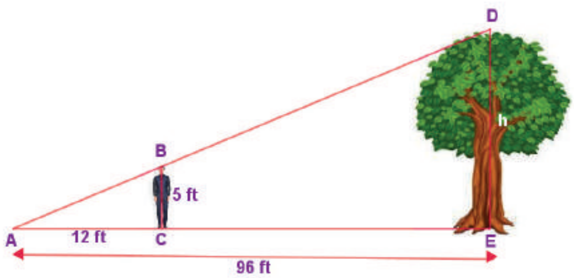


Fig. 4.15

Solution

Here, $\Delta ABC \sim \Delta ADE$ (given)

$\therefore$ Their corresponding sides are proportional (by RHS similarity).

$$\therefore \quad \frac{AC}{AE} = \frac{BC}{DE}$$

$$\Rightarrow \frac{12}{96} = \frac{5}{h}$$

$$\Rightarrow h = \frac{5 \times 96}{12} = 40 \ feet$$

$\therefore$ The height of the tree is 40 *feet.*

R3PP93

4.3 Congruent Triangles

Congruent figures are exactly the same in shape and size. In other words, shapes are congruent if one fits exactly over the other.

Here, two given triangles PQR and ABC are congruent(|||) because PQ=AB, QR=BC and PR=AC. Both triangles match exactly one on the other. This is denoted as $\Delta PQR \equiv \Delta ABC$.

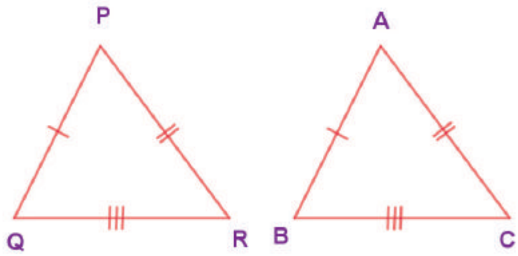


Fig. 4.16

Some examples where congruent triangles are seen and used in real life are :

(i) in the construction of structures like railway bridges to make them strong and stable against strong winds and when under load.

(ii) in buildings where it can protect from the sun by reflecting off opposite triangular faces.

(iii) used in kite making by the children and also in the playground equipment Geodesic dome.

There are 4 ways by which one can prove that two triangles are congruent. They are:

(i) SSS (Side – Side – Side)

(ii) SAS (Side – Angle – Side)

(iii) ASA (Angle – Side –Angle)

(iv) RHS (Right Angle–Hypotenuse– Side)

(i) SSS (Side – Side – Side) Congruence

If the three sides of a triangle are congruent to the three sides of another triangle, then the triangles are congruent. That is $AB = PQ$, $BC = QR$ and $AC = PR$

$$\Rightarrow \Delta\, ABC \equiv \Delta\, PQR.$$

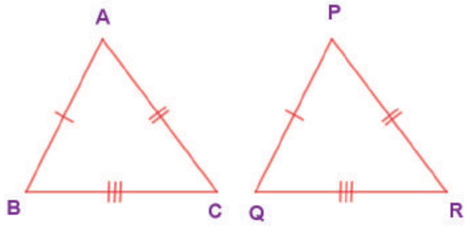


Fig. 4.17

(ii) SAS (Side – Angle – Side) Congruence

If two sides and the included angle (the angle between them) of a triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. Here, $AC = PQ$, $\angle A = \angle P$ and $AB = PR$ and hence $\Delta\, ACB \equiv \Delta\, PQR$.

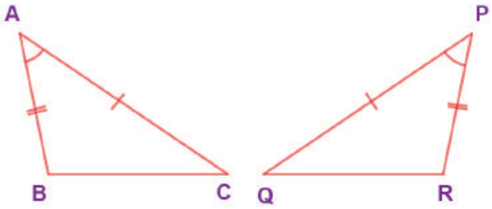


Fig. 4.18

(iii) ASA (Angle-Side-Angle) Congruence

If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. Here, $\angle A = \angle R$, CA=PR and $\angle C = \angle P$ and hence $\Delta\, ABC \equiv \Delta\, RQP$

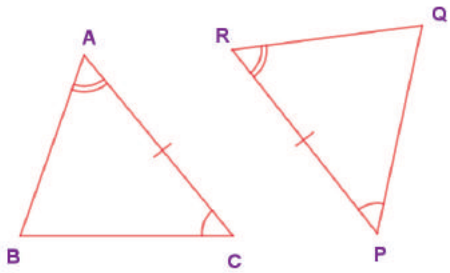


Fig. 4.19

(iv) RHS (Right Angle – Hypotenuse – Side)

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Here, $BC = QR$, $AB = PQ$ and $AC = PR$

(leg) (leg) (hypotenuse)

and hence $\Delta\, ABC \equiv \Delta\, PQR$.

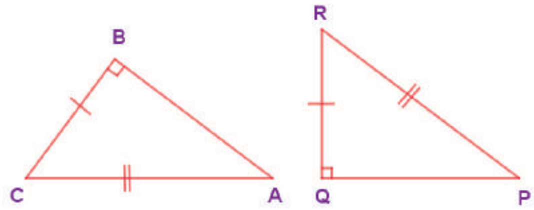


Fig. 4.20

Note

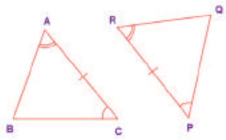
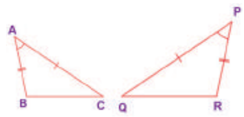
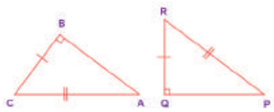
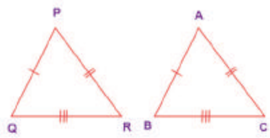
- Any segment or angle is congruent to itself! This is called Reflexive Property
- If two triangles are congruent, then their corresponding parts are congruent. This is called **CPCTC (Corresponding parts of Congruent Triangles are Congruent).**
- *If angles then sides* means if two angles are equal in a triangle, then the sides opposite to them are equal.
- *If sides then angles* means if two sides are equal in a triangle, then the angles opposite to them are equal.

Activity-3

The teacher cuts many triangles that are similar or congruent from a card board (or) chart sheet. The students are asked to find which pair of triangles are similar or congruent based on the measures indicated in the triangles.

Try this

1. Match the following by their congruence property

S.No.	A	B
1		RHS
2		SSS
3		SAS
4		ASA

Example 4.11 (Illustrating SSS and SAS Congruence)

If $\angle E = \angle S$ and G is the midpoint of ES, prove that $\Delta\ GET \equiv \Delta\ GST$.

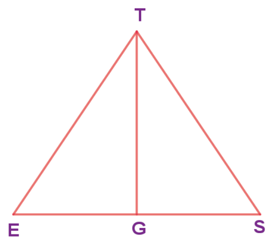
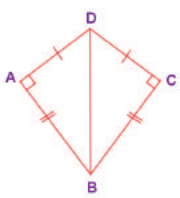


Fig. 4.21

Proof

	Statements	Reasons
1	$\angle E \equiv \angle S$	given
2	$ET \equiv ST$	if angles, then sides
3	G is the midpoint of ES	given
4	$EG \equiv SG$	follows from 3
5	$TG \equiv TG$	reflexive property
6	$\Delta\ GET \equiv \Delta\ GST$	by SSS (2,4,5) & also by SAS (2,1,4)

Think



In the figure, $DA = DC$ and $BA = BC$. Are the triangles DBA and DBC congruent? Why?

Example 4.12 (Illustrating SAS Congruence)

If CW and CT trisect OA and $CO \equiv CA$, prove that $\Delta\, COW \equiv \Delta\, CAT$

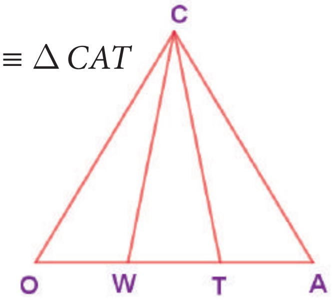


Fig. 4.22

Proof:

	Statements	Reasons
1	$CO \equiv CA$	given
2	$\angle A \equiv \angle O$	if sides, then angles
3	CW and CT trisect OA	given
4	$OW \equiv AT \equiv WT$	trisection definition
5	$\Delta\, COW \equiv \Delta\, CAT$	by SAS (1,2,4)

Example 4.13 (Illustrating AAS and ASA Congruence)

If $\angle YTB \equiv \angle YBT$ and $\angle BOY \equiv \angle TRY$,

prove that $\Delta\, BOY \equiv \Delta\, TRY$

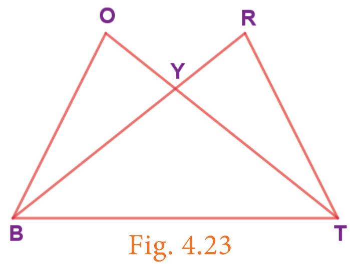


Fig. 4.23

Proof:

	Statements	Reasons
1	$\angle YTB \equiv \angle YBT$	given
2	$BY \equiv TY$	if angles, then sides
3	$\angle BYO \equiv \angle TYR$	vertical angles are congruent
4	$\angle BOY \equiv \angle TRY$	given
5	$\Delta\, BOY \equiv \Delta$ TRY	by AAS (4,3,2)
6	$\angle OBY \equiv \angle RTY$	follows from 3 and 4
7	$\Delta\, BOY \equiv \Delta$ TRY	by ASA (6,2,3)

Example 4.14 (Illustrating RHS Congruence)

If TAP is an isosceles triangle with $TA = TP$ and $\angle TSA = 90°$.

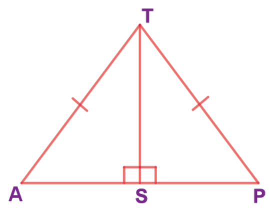


Fig. 4.24

(i) Is $\triangle TAS \equiv \triangle TAP$? Why?

(ii) Is $\angle P = \angle A$? Why?

(iii) Is $AS = PS$? Why?

Proof:

(i) TA = TP (hypotenuse) and $\angle TSA = 90°$.

TS is common (leg)

Hence, by RHS congruence, $\triangle TAS \equiv \triangle TAP$

(ii) Given TA = TP

$\therefore \quad \angle P = \angle A$ (if angles then sides)

(iii) $\angle TSA = 90°$

$\angle P = \angle A$ as TA = TP

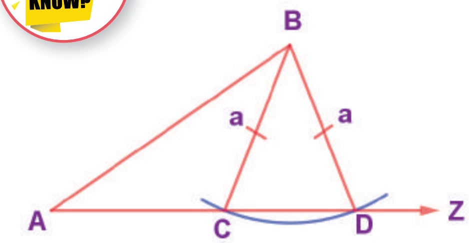
$\therefore$ $\angle ATS = \angle PTS$

$AS = PS$ (if sides then angles)

Note

- If two angles are both congruent and supplementary then, they are right angles.
- All congruent triangles are similar.

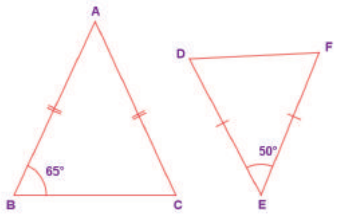
DO YOU KNOW?



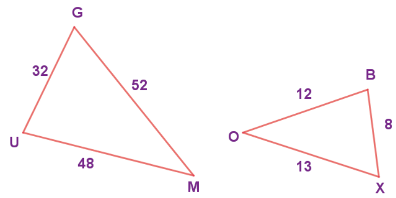
SSA and ASS properties are not sufficient to prove that two triangles are congruent. This is explained in the given figure. By construction, in triangles ABD and ABC, BC = BD = a. Also, AB and $\angle$BAZ are common. But AC $\neq$ AD. So, Δ ABD is not congruent to Δ ABC and so SSA fails.

Exercise 4.1

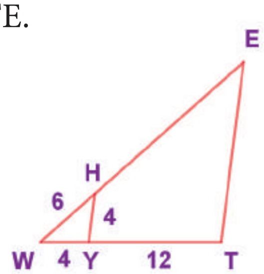
1. From the given figure, prove that $\Delta ABC \sim \Delta DEF$



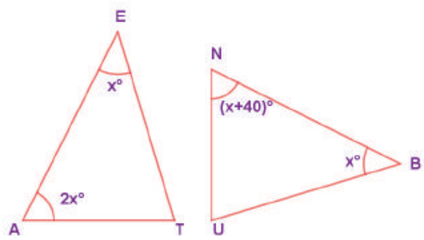
2. In the given figure prove that Δ GUM $\sim$ Δ BOX.



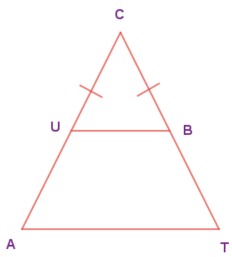
3. In the given figure $YH \parallel TE$. Prove that $\Delta WHY \sim \Delta WET$ and also find HE and TE.



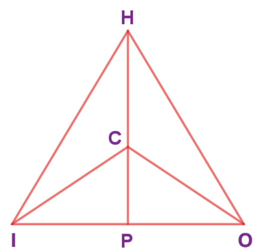
4. In the given figure, if $\Delta EAT \sim \Delta BUN$, find the measure of all angles.



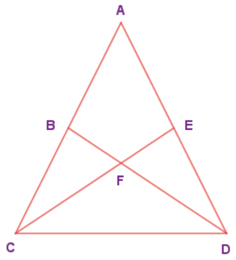
5. In the given figure, $UB \parallel AT$ and $CU \equiv CB$ Prove that $\Delta CUB \sim \Delta CAT$ and hence ΔCAT is isosceles.



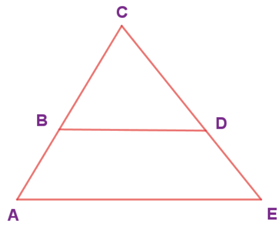
6. In the given figure, $\angle CIP \equiv \angle COP$ and $\angle HIP \equiv \angle HOP$. Prove that $IP \equiv OP$.



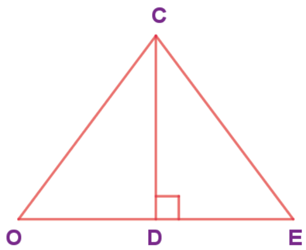
7. In the given figure, $AC \equiv AD$ and $\angle CBD \equiv \angle DEC$. Prove that $\Delta BCF \equiv \Delta EDF$.



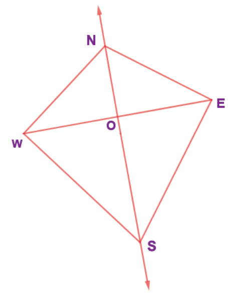
8. In the given figure, ΔBCD is isosceles with base BD and $\angle BAE \equiv \angle DEA$. Prove that $AB \equiv ED$.



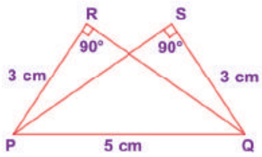
9. In the given figure, D is the midpoint of OE and $\angle CDE = 90°$. Prove that $\Delta ODC \equiv \Delta EDC$



10. In the given figure, if $SW \equiv SE$ and $\angle NWO \equiv \angle NEO$. then, prove that NS bisects WE and $\angle NOW = 90°$



11. Is $\triangle PRQ \equiv \triangle QSP$? Why?



12. Fill in the blanks with the correct term from the given list.
(in proportion, similar, corresponding, congruent, shape, area, equal)
   (i) Corresponding sides of similar triangles are ________.
   (ii) Similar triangles have the same __________ but not necessarily the same size.
   (iii) In similar triangles, _______ sides are opposite to equal angles.
   (iv) The symbol $\equiv$ is used to represent ________ triangles.
   (v) The symbol $\sim$ is used to represent _________ triangles.

Objective Type Questions

13. Two similar triangles will always have _________angles
(A) acute (B) obtuse (C) right (D) matching

14. If in triangles PQR and XYZ, $\frac{PQ}{XY} = \frac{QR}{ZX}$ then they will be similar if
(A) $\angle Q = \angle Y$ (B) $\angle P = \angle X$ (C) $\angle Q = \angle X$ (D) $\angle P = \angle Z$

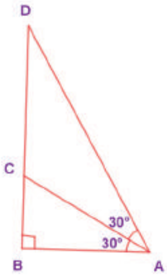
15. A flag pole 15 m high casts a shadow of 3 m at 10 a.m. The shadow cast by a building at the same time is 18.6 m. The height of the building is
(A) 90 m (B) 91 m (C) 92 m (D) 93 m

16. If $\Delta ABC \sim \Delta PQR$ in which $\angle A = 53^\circ$ and $\angle Q = 77^\circ$, then $\angle R$ is

(A) 50° (B) 60° (C) 70° (D) 80°

17. In the figure, which of the following statements is true?

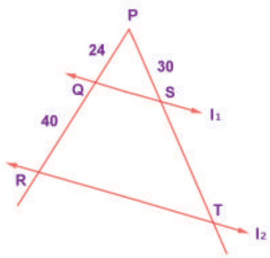
(A) $AB = BD$ (B) $BD < CD$ (C) $AC = CD$ (D) $BC = CD$



Exercise 4.2

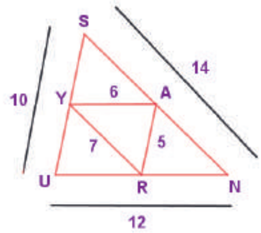
Miscellaneous Practice Problems

1. In the given figure, find PT given that $l_1 \parallel l_2$.

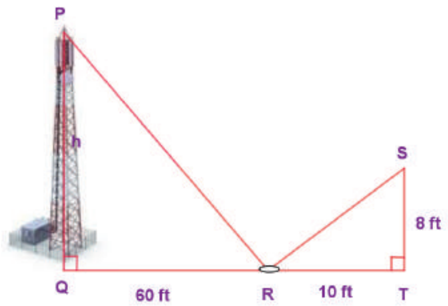


X214L1

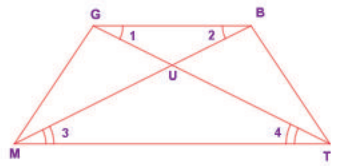
2. From the figure, prove that $\Delta\ SUN \sim \Delta\ RAY$.



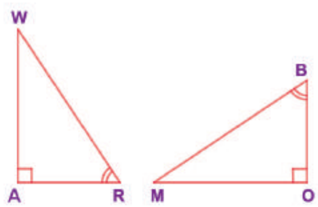
3. The height of a tower is measured by a mirror on the ground at R by which the top of the tower's reflection is seen. Find the height of the tower.



4. In the figure, given that $\angle 1 = \angle 2$ and $\angle 3 \equiv \angle 4$.
Prove that $\Delta\ MUG \equiv \Delta\ TUB$.

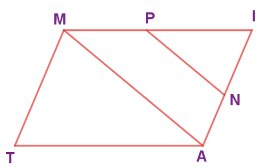


5. If $\Delta WAR \equiv \Delta MOB$, name the additional pair of corresponding parts. Name the criterion used by you.

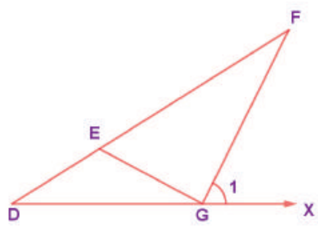


Challenging problems

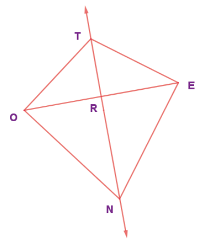
6. In the figure, $\angle TMA \equiv \angle IAM$ and $\angle TAM \equiv \angle IMA$. P is the midpoint of MI and N is the midpoint of AI. Prove that $\Delta\ PIN \sim \Delta\ ATM$.



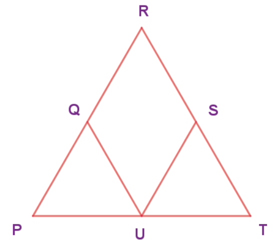
7. In the figure, if $\angle FEG \equiv \angle 1$ then, prove that $DG^2 = DE.DF$.



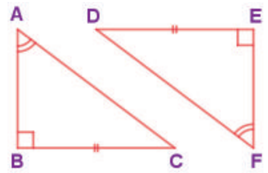
8. In the figure, $\angle TEN \equiv \angle TON = 90^\circ$ and $TO \equiv TE$. Prove that $\angle ORN \equiv \angle ERN$.



9. In the figure, $PQ \equiv TS$, Q is midpoint of PR, S is the midpoint TR and $\angle PQU \equiv \angle TSU$. Prove that $QU \equiv SU$.



10. In the figure $\triangle TOP \equiv \triangle ARM$. Explain why?



4.4 Construction of Quadrilaterals

We have already learnt how to draw triangles in the earlier classes. A polygon that has got 3 sides is a triangle. To draw a triangle, we need 3 independent measures. Also, there is only one way to construct a triangle, given its 3 sides. For example, to construct a triangle with sides 3*cm*, 5*cm* and 7*cm*, there is only one way to do it.

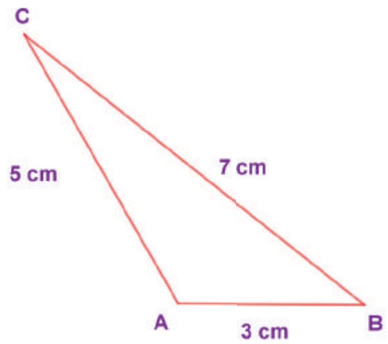


Fig. 4.25

Now, let us move on to quadrilaterals. A polygon that is formed by 4 sides is called a quadrilateral. Isn't it? But, a quadrilateral can be of different shapes. They need not look like the same for the given 4 measures. For example, some of the quadrilaterals having their sides as 4 *cm*, 5 *cm*, 7 *cm* and 9 *cm* are given below.

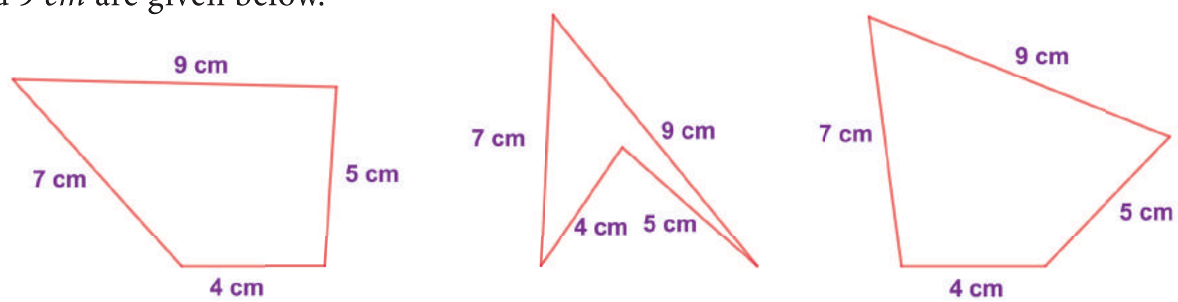


Fig. 4.26

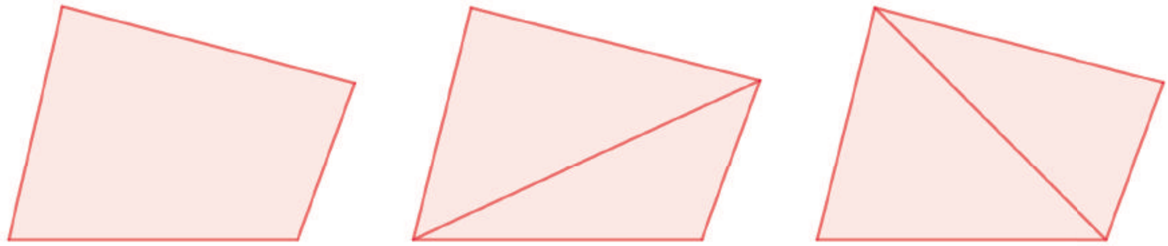
So, to construct a particular quadrilateral, we need a 5^{th} measure. That can be its diagonal or an angle measure. Moreover, even if 2 or 3 sides are given, using the measures of the diagonals and angles, we can construct quadrilaterals.

We shall now see a few types on constructing these quadrilaterals when its:

(i) 4 sides and a diagonal are given.	(ii) 3 sides and 2 diagonals are given.
(iii) 4 sides and an angle are given.	(iv) 3 sides and 2 angles are given.
(v) 2 sides and 3 angles are given.	

DO YOU KNOW?

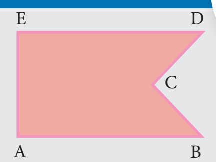
We can split any quadrilateral into two triangles by drawing a diagonal.



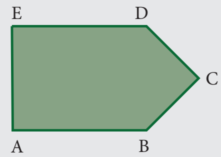
In the above figures, a quadrilateral is split in two ways by its diagonals. So, if a diagonal is given, first draw the lower triangle with two sides and one diagonal. Then, draw the upper triangle with other two measures.

Note

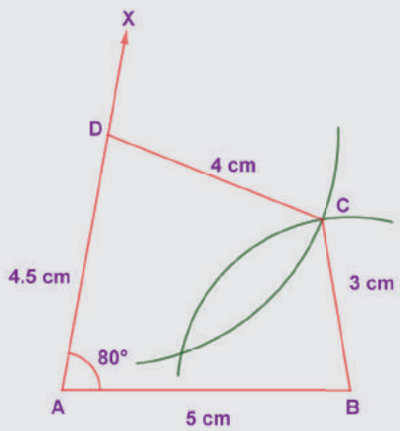
(i) • A polygon in which atleast one interior angle is more than 180°, is called a concave polygon. In the given polygon, interior angle at C is more than 180°.



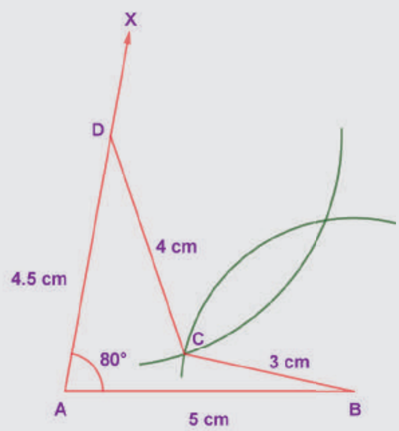
• A polygon in which each interior angle is less than 180°, is called a convex polygon. In the given polygon, all interior angles are less than 180°.



(ii) Look at the following quadrilaterals.



Convex quadrilateral



Concave quadrilateral

Though, we can construct a quadrilateral in two ways as shown above, we do not take into account the concave quadrilaterals in this chapter. Hence, the construction of only convex quadrilaterals are treated here.

Think

Is it possible to construct a quadrilateral $PQRS$ with $PQ = 5$ cm, $QR = 3$ cm, $RS = 6$ cm, $PS = 7$ cm and $PR = 10$ cm. If not, why?

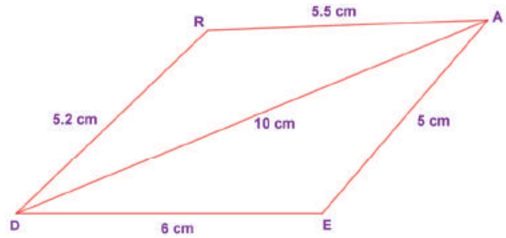
4.4.1 Constructing a quadrilateral when its 4 sides and a diagonal are given

Example 4.15

Construct a quadrilateral $DEAR$ with $DE=6\ cm$, $EA = 5\ cm$, $AR = 5.5$cm, $RD = 5.2\ cm$ and $DA = 10\ cm$. Also find its area.

Solution:

Given:
$DE = 6\ cm$, $EA = 5\ cm$, $AR = 5.5\ cm$, $RD = 5.2\ cm$ and a diagonal $DA = 10\ cm$



Rough diagram

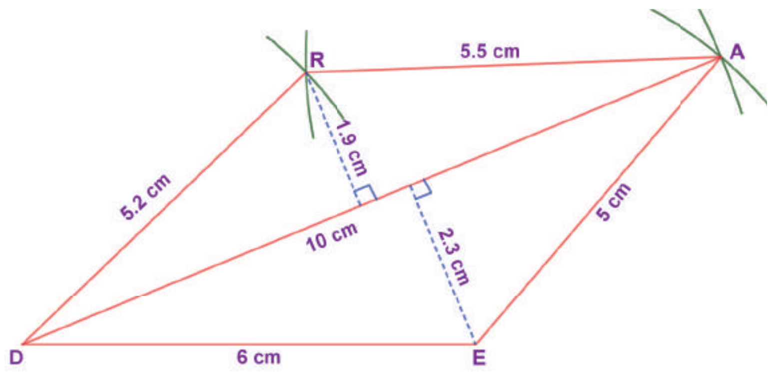


Fig. 4.27

Steps:

1. Draw a line segment $DE = 6\ cm$.
2. With D and E as centres, draw arcs of radii 10 cm and 5 cm respectively and let them cut at A.
3. Join DA and EA.
4. With D and A as centres, draw arcs of radii 5.2 cm and 5.5 cm respectively and let them cut at R.
5. Join DR and AR .
6. $DEAR$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $DEAR$ $= \frac{1}{2}\times d\times(h_1+h_2)$ *sq. units*

$= \frac{1}{2}\times 10\times(1.9+2.3)$

$= 5 \times 4.2 = 21\ cm^2$

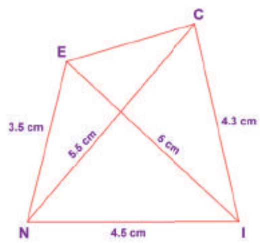
4.4.2 Construct a quadrilateral when its 3 sides and 2 diagonals are given

Example 4.16

Construct a quadrilateral *NICE* with NI=4.5 *cm*, IC= 4.3 *cm*, NE= 3.5 *cm*, NC= 5.5 *cm* and IE = 5 *cm*. Also find its area.

Solution:

Given:
NI = 4.5 *cm*, IC= 4.3 *cm*,
NE = 3.5 *cm* and two diagonals,
NC = 5.5 *cm* and IE = 5 *cm*



Rough diagram

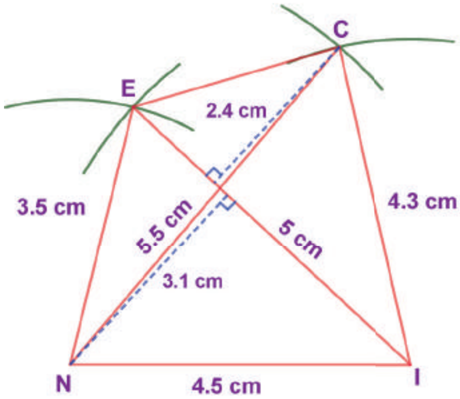


Fig. 4.28

Steps:

1. Draw a line segment NI = 4.5 *cm*.
2. With N and I as centres, draw arcs of radii 5.5 *cm* and 4.3 *cm* respectively and let them cut at C.
3. Join NC and IC.
4. With N and I as centres, draw arcs of radii 3.5 *cm* and 5 *cm* respectively and let them cut at E.
5. Join NE, IE and CE .
6. *NICE* is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral *NICE* $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 5 \times (2.4 + 3.1)$

$= 2.5 \times 5.5 = 13.75\ cm^2$

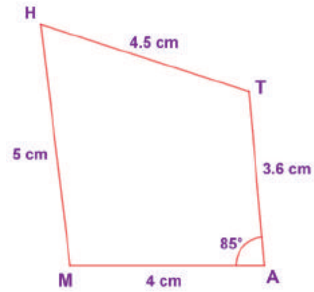
4.4.3 Construct a quadrilateral when its 4 sides and one angle are given

Example 4.17

Construct a quadrilateral $MATH$ with MA=4 cm, AT= 3.6 cm, TH = 4.5 cm, MH= 5 cm and $\angle A = 85°$. Also find its area.

Solution:

Given:
MA=4 cm, AT= 3.6 cm,
TH = 4.5 cm, MH= 5 cm and $\angle A = 85°$



Rough diagram

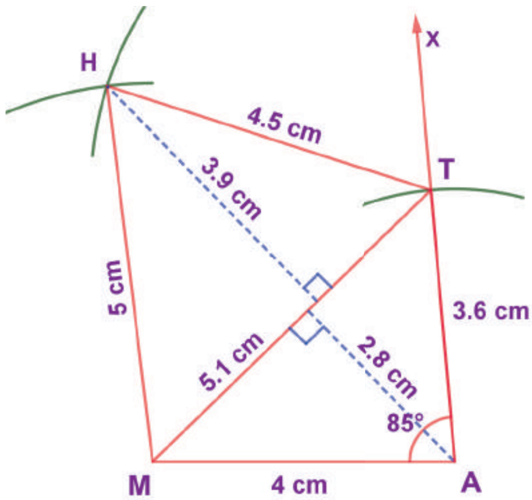


Fig. 4.29

Steps:

1. Draw a line segment MA = 4 cm.
2. Make $\angle A = 85°$.
3. With A as centre, draw an arc of radius 3.6 cm. Let it cut the ray AX at T.
4. With M and T as centres, draw arcs of radii 5 cm and 4.5 cm respectively and let them cut at H.
5. Join MH and TH.
6. $MATH$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $MATH$ $= \frac{1}{2} \times d \times (h_1 + h_2)$ $sq.\ units$

$= \frac{1}{2} \times 5.1 \times (3.9 + 2.8)$

= 2.55 × 6.7 = 17.09 cm^2

4.4.4 Construct a quadrilateral when its 3 sides and 2 angles are given

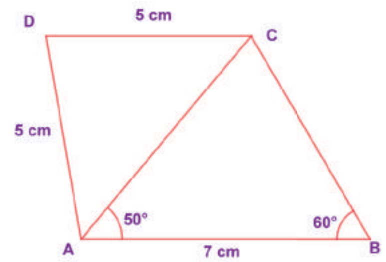
Example 4.18

Construct a quadrilateral $ABCD$ with $AB=7\ cm$, $AD= 5\ cm$, $CD = 5\ cm$, $\angle BAC = 50°$ and $\angle ABC = 60°$. Also find its area.

Solution:

Given:

$AB=7\ cm$, $AD= 5\ cm$, $CD = 5\ cm$ and two angles $\angle BAC = 50°$ and $\angle ABC = 60°$



Rough diagram

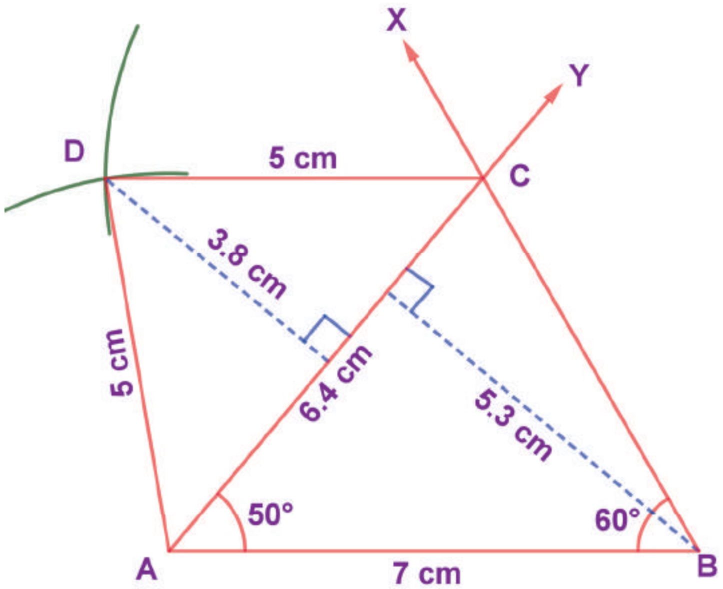


Fig. 4.30

Steps:

1. Draw a line segment $AB = 7\ cm$.
2. At A on AB, make $\angle BAY = 50°$ and at B on AB, make $\angle ABX = 60°$. Let them intersect at C.
3. With A and C as centres, draw arcs of radius 5 cm. each. Let them intersect at D.
4. Join AD and CD.
5. $ABCD$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $ABCD$ $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 6.4 \times (3.8 + 5.3)$

$= 3.2 \times 9.1 = 29.12\ cm^2$

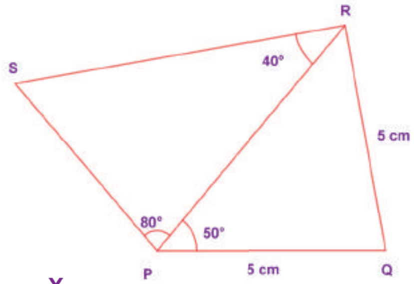
4.4.5 Construct a quadrilateral when its 2 sides and 3 angles are given

Example 4.19

Construct a quadrilateral $PQRS$ with $PQ=QR=5\ cm$, $\angle QPR = 50^\circ$, $\angle PRS = 40^\circ$ and $\angle RPS = 80^\circ$. Also find its area.

Solution:

Given:
$PQ=5\ cm$, $QR=5\ cm$, $\angle QPR = 50^\circ$,
$\angle PRS = 40^\circ$ and $\angle RPS = 80^\circ$



Rough diagram

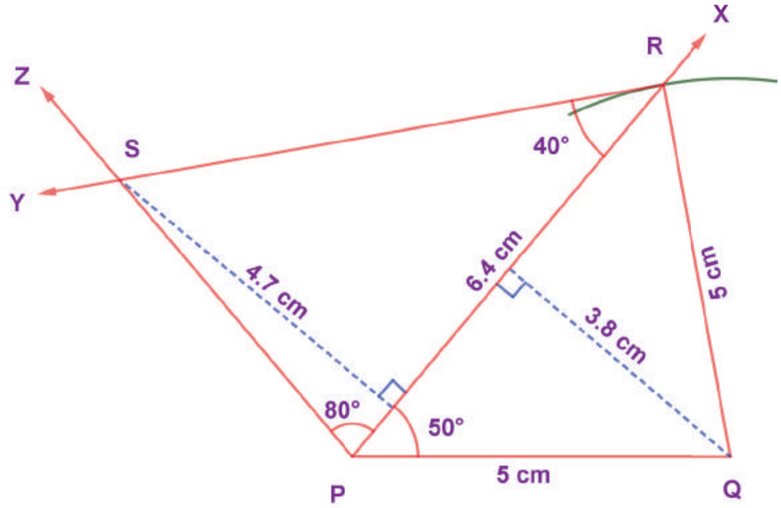


Fig. 4.31

Steps:

1. Draw a line segment $PQ = 5\ cm$.
2. At P on PQ, make $\angle QPX = 50^\circ$.
3. With Q as centre, draw an arc of radius 5 cm. Let it cut PX at R.
4. At R on PR, make $\angle PRS = 40^\circ$ and at P on PR, make $\angle RPS = 80^\circ$. Let them intersect at S.
5. $PQRS$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $PQRS$ $= \frac{1}{2} \times d \times (h_1 + h_2)\ sq.\ units$

$= \frac{1}{2} \times 6.4 \times (4.7 + 3.8)$

$= 3.2 \times 8.5 = 27.2\ cm^2$

Exercise 4.3

Construct the following quadrilaterals with the given measurements and also find their area.

1. ABCD, AB = 5 cm, BC = 4.5 cm, CD = 3.8 cm, DA = 4.4 cm and AC= 6.2 cm.
2. KITE, KI= 5.4 cm, IT = 4.6 cm, TE= 4.5 cm, KE = 4.8 cm and IE = 6 cm.
3. PLAY, PL= 7 cm, LA = 6 cm, AY= 6 cm, PA = 8 cm and LY = 7 cm.
4. LIKE, LI = 4.2 cm, IK = 7 cm, KE= 5 cm, LK = 6 cm and IE = 8 cm.
5. PQRS, PQ=QR= 3.5 cm, RS= 5.2 cm, SP = 5.3 cm and $\angle Q = 120^{\circ}$.
6. EASY, EA= 6 cm, AS = 4 cm, SY = 5 cm, EY = 4.5 cm and ∠E = 90°.
7. MIND, MI =3.6 cm, ND = 4 cm, MD= 4 cm, ∠M = 50° and ∠D = 100°.
8. WORK, WO= 9 cm, OR = 6 cm, RK = 5 cm, ∠O = 100° and ∠R = 60°.
9. AGRI, AG= 4.5 cm, GR = 3.8 cm, ∠A = 60°, ∠G = 110° and ∠R = 90°.
10. YOGA, YO = 6 cm, OG = 6 cm, ∠O = 55°, ∠G = 35° and ∠A = 100°.

ICT CORNER

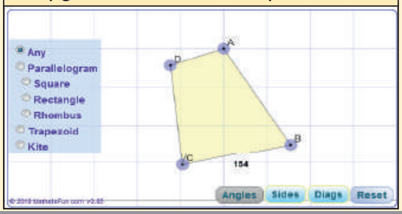
Step-1 Open the Browser and type the URL given below.

Step-2 You can see Interactive Quadrilaterals games.

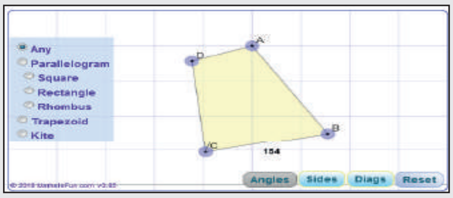
Step-3 You can see the topics like Quadrilaterals, Parallelograms, Squares, Rectangles etc... You can see Angles, Sides, Diagonals and Reset options too.

Step-4 For example, click on the Quadrilaterals, you can see the diagrams of it. Likewise you can view the details of the other topics.

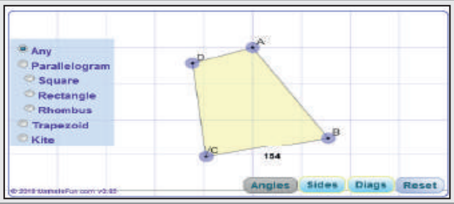
Through this activity, you will know about the Interactive Quadrilaterals, Interactive Polygons and Geometry Index .



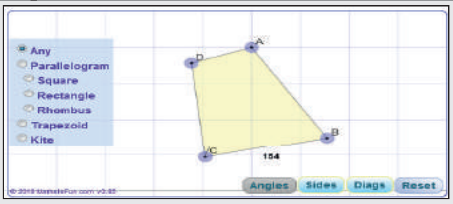
Step 1



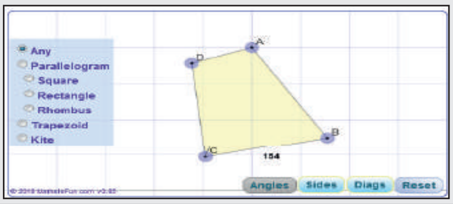
Step 2



Step 3



Step 4



Web URL practical geometry (quardilateral links)
https://www.mathsisfun.com/geometry/quadrilaterals-interactive.html
*Pictures are indicatives only.
*If browser requires, allow Flash Player or Java Script to load the page.

B358_8_MATHS_EM

Summary

- Similar figures have the same shape but different sizes.
- Two triangles are similar if two angles of one triangle are equal respectively to two angles of the other triangle. This is AA Similarity.
- Two triangles are similar if two sides of one triangle are proportional to two sides of the other triangle and the included angles are equal. This is SAS Similarity.
- Two triangles are similar if the corresponding sides of two triangles are in the same ratio. This is SSS Similarity.
- Two right triangles are similar if the hypotenuse and a leg of one triangle are respectively proportional to the hypotenuse and a leg of the other triangle. This is RHS Similarity.
- If two triangles are similar, then the corresponding angles are respectively equal and their corresponding sides are proportional.
- Congruent figures are exactly the same in shape and size.
- There are four ways to prove that two triangles are congruent. They are SSS, SAS, ASA and RHS.
- If the three sides of a triangle are congruent to the three sides of another triangle, then the triangles are congruent. This is SSS Congruence.
- If two sides and the included angle (the angle between them) of a triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. This is SAS Congruence.
- If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. This is ASA Congruence.
- If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. This is RHS Congruence.
- If two triangles are congruent, then their corresponding parts are congruent. This is called CPCTC (Corresponding parts of Congruent Triangles are Congruent).
- To construct a quadrilateral, 5 independent measures are needed.
- The 5 independent measures can be a combination of sides, diagonals and angles of a quadrilateral.
- Area of a quadrilateral = $\frac{1}{2} \times d \times (h_1 + h_2)$.

5 INFORMATION PROCESSING

Learning Outcomes

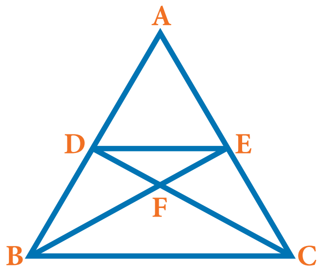
- To determine the number of possible orderings of an arbitrary number of objects following some procedures.
- To reinforce the idea of sameness and distinctness of the SET game .
- To investigate the role of map colouring in representing and modelling of mathematical ideas and problem solving through graph colouring.

Recap

Answer the following questions, recalling how to write a list of all the possibilities and then to count them, which you have studied in the earlier classes.

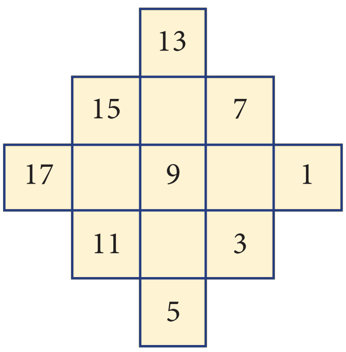
1. Find the number of all possible triangles that can be formed from the triangle given below.

3ZBIXY



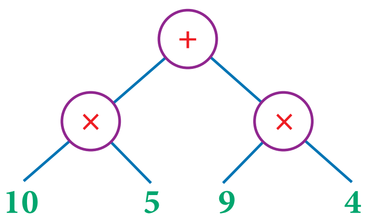
Ans:______________

2. Use the numbers given in the figure to form a 3 x 3 magic square.



Ans:

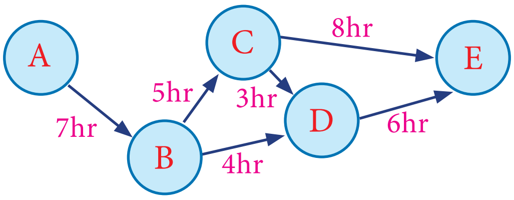
3. Convert the tree diagram into a numeric expression.



Ans:______________

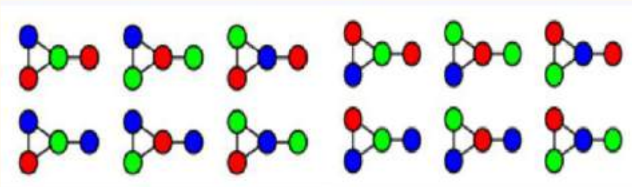
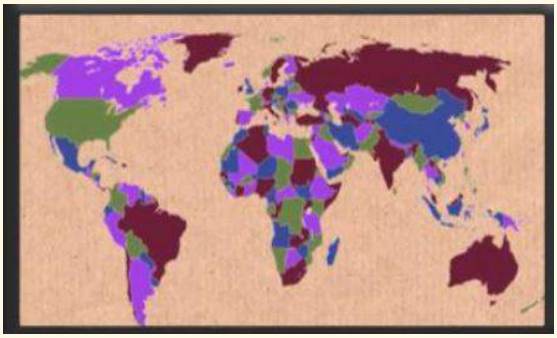
4. (i) Find the total time taken by the bus to reach from A to E via B , C and D.

(ii) Find which is the shortest route from A to E.



(i) ....hrs.

(ii) A → ... → ... → E

MATHEMATICS ALIVE	INFORMATION PROCESSING IN REAL LIFE
 A pattern which uses only 3 colours in 12 different ways	 The world map with minimum colours.

5.1 Introduction

Success in mathematics begins with the development of the number sense through counting and quantity.

There are some basic counting techniques which will be useful in determining the number through different ways of arranging or selecting objects. The basic counting principles are given below.

5.2 Principles of counting

5.2.1 Addition principle

If there are two operations such that they can be done independently in **m** ways and **n** ways respectively, then either of the two operations can be done in (**m** + **n**) ways.

Let us learn about this addition principle of counting as given below :

Situation 1

In class VIII, there are 11 boys and 9 girls. The teacher wants to select either a boy or a girl as the class leader. Let us see, in how many ways can the teacher select the class leader.

Fig. 5.1

The teacher can select the class leader in any one of the following ways.

(i) In the first choice, the teacher can select **a boy among 11 boys in 11 ways** (who ever may be of the 11 boys).

(ii) In the second choice, the teacher can select **a girl among 9 girls in 9 ways** (who ever may be of the 9 girls).

Hence, the teacher can select the class leader who is **a boy** or **a girl** in **20 different ways (11 boys + 9 girls).**

Situation 2

Consider, if you are going to a hotel to have food and the hotel offers different food items as shown in pictures below . Let us find how many ways are possible to have food.

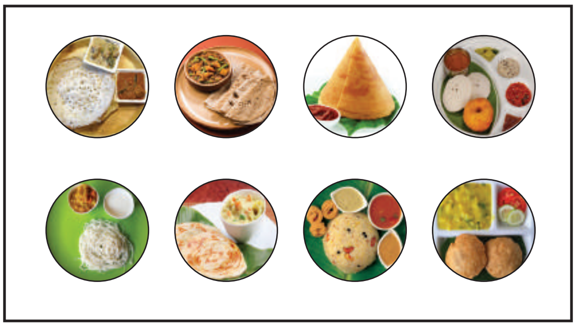


Fig. 5.2

Fig. 5.3

For tiffin, you have 8 choices and for meals, you have 9 choices.

Therefore, there are $8 + 9 = 17$ different ways by which you can choose your lunch.

Thus, we come to know, if an operation A can occur in m ways and another operation B can occur in n ways, and suppose that both cannot occur together, then A or B can occur in $(m + n)$ ways. Let us see some more examples.

Example 5.1

How many possible outcomes are there in tossing a coin  ?

Solution:

Fig. 5.4

There are 2 different outcomes in tossing a coin namely **a Head and a Tail**. Both events cannot occur simultaneously.

Therefore, **Head and Tail** are 2 different possible outcomes in tossing a coin.

Example 5.2

What are the possible outcomes if a fair die is  rolled ?

Solution:

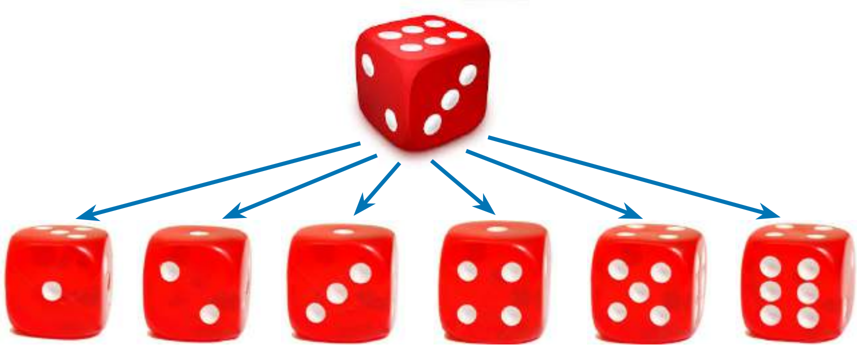


Fig. 5.5

There are 6 different outcomes, namely **1, 2, 3, 4, 5 and 6.** All the 6 outcomes cannot occur simultaneously. Hence, there are 6 different possible outcomes in rolling a die.

Example 5.3

The teacher has trained 11 boys and 6 girls and wants to select one of them to participate in the Inter School Quiz competition. In how many ways can the teacher make this selection?

Fig. 5.6

Solution:

The teacher can select one of them in any one of the following ways.

The first way of selecting a **boy among 11 boys** can be done in **11 ways** and the second way of selecting a **girl among 6 girls** can be done in **6 ways**.

Therefore, the teacher can make the selection of either a boy or a girl in **17(11 +6)** different ways.

5.2.2 Multiplication principle

If an operation can be performed in m ways, following which another operation can be performed in n ways , and both the operations are dependent on each other then, the two operations can be performed in exactly ($m \times n$) different ways.

Now, we shall learn about multiplication principle of counting.

Situation 1

There are 3 places in a city namely A,B and C. There are 3 routes a1, a2 and a3 from A to B. There are 2 different routes, b1 and b2 from B to C as shown in the picture given below.

Suppose a person wants to travel from A to C via B. Lets us see, the number of ways he can go from place A to C via B.

(i) In the first operation, he can go from **A to B in 3 routes** and

(ii) In the Second operation, he can go from **B to C in 2 different routes.**

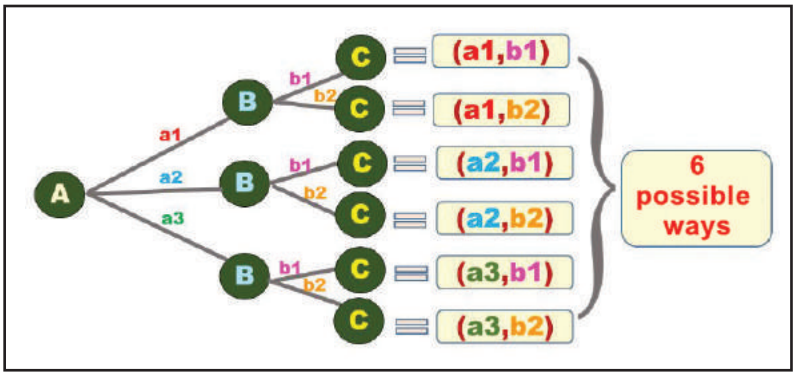


Fig. 5.7

Therefore, the total number of ways in which he can travel is **3 × 2 = 6** routes as shown in fig 5.7.

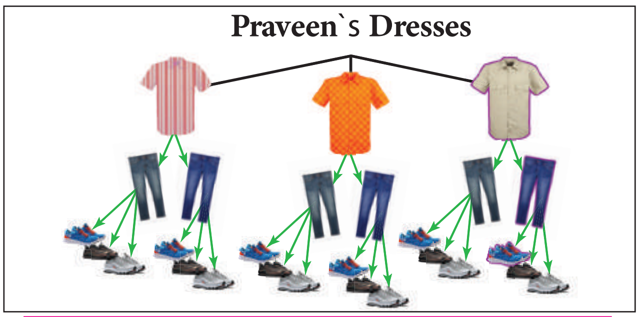
Situation 2

Praveen bought 3 shirts , 2 jeans and 3 pairs of shoes for his birthday. The picture given below shows the different ways of wearing the dress. Let us find, in how many different ways can Praveen wear a dress on his birthday.

Fig. 5.8

Here, Praveen has 3 shirts  , 2 jeans  and  3 pairs of shoes.

He can wear a dress either this way  or he can have the choices as shown in the Fig. 5.9.



3 shirts × 2 jeans × 3 pairs of shoes = 18 ways

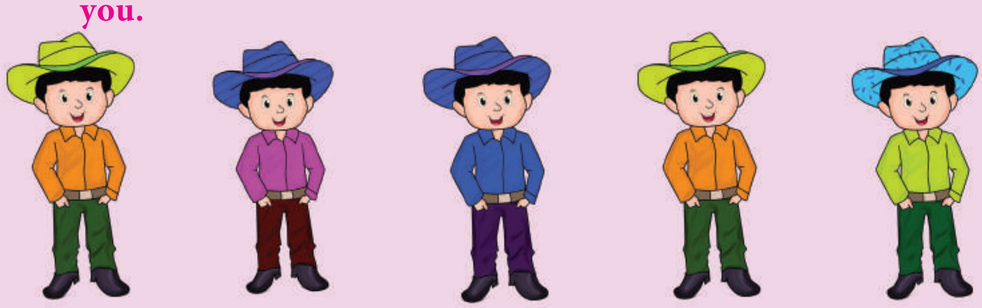
Fig. 5.9

Therefore, Praveen can wear his dress in **18 (3×2×3)** different possible ways on his birthday.

Try this

Kumar's Hats

Kumar has to use four different hats as given above. He always wears a hat on weekdays for the whole day. Sometimes he wears the same hat more than once in a week as in the figure. In how many different ways can he wear his hats in one week? One answer is given for you.



Example 5.4

In how many ways, can the students answer 3 questions which are true or false type in a slip test?

Solution:

Assuming that the question Q_1 is answered True, questions Q_2 and Q_3 can be answered as TT, TF, FT and FF in 4 ways.

Similarly, assuming that the question Q_2 is answered False, Q_2 and Q_3 can also be answered as TT, TF, FT, and FF in 4 ways.

Thus, as each question has only two options (True or False), the number of ways of answering these 3 questions in a slip test is **2x2x2 = 8** possible ways.

Number of ways of completing the questions

	Questions		
	Q_1	Q_2	Q_3
Answer1	**T**	**T**	**T**
Answer2	**T**	**T**	**F**
Answer3	**T**	**F**	**T**
Answer4	**T**	**F**	**F**
Answer5	**F**	**T**	**T**
Answer6	**F**	**T**	**F**
Answer7	**F**	**F**	**T**
Answer8	**F**	**F**	**F**

Fig. 5.10

Example 5.5

In class VIII, a math club has four members M,A,T and H. Find the number of different ways, the club can elect

(i) a leader,

(ii) a leader and an assistant leader.

Solution:

(i) To elect a leader

In class VIII, a math club has four members namely M, A, T and H,

Therefore, there are 4 different ways by which they can be elected a leader.

(ii) To elect a leader and an assistant leader

In the Fig. 5.11, the pink shaded boxes show that same member comes twice. As, one person cannot have two leadership. Therefore, the red shaded boxes cannot be counted. There are only 12 different ways (either shown in yellow boxes or green boxes) to choose a leader and an assistant leader for a math club.

		ASSISTANT LEADER			
		M	A	T	H
LEADER	M	MM	MA	MT	MH
	A	AM	A A	AT	AH
	T	TM	T A	TT	TH
	H	HM	H A	HT	HH

Fig. 5.11

DO YOU KNOW?

A password using 6 characters is created where the first 2 characters are any of the alphabets, the third character is a special character like @, #, $, %, &, _,+,~, * and - and the last 3 characters are any of the numbers from 0 to 9. For that, there are $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 67{,}60{,}000$ number of different ways possible to create that password.

Activity-1

1. Determine the number of two digit numbers that can be formed using the digits 1, 3 and 5 with repetition of digits allowed.

The activity consists of two parts

(i) Choose a ones digit.

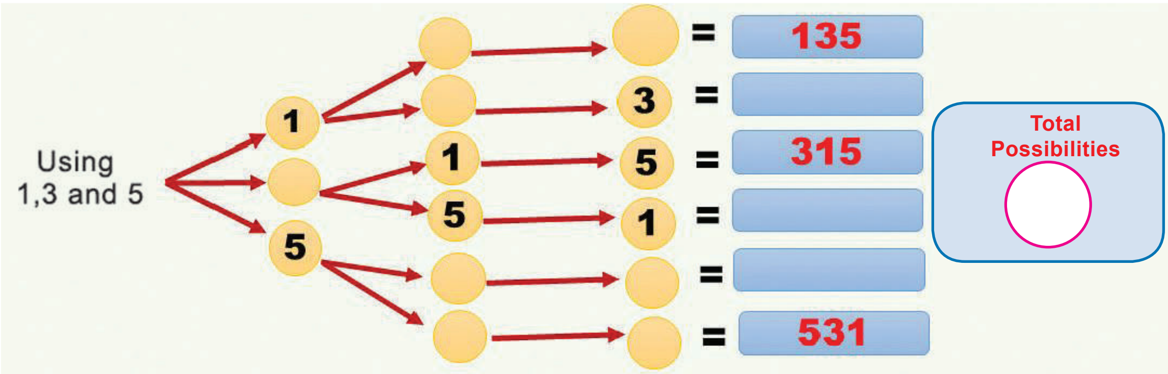
(ii) Choose a tens digit.

Complete the table given beside

		Ones Digit		
		1	3	5
Tens Digit	1			15
	3		33	
	5	51		

2. Find the number of three digit numbers that can be formed using the digits 1, 3 and 5 without repetition of digits.

Complete the tree diagram given below to the numbers.



Example 5.6

Madhan wants to a buy a new car. The following choices are available for him.

- There are 2 types of cars as shown in the Fig. 5.12
- There are 5 colours available in each type as shown in Fig. 5.12.
- There are 3 models available in each colour

 (i) GL (standard model)

 (ii) SS (sports model)

 (iii) SL (luxury model)

Fig. 5.12

(i) In how many different ways can Madhan buy any one of the new car?

(ii) If the white colour is not available in Type 2, then in how many ways can Madan buy a new car among in given option?

Solution:

(i) Here, we have **2 types** of car with **5 different colours** and **3 models** in each colour.

Fig. 5.13

The Total number of different ways to buy a new car by Madhan $= \mathbf{2 \times 5 \times 3 = 30.}$

(You can also find the number of ways from the figure shown above).

(ii) If the white colour is not available in Type 2, then...

Fig. 5.14

For **Type 1,** we have 5 colours and 3 models and hence there are **$5 \times 3 = 15$ choices.**

For **Type 2,** we have only 4 colours and 3 models and hence there are **$4 \times 3 = 12$ choices.**

Therefore, the total number of different ways **$15 + 12 = 27$ ways.**

The above example illustrates both the addition and multiplication principles.

Exercise 5.1

I. Choose the best answer.

1. How many outcomes can you get when you toss three coins once?

   (A) 6 (B) 8 (C) 3 (D) 2

2. In how many ways can you answer 3 multiple choice questions, with the choices A,B,C and D ?

   (A) 4 (B) 3 (C) 12 (D) 64

3. How many 2 digit numbers contain the number 7 ?

   (A) 10 (B) 18 (C) 19 (D) 20

II. Answer the following.

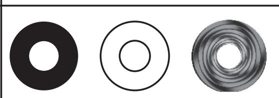
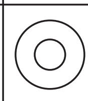
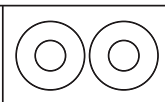
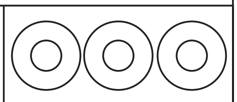
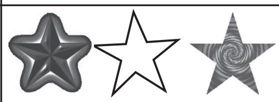
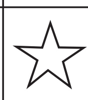
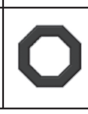
1. You are going to have an ice cream or a cake. There are three flavours (chocolate, strawberry, vanilla) in ice creams, and two flavours (orange or red velvet) in the cake. In how many possible ways can you choose an ice cream or the cake?
2. In how many ways, can the teacher choose 3 students in all, one each from 10 students in VI std, 15 students in VII std and 20 students in VIII std to go to an excursion ?
3. If you have 2 school bags and 3 water bottles then, in how many different ways can you carry both a school bag and a water bottle, while going to school ?
4. Roll numbers are created with a letter followed by 3 digits in it, from the letters A, B, C, D and E and any 3 digits from 0 to 9. In how many possible ways can the roll numbers be generated ?
5. A safety locker in a jewel shop requires a 4 digit unique code. The code has the digits from 0 to 9. How many unique codes are possible ?

5.3 SET - Game

Any game that uses features can be used to stimulate logical thinking and it provides an interesting and challenging context for exploring ideas in discrete mathematics.

A SET game proves to be an excellent extension for activities that involve organizing the objects by attributes. The SET game builds the cognitive, logical, spatial reasoning as well as visual perception skills.

The SET game is a puzzle that uses cards which have four features on them. They are shapes, colours, shades and the number of shapes as given below.

Shape	Colour	Shade	Number		
CIRCLE (◎)					
STAR (☆)					
HEXAGON (O)					

A set consists of three cards should satisfy all the 4 following conditions:

(i) All the three cards have the **same shape** or have **three different shapes**.

(ii) All the three cards have the **same colour** or have **three different colours**.

(iii) All the three cards have the **same shade** or have **three different shades**.

(iv) All the three cards have the **same number** or have **three different numbers**.

One card can be chosen from 3 different colours in 3 ways and since each colour has 3 different shapes, totally 9 cards can be chosen. Each of these 9 cards have 3 different shades and hence totally we will have **$9 \times 3 = 27$** cards. Also, 27 cards can be paired 3 with different shapes. Now, we will have **$27 \times 3 = 81$** cards in the deck.

Situation 1

The teacher displays the deck of cards and gives students 30 seconds to search and asks them to form a **SET.** Now, the teacher lays down **9 cards** as shown in Fig. 5.15 and explains using 2 cards  and  taken from them. Now, follow the step by step procedure to figure out the third card to complete this **SET** is as follows**.**

Fig. 5.15

Remember, a SET consists of 3 cards.

If you look at the **shape** then, **one is star** and the other one is also **star**. These two cards have the **same shape**. So, the last card also should have the **same shape**. []

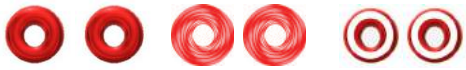
If you look at the **colour** then , **other one is green**  and the **other is red**  These two cards have **different colours**. So, the last card also should have a **different colour** that is **blue.** 

If you look at the **shade** then, **one is solid** and the other one is also **solid**. These two cards have the **same shade**. So, the last card also should have the **same shade**. []

If you look at the **number** then, one card has **one** and the other card has **two**. These two cards have **different numbers.** So, the last card also should have a **different number three**. []

Now, this completes the rules for a SET.

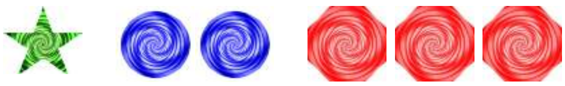
Now, the teacher asks the students to find two more **SET**s from Fig.5.15. Let us check the **SET** again.

1. 

Shape : All same ✓ Colour : All same ✓

Shade : All different ✓ Number : All same ✓

Yes, This is a SET .

2. 

Shape : All different ✓ Colour : All different ✓

Shade : All same ✓ Number : All different ✓

Yes, This is a SET.

Again , the teacher makes a **SET** and asks them to check **whether, it is a SET**?

Shape : All same or all different ✕ Colour : All same or all different ✕

Shade : All same or all different ✕ Number : All same ✓

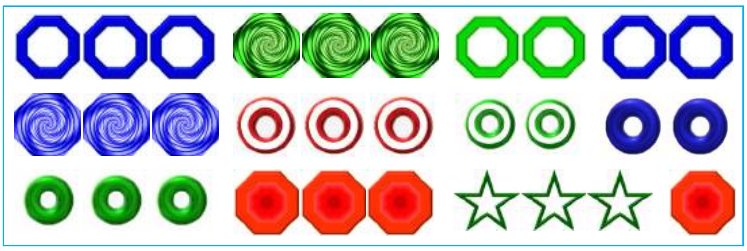
No, this is not a SET .

Hence, the above **SET** does not complete the rule.

Thus, a **SET** consists of **3 cards** in which each individual feature is either **all same** on each card or **all different** on each card.

DO YOU KNOW?

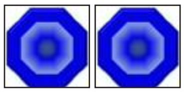
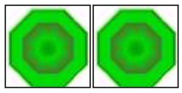
It is proved that there are no **SET**s among these set of cards in columns A and B given below.

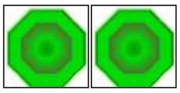
A	B
	

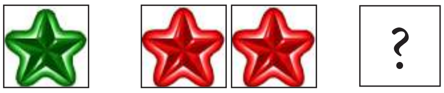
Activity-2

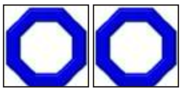
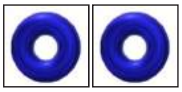
Choose the correct card from the bottom row that will complete the **SET** in the top row to make perfect **SET**s. One is done for you.

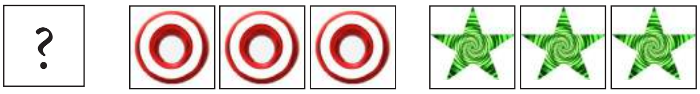
1. 

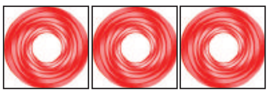
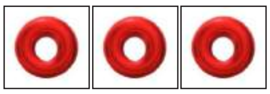
(a)  (b)  (c) 

Ans : 

2. 

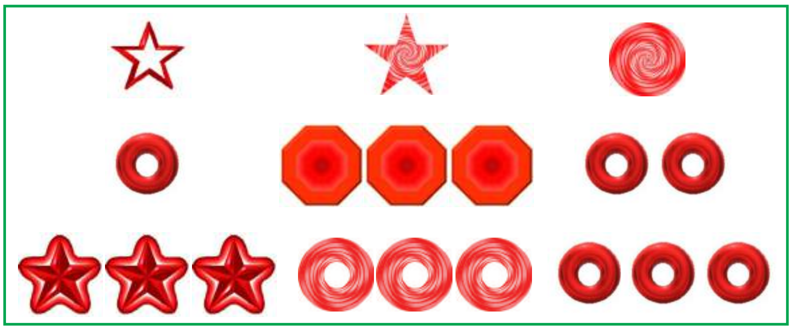
(a)  (b)  (c) 

3. 

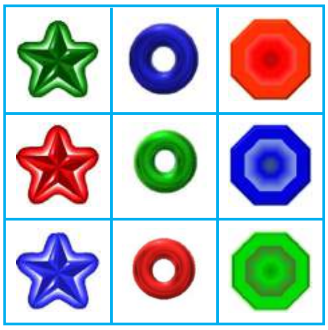
(a)  (b)  (c) 

Try these

1. Find any three **SET**s among these set of cards.



2. This is an example for a magic square in **SET**s. can you make another one?



5.4 Map colouring

Map colouring is the act of assigning different colours to different features on a map. In Mathematics, the problem is to determine the minimum number of colours required to colour a map,so that no two adjacent regions have the same colour. Let us learn about the role of map colouring.

Situation 1:

The teacher divides the class into two groups and instructs one group to use as many colours possible and another group to use minimum number of colours for the given patterns. The only rule is that, shaded regions cannot share the same colour at edges, although it is allowed to meet at a corner.

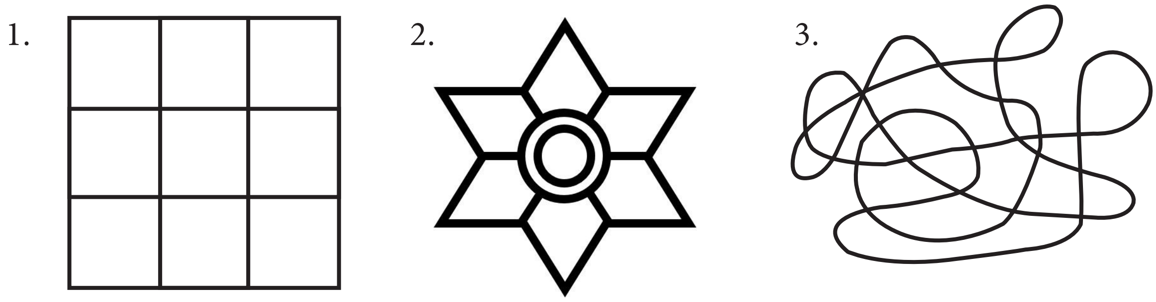


Fig. 5.16

Fig. 5.17 , shows how each group is coloured

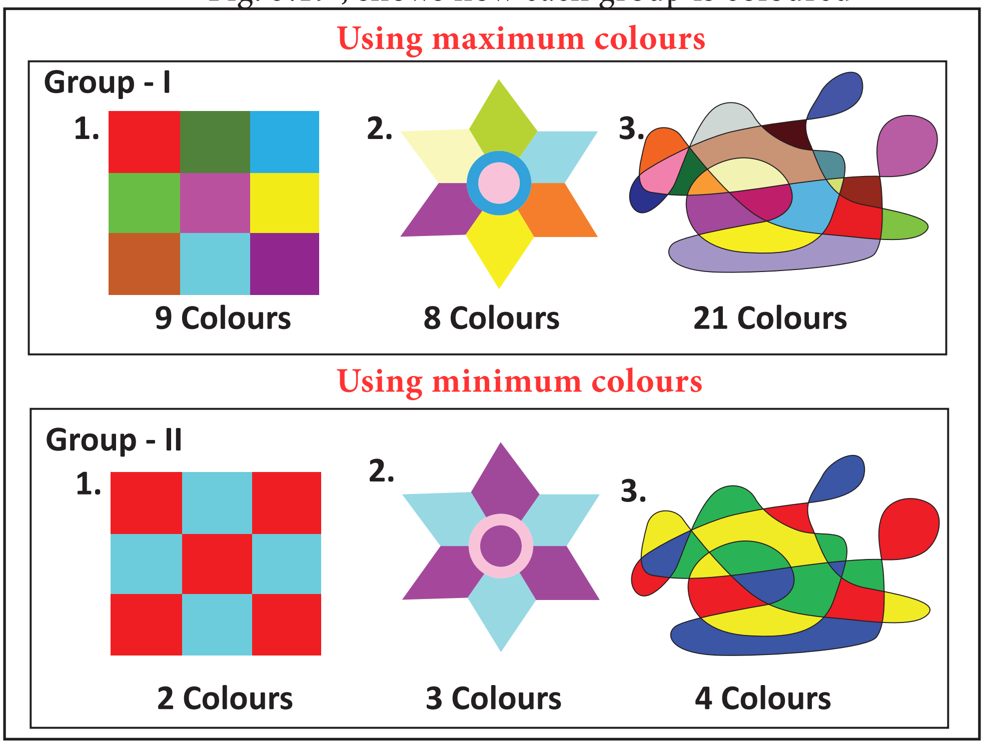


Fig. 5.17

From this investigation, we will get some interesting results. This could get more interesting , if we want to colour a map.

Many optimization problems involve situations where certain events cannot occur at the same time like

- Airlines need to allocate their aircraft to flights (like railway allocate platforms to trains, transport company allocate platforms to their buses, car parking and so on.) so that, no two aircrafts can be parked in the same place .
- Sports organizations need to schedule sporting events such as indoor and outdoor games so that the matches of the individual players or teams does not clash.
- The conditions that are taken into account for framing any type of time tables uses.

Map colouring in which colours must be chosen for regions in a map, that makes bordering regions with different colours. Let us learn more from following examples.

Example 5.7

There are 3 blue tiles , 3 green tiles  and 3 red tiles . Put them together to form a square, so that no two tiles of the same colour are adjacent to each other.

Solution:

This is one of the solutions. Try for more.

Example 5.8

Colour a map of South India (Fig. 5.18) with the fewest number of colours.

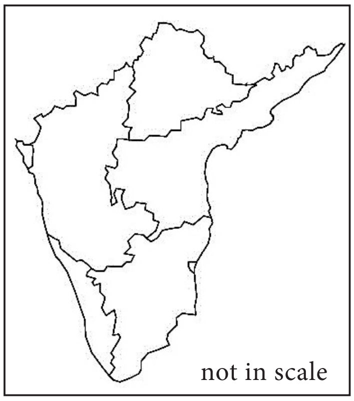


Fig. 5.18

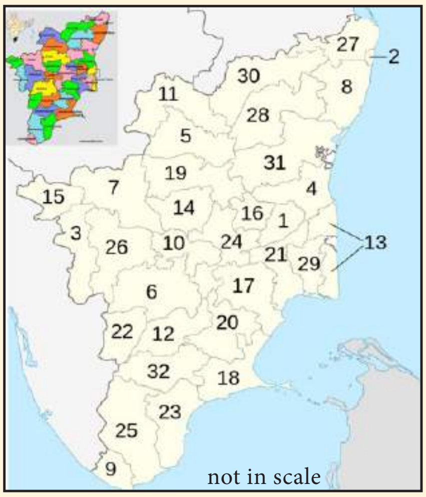
Solution:

Fig. 5.19

This is one of the solutions. Try for more solutions.

Activity-3

Try to colour the Tamilnadu District map with the fewest number of colours.

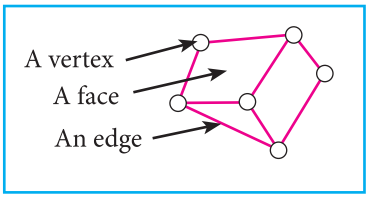


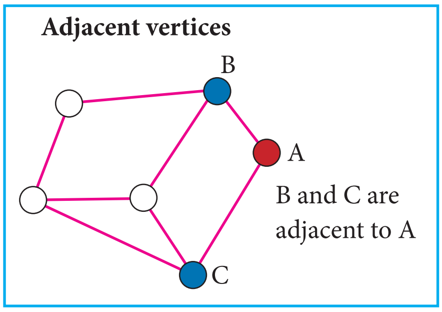
5.5 Graph Colouring

The problem of colouring geographical and political maps has historically been associated with a general class of problems known as "graph colouring" in mathematics.

The objective of graph colouring is to assign minimum number of colours to the vertices so that the adjacent vertices do not have the same colour.

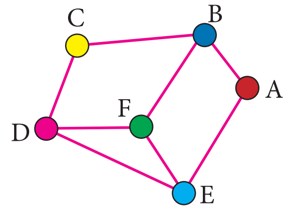
Here, a graph is given with 6 vertices 8 edges and 3 faces. Note that, 2 vertices are adjacent if they are connected by an edge.





The adjacent vertices of a vertex, for example **A** are the vertices that connect **A** by a single edge.

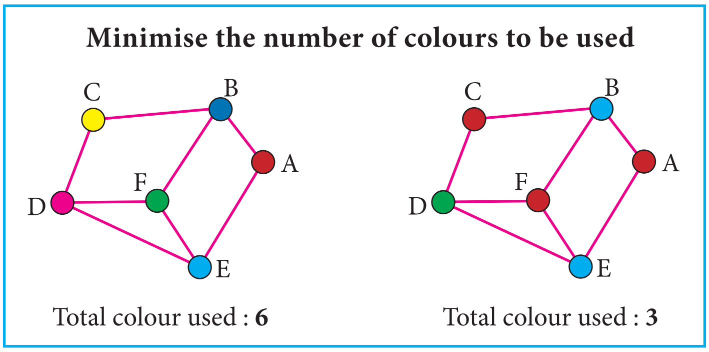
Since B and C are connected to **A,** they are adjacent vertices. Does A have any other adjacent vertices ? No.



A valid graph colouring is that the adjacent vertices do not have the same colour as shown in the figure beside. Notice that they all are different.

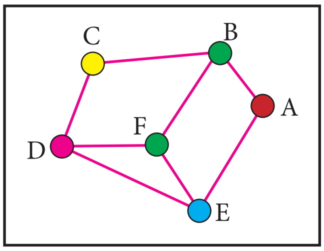
Now our objective is to **minimize the number of colours used**. Let us look at the picture given below.

For example, this picture uses a total of 6 colours. Can we do better than that without any conflict ? Yes, this example can use only 3 colours. Can we do any better than that ? Probably not.



Think

Think why this graph colouring is invalid?



The proof involved in turning the map into a graph, with each region represented by a vertex, and two vertices is linked if the corresponding regions are neighbours. There is atleast one region in a map that is either connected to one or two or three or four or more regions, as shown below in the given figure.

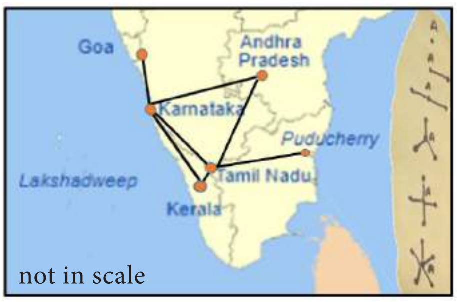


Fig. 5.20

Let us see an example for graph colouring in a map.

Example 5.9

Use graph colouring to determine the minimum number of colours that can be used. The adjacent regions should not have the same colour. Use the graph given below such that,

Fig. 5.21

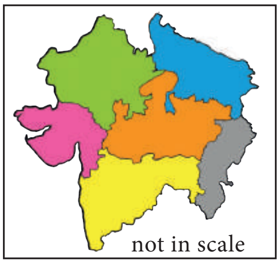
not in scale (i) each state is assigned a coloured vertex.

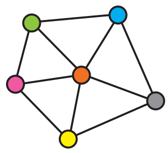
(ii) edges are used to connect the vertices of regions.

Solution:

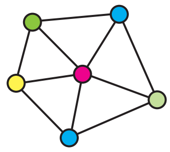
(i)

Given map



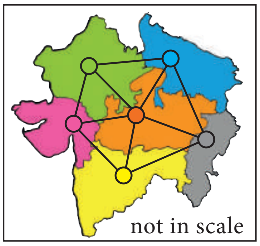


vertices with maximum colours

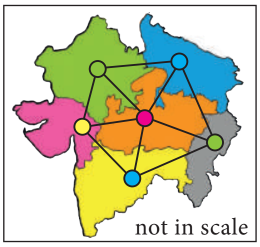


vertices with minimum colours

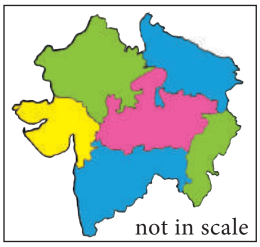
(ii)



map with maximum colours



fixing minimum colours



map with minimum colours (Required map)

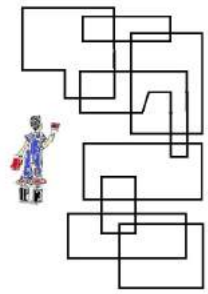
This is one of the solution. Try for more

Try this

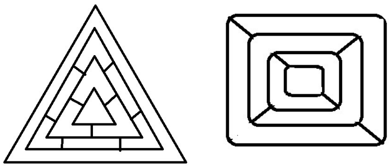
Draw your school building map showing the HM room, class rooms, staff room, science lab, PET room, computer lab, office room, etc., and use graph colouring to detetermine the minimum number of colours that can be used to colour the map.

Exercise 5.2

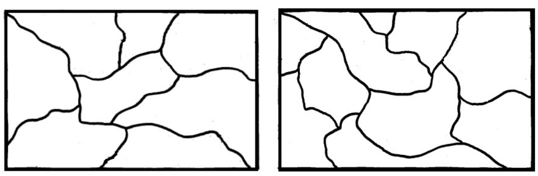
1. Colour the following patterns with as few colours as possible but make sure that no two adjacent sections are of the same colour.



2. Ramya wants to paint a pattern in her living room wall with minimum budget. Help her to colour the pattern with 2 colours but make sure that no two adjacent boxes are the same colour. The pattern is shown in the picture.



3. Colour the countries in the following maps with as few colours as possible but make sure that no two adjacent countries are of the same colour.



Exercise 5.3

Miscellaneous Practice Problems

1. Shanthi has 5 chudithar sets and 4 frocks. In how many possible ways, can she wear either a chudithar or a frock ?

2. In a Higher Secondary School, the following types of groups are available in XI standard

   I. Science Group:

      (i) Physics, Chemistry, Biology and Mathematics
      (ii) Physics, Chemistry, Mathematics and Computer Science
      (iii) Physics, Chemistry, Biology and Home Science

   II. Arts Group:

      (i) Accountancy, Commerce, Economics and Business Maths
      (ii) Accountancy, Commerce, Economics and Computer Science
      (iii) History, Geography, Economics and Commerce

   III. Vocational Group:

      (i) Nursing – Biology, Theory, Practical I and Practical II
      (ii) Textiles and Dress Designing – Home Science, Theory, Practical I and Practical II

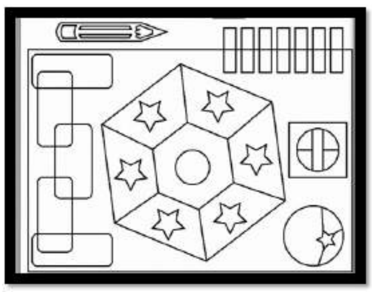
   In how many possible ways, can a student choose the group?

3. An examination paper has 3 sections, each with five questions and students are instructed to answer one question from each section. In how many different ways of can the questions be answered?

4. On sports day, students must take also part in one of the one track events 100m Running and 4×100 m Relay. He must take part of any of the field events Long Jump, High Jump and Javelin Throw. In how many different ways can the student take part in the given events?

5. The given spinner is spun twice and the two numbers got are used to form a 2 digit number. How many different 2 digits numbers are possible?

6. Colour the following pattern with as few colours as possible but make sure that no two adjacent sections are of the same colour.



ANSWERS

1. RATIONAL NUMBERS

Exercise -1.1

1. (i) –4 and –3 (ii) $\frac{11}{25}$ (iii) $\frac{-29}{39}$ (iv) $\frac{1}{20}$ (v) 1

2.(i) False (ii) True (iii) True (iv) False (v) False

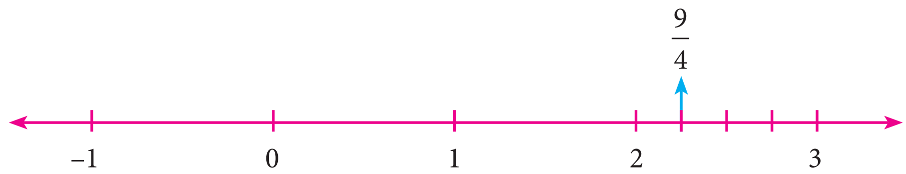
3. (i) $-2,\ 0 \rightarrow \frac{-19}{10}, \frac{-18}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}$ (ii) $\frac{-1}{2}, \frac{3}{5} \rightarrow \frac{-3}{10}, \frac{-1}{10}, 0, \frac{1}{10}, \frac{2}{10}, \frac{5}{10}$

(iii) $0.25,\ 0.35 \rightarrow \frac{26}{100}, \frac{27}{100}, \frac{30}{100}, \frac{32}{100}, \frac{33}{100}$ (iv) $-1.2,\ -2.3 \rightarrow \frac{-21}{10}, \frac{-20}{10}, \frac{-15}{10}, \frac{-14}{10}, \frac{-13}{10}$

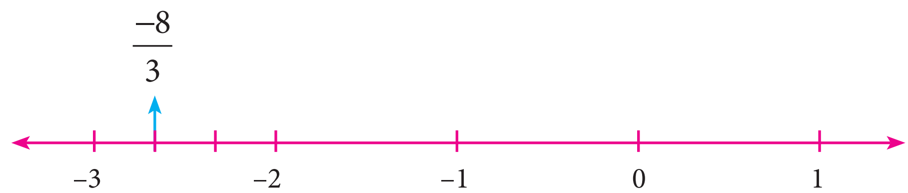
4.(i) $\frac{-3}{5} \rightarrow \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \frac{-15}{25}$ (ii) $\frac{7}{-6} \rightarrow \frac{14}{-12}, \frac{21}{-18}, \frac{28}{-24}, \frac{35}{-30}$ (iii) $\frac{8}{9} \rightarrow \frac{16}{18}, \frac{24}{27}, \frac{32}{36}, \frac{40}{45}$

5.

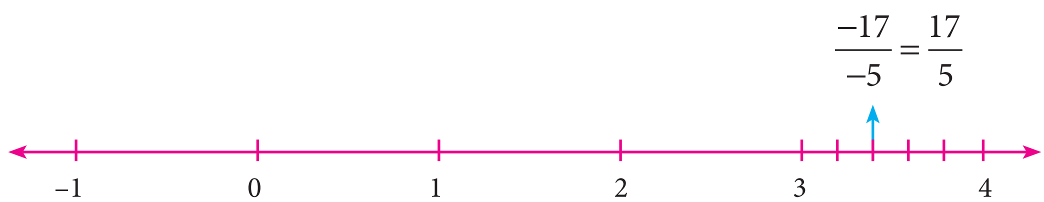
(i)



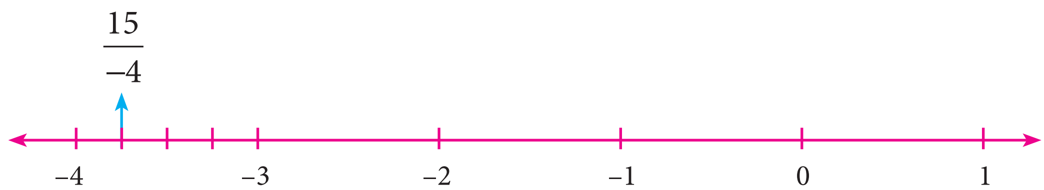
(ii)



(iii)



(iv)



6. (*i*) $-\frac{11}{3}$ (*ii*) $\frac{-2}{5}$ (*iii*) $\frac{7}{4}$ 7. $\frac{61}{15}, \frac{103}{30}, \frac{141}{30}$ or any 3 of your choice 9. $\frac{-6}{7}$

10. $\frac{58}{45}$ 11. $\frac{-15}{11}$ 12. (*i*) $\frac{-33}{2}$ (*ii*) $\frac{8}{45}$ 13. (*i*) -6 (*ii*) $\frac{1}{13}$ (*iii*) 5

14. $6\frac{1}{3}$ and hence lies between 6 and 7 .

15. All integers are rational numbers. So, the integers −7, −6, −5, −4, −3 are rational numbers less than −2. (or) any answer of your choice.

16. (*i*) $\frac{-11}{5} > \frac{-21}{8}$ (*ii*) $\frac{3}{-4} < \frac{-1}{2}$ (*iii*) $\frac{2}{3} < \frac{4}{5}$

17. (i) Ascending Order: $\frac{-11}{8}, \frac{-15}{24}, \frac{-5}{12}, \frac{12}{36}, \frac{-7}{-9}$ Descending Order: $\frac{-7}{-9}, \frac{12}{36}, \frac{-5}{12}, \frac{-15}{24}, \frac{-11}{8}$

(ii) Ascending Order: $\frac{-17}{10}, \frac{-7}{5}, \frac{-19}{20}, \frac{-2}{4}, 0$ Descending Order: $0, \frac{-2}{4}, \frac{-19}{20}, \frac{-7}{5}, \frac{-17}{10}$

18. (*B*) $\frac{-142}{99}$ 19. (*A*) $\frac{-17}{24}$ 20.(*C*) -1 and -2 21. (D) 1 22. (C) 6

23. (D) all of these 24. (B) $\frac{16}{-30}, \frac{-8}{15}$ 25. (B) $\frac{2}{3}$

Exercise 1.2

1. (i) $\frac{5}{13}$ (ii) −3 (iii) $\frac{3}{5}, \frac{-4}{9}$ and $\frac{15}{17}$ (iv) Multiplicative Inverse (v) −1

2. (i) False (ii) True (iii) False (iv) True (v) False

7. 0 8. (i) 0 (ii) $\frac{-51}{160}$ 10. (D) does not exist 11.(A) $\frac{1}{8} - \frac{1}{8} = 0$

12. (C) 0 13. (D) associative 14. (B) $\frac{3}{4}$

Miscellaneous Practice Problems

Exercise 1.3

1. (i) associative property of addition
   (ii) distributive property of multiplication over addition.
   (iii) multiplicative inverse property (iv) does not exist (v) a rational number

2. (i) closure, since $\frac{-1}{4} + \frac{3}{2} = \frac{-1+6}{4} = \frac{5}{4}$, is rational.

(ii) commutative fails as $\frac{1}{3} - \frac{2}{4} \neq \frac{2}{4} - \frac{1}{3}$

(iii) associative fails as $= \frac{1}{2} - \left(\frac{1}{3} - \frac{1}{4}\right) \neq \left(\frac{1}{2} - \frac{1}{3}\right) - \frac{1}{4}$

(iv) identity fails $5-0 \neq 0-5$ (v) inverse also fails.

3. $\frac{1}{6}$ 4. $\frac{1}{10}$ 5. 4 *kg* 300 *gm* 6. $10\frac{2}{5}$ *litre* 7. 6 : 11

8. both are correct 9. 10 *cm* 10. 16 *m*

Challenging Problems

2. $\frac{47}{20}$ *km*, $\frac{23}{20}$ *km* 3. 900 *km* 5. 320 *gm* 6. $\frac{17}{12}$ 7. $\frac{9}{20}$ 8. $x = 3$ 9. 225 10. $\frac{12}{11}$

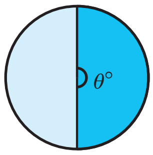
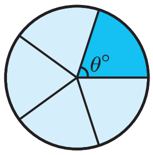
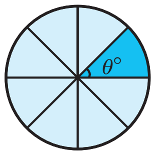
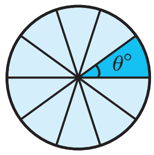
2. MEASUREMENTS

Exercise 2.1

1. (*i*) π (ii) chord (iii) diameter (iv) 12 *cm* (v) an arc

2. (i) c (ii) d (iii) e (iv) b (v) a

3.

Sectors				
Central angle of each sector ($\theta°$)	180°	72°	45°	36°

4.

	Length of arc (approx.)	Area of sector (approx.)	Perimeter of sector (approx.)
i	12.56 *cm*	100.48 cm^2	44.56 *cm*
ii	13.19 *cm*	41.54 cm^2	25.79 *cm*
iii	37.68 *cm*	678.24 cm^2	109.68 *cm*
iv	6.28 *cm*	15.70 cm^2	16.28 *cm*

5. (i) 240 m^2 (ii) 37.5 cm^2 (iii) 337.5 cm^2

6. (*i*) $\theta = 120°$ (*ii*) $\theta = 30°$ (*iii*) $\theta = 72°$ (*iv*) $\theta = 12°$

7. (*i*) $30\pi\, m$ (*ii*) $980\pi\, cm^2$ 8. 350 mm^2 9. 220 cm^2 10. 12.8 *cm*

11. 3.14 *sq. feet* (approximately) 12. 706.5 cm^2 (approximately)

13. 1232 cm^2 (approximately)

Exercise 2.2

1. (i) 38 *cm*, 50.75 cm^2 (approximately) (ii) 30 *cm*,40.25 cm^2 (approximately)

2. (i) 21.5 cm^2 (approximately) (ii) 27.93 cm^2 (approximately)

3. 48 cm^2 4. 41.13 cm^2 (approximately) 5. 5600 cm^2 6. 976.5 cm^2 (approximately)

7. 128 cm^2 8. 3500 cm^2 9. 1020 cm^2 10. (i) 17500 m^2 (ii) 244 m^2

Exercise-2.3

1. (i) length, breadth and height (ii) Vertex (iii) Six (iv) Circle (v) Cube

2. (i) (b) (ii) (a) (iii) (d) (iv) (c)

3. (i) Cube (ii) Cuboid (iii) Triangular Prism (iv) Square Pyramid (v) Cylinder

4. (i) F, T, S (ii) T, S, F (iii) S, F, T

5. (i) Yes (ii) No (iii) Yes (iv) No (v) Yes

6. (i) 24 *sq. units* (ii) 62 *sq. units*

7. No

Miscellaneous Practice Problems

Exercise-2.4

1. 9.42 *feet* 2. 314 *m* 3. 128 cm^2

4. (i) Top View Front View Side View (ii) Top View Front View Side View

Challenging Problems

6. double door requires the minimum area 7. 63.48 m^2 (approximately)

8. 1.46 cm^2 (approximately) 9. 94 cm^2 10. (i) F = 10 (ii) V = 4 (iii) E= 28

3. ALGEBRA

Exercise 3.1

1.(*i*) $24x$ (*ii*) $-21xy$ (*iii*) $250m^5$ (*iv*) $-4a^5b$ (*v*) $-18p^3q^5$

4.

x	$2x^2$	$-2xy$	x^4y^3	$2xyz$	$-5xz^2$
x^4	$2x^6$	$-2x^5y$	x^8y^3	$2x^5yz$	$-5x^5z^2$
$4xy$	$8x^3y$	$-8x^2y^2$	$4x^5y^4$	$8x^2y^2z$	$-20x^2yz^2$
$-x^2y$	$-2x^4y$	$2x^3y^2$	$-x^6y^3$	$-2x^3y^2z$	$5x^3yz^2$
$2y^2z$	$4x^2y^2z$	$-4xy^3z$	$2x^4y^5z$	$4xy^3z^2$	$-10xy^2z^3$
$-3xyz$	$-6x^3yz$	$6x^2y^2z$	$-3x^5y^4z$	$-6x^2y^2z^2$	$15x^2yz^3$
$-7z$	$-14x^2z$	$14xyz$	$-7x^4y^3z$	$-14xyz^2$	$35xz^3$

3. (*i*) $24m^4n^2$ (*ii*) $-9x^5y^6$ 4. $-24p^6q^6$

5. (*i*) $10xy-15x$ (*ii*) $-10p^3+6p^2-14p$ (*iii*) $3m^4n^4-15m^3n^2+21m^2n^3$

(*iv*) $x^3+y^3-z^3+x^2y+x^2z+xy^2+zy^2+xz^2-yz^2$

6. (*i*) $4x^2-2x-12$ (*ii*) $2y^4+3y^3-8y^2-12y$ *iii*) $5m^4n^2-m^2n^2-5m^2n^3+n^3$ *iv*) $6x^2-36x+30$

7. (*i*) $-2x^2$ *ii*) $-3mp$ *iii*) $2y(5x^2y-x+3y^2)$

8. (C) iv, v, ii, iii, i 9. $xy+2x+30y+60$

10. (B) $28p^7$ 11. (A) mn^2, 27 12. (c) $6x^2y$ 13.(A) 6 mn 14. (B) $(a+b)$

Exercise 3.2

1. (i) $\frac{18m^4(n^8)}{2m^{(3)}n^3}=9mn^5$ (ii) $\frac{l^4m^5n^{(7)}}{2lm^{(3)}n^6}=\frac{l^3m^2n}{2}$ (iii) $\frac{42a^4 6^5(c^2)}{6(a)^4(b)^2}=(7)b^3c^2$

2. (i) True (ii) False 3. (i) $9y^2$ (ii) xy (iii) $-3x^2yz^3$ (iv) xy

4. (i) $5m$ (ii) $7p^3q^2-4p^3q$ 5. (i) $16y-4z$ (ii) $2mn^2+8m^3n-\frac{1}{2}$ (iii) 8

(iv) $=9p^2r+18pq-45$ 6. $25mn$

7. (i) $9y^2$ (ii) $9xy$ (iii) $4m^2-3m$ (iv) $16n^2-2n+3$ (v) x^2+1x-6

Exercise 3.3

1. (i) $9m^2+30m+25$ (ii) $25p^2-10p+1$ (iii) $4n^2+4n-3$

(iv) $(2p+5q)(2p-5q)$

2. (i) $m^3+9m^2+27m+27$ (ii) $8a^3+60a^2+150a+125$ (iii) $27p^3+108p^2q+144pq^2+64q^3$

(iv) 14,0608 (v) 1124864

3. (i) $125-75x+15x^2-x^3$ (ii) $8x^3-48x^2y+96xy^2-64y^3$ (iii) $a^3b^3-3a^2b^2c+3abc^3-c^3$

(iv) 110, 592 (v) $912673x^3y^3$

4. (i) $125y^3+150y^2+55y+6$ (ii) p^3-5p^2+2p+8

5. x^3+3x^2+3x+1 6. x^3-2x^2-5x+6

Exercise 3.4

1. (i) $6y(3x-2z)$ (ii) $3x^2y(3x^3y^2+2xy-6)$ (iii) $(b-2c)(x+y)$

(iv) $(x+y)(a+b)$ (v) $(4x-1)(2x^2-1)$ (vi) $(x-2)[3y(x-2)+2]$

(vii) $2y(9x-2y-zx)$ ($viii$) $(a-3)(a^2+1)$ (ix) $3y(y+4)(y-4)$ (x) $(b-1)(ab-c^2)$

2. (i) $(x+7)^2$ (ii) $(y-5)^2$ (iii) $(c+2)(c-6)$

(iv) $(m+9)(m-8)$ (v) $(2x-3)(2x-1)$

3. (i) $(h+k)(h^2-hk+k^2)$ (ii) $2[(a+2)(a^2-2a+4)]$ (iii) $(xy+3)[(x^2y^2-3xy+9)]$

(iv) $(4m+n)(16m^2-4mn+n^2)$ (v) $r[(r+3p)(r^2-3rp+9p^2)]$

4. (i) $(y-3)[(y^2+3y+9)]$ (ii) $3[(b-4c)(b^2+4bc+16c^2)]$ (iii) $-2[(2y-x)(4y^2-2yx+x^2)]$

(v) $(xy-7)(x^2y^2+7xy+49)$ (v) $(c-3ab)[(c^2+3abc+9a^2b^2)$

Miscellaneous Practice Problems

Exercise 3.5

1. $7x^2y^5 + 4x^4y^3 + 60x^2y^2$ 2. $12x^3 - 8x^2 + 27x - 18$ 3. $S.I = \frac{7}{5}a^3b^4$

4. $\frac{1}{2}a + 2b + 4$ 5. $(y-3)(7y+2)$

Challenging Problems

1. $4x+3$ 2. $x+2$ 3. $(y+7)(y+8)$ 4. $(4p^2+1)(2p+1)(2p-1)$

5. $(x^2-4y)(x^4+4x^2y+16y^2)$

4. GEOMETRY

Exercise 4.1

3. $HE = 18, TE = 16$ 4. $\angle T = \angle N = 75°, \angle E = \angle B = 35°, \angle A = \angle U = 70°$

11. Yes,RHS Congruence

12. (i) in proportion (ii) shape (iii) equal (iv) congruent (v) similar

13. (D) matching 14. (C) $\angle Q = \angle X$ 15. (D) 93 *m*

16. (A) 50° 17. (C) AC = CD

Exercise 4.2

1. PT = 80 3. 48 *feet*

5. INFORMATION PROCESSING

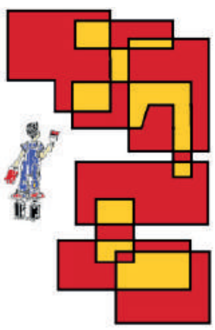
Exercise 5.1

1. (B) 8 2. (D) 64 3. (C) 19

2. (i) 5 (ii) 45 (iii) 6 (iv) 5000 (v) 10000

Exercise 5.2

1. (i)



more possible ways

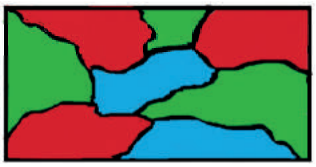
2. (i)

more possible ways

(ii)

more possible ways

3. (i)



more possible ways

(ii)

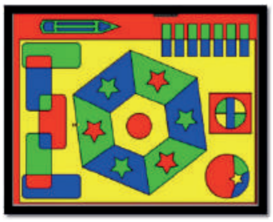
more possible ways

Miscellaneous Practice Problems

Exercise 5.3

1. (i) 9 (ii) 8 (iii) 15 (iv) 6 (v) 20

(vi) 16 (vii)



more possible ways

MATHEMATICAL TERMS

Algebra	இயற்கணிதம்
Algebraic expression	இயற்கணிதக் கோவை
Altitude	குத்துக்கோடு
Angle	கோணம்
Approximately	தோராயமாக
Arbitrary	தன்னிச்சையான
Associative property	சேர்ப்புப் பண்பு
Average	சராசரி
Binomial	ஈருறுப்புக் கோவை
Bisector	இரு சமவெட்டி
Central angle	மையக்கோணம்
Chord	நாண்
Circular arc	வட்ட வில்
Circular sector	வட்டக் கோணப் பகுதி
Circumference	பரிதி
Closure property	அடைவுப் பண்பு
Cognitive	அறிவாற்றல்
Combined shapes	கூட்டு வடிவங்கள்
Common errors	பொதுவான தவறுகள்
Common factor	பொதுக்காரணி
Commutative property	பரிமாற்றுப் பண்பு
Complementary angles	நிரப்புக் கோணங்கள்
Congruent	சர்வ சமம்
Converse	மறுதலை
Corresponding /Similar	ஒத்த
Cube	கனச்சதுரம்
Cubic identities	கன முற்றொருமைகள்
Cuboid	கனச்செவ்வகம்
Decimal	தசம / பதின்மான
Denominator	பகுதி
Diameter	விட்டம்
Distributive property	பங்கீட்டுப் பண்பு
Edges	விளிம்புகள்
Equation	சமன்பாடு
Equivalent	சமான
Exterior	வெளிப்புற
Faces	முகங்கள்
Factorization	காரணிப்படுத்துதல்
Front view	முகப்புத்தோற்றம்
Geometrical proof	வடிவியல் நிரூபணம்

Hypotenuse	கர்ணம்
Identity	சமனி
Integers	முழுக்கள்
Interior	உள்புற / உட்புற
Inverse	நேர்மாறு
Irregular polygon	ஒழுங்கற்ற பலகோணம்
Logical	தருக்கரீதியான
Measure	அளவு
Median	நடுக்கோடு
Midpoint	மையப்புள்ளி/ நடுப்புள்ளி
Monomial	ஓருறுப்புக் கோவை
Net	வலை
Numerator	தொகுதி
Operation	செயல்பாடு
Perfect cube numbers	முழு கன எண்கள்
Polyhedron	பன்முக வடிவம்
Polynomial	பல்லுறுப்புக் கோவை
Prism	பட்டகம்
Radius	ஆரம்
Rational	விகித முறு
Real number line	மெய்யெண் கோடு
Reciprocal	தலைகீழி/பெருக்கல் நேர்மாறு
Regular polygon	ஒழுங்குப் பலகோணம்
Repeating	மீண்டும் / திரும்ப
Circular Segment	வட்டத்துண்டு
Set	கணம்
Shaded region	நிழலிட்ட பகுதி
Side view	பக்கவாட்டுத்தோற்றம்
Simultaneously	ஒரேநேரத்தில்
Solid shapes	திண்ம வடிவங்கள்
Spatial	இடம் சார்ந்த
Standard form	நிலையான வடிவு
Stimulate	தூண்டுதல்
Supplementary angles	மிகை நிரப்புக் கோணங்கள்
Terminating	முடிவுறு / முற்றுபெற்ற
Three dimensional shapes	முப்பரிமாண வடிவங்கள்
Top view	மேற்பக்கத் தோற்றம்
Trinomial	மூவுறுப்புக் கோவை
Vertically opposite	குத்தெதிர்
Vertices	உச்சிகள்

Mathematics - Class 8 Term I
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