DAY TWENTY SIX

Electromagnetic Waves

Learning & Revision for the Day

- Electromagnetic Waves and their Characteristics
- Maxwell's Equations
- Properties of Electromagnetic Waves
- Transverse Nature of Electromagnetic Waves
- Spectrum of Electromagnetic Radiation

Electromagnetic Waves and their Characteristics

Electromagnetic waves are those waves, in which electric and magnetic fields vary sinusoidally in space and with time. The electric and magnetic fields are mutually perpendicular to each other and each field is perpendicular to the direction of propagation of the wave.

- Maxwell's theory predicted that electromagnetic waves of all frequencies (and hence all wavelengths) propagate in vacuum, with a speed given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

where, μ_0 is the magnetic permeability and ε_0 is the electric permittivity of vacuum.
Now, for the vacuum, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1} and $\varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-2}$.
Substituting these values in the above relation, we have

$$c = \frac{1}{[(4\pi \times 10^{-7})(8.85 \times 10^{-12})]^{1/2}} \simeq 3.0 \times 10^8 \text{ms}^{-1}$$

- All the electromagnetic waves are of the transverse nature whose speed depends upon the medium, but their frequency does not depend on the medium.
- Transverse waves can be polarised.
- Energy is being transported with the electromagnetic waves.

Conduction Current

It is a current in the electric circuit, which arises due to the flow of electrons in the connecting wires of the circuit, in a definite closed path.

Maxwell's Displacement Current

It is that current which comes into play in the region, whenever the electric field and hence the electric flux is changing with it.

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

The generalised form of the Ampere's law is

$$\oint \mathbf{B} \cdot d\,\mathbf{l} = \mu_0 (i + i_d) = \mu_0 \left(i + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

Maxwell's Equations

Maxwell in 1862, gave the basic laws of electricity and magnetism in the form of four fundamental equations, which are known as Maxwell's equations.

- **Gauss's law for electrostatics** This law states that the electric lines of force start from positive charge and end at negative charge i.e. the electric lines of force do not form a continuous closed path.

  Mathematically, $\oint_S \mathbf{E} \cdot d\mathbf{S} = \dfrac{q}{\varepsilon_0}$
- **Gauss's law for magnetism** This law also predicts that the isolated magnetic monopole does not exist.

  Mathematically, $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$
- **Faraday's law of electromagnetic induction** This law tells that the changing magnetic field is the source of electric field. Mathematically, $\oint \mathbf{E} \cdot d\mathbf{l} = -\dfrac{d\phi_B}{dt}$
- **Ampere-Maxwell's law** At an instant, in a circuit, the conduction current is equal to displacement current.

  Mathematically, $\oint \mathbf{E} \cdot d\mathbf{l} = \mu_0 \left(I_c + \varepsilon_0 \dfrac{d\phi_E}{dt} \right)$

  These equations are collectively called Maxwell's equations.

Properties of Electromagnetic Waves

- If the electromagnetic wave is travelling along the positive direction of the X-axis, the electric field is oscillating parallel to the Y-axis and the magnetic field is oscillating parallel to the Z-axis.

  $$E = E_0 \sin(kx - \omega t) \Rightarrow B = B_0 \sin(kx - \omega t)$$

  In this, E_0 and B_0 are the amplitudes of the fields.

  Further, $c = \dfrac{E_0}{B_0} = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$ = speed of light in vacuum
- The rate of flow of energy in an electromagnetic wave, is described by the vector **S** called the **Poynting vector**, which is defined by the expression,

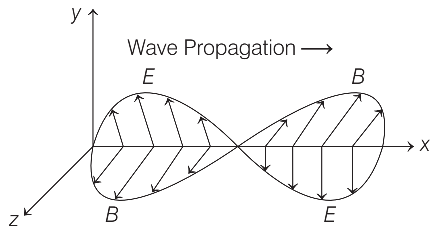
  $$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$
- Energy density of electromagnetic wave,

  $$u_e = \frac{1}{2} \varepsilon_0 E^2_{u_B} = \frac{1}{2} \frac{B^2}{\mu_0}$$
- Momentum delivered, $p = \dfrac{u}{c}$ (absorbing surface)

  $p = \dfrac{2u}{c}$ (reflecting surface)
- Energy of wave $= \dfrac{hc}{\lambda} = h\nu$

Transverse Nature of Electromagnetic Waves

According to Maxwell, electromagnetic waves consist of time varying electric and magnetic fields, which are perpendicular to each other, as well as direction of wave propagation.



Spectrum of Electromagnetic Radiation

The array obtained on arranging all the electromagnetic waves in an order on the basis of their wavelength is called the electromagnetic spectrum.

The Electromagnetic Spectrum

Name	Frequency Range (Hz)	Wavelength Range (m)	Source
Radio waves	10^4 to 10^8	0.1 to 600	Oscillating electric circuits
Microwaves	10^9 to 10^{12}	10^{-3} to 0.3	Oscillating current in special vacuum tubes
Infrared	10^{11} to 5×10^{14}	10^{-6} to 5×10^{-3}	Outer electrons in atoms and molecules
Visible light	4×10^{14} to 7×10^{14}	4×10^{-7} to 8×10^{-7}	Outer electrons in atoms
Ultraviolet	10^{15} to 10^{17}	1.5×10^{-7} to 3.5×10^{-7}	Outer electrons in atoms
X-rays	10^{18} to 10^{20}	10^{-11} to 10^{-8}	Inner electrons in atoms and sudden deacceleration of high energy free electrons
Gamma rays	10^{19} to 10^{24}	10^{-16} to 10^{-13}	Nuclei of atoms and sudden deacceleration of high energy free electrons

Various Electromagnetic Radiations

- **Gamma rays** The main sources of gamma rays are the natural and artificial radioactive substances. These rays affect the photographic plate and mainly used in the treatment of cancer disease.
- **X-rays** X-rays are produced, when highly energetic cathode rays are stopped by a metal target of high melting point. They affect the photographic plate and can penetrate through the transparent materials. They are mainly used in detecting the fracture of bones, hidden bullet, needle, costly material etc. inside the body, and also used in the study of crystal structure.
- **Ultraviolet Rays** The major part of the radiations received from sun consists of the ultraviolet radiation. Its other sources are the electric discharge tube, carbon arc, etc.

  These radiations are mainly used in excitation of photoelectric effect and to kill the bacteria of many diseases.
- **Visible Light** Visible light is obtained from the glowing bodies, while they are white hot. The light obtained from the electric bulbs, sodium lamp, fluorescent tube is the visible light.
- **Thermal or Infrared Waves** A body on being heated, emits out the infrared waves. These radiations have the maximum heating effect. The glass absorbs these radiations, therefore for the study of these radiations, rock salt prism is used instead of a glass prism. These waves are mainly used for therapeutic purpose by the doctors because of their heating effect.
- **Microwaves** These waves are produced by the spark discharge or magnetron valve. They are detected by the crystal or semiconductor detector. These waves are used mainly in radar and long distance communication.
- **Radio waves** They can be obtained by the flow of high frequency alternating current in an electric conductor.

  These waves are detected by the tank circuit in a radio receiver or transmitter.

Applications of Electromagnetic Spectrum

- Radio waves are used in radar and radio broadcasting.
- Microwaves are used in long distance wireless communications *via* satellites.
- Infrared, visible and ultraviolet radiations are used to know the structure of molecules.
- Diffraction of X-rays by crystals, gives the details of the structure of crystals.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The speed of electromagnetic waves in a vacuum, is given by

(a) $\frac{1}{\mu_0\varepsilon_0}$ (b) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ (c) $\mu_0\varepsilon_0$ (d) $\sqrt{\mu_0\varepsilon_0}$

2 An electric field **E** and magnetic field **B** exist in a region. If these fields are not perpendicular to each other, then the electromagnetic wave

(a) will not pass through the region
(b) will pass through the region
(c) may pass through the region
(d) Nothing is definite

3 Which of the following statement is false for the properties of electromagnetic waves ? ➜ CBSE AIPMT 2010

(a) Both electric and magnetic field vectors attain the maxima and minima at the same place and same time
(b) The energy in electromagnetic wave is divided equally between electric and magnetic vectors
(c) Both electric and magnetic field vectors are parallel to each other and perpendicular to the direction of propagation of wave
(d) These waves do not require any material medium for propagation

4 The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to ➜ CBSE AIPMT 2013

(a) the speed of light in vacuum
(b) reciprocal of speed of light in vacuum
(c) the ratio of magnetic permeability to the electric susceptibility of vacuum
(d) unity

5 Electromagnetic waves are transverse in nature, is evident by

(a) polarisation (b) interference
(c) reflection (d) diffraction

6 Maxwell's modified form of Ampere's circuital law is

(a) $\oint \mathbf{B}\cdot d\mathbf{S} = 0$ (b) $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_c$
(c) $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_c + \frac{1}{\varepsilon_0}\frac{dq}{dt}$ (d) $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_c + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$

7 In an electromagnetic wave, the electric and magnetic fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is

(a) 26.5 Wm^{-2} (b) 36.5 Wm^{-2} (c) 46.7 Wm^{-2} (d) 765 Wm^{-2}

8 Which of the following pairs of space and time varying $E = (\hat{\mathbf{i}} E_x + \hat{\mathbf{j}} E_y + \hat{\mathbf{k}} E_z)$ and $B = (\hat{\mathbf{i}} B_x + \hat{\mathbf{j}} B_y + \hat{\mathbf{k}} B_z)$ would generate a plane electromagnetic wave travelling in the z-direction?

(a) E_x, B_z (b) E_y, B_z (c) E_z, B_x (d) E_x, B_y

9 Consider the following two statements, regarding a linearly polarised plane electromagnetic wave.

I. Electric field and the magnetic field have equal average values.
II. Electric energy and the magnetic energy have equal average values.

(a) I is true (b) II is true
(c) Both statements are true (d) Both statements are false

10 In an apparatus, the electric field was found to oscillate with an amplitude of 18 Vm^{-1}. The magnitude of the oscillating magnetic field will be

(a) 4×10^{-6} T (b) 6×10^{-8} T
(c) 9×10^{-9} T (d) 11×10^{-11} T

11 An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t); B = B_0 \sin(kx - \omega t)$. Which of the following equations is true ?

(a) $E_0 k = B_0 \omega$ (b) $E_0 \omega = B_0 k$
(c) $E_0 B_0 = \omega k$ (d) None of these

12 The electric and the magnetic field associated with an electromagnetic wave, propagating along the $+Z$-axis, can be represented by ➜ CBSE AIPMT 2011

(a) $\mathbf{E} = E_0 \hat{\mathbf{k}}, \mathbf{B} = B_0 \hat{\mathbf{i}}$ (b) $\mathbf{E} = E_0 \hat{\mathbf{j}}, \mathbf{B} = B_0 \hat{\mathbf{j}}$
(c) $\mathbf{E} = E_0 \hat{\mathbf{j}}, \mathbf{E} = B_0 \hat{\mathbf{k}}$ (d) $\mathbf{E} = E_0 \hat{\mathbf{i}}, \mathbf{B} = B_0 \hat{\mathbf{j}}$

13 A radio can tune into any station in the 7.5 MHz to 12 MHz band. The corresponding wavelength band is

(a) 5 m to 15 m (b) 2 m to 16 m
(c) 25 m to 40 m (d) 30 m to 45 m

14 If a source is transmitting electromagnetic wave of frequency 8.2×10^6 Hz, then wavelength of electromagnetic waves transmitting from the source will be

(a) 36.6 m (b) 40.5 m
(c) 42.3 m (d) 50.9 m

15 An electric charge oscillating with a frequency of 1 kilo cycle/s can radiate electromagnetic wave of wavelength?

(a) 100 km (b) 200 km
(c) 300 km (d) 400 km

16 The wave of wavelength 5900Å emitted by any atom or molecule must have some finite total length which is known as the coherence length. For sodium light, this length is 2.4 cm. The number of oscillations in this length will be

(a) 4.068×10^8 (b) 4.068×10^4
(c) 4.068×10^6 (d) 4.068×10^5

17 21 cm radio wave emitted by hydrogen in intersteller space is due to the interaction called the hyperfine interaction in atomic hydrogen, the energy of the emitted wave is nearly

(a) 10^{-17} J (b) 1 J
(c) 7×10^{-8} J (d) 10^{-24} J

18 The condition under which a microwave oven heats up a food item containing water molecules most efficiently is ➜ NEET 2013

(a) the frequency of the microwave must match the resonant frequency of the water molecules
(b) the frequency of the microwave has no relation with natural frequency of water molecules
(c) microwave are heat waves, so always produce heating
(d) infrared waves produce heating in a microwave oven

19 The decreasing order of wavelength of infrared, microwave, ultraviolet and gamma rays is ➜ CBSE AIPMT 2011

(a) gamma rays, ultraviolet, infrared, microwaves
(b) microwaves, gamma rays, infrared, ultraviolet
(c) infrared, microwave, ultraviolet, gamma rays
(d) microwave, infrared, ultraviolet, gamma rays

20 On required 11eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in

(a) visible region (b) infrared region
(c) ultraviolet region (d) microwave region

21 The energy of an electromagnetic waves is of the order of 15 keV. To which part of the spectrum does it belong? ➜ CBSE AIPMT 2015

(a) X-rays (b) Infrared rays
(c) Ultraviolet rays (d) γ-rays

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 Out of the following options which one can be used to produce a propagating electromagnetic wave? → NEET 2016

(a) A stationary charge
(b) A chargeless particle
(c) An accelerating charge
(d) A charge moving at constant velocity

2 An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$.

Which of the following is independent of wavelength?

(a) k (b) ω (c) k/ω (d) $k\omega$

3 Light with an energy flux of 18 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm^2, the average force exerted on the surface during a span 30 min is

(a) 1.2×10^{-6} N (b) 10^{-3} N
(c) 4×10^{-7} N (d) 5×10^{-4} N

4 The electric field associated with an electromagnetic wave in vacuum, is given by $\mathbf{E} = \hat{\mathbf{i}}\, 40 \cos(kz - 6\times 10^8 t)$, where E, z and t are in Volt/m, metre and second, respectively. The value of wave vector k is → CBSE AIPMT 2012

(a) 2 m^{-1} (b) 0.5 m^{-1} (c) 6 m^{-1} (d) 3 m^{-1}

5 An EM wave is propagating in a medium with a velocity $\mathbf{v} = v\hat{\mathbf{i}}$. The instantaneous oscillating electric field of this EM wave is along $+Y$-axis. Then, the direction of oscillating magnetic field of EM wave will be along → NEET 2018

(a) $-y$-direction (b) $+z$-direction
(c) $-z$-direction (d) $-x$-direction

6 A linearly polarised electromagnetic wave given as $\mathbf{E} = E_0\, \hat{\mathbf{i}} \cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as

(a) $\mathbf{E}_r = -E_0\, \hat{\mathbf{i}} \cos(kz - \omega t)$ (b) $\mathbf{E}_r = E_0\, \hat{\mathbf{i}} \cos(kz + \omega t)$
(c) $\mathbf{E}_r = -E_0 \hat{\mathbf{i}} \cos(kz + \omega t)$ (d) $\mathbf{E}_r = E_0 \hat{\mathbf{i}} \sin(kz - \omega t)$

7 In an electromagnetic wave in free space, the root mean square value of the electric field is $E_{rms} = 6$ V/m. The peak value of the magnetic field is → NEET 2017

(a) 1.41×10^{-8} T (b) 2.83×10^{-8} T
(c) 0.70×10^{-8} T (d) 4.23×10^{-8} T

8 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2×10^{10} Hz and amplitude 48 Vm^{-1}. Then, which one of the following statement is true?

(a) Wavelength of the wave is 2×10^5 m
(b) Amplitude of oscillating magnetic field is 48 T
(c) Average energy density of electric field equals the average energy density of magnetic field
(d) None of the above

9 The electric field part of an electromagnetic wave in a medium is represented by $E_x = 0$;

$$E_y = 2.5 \frac{N}{C} \cos\left[\left(2\pi \times 10^6 \frac{\text{rad}}{\text{m}}\right) t - \left(\pi \times 10^{-2} \frac{\text{rad}}{s}\right) x\right];$$

$E_z = 0$. The wave is → CBSE AIPMT 2009

(a) moving along y-direction with frequency $2\pi \times 10^6$ Hz and wavelength 200 m
(b) moving along x-direction with frequency 10^6 Hz and wavelength 100 m
(c) moving along x-direction with frequency 10^6 Hz and wavelength 200 m
(d) moving along $-x$-direction with frequency 10^6 Hz and wavelength 200 m

10 A uniform electric field and an uniform magnetic field are acting along the same direction in a certain region. If an electron is projected in the region, such that its velocity is pointed along the direction of fields, then the electron → CBSE AIPMT 2011

(a) speed will decrease
(b) speed will increase
(c) will turn towards left of direction of motion
(d) will turn towards right of direction of motion

ANSWERS

SESSION 1	**1** (b)	**2** (c)	**3** (c)	**4** (b)	**5** (a)	**6** (d)	**7** (a)	**8** (d)	**9** (b)	**10** (b)
	11 (a)	**12** (d)	**13** (c)	**14** (a)	**15** (c)	**16** (b)	**17** (d)	**18** (a)	**19** (d)	**20** (c)
	21 (a)									
SESSION 2	**1** (c)	**2** (c)	**3** (a)	**4** (a)	**5** (b)	**6** (b)	**7** (b)	**8** (c)	**9** (c)	**10** (a)

Hints and Explanations

SESSION 1

1 Speed of electromagnetic waves in vacuum $= \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

2 The electromagnetic wave being packets of energy moving with speed of light may pass through the region.

3 The time varying electric and magnetic fields are mutually perpendicular to each other and also perpendicular to the direction of propagation of the wave.

4 As, $B_0 = \dfrac{E_0}{c} \Rightarrow \dfrac{B_0}{E_0} = \dfrac{1}{c}$

i.e. reciprocal of speed of light in vacuum.

5 As electromagnetic waves are transverse in nature, i.e. have oscillating electric and magnetic vector propagating perpendicular to each other and also to wave propagation. Hence, can be proved by polarisation.

6 The modified form of Ampere's circuital law is

$$\oint \mathbf{B} \cdot d\,\mathbf{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

7 Maximum rate of energy flow,

$S = E_0 \times B_0 = 100 \times 0.265$
$= 26.5\ \text{Wm}^{-2}$

8 Direction of propagation of electromagnetic waves is in the direction of cross product of **E** and **B** (i.e. $\mathbf{E} \times \mathbf{B}$). So, if wave is propagating in two direction, then

$$\mathbf{E} = E_x\, \hat{\mathbf{i}} \text{ and } \mathbf{B} = B_y\, \hat{\mathbf{j}}.$$

Then, $\mathbf{E} \times \mathbf{B}$ point in $+z$-direction.

9 Average energy is equally shared by electric field and magnetic field. However, their average magnitudes are not equal.

As, $|B_0| = \dfrac{1}{C} \cdot |E_0|$

10. $\therefore B_0 = \dfrac{E_0}{c} = \dfrac{18}{3 \times 10^8} = 6 \times 10^{-8}$ T

11. We know $E_0 = c\, B_0$, where c is the velocity of light.

$\therefore \quad c = \nu\lambda = \dfrac{\omega}{2\pi}\lambda = \dfrac{\omega}{k}$

Thus, $E_0 = \dfrac{\omega}{k} B_0$

or $E_0 k = B_0 \omega$

12 $\mu = \mathbf{E} \times \mathbf{B} = E_0\, \hat{\mathbf{i}} \times B_0\, \hat{\mathbf{j}} = E_0 B_0\, \hat{\mathbf{k}}$

$\mathbf{E} \times \mathbf{B}$ points in the direction of wave propagation.

13 $\therefore \lambda = \dfrac{c}{f}$

When $f = 7.5$ MHz $= 7.5 \times 10^6$ Hz,

then $\lambda = \dfrac{3 \times 10^8}{7.5 \times 10^6} = 40$ m

When $f = 12$ MHz $= 12 \times 10^6$ Hz,

then $\lambda = \dfrac{3 \times 10^8}{12 \times 10^6} = 25$ m

$\therefore$ Wavelength band of stations is 25 m to 40 m.

14 $\therefore \lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^8}{8.2 \times 10^6} = 36.58 \approx 36.6$ m

15 $\therefore \lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^8}{1000} = 3 \times 10^5$ m $= 300$ km

16 Number of oscillations in coherence length $= \dfrac{l}{\lambda} = \dfrac{0.024}{5900 \times 10^{-10}}$

$= 4.068 \times 10^4$

17 $\therefore E = \dfrac{hc}{\lambda} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8}{21 \times 10^{-2}}$

$\approx 10^{-24}$ J

18 It is an electromagnetic wave. The frequency of the microwave oven must match the resonant frequency of the water molecules.

19 Decreasing order of wavelength of various rays is Microwave > Infrared > Ultraviolet > Gamma rays

20 Given, $E = 11$ eV $= 11 \times 1.6 \times 10^{-19}$ V,

$E = h\nu$

or $\nu = \dfrac{11 \times 1.6 \times 10^{-19}}{h}$

$= \dfrac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 2.65 \times 10^{15}$ Hz

This frequency radiation belongs to ultraviolet region.

21 Given, energy of an electromagnetic waves is of the order of 15 keV,

i.e. $E = h\nu = h \times \dfrac{c}{\lambda}$

$\Rightarrow \lambda = \dfrac{h \times c}{E} = \dfrac{6.624 \times 10^{-34} \times 3 \times 10^8}{15 \times 10^3 \times 1.6 \times 10^{-19}}$

$= \dfrac{1.3248 \times 10^{-29}}{1.6 \times 10^{-19}} = 0.828 \times 10^{-10}$ m

$= 0.828$ Å $\quad [\because 1\text{Å} = 10^{-10}\text{ m}]$

$\lambda = 0.828$ Å

Thus, this spectrum is a part of X-rays.

SESSION 2

1 A particle is known that an electric charge at rest has electric field in the region around it, but no magnetic field. A moving charge produces both the electric and magnetic fields. If a charge is moving with a constant velocity, the electric and magnetic fields will not change with time, hence no EM wave will be produced. But, if the charge is moving with a non-zero acceleration, both the electric and magnetic field will change with space and time, it then produces EM wave. This shows that accelerated charge emits electromagnetic waves.

2 Here, $kx = \theta_0$ or $k = \theta_0 / x$ and $\omega t = \theta_0$ or $\omega = \theta_0 / t$

$\therefore \quad \dfrac{k}{\omega} = \dfrac{t}{x}$

$= \dfrac{1}{(x/t)} = \dfrac{1}{v}$

where, v is the velocity of electromagnetic wave, which is independent of wavelength of wave.

3 Total energy falling on the surface,

$U = (18\,\text{W/cm}^2)(30 \times 60\text{ s})(20\text{ cm}^2)$

$U = 6.48 \times 10^5$ J

Total momentum,

$p = \dfrac{U}{c} = \dfrac{6.48 \times 10^5}{3 \times 10^8}$ kg m/s

$\Rightarrow \quad p = 2.16 \times 10^{-3}$ kg m/s

Average force exerted on the surface is

$F = \dfrac{p}{t} = \dfrac{2.16 \times 10^{-3}}{30 \times 60}$

$= 1.2 \times 10^{-6}$ N

4 Electromagnetic wave equation,

$E = E_0 \cos(kz - \omega t) \quad$ ...(i)

Speed of electromagnetic wave,

$v = \dfrac{\omega}{k}$

Given equation,

$\mathbf{E} = \hat{\mathbf{i}}\ 40\cos(kz - 6 \times 10^8 t) \quad$ ...(ii)

On comparing Eqs. (i) and (ii), we get $\omega = 6 \times 10^8$ and $E_0 = 40\,\hat{\mathbf{i}}$

Here, wave factor,

$k = \dfrac{\omega}{v} = \dfrac{6 \times 10^8}{3 \times 10^8} = 2\ \text{m}^{-1}$

5 Here, velocity of EM wave, $\mathbf{v} = v\hat{\mathbf{i}}$

Instantaneous oscillating electric field,

$\mathbf{E} = E\hat{\mathbf{j}}$

As we already know that, during the propagation of EM waves through a medium oscillating electric and magnetic field vectors are mutually perpendicular to each other and to the direction of each other and to the direction of propagation of the wave $(\mathbf{E} \times \mathbf{B})$.

i.e. $\mathbf{E} \times \mathbf{B} = v \Rightarrow (E\hat{\mathbf{j}}) \times \mathbf{B} = v\hat{\mathbf{i}}$ ...(i)

As we know that from vector algebra,

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\mathbf{B} = B\hat{\mathbf{k}},$$

where, B (say) be the magnitude of magnetic field.

Thus, we can say that the direction of oscillating magnetic field of the EM wave will be along $+z$-direction.

6 When a wave is reflected from denser medium, the reflected wave is without change in type of wave, but with a change in phase by $180°$ or π radian. Therefore, for the reflected wave, we use $z = -z$, $\hat{\mathbf{i}} = -\hat{\mathbf{i}}$ and additional phase of π in the incident wave. The incident of electromagnetic wave is, $E = E_0 \hat{\mathbf{i}} \cos(kz - \omega t)$. The reflected electromagnetic wave is

$$\mathbf{E}_r = E_0(-\hat{\mathbf{i}})\cos[k(-z) - \omega t + \pi]$$

$$= -E_0 \hat{\mathbf{i}} \cos[-(kz + \omega t) + \pi]$$

$$= E_0 \hat{\mathbf{i}} \cos[-(kz + \omega t)] \quad [\because \cos(\theta + \pi) = -\cos\theta]$$

$$= E_0 \hat{\mathbf{i}} \cos(kz + \omega t) \quad [\because \cos(-\theta) = \cos\theta]$$

7 Given, root mean square value of electric field,

$$E_{rms} = 6 \text{ V/m}$$

We know that, peak value of electric field,

$$E_0 = \sqrt{2}\, E_{rms}$$

$$\Rightarrow \quad E_0 = \sqrt{2} \times 6 \text{ V/m}$$

Also, we know that, $c = \dfrac{E_0}{B_0}$

where, c = speed of light in vacuum

B_0 = peak value of magnetic field

$$\Rightarrow \quad B_0 = \frac{E_0}{c}$$

$$\Rightarrow \quad B_0 = \frac{\sqrt{2} \times 6}{3 \times 10^8}$$

$$\Rightarrow \quad B_0 = \frac{8.48}{3} \times 10^{-8}$$

$$\Rightarrow \quad B_0 = 2.83 \times 10^{-8} \text{ T}$$

8 $\therefore$ Wavelength,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$$

Hence, option (a) is false.

$$\therefore \quad B_0 = \frac{E_0}{c} = \frac{48}{3 \times 10^8}$$

$$= 16 \times 10^{-8} \text{ T}$$

$$= 1.6 \times 10^{-7} \text{ T}$$

Energy density of electric field is

$$U_C = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

Energy density of magnetic field is

$$U_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0}\left(\frac{B_0}{\sqrt{2}}\right)^2$$

$$= \frac{1}{4\mu} \cdot B_0^2 = \frac{B_0^2}{4\mu_0} \quad \left[\therefore B_0 = \frac{E_0}{c}\right]$$

$$= \frac{\left(\frac{E_0}{c}\right)^2}{4\mu_0} = \frac{1}{4\mu_0^2}\frac{E_0^2}{c^2}$$

$$= \frac{E_0^2}{4\mu_0\left(\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)^2} \quad \left[\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

$$= \frac{\mu_0\varepsilon_0}{4\mu_0} E_0^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

Hence, option (c) is true.

9 Comparing the given equation,

$$E_y = 2.5 \frac{N}{C} \cos\left[\left(2\pi \times 10^6 \frac{\text{rad}}{\text{m}}\right)t - \left(\pi \times 10^{-2} \frac{\text{rad}}{\text{s}}\right)x\right]$$

With the standard equation

$E_y = E_0 \cos(\omega t - kx)$, we get

$$\omega = 2\pi f = 2\pi \times 10^6$$

$$\therefore \quad f = 10^6 \text{ Hz}$$

Moreover, we know that

$$\frac{2\pi}{\lambda} = k = \pi \times 10^{-2} \text{ m}^{-1} \Rightarrow \lambda = 200 \text{ m}$$

As direction of field $\mathbf{E}$ of electromagnetic wave is in y-direction, so the wave is moving along positive x-direction with frequency 10^6 Hz and wavelength 200 m.

10 Field B will not apply any force. Field E will apply a force opposite to velocity of the electron, hence speed will decreases.