

Differentiate the functions given in Exercise 1 to 11 w.r.t x .

QNo. 1 $\cos x, \cos 2x, \cos 3x$.

Sol Let $y = \cos x \cos 2x \cos 3x$

Taking log on both sides

$$\log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$$

Differentiating both sides

$$(\because \log(mn) = \log m + \log n)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x) + \frac{1}{\cos 2x} \cdot \frac{d}{dx}(\cos 2x) + \frac{1}{\cos 3x} \cdot \frac{d}{dx}(\cos 3x)$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{\cos x} (-\sin x) + \frac{1}{\cos 2x} (-\sin 2x \cdot 2) + \frac{1}{\cos 3x} (-\sin 3x \cdot 3)$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -\tan x - 2\tan 2x - 3\tan 3x$$

$$\Rightarrow \frac{dy}{dx} = -y (\tan x + 2\tan 2x + 3\tan 3x)$$

$$= -\cos x \cdot \cos 2x \cos 3x (\tan x + 2\tan 2x + 3\tan 3x)$$

QNo 2

Sol

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} = \left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)} \right]^{\frac{1}{2}}$

Taking log on both sides

$$\log y = \frac{1}{2} [\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$$

Differentiating both sides w.r.t x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2} \left[\frac{1}{(x-1)} (1-0) + \frac{1}{(x-2)} (1-0) - \frac{1}{x-3} (1-0) - \frac{1}{x-4} (1-0) - \frac{1}{x-5} (1-0) \right]$$

$$\frac{dy}{dx} = \frac{y}{2} \left[\left(\frac{1}{x-1} + \frac{1}{x-2} \right) - \left(\frac{1}{x-3} + \frac{1}{x-4} + \frac{1}{x-5} \right) \right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \left[\left(\frac{1}{x-1} + \frac{1}{x-2} \right) - \left(\frac{1}{x-3} + \frac{1}{x-4} + \frac{1}{x-5} \right) \right]$$

Q No. 3

Sol.

$$(\log x)^{\cos x}$$

$$\text{Let } y = (\log x)^{\cos x}$$

Taking log on both sides

$$\log y = \cos x \log (\log x)$$

Differentiating both sides

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \cdot \frac{d}{dx} \log (\log x) + \log (\log x) \cdot \frac{d}{dx} (\cos x)$$

$$= \cos x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log (\log x) (-\sin x)$$

$$= \frac{\cos x}{x \log x} - \sin x \log (\log x)$$

$$\Rightarrow \frac{dy}{dx} = y \left(\frac{\cos x}{x \log x} - \sin x \log (\log x) \right)$$

$$= (\log x)^{\cos x} \left[\frac{\cos x}{x \log x} - \sin x \log (\log x) \right]$$

Q No. 4

$$x^x - 2^{\sin x}$$

Sol

$$\text{Let } y = u - v \quad \text{where } u = x^x \quad v = 2^{\sin x}$$

$$\text{So that } \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} \quad \dots (1)$$

$$\text{Now } u = x^x$$

Taking log on both sides we get

$$\log u = x \log x$$

Differentiating both sides, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\therefore \frac{du}{dx} = u [1 + \log x] = x^x [1 + \log x]$$

$$\text{Also } \frac{dv}{dx} = 2^{\sin x} \cdot \log 2 \cdot \frac{d}{dx} (\sin x) = \log 2 \cdot 2^{\sin x} \cdot \cos x$$

$$\therefore \text{From (1)} \quad \frac{dy}{dx} = x^x(1+\log x) - 2^{\sin x} \cos x \log 2.$$

4.

QNo.5: $(x+3)^2(x+4)^3(x+5)^4$

Sol. Let $y = (x+3)^2(x+4)^3(x+5)^4$

Taking log on both sides

$$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$$

$$\text{i.e. } \log y = 2 \log(x+3) + 3 \log(x+4) + 4 \log(x+5)$$

Differentiating both sides w.r.t x

$$\frac{1}{y} \frac{dy}{dx} = 2 \cdot \frac{1}{x+3} + 3 \cdot \frac{1}{x+4} + 4 \cdot \frac{1}{x+5}$$

$$\text{i.e. } \frac{1}{y} \frac{dy}{dx} = \frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5} \right] = (x+3)^2(x+4)^3(x+5)^4 \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5} \right]$$

QNo.6

$$\left(x + \frac{1}{x}\right)^x + \left(x^{(1+\frac{1}{x})}\right)$$

Sol. Let $y = u+v$ where $u = \left(x + \frac{1}{x}\right)^x$; $v = x^{(1+\frac{1}{x})}$

$$\text{So that } \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{--- (1)}$$

Now

$$u = \left(x + \frac{1}{x}\right)^x$$

Taking log on both sides

$$\log u = \log \left(x + \frac{1}{x}\right)^x = x \log \left(x + \frac{1}{x}\right)$$

Differentiating both sides w.r.t x

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{d}{dx} \log \left(x + \frac{1}{x}\right) + \log \left(x + \frac{1}{x}\right) \cdot \frac{d}{dx} (x)$$

$$\text{i.e. } \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{x + \frac{1}{x}} \cdot \frac{d}{dx} \left(x + \frac{1}{x}\right) + \log \left(x + \frac{1}{x}\right) \cdot 1$$

$$= \frac{x^2}{x^2+1} \left(1 - \frac{1}{x^2}\right) + \log \left(x + \frac{1}{x}\right) \quad \left[\because \frac{d}{dx} \left(\frac{1}{x}\right) = -\frac{1}{x^2} \right]$$

$$= \frac{x^2-1}{x^2+1} + \log \left(x + \frac{1}{x}\right) = \frac{x^2-1}{x^2+1} + \log \left(\frac{x^2+1}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = u \left[\frac{x^2-1}{x^2+1} + \log\left(\frac{x^2+1}{x}\right) \right] = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(\frac{x^2+1}{x}\right) \right]$$

And $v = x^{(1+\frac{1}{x})}$

Taking log on both sides

$$\log v = \log x^{(1+\frac{1}{x})} = \left(1 + \frac{1}{x}\right) \log x$$

Differentiating both sides w.r.t. x.

$$\frac{1}{v} \cdot \frac{dv}{dx} = \left(1 + \frac{1}{x}\right) \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}\left(1 + \frac{1}{x}\right)$$

$$= \left(1 + \frac{1}{x}\right) \cdot \frac{1}{x} + \log x \cdot \left(0 - \frac{1}{x^2}\right)$$

$$= \frac{x+1}{x^2} - \frac{\log x}{x^2} = \frac{x+1}{x^2} - \frac{1}{x^2} \log x$$

$$\therefore \frac{dv}{dx} = v \cdot \left(\frac{x+1}{x^2} - \frac{1}{x^2} \log x \right) = x^{(1+\frac{1}{x})} \left[\frac{x+1}{x^2} - \frac{1}{x^2} \log x \right]$$

$\therefore$ From (i) $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$\Rightarrow \frac{dy}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(\frac{x^2+1}{x}\right) \right] + x^{(1+\frac{1}{x})} \left[\frac{x+1}{x^2} - \frac{1}{x^2} \log x \right]$$

Q No 7

$$(\log x)^x + x^{\log x}$$

Sol: Let $y = u + v$ where $u = (\log x)^x$ and $v = x^{\log x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots (1)$$

Now $u = (\log x)^x$

$$\Rightarrow \log u = x \log(\log x)$$

[Taking log on both side]

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \frac{d}{dx}(\log(\log x)) + \log(\log x) \cdot \frac{d}{dx}(x)$$

$$= \frac{du}{dx} = u \left[x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log(\log x) \cdot 1 \right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right]$$

And $v = (x)^{\log x}$

$$\Rightarrow \log v = \log x \cdot \log x \quad \left[\text{Taking log} \right]$$

$$\Rightarrow \log v = (\log x)^2$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = 2 \cdot \log x \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = v \left(\frac{2 \log x}{x} \right) = 2 \left(\frac{\log x}{x} \right) \cdot x^{\log x}$$

$$\therefore \text{from (1)} \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + 2 \left(\frac{\log x}{x} \right) \cdot x^{\log x}$$

QNo. 8: $(\sin x)^x + \sin^{-1}(\sqrt{x})$

NOTE We can also use the result $\frac{d}{dx}(u^v) = u^v \frac{d}{dx}(v \log u)$

$$\therefore \frac{d}{dx} \left[(\sin x)^x + \sin^{-1} \sqrt{x} \right] = \frac{d}{dx} (\sin x)^x + \frac{d}{dx} (\sin^{-1}(x)^{1/2})$$

$$= (\sin x)^x \cdot \frac{d}{dx} (x \log \sin x) + \frac{1}{\sqrt{1-(x^{1/2})^2}} \cdot \frac{d}{dx} (x)^{1/2}$$

$$= (\sin x)^x \left[x \cdot \frac{1}{\sin x} \cos x + \log(\sin x) \cdot 1 \right] + \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-1/2}$$

$$= (\sin x)^x \left[x \cdot \cot x + \log(\sin x) \right] + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

QNo 9: $x^{\sin x} + (\sin x)^{\cos x}$

$$\therefore \frac{d}{dx}(u^v) = u^v \cdot \frac{d}{dx} [v \log u]$$

$$\therefore \frac{d}{dx} \left[x^{\sin x} + (\sin x)^{\cos x} \right] = \frac{d}{dx} (x^{\sin x}) + \frac{d}{dx} (\sin x)^{\cos x}$$

$$\begin{aligned}
 &= x^{\sin x} \left[\sin x \cdot \log x \right] + (\sin x)^{\cos x} \left[\cos x \log (\sin x) \right] \quad (7) \\
 &= x^{\sin x} \left[\sin x \cdot \frac{1}{x} + \log x \cdot \cos x \right] + (\sin x)^{\cos x} \left[\cos x \cdot \frac{1}{\sin x} \cdot \cos x + \log (\sin x) (-\sin x) \right] \\
 &= x^{\sin x} \left[\frac{\sin x}{x} + (\log x)(\cos x) \right] + (\sin x)^{\cos x} \left[\cos x \cot x - (\sin x) \log (\sin x) \right]
 \end{aligned}$$

Q No. 10

$$x^{x \cos x} + \frac{x^2+1}{x^2-1}$$

$$\therefore \frac{d}{dx} \left[x^{x \cos x} + \frac{x^2+1}{x^2-1} \right] = \frac{d}{dx} \left[x^{x \cos x} \right] + \frac{d}{dx} \left[\frac{x^2+1}{x^2-1} \right]$$

$$= x^{x \cos x} \left[x \cos x \cdot \log x \right] + \frac{(x^2-1) \frac{d}{dx}(x^2+1) - (x^2+1) \frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= x^{x \cos x} \left[x \cdot \cos x \cdot \frac{1}{x} + \log x [x(-\sin x) + \cos x \cdot 1] \right] + \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$= x^{x \cos x} \left[\frac{x \cos x}{x} + \log x (-x \sin x + \cos x) \right] + \frac{-4x}{(x^2-1)^2}$$

Q No. 11 $\frac{d}{dx} \left[(x \cos x)^x + (x \sin x)^{1/x} \right] = \frac{d}{dx} (x \cos x)^x + \frac{d}{dx} (x \sin x)^{1/x}$

$$= (x \cos x)^x \frac{d}{dx} \left[x \cdot \log (x \cos x) \right] + (x \sin x)^{1/x} \frac{d}{dx} \left[\frac{1}{x} \log (x \sin x) \right]$$

$$\left[\because \frac{d}{dx} (u^v) = u^v \cdot \frac{d}{dx} (v \log u) \right]$$

$$\begin{aligned}
 &= (x \cos x)^x \left[x \cdot \frac{1}{x \cos x} [x(-\sin x) + \cos x \cdot 1] + \log (x \cos x) \cdot 1 \right] + (x \sin x)^{1/2} \left[\frac{1}{x} \cdot \frac{1}{x \sin x} [x \cos x + \sin x \cdot 1] + \log (x \sin x) (-1/x^2) \right] \\
 &= (x \cos x)^x \left[\frac{\cos x - x \sin x}{\cos x} + \log (x \cos x) \right] + (x \sin x)^{1/2} \left[\frac{\sin x + x \cos x}{x^2 \sin x} - \frac{\log (x \sin x)}{x^2} \right]
 \end{aligned}$$

Find $\frac{dy}{dx}$ of the functions given in exercise 12 to 15

(8)

QNo 12 $x^y + y^x = 1$

differentiating both sides we get.

$$\frac{d}{dx}(x^y + y^x) = 0 \Rightarrow \frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = 0$$

$$\Rightarrow x^y \cdot \frac{d}{dx}(y \log x) + y^x \cdot \frac{d}{dx}(x \log y) = 0 \quad \left[\because \frac{d}{dx}(u^v) = u^v \cdot \frac{d}{dx}(v \log u) \right]$$

$$\Rightarrow x^y \left[y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} \right] + y^x \left[x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \right] = 0$$

$$\Rightarrow y x^{y-1} + x^y \log x \cdot \frac{dy}{dx} + x y^{x-1} \frac{dy}{dx} + y^x \log y = 0$$

$$\Rightarrow (x^y \log x + x y^{x-1}) \frac{dy}{dx} = -y x^{y-1} - y^x \log y$$

$$\Rightarrow \frac{dy}{dx} = - \left(\frac{y x^{y-1} + y^x \log y}{x^y \log x + x y^{x-1}} \right)$$

QNo 13

Given $y^x = x^y$

Taking $\log$ we get $x \log y = y \log x$

Differentiating both sides

$$x \left(\frac{1}{y} \right) \frac{dy}{dx} + \log y \cdot 1 = y \cdot \frac{1}{x} + (\log x) \frac{dy}{dx}$$

$$\Rightarrow \frac{x}{y} \frac{dy}{dx} + (\log y) \frac{dy}{dx} = \frac{y}{x} + \log x$$

$$\Rightarrow \left(\frac{x}{y} + \log y \right) \frac{dy}{dx} = \frac{y}{x} + \log x \Rightarrow \left(\frac{x + y \log y}{y} \right) \frac{dy}{dx} = \frac{y + x \log x}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y + x \log y}{x} \times \frac{y}{y + x \log x} = \frac{y}{x} \left(\frac{y + x \log y}{y + x \log x} \right)$$

QNo.14

$$(\cos x)^y = (\cos y)^x$$

$$\Rightarrow y \log(\cos x) = x \log(\cos y) \quad \left[\text{Taking log on both sides} \right]$$

Differentiating both sides w.r.t. x

$$y \left(\frac{1}{\cos x} \right) (-\sin x) + \log(\cos x) \cdot \frac{dy}{dx} = x \left(\frac{1}{\cos y} \right) (-\sin y) \cdot \frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow \log(\cos x) \frac{dy}{dx} + x \tan y \frac{dy}{dx} = \log(\cos y) + y \tan x$$

$$\Rightarrow \left(\log(\cos x) + x \tan y \right) \frac{dy}{dx} = \log(\cos y) + y \tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log(\cos y) + y \tan x}{\log(\cos x) + x \tan y}$$

QNo.15

$$xy = e^{x-y}$$

$$\Rightarrow \frac{d}{dx}(xy) = \frac{d}{dx}(e^{x-y}) \Rightarrow \left(x \frac{dy}{dx} + y \cdot 1 \right) = e^{x-y} \frac{d}{dx}(x-y)$$

$$\Rightarrow x \frac{dy}{dx} + y = e^{x-y} \left(1 - \frac{dy}{dx} \right)$$

$$\text{or } x \frac{dy}{dx} + e^{x-y} \frac{dy}{dx} = e^{x-y} - y$$

$$\text{or } \frac{dy}{dx} (x + e^{x-y}) = e^{x-y} - y$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{e^{x-y} - y}{x + e^{x-y}} \right] \Rightarrow \frac{dy}{dx} = \left[\frac{xy - y}{x + xy} \right] \quad \left[\because e^{x-y} = xy \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$$

QNo.16: Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$

Sol: Taking log on both sides we get

$$\log(f(x)) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$

$$\left[\because \log(mn) = \log m + \log n \right]$$

Differentiating both sides, we get.

$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{1+x}(0+1) + \frac{1}{1+x^2}(0+2x) + \frac{1}{1+x^4}(0+4x^3) + \frac{1}{1+x^8}(0+8x^7)$$

$$\therefore f'(x) = f(x) \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} \right]$$

$$\therefore f'(1) = f(1) \left[\frac{1}{1+1} + \frac{2 \times 1}{1+1^2} + \frac{4(1)^3}{1+(1)^4} + \frac{8(1)^7}{1+(1)^8} \right]$$

$$= \left((1+1)(1+1^2)(1+1^4)(1+1^8) \right) \left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2} \right]$$

$$= (2 \times 2 \times 2 \times 2) \left(\frac{1+2+4+8}{2} \right) = 2 \times 2 \times 2 \times 15 = 120.$$

Q No. 17 Differentiate $(x^2-5x+8)(x^3+7x+9)$ in three ways as below:

(i) By using product rule.

(ii) By expanding the product to obtain a single polynomial.

(iii) By logarithmic differentiation.

Do they all give the same answer?

Sol.

$$\text{Let } y = (x^2-5x+8)(x^3+7x+9)$$

$$(i) \text{ Using product rule } \frac{dy}{dx} = (x^2-5x+8) \left(\frac{d}{dx}(x^3+7x+9) \right) + (x^3+7x+9) \frac{d}{dx}(x^2-5x+8)$$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= (x^2-5x+8)(3x^2+7) + (x^3+7x+9)(2x-5) \\ &= 3x^4 - 15x^3 + 24x^2 + 7x^2 - 35x + 56 + 2x^4 + 14x^2 + 18x \\ &\quad - 5x^3 - 35x - 45 \\ &= 5x^4 - 20x^3 + 45x^2 - 52x + 11 \end{aligned}$$

$$(ii) \quad y = (x^2-5x+8)(x^3+7x+9) = x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$$

$$\therefore \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$$

(iii) Taking log on both sides

$$\log y = \log(x^2 - 5x + 8) + \log(x^3 + 7x + 9)$$
$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{2x-5}{x^2-5x+8} + \frac{3x^2+7}{x^3+7x+9} \Rightarrow \frac{dy}{dx} = y \left[\frac{(2x-5)(x^3+7x+9) + (3x^2+7)(x^2-5x+8)}{(x^2-5x+8)(x^3+7x+9)} \right]$$

$$= (2x-5)(x^3+7x+9) + (3x^2+7)(x^2-5x+8)$$
$$= 2x^4 + 14x^2 + 18x - 5x^3 - 35x - 45 + 3x^4 - 15x^3 + 24x^2 + 7x^2 - 35x + 56$$
$$= 5x^4 - 20x^3 + 45x^2 - 52x + 1$$

We find that result obtained is same in every case.

Q.No.18: If u, v, w are functions of x , then show that

$$\frac{d}{dx}(u \cdot v \cdot w) = \frac{du}{dx}(v \cdot w) + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx} \text{ in two ways,}$$

- first by repeated application of product rule, second by logarithmic differentiation.

Sol. First we use Product Rule and find that

$$\frac{d}{dx}(uvw) = \frac{d}{dx}[(uv) \cdot w] = uv \cdot \frac{dw}{dx} + w \cdot \frac{d}{dx}(uv)$$
$$= uv \cdot \frac{dw}{dx} + w \left[u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx} \right]$$
$$= uv \cdot \frac{dw}{dx} + w \cdot u \cdot \frac{dv}{dx} + w \cdot v \cdot \frac{du}{dx} \quad \dots (1)$$

Second by taking log on both sides of $y = uvw$

$$\Rightarrow \log y = \log u + \log v + \log w.$$

$$\therefore \frac{1}{y} \frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx} + \frac{1}{v} \cdot \frac{dv}{dx} + \frac{1}{w} \cdot \frac{dw}{dx}$$

$$\begin{aligned}
 \Rightarrow \frac{dy}{dx} &= y \left[\frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx} \right] \\
 &= (uvw) \left[\frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx} \right] \\
 &= (vw) \frac{du}{dx} + (uw) \frac{dv}{dx} + (uv) \frac{dw}{dx}
 \end{aligned}$$

$$\therefore \frac{d}{dx}(uvw) = uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx} \dots (2)$$

From (i) and (ii) we find that Result obtained is same in both the cases.

