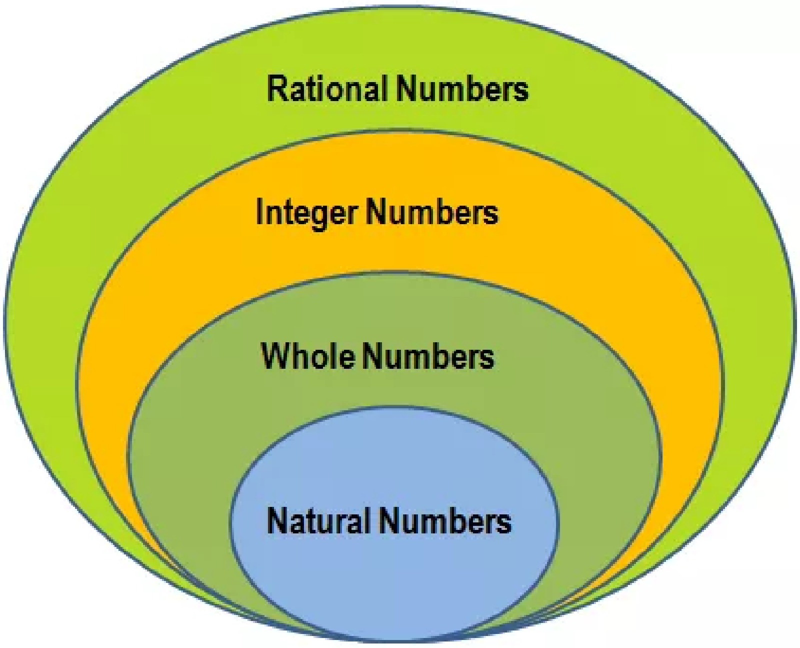
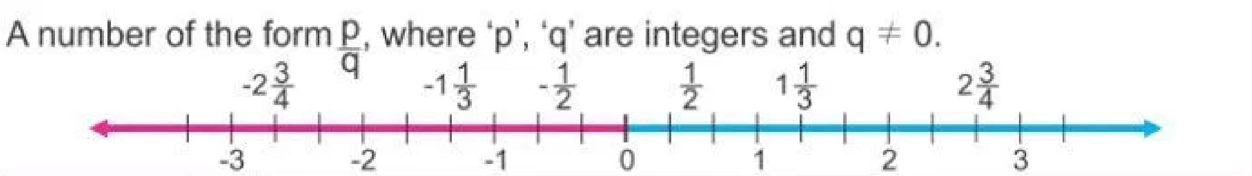
10. Rational and Irrational Numbers

What is a Rational Number?

A rational number is a number which can be put in the form $\frac{p}{q}$, where p and q are both integers and q ≠ 0.
p is called numerator (N^r) and q is called denominator (D^r).



- A rational number is either a terminating or non-terminating but recurring (repeating) decimal.
- A rational number may be positive, negative or zero.
  Examples:
  $3, 4, \frac{7}{3}, \frac{5}{2}, -\frac{3}{7}, 2.7, 3.923, 1.42\overline{7}, 1.2343434$, etc.
- The sum, difference and the product of two rational numbers is always a rational number.
- The quotient of a division of one rational number by a non-zero rational number is a rational number. Rational numbers satisfy the closure property under addition, subtraction, multiplication and division.



Property	Operations on Rational Numbers			
Name	Addition	Subtraction	Multiplication	Division*
Closure	a + b ∈ Q	a - b ∈ Q	a × b ∈ Q	a ÷ b ∈ Q
Commutative	a + b = b + a	a - b ≠ b - a	a × b = b × a	a ÷ b ≠ b ÷ a
Associative	(a + b) + c = a + (b + c)	(a - b) - c ≠ a - (b - c)	(a × b) × c = a × (b × c)	(a ÷ b) ÷ c ≠ a ÷ (b ÷ c)
Distributive	a × (b + c) = ab + ac	a × (b - c) = ab - ac	Not applicable	Not applicable
Where a, b, c ∈ Q (set of rational numbers), *b is a non-zero rational number				

Results

Since every number is divisible by 1, we can say that :

1. Every natural number is a rational number, but every rational number need not be a natural number.

   For example, 3 = $\frac{3}{1}$, 5 = $\frac{5}{1}$, 9 = $\frac{9}{1}$ and so on.

   but, $\frac{7}{9}, \frac{11}{13}, \frac{5}{7}$ are rational numbers but not natural numbers.
2. Zero is a rational number because $\left(0 = \frac{0}{1} = \frac{0}{2} = \ldots\right)$.
3. Every integer is a rational number, but every rational number may not be an integer.

   For example $\frac{-2}{1}, \frac{-5}{1}, \frac{0}{1}, \frac{3}{1}, \frac{5}{1}$, etc. are all rationals, but rationals like $\frac{3}{2}, \frac{-5}{2}$ etc. are not integers.
4. Rational numbers can be positive and negative.

   Eg : $\frac{2}{3}, \frac{-7}{-8}, \frac{8}{11}, \frac{-9}{-3}$ etc. are positive rational numbers

   & $\frac{-2}{3}, \frac{7}{-8}, \frac{-8}{11}, \frac{11}{-20}$ etc. are negative rational numbers.
5. Every positive rational number is greater than zero.
6. Every negative rational number is less than zero.
7. Every positive rational number is greater than every negative rational number.
8. Every negative rational number is smaller than every positive rational number.

Equivalent Rational Numbers

∵ Rational no. can be written with different N^r and D^r.

Eg :

$\frac{-5}{7} = \frac{-5\times 2}{7\times 2} = \frac{-10}{14}$ ∴ $\frac{-5}{7}$ is same as $\frac{-10}{14}$

$\frac{-5}{7} = \frac{-5\times 3}{7\times 3} = \frac{-15}{21}$ $\frac{-5}{7}$ is same as $\frac{-15}{21}$

$\frac{-5}{7} = \frac{-5\times -1}{7\times -1} = \frac{5}{-7}$ $\frac{-5}{7}$ is same as $\frac{5}{-7}$

Such rational number that are equal to each other are said to be equivalent to each other.

Example: Write $\frac{2}{5}$ in an equivalent form so that the numerator is equal to −56.

Solution:

Multiplying both the numerator and denominator of $\frac{2}{5}$ by −28, we have

$$\frac{2\times(-28)}{5\times(-28)} = \frac{-56}{-140}.$$

Lowest Form of a Rational Number

A rational number is said to be in lowest form if the numerator and the denominator have no common factor other than 1.

Example: Write the following rational numbers in the lowest form :

(i) $\frac{-36}{180}$ (ii) $\frac{-64}{256}$

Solution:

(i) Here, HCF of 36 and 180 is 36, therefore, we divide the numerator and denominator of $\frac{-36}{180}$ by 36, we have

$$\frac{-36 \div 36}{180 \div 36} = \frac{-1}{5}$$

So, the lowest form of $\frac{-36}{180}$ is $\frac{-1}{5}$

(ii) Here, HCF of 64 and 256 is 64.

Dividing the numerator and denominator of $\frac{-64}{256}$ by 64, we have

$$\frac{-64 \div 64}{256 \div 64} = \frac{-1}{4}$$

So, the lowest form of $\frac{-64}{256}$ is $\frac{-1}{4}$.

Standard Form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in its standard form if
(i) its denominator 'q' is positive
(ii) the numerator and denominator have no common factor other than 1.
For example : $\frac{3}{2}$, $\frac{-5}{2}$, $\frac{1}{7}$, etc.

Example: Express the rational number $\frac{14}{-21}$ in standard form.

Solution:

The given rational number is $\frac{14}{-21}$.

1. Its denominator is negative. Multiply both the numerator and denominator by −1 to change it to positive, i.e.,

$$\frac{14}{-21} = \frac{14 \times (-1)}{(-21) \times (-1)} = \frac{-14}{21}$$

2. The greatest common divisor of 14 and 21 is 7. Dividing both numerator and denominator by 7, we have

$$\frac{-14}{21} = \frac{(-14) \div 7}{21 \div 7} = \frac{-2}{3}$$

which is the required answer.

Equality of Rational Numbers

Method-1: If two or more rational numbers have the same standard form, we say that the given rational numbers are equal.

Example: Are the rational numbers $\frac{8}{-12}$ and $\frac{-50}{75}$ equal?

Solution: We first express these given rational numbers in the standard form.

The first rational number is $\frac{8}{-12}$.

(i) Multiplying both the numerator and denominator by −1.

$$\text{We have, } \frac{8}{-12} = \frac{8 \times (-1)}{(-12) \times (-1)} = \frac{-8}{12}$$

(ii) Dividing both the numerator and denominator by the greatest common divisor of 8 and 12, which is 4.

$$\text{We have, } \frac{8}{-12} = \frac{(-8) \div 4}{12 \div 4} = \left[\frac{-2}{3}\right]$$

Again, the second rational number is $\frac{-50}{75}$.

(i) The denominator is positive.

(ii) Dividing both numerator and denominator by the greatest common divisor of 50 and 75, which is 25.

$$\text{We have, } \frac{-50}{75} = \frac{(-50) \div 25}{75 \div 25} = \left[\frac{-2}{3}\right]$$

Clearly, both the rational numbers have the same standard form.

Therefore, $\frac{8}{-12} = \frac{-50}{75}$

Method-2: In this method, to test the equality of two rational numbers, say $\frac{a}{b}$ and $\frac{c}{d}$, we use cross multiplication in the following way : $\frac{a}{b} = \frac{c}{d}$

Then a × d = b × c

If a × d = b × c, we say that the two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

Example: Check the equality of the rational numbers $\frac{-7}{21}$ and $\frac{3}{-9}$.

Solution:

The given rational numbers are

$\frac{-7}{21}$ and $\frac{3}{-9}$.

By cross multiplication, we get

$$(-7) \times (-9) = 21 \times 3$$

i.e., $63 = 63$.

Clearly, both sides are same. Thus, we can say that $\frac{-7}{21} = \frac{3}{-9}$.

Comparison of Rational Numbers

Comparing fraction. We compare two unequal fractions, each is written as another equal fraction so that both have the same denominators. Then the fraction with greater numerator is greater.

Example : To compare $\frac{7}{6}$ and $\frac{5}{8}$, find the L.C.M. of 6 and 8 (it is 24) and

$$\frac{7}{6} = \frac{7 \times 4}{6 \times 4} = \frac{28}{24}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

As $\frac{28}{24} > \frac{15}{24}$ (as $28 > 15$)

$\Rightarrow \frac{7}{6} > \frac{5}{8}$

> Quicker method of comparison of $\frac{a}{b}$ and $\frac{c}{d}$ is that $\frac{a}{b} > \frac{c}{d}$ if $ad > bc$.
>
> $\frac{7}{6} > \frac{5}{8}$ as $(7 \times 8 > 6 \times 5)$

To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

For example,

$\frac{-9}{13}$ and $\frac{-5}{3}$,

we first compare $\frac{9}{13}$ and $\frac{5}{3}$.

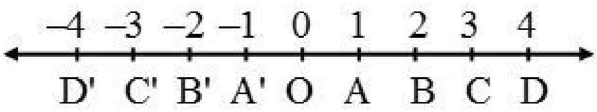
$$\frac{9}{13} < \frac{5}{3} \qquad (\because 9 \times 3 < 13 \times 5 \Rightarrow 27 < 65)$$

and conclude that $\frac{-9}{13} > \frac{-5}{3}$.

Note : Every positive rational number is greater than negative rational number.

Representation of Rational Numbers on Number Line

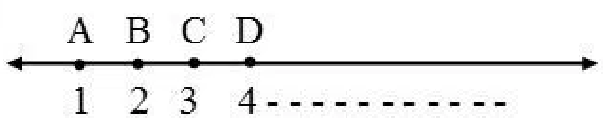
We know that the natural numbers, whole numbers and integers can be represented on a number line. For representing an integer on a number line, we draw a line and choose a point O on it to represent '0'. We can represent this point 'O' by any other alphabet also. Then we mark points on the number line at equal distances on both sides of O. Let A, B, C, D be the points on the right hand side and A′, B′, C′, D′ be the points on the left of O as shown in the figure.



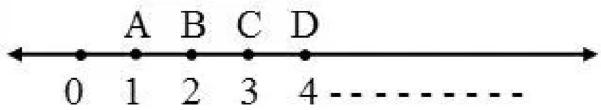
The points on the left side of O, i.e., A′, B′, C′, D′, etc. represent negative integers −1, −2, −3, −4 whereas, points on the right side of O, i.e., A, B, C, D represent positive integers 1, 2, 3, 4 etc. Clearly,

the points A and A′ representing the integers 1 and −1 respectively are on opposite sides of O, but at equal distance from O. Same is true for B and B′ ; C and C′ and other points on the number line.

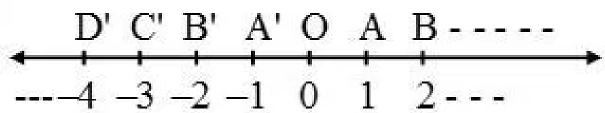
(1) Natural Numbers



(2) Whole Numbers



(3) Integers



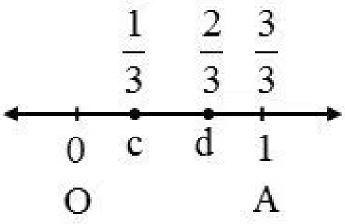
Negative numbers are in left side of zero (0) & positive numbers are in right side.
∵ negative numbers are less than positive numbers
∴ If we move on number line from right to left we are getting smaller numbers.
Also OA = distance of 1 from 0
OD′ = distance of −4 from 0
D′A = distance between −4 and 1. etc.

(4) Rational Numbers

(a) If $N^r < D^r$:

We divide line segment OA (i.e. distance between 0 & 1) in equal parts as denominator (Dr).

Eg : $\frac{1}{3}$ and $\frac{2}{3}$ =



$\because$ D^r is 3, so we divide OA in three equal parts by points c and d.

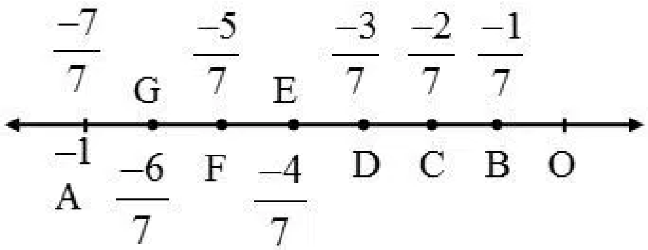
$\therefore \; c = \frac{1}{3}$, $d = \frac{2}{3}$ and $A = \frac{3}{3} = 1$

Eg : $\frac{-1}{7}$ and $\frac{-4}{7}$

$\because$ D^r is 7 $\therefore$ we divide OA in 7 equal parts by points B, C, D, E, F, G. So, these points represent

$\frac{-1}{7}, \frac{-2}{7}, \frac{-3}{7}, \frac{-4}{7}, \frac{-5}{7}, \frac{-6}{7}, \frac{-7}{7}$ respectively.

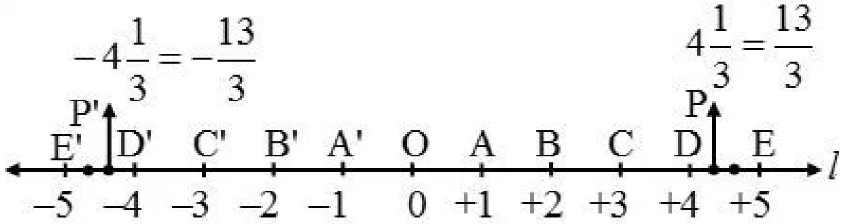
$\therefore$ $\frac{-1}{7}$ by B and $\frac{-4}{7}$ by E.



(b) If $N^r > D^r$:

Example: Represent $\frac{13}{3}$ and $-\frac{13}{3}$ on number line.

Solution:



Draw a line l and mark zero on it

$$\frac{13}{3} = 4\frac{1}{3} = 4 + \frac{1}{3} \quad \text{and} \quad \frac{-13}{3} = -\left(4 + \frac{1}{3}\right)$$

Therefore, from O mark OA, AB, BC, CD and DE to the right of O such that OA = AB = BC = CD = DE = 1 unit.
Clearly,
Point A,B,C,D,E represents the Rational numbers 1, 2, 3, 4, 5 respectively.

Since we have to consider 4 complete units and a part of the fifth unit, therefore divide the fifth unit DE into 3 equal parts. Take 1 part out of these 3 parts. Then point P is the representation of number $\frac{13}{3}$ on the number line. Similarly, take 4 full unit lengths to the left of 0 and divide the fifth unit D'E' into 3 equal parts. Take 1 part out of these three equal parts. Thus, P' represents the rational number $-\frac{13}{3}$.

Rational Number Example Problems With Solutions

Example 1: Is zero a rational number? can you write it in the form , where p and q are integers and q ≠ 0?

Solution:

Yes, zero is a rational number.

It can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc.

where denominator q ≠ 0, it can be negative also.

Example 2: Find five rational numbers between 3/5 and 4/5.

Solution:

A rational number between r and s is $\frac{r+s}{2}$.

A rational number between

$\frac{3}{5}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{7}{10}$.

And a rational number between

$\frac{3}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{3}{5} + \frac{7}{10}\right) = \frac{13}{20}$

Similarly; $\frac{5}{8}, \frac{27}{40}, \frac{31}{40}$ are between $\frac{3}{5}$ and $\frac{4}{5}$.

So, five rational number between

$\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{5}{8}, \frac{13}{20}, \frac{7}{10}, \frac{31}{40}, \frac{27}{40}$

Example 3: Find six rational numbers between 3 and 4.

Solution: We can solve this problem in two ways.

Method 1:

A rational number between r and s is $\frac{r+s}{2}$.

Therefore, a rational number between 3 and 4 = $\frac{1}{2}(3+4) = \frac{7}{2}$

A rational number between 3 and $\frac{7}{2} = \frac{1}{2}\ \frac{6+7}{2} = \frac{13}{4}$.

We can accordingly proceed in this manner

to find three more rational numbers between 3 and 4.

Hence, six rational numbers between 3 and 4

are $\frac{15}{8}, \frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}$.

Method 2 :

Since, we want six numbers,

we write 3 and 4 as rational numbers with denominator 6 + 1,

i.e., $3 = \frac{21}{7}$ and $4 = \frac{28}{7}$.

Then we can check that $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$,

and $\frac{27}{7}$ are all between 3 and 4.

Hence, the six numbers between 3 and 4

are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$, and $\frac{27}{7}$

Example 4: Find two rational & two irrational numbers between 4 and 5.

Solution:

Rational numbers $\frac{4+5}{2} = 4.5$ **Ans.**

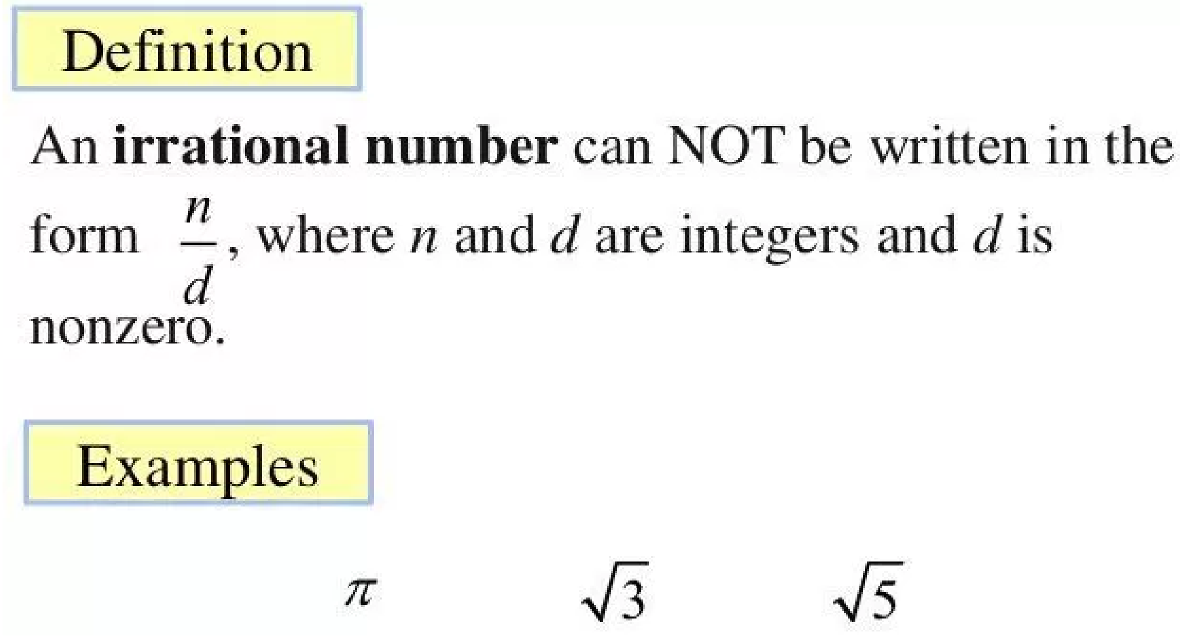
& $\frac{4.5+4}{2} = \frac{8.5}{2} = 4.25$ **Ans.**

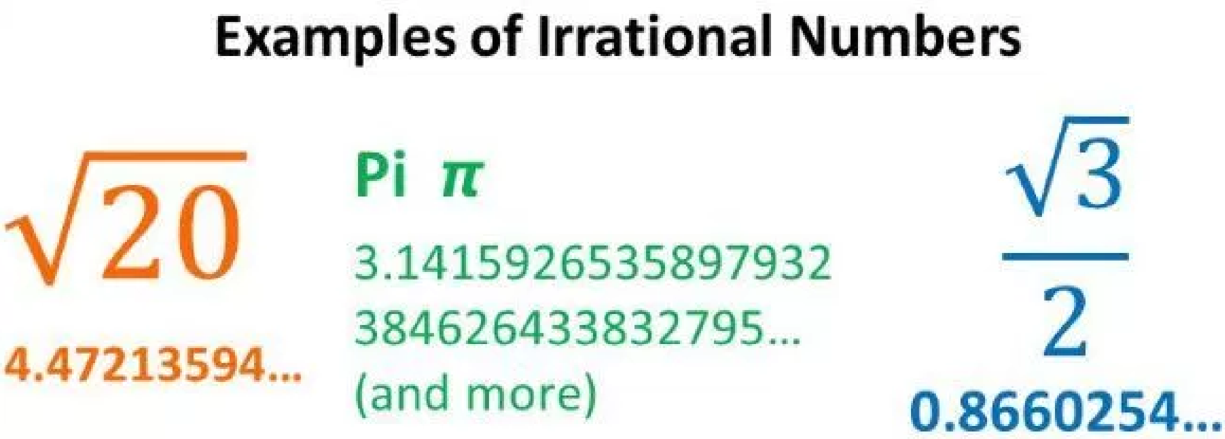
Irrational numbers 4.12316908………

4.562381032…….

What Is Irrational Number

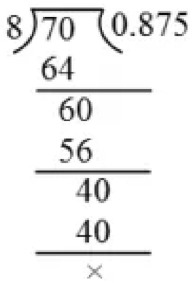
1. A number is irrational if and only if its decimal representation is non-terminating and non-repeating. e.g.$\sqrt{2}, \sqrt{3}, \Pi$............... etc.





2. Rational number and irrational number taken together form the set of real numbers.
3. If a and b are two real numbers, then either
   (i) a > b or (ii) a = b or (iii) a < b
4. Negative of an irrational number is an irrational number.
5. The sum of a rational number with an irrational number is always irrational.
6. The product of a non-zero rational number with an irrational number is always an irrational number.
7. The sum of two irrational numbers is not always an irrational number.
8. The product of two irrational numbers is not always an irrational number.
9. In division for all rationals of the form $\frac{p}{q}$(q ≠ 0), p & q are integers, two things can happen either the remainder becomes zero or never becomes zero.

Type (1) Example: $\frac{7}{8}$ = 0.875



This decimal expansion 0.875 is called **terminating**.

∴ If remainder is zero then decimal expansion ends (terminates) after finite number of steps. These decimal expansion of such numbers terminating.

Type (2) Example: $\frac{1}{3}$ = 0.333……… = $0.\overline{3}$

$3\overline{)10}(0.33\ldots\ldots$
9
10
9
1……

or $\frac{1}{7}$ = 0.142857142857….. = $0.\overline{142857}$

$7\overline{)10}(0.14285\ldots$
7
30
28
20
14
60
56
40
35
50
49
1….

In both examples remainder is never becomes zero so the decimal expansion is never ends after some or infinite steps of division. These type of decimal expansions are called **non terminating.**

In above examples, after Ist step & 6 steps of division (respectively) we get remainder equal to dividend so decimal expansion is repeating (recurring).
So these are called **non terminating recurring decimal expansions**.
Both the above types (1 & 2) are rational numbers.

Types (3) Example: The decimal expansion 0.327172398……is not ends any where, also there is no arrangement of digits (not repeating) so these are called **non terminating not recurring**. These numbers are called **irrational numbers**.

Example:

0.1279312793	rational	terminating
0.1279312793…. or $0.\overline{12793}$	rational	non terminating recurring
0.32777	rational	terminating
or $0.32\overline{7}$ 0.32777…….	rational	non terminating & recurring
0.5361279	rational	terminating
0.3712854043….	irrational	non terminating non recurring
0.10100100010000	rational	terminating
0.10100100010000….	irrational	non terminating non recurring.

Irrational Number Example Problems With Solutions

Example 1: Insert a rational and an irrational number between 2 and 3.

Sol. If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b. Also, if a,b are rational numbers, then $\frac{a+b}{2}$ is a rational number between them.
∴ A rational number between 2 and 3 is
$\frac{2+3}{2}$ = 2.5

An irrational number between 2 and 3 is
= $\sqrt{2 \times 3}$ = $\sqrt{6}$

Example 2: Find two irrational numbers between 2 and 2.5.

Sol. If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then is an irrational number lying between a and b.
∴ Irrational number between 2 and 2.5 is
= $\sqrt{2 \times 2.5}$ = $\sqrt{5}$
Similarly, irrational number between 2 and $\sqrt{5}$ is $\sqrt{2 \times \sqrt{5}}$
So, required numbers are $\sqrt{5}$ and $\sqrt{2 \times \sqrt{5}}$

Example 3: Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol. We know that, if a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.
∴ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is = $\sqrt{\sqrt{2} \times \sqrt{3}}$ = $6^{1/4}$
Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2} \times 6^{\frac{1}{4}}}$ = $2^{1/4} \times 6^{1/8}$.
Hence required irrational number are $6^{1/4}$ and
$2^{1/4} \times 6^{1/8}$.
Example 4: Find two irrational numbers between 0.12 and 0.13.

Sol. Let a = 0.12 and b = 0.13. Clearly, a and b are rational numbers such that a < b.
We observe that the number a and b have a 1 in the first place of decimal. But in the second place of decimal a has a 2 and b has 3. So, we consider the numbers
c = 0.1201001000100001 ……
and, d = 0.12101001000100001…….
Clearly, c and d are irrational numbers such that a < c < d < b.

Example 5: Prove that is $\sqrt{2}$ irrational number

Sol. Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers r and s (≠0) such that $\sqrt{2} = \frac{r}{s}$. Suppose r and s not having a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$ where a and b are coprime.
So, b$\sqrt{2}$ = a.
Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2. Now, by Theorem it following that 2 divides a.
So, we can write a = 2c for some integer c.
Substituting for a, we get $2b^2 = 4c^2$, that is,
$b^2 = 2c^2$.
This means that 2 divides b^2, and so 2 divides b (again using Theorem with p = 2).
Therefore, a and b have at least 2 as a common factor.
But this contradicts the fact that a and b have no common factors other than 1.
This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.
So, we conclude that $\sqrt{2}$ is irrational.

Example 6: Prove that is $\sqrt{3}$ irrational number.

Sol. Let us assume, to contrary, that is rational. That is, we can find integers a and b (≠0) such that $\sqrt{2} = \frac{a}{b}$. Suppose a and b not having a common factor other than 1, then we can divide by the common factor, and assume that a and b are coprime.
So, b$\sqrt{3}$ = a.
Squaring on both sides, and rearranging, we get $3b^2 = a^2$.
Therefore, a^2 is divisible by 3, and by Theorem, it follows that a is also divisible by 3.
So, we can write a = 3c for some integer c.
Substituting for a, we get $3b^2 = 9c^2$, that is,
$b^2 = 3c^2$.

This means that b^2 is divisible by 3, and so b is also divisible by 3 (using Theorem with p = 3).
Therefore, a and b have at least 3 as a common factor.
But this contradicts the fact that a and b are coprime.
This contradicts the fact that a and b are coprime.
This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.
So, we conclude that $\sqrt{3}$ is irrational.

Example 7: Prove that $7-\sqrt{3}$ is irrational

Sol. Method I :
Let $7-\sqrt{3}$ is rational number
∴ $7-\sqrt{3} = \frac{p}{q}$ (p, q are integers, q ≠ 0)
∴ $7 - \frac{p}{q} = \sqrt{3}$
⇒ $\sqrt{3} = \frac{7q-p}{q}$
Here p, q are integers
∴ $\frac{7q-p}{q}$ is also integer
∴ LHS = $\sqrt{3}$ is also integer but this $\sqrt{3}$ is contradiction that is irrational so our assumption is wrong that $7-\sqrt{3}$ is rational
∴ $7-\sqrt{3}$ is irrational proved.

Method II :

Let $7-\sqrt{3}$ is rational
we know sum or difference of two rationals is also rational
∴ $7-7-\sqrt{3}$
= $\sqrt{3}$ = rational
but this is contradiction that $\sqrt{3}$ is irrational
∴ $7-\sqrt{3}$ is irrational proved.

Example 8: Prove that $\frac{\sqrt{5}}{3}$ is irrational.

Sol. Let $\frac{\sqrt{5}}{3}$ is rational
∴ $3\left(\frac{\sqrt{5}}{3}\right) = \sqrt{5}$ is rational
(∵ Q product of two rationals is also rational)
but this is contradiction that $\sqrt{5}$ is irrational
∴ $\frac{\sqrt{5}}{3}$ is irrational proved.

Example 9: Prove that $2\sqrt{7}$ is irrational.

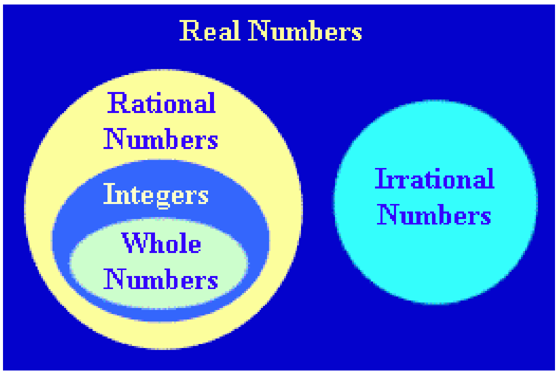
Sol. Let is rational
∴ $2\sqrt{7} \times \left(\frac{1}{2}\right) = \sqrt{7}$
(∵ Q division of two rational no. is also rational)
∴ $\sqrt{7}$is rational
but this is contradiction that is irrational
∴ $2\sqrt{7}$ is irrational

Example 10: Find 3 irrational numbers between 3 & 5.

Solution: ∵ 3 and 5 both are rational
The irrational are 3.127190385..............
3.212325272930.........
3.969129852937...........

Rational and Irrational Numbers

Both rational and irrational numbers are real numbers.



This Venn Diagram shows the relationships between sets of numbers. Notice that rational and irrational numbers are contained in the large blue rectangle representing the set of Real Numbers.

1. A **rational number** is a number that can be expressed as a fraction or ratio. The numerator and the denominator of the fraction are both integers.
2. When the fraction is divided out, it becomes a terminating or repeating decimal. (The repeating decimal portion may be one number or a billion numbers.)
3. Rational numbers can be ordered on a number line.

Examples of rational numbers are:

6 or $\frac{6}{1}$	can also be written as	**6.0**
-2 or $\frac{-2}{1}$	can also be written as	**-2.0**
$\frac{1}{2}$	can also be written as	**0.5**
$\frac{-5}{4}$	can also be written as	**-1.25**
$\frac{2}{3}$	can also be written as	**0.666666666...** $0.\overline{6}$
$\frac{21}{55}$	can also be written as	**0.38181818...** $0.3\overline{18}$
$\frac{53}{83}$	can also be written as	**0.62855421687...** the decimals will repeat after 41 digits
Be careful when using your calculator to determine if a decimal number is irrational. The calculator may not be displaying enough digits to show you the repeating decimals, as was seen in the last example above.		

Hint: When given a rational number in decimal form and asked to write it as a fraction, it is often helpful to "say" the decimal out loud using the place values to help form the fraction.

2	.	3	4	5	6
o	a	t	h	t	ten-
n	n	e	u	h	t
e	d	n	n	o	h
s		t	d	u	o
		h	r	s	u
		s	e	a	s
			d	n	a
			t	d	n
			h	t	d
			s	h	t
				s	h
					s

Examples: Write each rational number as a fraction:

Rational number in decimal form	Rational number in fractional form
1. 0.3	$\frac{3}{10}$
2. 0.007	$\frac{7}{1000}$
3. -5.9	$-5\frac{9}{10} = -\frac{59}{10}$

Hint: When checking to see which fraction is larger, change the fractions to decimals by dividing and compare their decimal values.

Examples:

	Which of the given numbers is greater?	Using full calculator display to compare the numbers.
1.	$\frac{2}{3}, \frac{1}{4}$	.6666666667 > .25
2.	$-\frac{7}{3}, -\frac{11}{3}$	-2.333333333 > -3.666666667

An **irrational number** cannot be expressed as a fraction.

1. Irrational numbers cannot be represented as terminating or repeating decimals.
2. Irrational numbers are non-terminating, non-repeating decimals.

3. **Examples of irrational numbers are:**

$$\pi = 3.141592654.....$$
$$\sqrt{2} = 1.414213562.....$$
$$\text{and } 0.12122122212...$$

Note: Many students think that π is the terminating decimal, 3.14, but it is not. Yes, certain math problems ask you to use π as 3.14, but that problem is rounding the value of to make your calculations easier. π is actually a non-ending decimal and is an irrational number.

Decimal Representation Of Rational Numbers

Example 1: Express $\frac{7}{8}$ in the decimal form by long division method.

Solution: We have,

```
8) 7.000 (0.875
   64
   ---
   60
   56
   ---
     40
     40
   ---
      0
```

$$\therefore \frac{7}{8} = 0.875$$

Example 2: Convert $\frac{35}{16}$ into decimal form by long division method.

Solution: We have,

```
16) 35.0000 (2.1875
    32
    ---
     30
     16
    ---
     140
     128
    ---
     120
     112
    ---
       80
       80
    ---
        0
```

$$\therefore \frac{35}{16} = 2.1875$$

Example 3: Express $\frac{2157}{625}$ in the decimal form.

Solution: We have,

$$\begin{array}{l} 625\overline{)2154.0000}(3.4512 \\ \underline{1875} \\ \;2820 \\ \underline{\;2500} \\ \;\;3200 \\ \underline{\;\;3125} \\ \;\;\;750 \\ \underline{\;\;\;625} \\ \;\;\;1250 \\ \underline{\;\;\;1250} \\ \;\;\;\;\;0 \end{array}$$

$$\therefore \quad \frac{2157}{625} = 3.4512$$

Example 4: Express $\frac{-17}{8}$ in decimal form by long division method.

Solution: In order to convert $\frac{-17}{8}$ in the decimal form, we first express $\frac{17}{8}$ in the decimal form and the decimal form of $\frac{-17}{8}$ will be negative of the decimal form of $\frac{17}{8}$
we have,

$$\begin{array}{l} 8\overline{)17.000}(2.125 \\ \underline{16} \\ 10 \\ \underline{\;8} \\ \;20 \\ \underline{\;16} \\ \;40 \\ \underline{\;40} \\ \;\;0 \end{array}$$

$$\therefore \quad \frac{-17}{8} = -2.125$$

Example 5: Find the decimal representation of $\frac{8}{3}$.

Solution: By long division, we have

$$\begin{array}{l} 3\overline{)8.0000}(2.6666 \\ \underline{6} \\ \;20 \\ \underline{\;18} \\ \;20 \\ \underline{\;18} \\ \;20 \\ \underline{\;18} \\ \;\;20 \\ \underline{\;\;18} \\ \;\;\;2 \end{array}$$

$$\therefore \quad \frac{8}{3} = 2.6666 \;\ldots = 2.\overline{6}$$

Example 6: Express $\frac{2}{11}$ as a decimal fraction.

Solution: By long division, we have

```
11)2.00 (0.181818
   1 1
   ----
     90
     88
   ----
     20
     11
   ----
     90
     88
   ----
     20
     11
   ----
     90
     88
   ----
      2
```

$$\therefore \quad \frac{2}{11} = 0.181818 \ldots.. = 0.\overline{18}$$

Example 7: Find the decimal representation of $\frac{-16}{45}$

Solution: By long division, we have

```
45)160 (0.3555
   135
   ----
    250
    225
   ----
    250
    225
   ----
    250
    225
   ----
     25
```

$$\therefore \quad \frac{16}{45} = 0.3555 \ldots. = 0.3\bar{5}$$

$$\text{Hence,} \quad \frac{-16}{45} = -0.3\bar{5}$$

Example 8: Find the decimal representation of $\frac{22}{7}$

Solution: By long division, we have

```
7)22 (3.142857142857
  21
  10
   7
  30
  28
  20
  14
  60
  56
  40
  35
  50
  49
  10
   7
  30
  28
  20
  14
  60
  56
  40
  35
  50
  49
   1
```

$\therefore \frac{22}{7} = 3.142857142857 \ldots.. = 3.\overline{142857}$

So division of rational number gives decimal expansion. This expansion represents two types

(A) Terminating (remainder = 0)

Ex. $\frac{6}{5}, \frac{8}{5}, \frac{7}{4}, \ldots\ldots\ldots$ are equal to 1.2, 1.6, 1.75 respectively,

So these are terminating and non repeating (recurring)

(B) Non terminating recurring (repeating)
(remainder ≠ 0, but equal to devidend)

Ex. $\frac{10}{3} = 3.333 \ldots\ldots\ldots$ or $3.\overline{3}$

$\frac{1}{7} = 0.1428514285 \ldots\ldots\ldots$ or $0.\overline{142857}$

$\frac{2320}{99} = 23.434343 \ldots\ldots.$ or $23.\overline{43}$

These expansion are not finished but digits are continusely repeated so we use a line on those digits, called bar $(\bar{a})$.
So we can say that rational numbers are of the form either terminating, non repeating or non terminating repeating (recurring).

How To Convert Decimal Number Into Rational Number

Conversion Of Decimal Numbers Into Rational Numbers Of The Form m/n

Case I: When the decimal number is of terminating nature.

Algorithm:

- **Step-1:** Obtain the rational number.
- **Step-2:** Determine the number of digits in its decimal part.
- **Step-3:** Remove decimal point from the numerator. Write 1 in the denominator and put as many zeros on the right side of 1 as the number of digits in the decimal part of the given rational number.
- **Step-4:** Find a common divisor of the numerator and denominator and express the rational number to lowest terms by dividing its numerator and denominator by the common divisor.

Case II: When decimal representation is of non-terminating repeating nature.
In a non terminating repeating decimal, there are two types of decimal representations

1. A decimal in which all the digit after the decimal point are repeated. These type of decimals are known as **pure recurring decimals**.
   For Example: $0.\overline{6},\ 0.\overline{16},\ 0.\overline{123}$ are pure recurring decimals.
2. A decimal in which at least one of the digits after the decimal point is not repeated and then some digit or digits are repeated. This type of decimals are known as **mixed recurring decimals**.
   For Example: $2.1\overline{6},\ 0.3\overline{5},\ 0.7\overline{85}$ are mixed recurring decimals.

Conversion Of A Pure Recurring Decimal To The Form p/q

Algorithm:

- **Step-1:** Obtain the repeating decimal and pur it equal to x (say)
- **Step-2:** Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits at least twice. For sample, write x = $0.\overline{8}$ as x = 0.888…. and x = $0.\overline{14}$ as x = 0.141414……
- **Step-3:** Determine the number of digits having bar on their heads.
- **Step-4:** If the repeating decimal has 1 place repetition, multiply by 10; a two place repetition, multiply by 100; a three place repetition, multiply by 1000 and so on.
- **Step-5:** Subtract the number in step 2 from the number obtained in step 4
- **Step-6:** Divide both sides of the equation by the coefficient of x.
- **Step-7:** Write the rational number in its simplest form.

Conversion Of A Mixed Recurring Decimal To The Form p/q

Algorithm:

- **Step-1 :** Obtain the mixed recurring decimal and write it equal to x (say)
- **Step-2 :** Determine the number of digits after the decimal point which do not have bar on them. Let there be n digits without bar just after the decimal point
- **Step-3 :** Multiply both sides of x by 10^n so that only the repeating decimal is on the right side of the decimal point.
- **Step-4 :** Use the method of converting pure recurring decimal to the form p/q and obtain the value of x

Conversion Of Decimal Numbers Into Rational Numbers Example Problems With Solutions

Example 1: Express each of the following numbers in the form p/q.
(i) 0.15 (ii) 0.675 (iii) –25.6875

Solution:

(i) $0.15 = \frac{15}{100}$

$= \frac{15 \div 5}{100 \div 5}$

$= \frac{3}{20}$

(ii) $0.675 = \frac{675}{1000}$

$= \frac{675 \div 25}{1000 \div 25} \Rightarrow = \frac{27}{40}$

(iii) $-25.6875 = \frac{-256875}{10000}$

$= \frac{-256875 \div 625}{10000 \div 625} = \frac{-411}{16}$

Example 2: Express each of the following decimals in the form p/q.

(i) $0.\overline{6}$ (ii) $0.\overline{35}$ (iii) $0.\overline{585}$

Solution:

(i) Let $x = 0.\overline{6}$

then, $x = 0.666.......$ ....(i)

Here, we have only one repeating digit,

So, we multiply both sides of (i) by 10 to get

$10x = 6.66....$ ....(ii)

Subtracting (i) from (ii), we get

$10x - x = (6.66 .....) - (0.66.....)$

$\Rightarrow 9x = 6 \quad \Rightarrow x = \frac{6}{9}$

$\Rightarrow x = \frac{2}{3}$ Hence $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.\overline{35}$

$\Rightarrow x = 0.353535....$ ....(i)

Here, we have two repeating digits after the decimal point.

So, we multiply sides of (i) by $10^2 = 100$ to get

$100x = 35.3535......$ ....(ii)

Subtracting (i) from (ii), we get

$100x - x = (35.3535....) - (0.3535....)$

$\Rightarrow 99x = 35$

$\Rightarrow x = \frac{35}{99}$

Hence, $0.\overline{35}$

(iii) Let $x = 0.\overline{585}$

$\Rightarrow \quad x = 0.585585585...$ ....(i)

Here, we have three repeating digits after the decimal point.

so, we multiple both sides of (i) by $10^3 = 1000$ to get

$1000\,x = 585.585585.........$ ....(ii)

Subtracting (i) from (ii), we get

$1000x - x = (585.585585...) - (0.585585585...)$

$1000x - x = 585$

$\Rightarrow 999x = 585$

$$\Rightarrow \quad x = \frac{585}{999} = \frac{195}{333} = \frac{65}{111}$$

The above example suggests us the following rule to convert a pure recurring decimal into a rational number in the form p/q.

Example 3: Convert the following decimal numbers in the form p/q.
(i) $5.\overline{2}$ (ii) $23.\overline{43}$

Solution:

(i) Let $x = 5.\overline{2}$

$\Rightarrow \; x = 5.2222$ ....(i)

Multiplying both sides of (i) by 10, we get

$10\,x = 52.222$ ....(ii)

Subtracting (i) from (ii), we get

$10x - x = (52.222...) - (5.222....)$

$\Rightarrow \; 9x = 47$

$$\Rightarrow \; x = \frac{47}{9}$$

(ii) Let $x = 23.\overline{43}$

$\Rightarrow \; x = 23.434343.....$

Multiplying both sides of (i) by 100, we get

$100\,x = 2343.4343.......$ ....(ii)

Subtracting (i) from (ii), we get

$100x - x = (2343.4343...) - (23.4343....)$

$\Rightarrow \; 99\,x = 2320$

$$\Rightarrow \; x = \frac{2320}{99}$$

Example 4: Express the following decimals in the form (i) $0.3\overline{2}$ (ii) $0.12\overline{3}$

Solution:

(i) Let $x = 0.3\overline{2}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore\ 10x = 3.\overline{2}$

$\Rightarrow\ 10x = 3 + 0.\overline{2} \quad \left[\because 0.\overline{2} = \frac{2}{9}\right]$

$\Rightarrow\ 10x = 3 + \frac{2}{9}$

$\Rightarrow\ 10x = \frac{9\times 3+2}{9} \Rightarrow\ 10x = \frac{29}{9}$

$\Rightarrow\ x = \frac{29}{90}$

(ii) Let $x = 0.12\overline{3}$

Clearly, there are two digits on the right side of the decimal point which are without bar. So, we multiply both sides of x by $10^2 = 100$ so that only the repeating decimal is left on on the right side of the decimal point.

$\therefore\ 100x = 12.\overline{3}$

$\Rightarrow\ 100x = 12 + 0.\overline{3}$

$\Rightarrow\ 100x = 12 + \frac{3}{9}$

$\Rightarrow\ 100x = \frac{12\times 9+3}{9}$

$\Rightarrow\ 100x = \frac{108+3}{9}$

$\Rightarrow\ 100x = \frac{111}{9}$

$\Rightarrow\ x = \frac{111}{900} = \frac{37}{300}$

Example 5: Express each of the following mixed recurring decimals in the form p/q

(i) $4.3\overline{2}$ (ii) $15.\overline{712}$

Solution:

(i) Let $x = 4.3\overline{2}$

$\Rightarrow 10x = 43.\overline{2}$ [Multiplying both sides of x by 10]

$\Rightarrow 10x = 43 + 0.\overline{2}$

$\Rightarrow 10x = 43 + \frac{2}{9}$

$\Rightarrow 10x = \frac{43 \times 9 + 2}{9}$

$\Rightarrow 10x = \frac{387 + 2}{9}$

$\Rightarrow 10x = \frac{389}{9}$

$\Rightarrow x = \frac{389}{90}$

(ii) Let $x = 15.7\overline{12}$. Then,

$10x = 157.\overline{12}$

$\Rightarrow 10x = 157 + 0.\overline{12}$

$\Rightarrow 10x = 157 + \frac{12}{99}$

$\Rightarrow 10x = 157 + \frac{4}{33}$

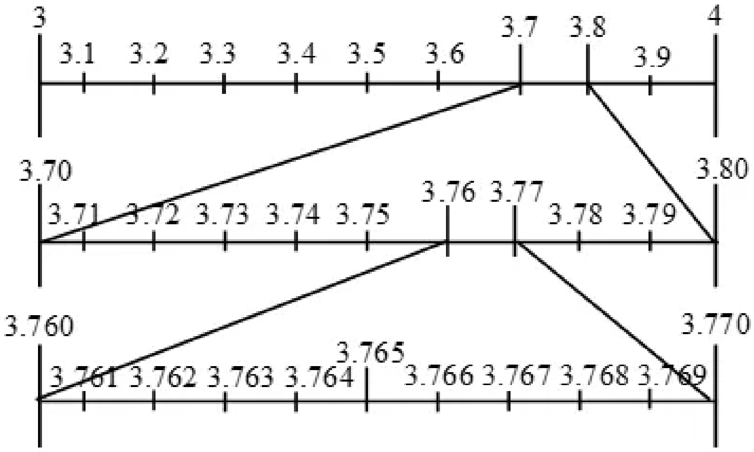
$\Rightarrow 10x = \frac{157 \times 33 + 4}{33}$

$\Rightarrow 10x = \frac{5181 + 4}{33}$

$\Rightarrow 10x = \frac{5185}{33} \Rightarrow x = \frac{5185}{330} = \frac{1037}{66}$

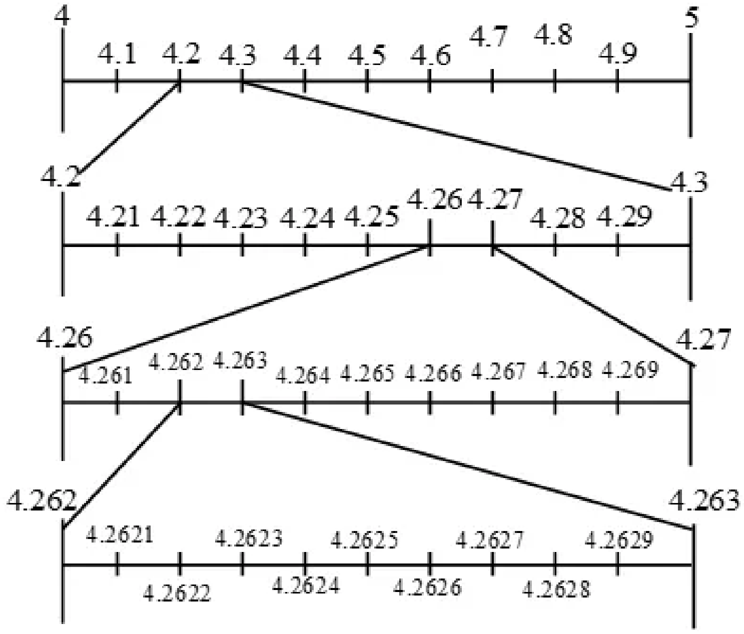
Example 6: Represent 3.765 on the number line.

Solution: This number lies between 3 and 4. The distance 3 and 4 is divided into 10 equal parts. Then the first mark to the right of 3 will represent 3.1 and second 3.2 and so on. Now, 3.765 lies between 3.7 and 3.8. We divide the distance between 3.7 and 3.8 into 10 equal parts 3.76 will be on the right of 3.7 at the sixth mark, and 3.77 will be on the right of 3.7 at the 7th mark and 3.765 will lie between 3.76 and 3.77 and soon.



To mark 3.765 we have to use magnifying glass

Example 7: Visualize $4.\overline{26}$ on the number line, upto 4 decimal places.
Solution: We have, $4.\overline{26}$ = 4.2626
This number lies between 4 and 5. The distance between 4 and 5 is divided into 10 equal parts. Then the first mark to the right of 4 will represent 4.1 and second 4.2 and soon. Now, 4.2626 lies between 4.2 and 4.3. We divide the distance between 4.2 and 4.3 into 10 equal parts 4.2626 lies between 4.26 and 4.27. Again we divide the distance between 4.26 and 4.27 into 10 equal parts. The number 4.2626 lies between 4.262 and 4.263. The distance between 4.262 and 4.263 is again divided into 10 equal parts. Sixth mark from right to the 4.262 is 4.2626.



Example 8: Express the decimal $0.00\overline{352}$ in the form p/q
Solution: Let x = $0.00\overline{352}$
Clearly, there is three digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10^3 = 1000 so that only the repeating decimal is left on the right side of the decimal point.

$$\therefore \quad 1000x = 3.\overline{52}$$
$$\Rightarrow \quad 1000x = 3 + 0.52$$
$$\Rightarrow \quad 1000x = 3 + \frac{52}{99}$$
$$\Rightarrow \quad 1000x = \frac{3 \times 99 + 52}{99} \Rightarrow 1000x = \frac{297 + 52}{99}$$
$$\Rightarrow \quad 1000x = \frac{349}{99} \Rightarrow x = \frac{349}{99000}$$

Example 9: Give an example of two irrational numbers, the product of which is
(i) a rational number
(ii) an irrational number

Solution: (i) The product of √27 and √3 is √81= 9, which is a rational number.
(ii) The product of √2 and √3 is √6, which is an irrational number.

Example 10: Insert a rational and an irrational number between 2 and 3.

Solution: If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b. Also, if a,b are rational numbers, then $\frac{a+b}{2}$ is a rational number between them.
∴ A rational number between 2 and 3 is
$\frac{2+3}{2}$ = 2.5

An irrational number between 2 and 3 is
$\sqrt{2 \times 3} = \sqrt{6}$

Example 11: Find two irrational numbers between 2 and 2.5.

Solution: If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.
∴ Irrational number between 2 and 2.5 is
$\sqrt{2 \times 2.5} = \sqrt{5}$

Similarly, irrational number between 2 and $\sqrt{5}$ is $\sqrt{2 \times \sqrt{5}}$

So, required numbers are $\sqrt{5}$ and $\sqrt{2 \times \sqrt{5}}$.

Example 12: Find two irrational numbers lying between √2 and √3.

Solution: We know that, if a and b are two distinct positive irrational numbers, then is an irrational number lying between a and b.
∴ Irrational number between √2 and √3 is $\sqrt{\sqrt{2} \times \sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$
Irrational number between √2 and $6^{1/4}$ is $\sqrt{\sqrt{2} \times 6^{1/4}} = 2^{1/4} \times 6^{1/8}$.
Hence required irrational number are $6^{1/4}$ and $2^{1/4} \times 6^{1/8}$

Example 13: Find two irrational numbers between 0.12 and 0.13.

Solution: Let a = 0.12 and b = 0.13. Clearly, a and b are rational numbers such that a < b.
We observe that the number a and b have a 1 in the first place of decimal. But in the second place of decimal a has a 2 and b has 3. So, we consider the numbers
c = 0.1201001000100001 ……
and, d = 0.12101001000100001…….
Clearly, c and d are irrational numbers such that a < c < d < b.

Example 14: Find two rational numbers between 0.232332333233332…. and 0.252552555255552……

Solution: Let a = 0.232332333233332…. and b = 0.252552555255552…..
The numbers c = 0.25 and d = 0.2525
Clearly, c and d both are rational numbers such that a < c < d < b.

Example 15: Find a rational number and also an irrational number between the numbers a and b given below:
a = 0.101001000100001…., b = 0.1001000100001…

Solution: Since the decimal representations of a and b are non-terminating and non-repeating. So, a and b are irrational numbers.
We observed that in the first two places of decimal a and b have the same digits. But in the third place of decimal a has a 1 whereas b has zero.
∴ a > b
Construction of a rational number between a and b : As mentioned above, first two digits after the decimal point of a and b are the same. But in the third place a has a 1 and b has a zero. So, if we consider the number c given by
c = 0.101
Then, c is a rational number as it has a terminating decimal representation.
Since b has a zero in the third place of decimal and c has a 1.
∴ b < c
We also observe that c < a, because c has zeros in all the places after the third place of decimal whereas the decimal representation of a has a 1 in the sixth place.
Thus, c is a rational number such that
b < c < a.
Hence , c is the required rational number between a and b.
Construction of an irrational number between a and b : Consider the number d given by
d = 0.1002000100001……
Clearly, d is an irrational number as its decimal representation is non-terminating and non-repeating.
We observe that in the first three places of their decimal representation b and d have the same digits but in the fourth place d and a 2 whereas b has only a 1.

∴ d > b
Also, comparing a and d, we obtain a > d
Thus, d is an irrational number such that
b < d < a.

Example 16: Find one irrational number between the number a and b given below :
a = 0.1111….. = $0.\bar{1}$ and b = 0.1101

Solution: Clearly, a and b are rational numbers, since a has a repeating decimal and b has a terminating decimal. We observe that in the third place of decimal a has a 1, while b has a zero.
∴ a > b
Consider the number c given by
c = 0.111101001000100001…..
Clearly, c is an irrational number as it has non-repeating and non-terminating decimal representation.
We observe that in the first two places of their decimal representations b and c have the same digits.
But in the third place b has a zero whereas c has a 1.
∴ b < c
Also, c and a have the same digits in the first four places of their decimal representations but in the fifth place c has a zero and a has a 1.
∴ c < a
Hence, b < c < a
Thus, c is the required irrational number between a and b.

How do you Add and Subtract Rational Numbers?

There are four basic operations on rational numbers :

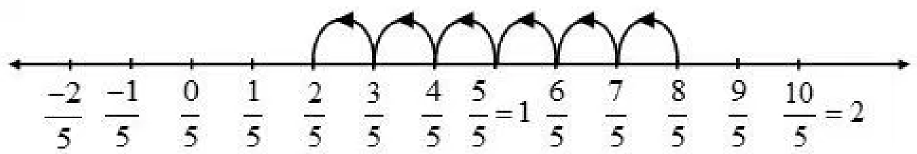
1. Addition
2. Subtraction
3. Multiplication
4. Division.

Addition of Rational Numbers

If two rational numbers are to be added, we first express each one of them as rational number with positive denominator.
There are two possibilities :
(1) Either they have same denominators, or
(2) They have different denominators.

Adding Rational Numbers with Same Denominator:

Let us add $\frac{8}{5}$ and $\frac{-6}{5}$
Represent the numbers on the number line.



Here, the distance between two consecutive points is $\frac{-6}{5}$. For $\frac{8}{5}$, move 6 steps to the left of $\frac{8}{5}$ and we reach at $\frac{2}{5}$.

$$\text{So,} \quad \frac{8}{5}+\left(\frac{-6}{5}\right)=\frac{8+(-6)}{5}=\frac{2}{5}$$

Example 1: Add : $\frac{-5}{9}$ and $\frac{-17}{9}$.

Solution:

Given rational numbers are $\frac{-5}{9}$ and $\frac{-17}{9}$.
Adding these two numbers, we have

$$\frac{(-5)}{9}+\frac{(-17)}{9}=\frac{(-5)+(-17)}{9}=\frac{-22}{9}$$

Which is the required answer.

Example 2: Add : $\frac{-23}{28}$ and $\frac{5}{-28}$.

Solution:

We first express $\frac{5}{-28}$ as a rational number with positive denominator.

$$\text{We have, } \frac{5\times(-1)}{(-28)\times(-1)}=\frac{-5}{28}$$

$$\text{Now, } \frac{-23}{28}+\left(\frac{-5}{28}\right)=\frac{-23}{28}-\frac{5}{28}$$

$$[\text{since } (+)\times(-)=-]$$

$$=\frac{-23-5}{28}=\frac{-28}{28}=-1$$

Addition of Rational numbers with Different Denominators:

In this case, we convert the given rational numbers to a common denominator and then add.

Examples:

1. Add $\frac{8}{-5}$ and $\frac{4}{-3}$.

Solution:

The given rational numbers are $\frac{8}{-5}$ and $\frac{4}{-3}$. Clearly, they have different denominators. Here, first we express the given rational numbers into standard forms.

i.e., $\frac{8}{-5} = \frac{8\times(-1)}{(-5)\times(-1)} = \frac{-8}{5}$

And, $\frac{4}{-3} = \frac{4\times(-1)}{(-3)\times(-1)} = \frac{-4}{3}$

Now, $\frac{8}{-5} + \frac{4}{-3} = \frac{(-8)}{5} + \frac{(-4)}{3}$

Converting to the same denominators,

we have

$$\frac{-8}{5} = \frac{(-8)\times 3}{5\times 3} = \frac{-24}{15}$$

And $\frac{-4}{3} = \frac{(-4)\times 5}{3\times 5} = \frac{-20}{15}$

So, $\frac{-8}{5} + \left(\frac{-4}{3}\right) = \frac{-24}{15} + \left(\frac{-20}{15}\right)$

$= \frac{(-24)+(-20)}{15} = \frac{-44}{15}$

Which is the required answer.

2. Add : $\frac{8}{10}$, 3.

Solution:

We have, 3 which can be written as $\frac{3}{1}$.

Multiplying both the numerator and denominator of $\frac{3}{1}$ by 10,

we get $\frac{3}{1} = \frac{3\times 10}{1\times 10} = \frac{30}{10}$

Therefore, $\frac{8}{10} + 3 = \frac{8}{10} + \frac{3}{1} = \frac{8}{10} + \frac{30}{10}$

$$= \frac{8+30}{10} = \frac{38}{10} = \frac{19}{5}$$

Which is the required answer.

3. Simplify : $\frac{4}{3} + \frac{3}{5} + \frac{-2}{5} + \frac{-11}{3}$

Solution:

$$\frac{4}{3}+\frac{3}{5}+\frac{-2}{5}+\frac{-11}{3}=\left(\frac{4}{3}+\frac{-11}{3}\right)+\left(\frac{3}{5}+\frac{-2}{5}\right)$$

$$=\frac{4-11}{3}+\frac{3-2}{5}=\frac{-7}{3}+\frac{1}{5}=\frac{-7\times5}{3\times5}+\frac{1\times3}{5\times3}$$

(changing them to same denominator)

$$=\frac{-35}{15}+\frac{3}{15}=\frac{-32}{15}.$$

4. Add $\frac{7}{9}$ and $\frac{-5}{9}$.

Solution:

$$\frac{7}{9}+\left(\frac{-5}{9}\right)=\frac{7+(-5)}{9}=\frac{7-5}{9}=\frac{2}{9}.$$

In case, if denominator of the rational number is negative, first we make it (denominator) Positive and then add.

5. Add : $\frac{6}{-5}$ and $\frac{4}{5}$.

Solution:

$$\frac{6}{-5}+\frac{4}{5}=\frac{-6}{5}+\frac{4}{5}=\frac{-6+4}{5}=\frac{-2}{5}.$$

6. Find the sum of $\frac{-8}{5}$ and $\frac{-5}{3}$

Solution:

LCM of 5 and 3 is 15.

$$\frac{-8}{5}=\frac{-8\times3}{5\times3}=\frac{-24}{15}$$

$$\Rightarrow \frac{-5}{3}=\frac{-5\times5}{3\times5}=\frac{-25}{15}$$

$$\text{So, } \frac{-8}{5}+\left(\frac{-5}{3}\right)=\frac{-24}{15}+\left(\frac{-25}{15}\right)=\frac{-24-25}{15}$$

$$\frac{-8}{5}+\left(\frac{-5}{3}\right)=\frac{-49}{15}$$

Note : Addition of rational numbers is closure (the sum is also rational) commutative (a + b = b + a) and associative(a + (b + c)) = ((a + b) + c).

Additive inverse:

The negative of a rational number is called additive inverse of the given number.

Note: Zero is the only rational no. which is its even negative or inverse.

Subtraction of Rational Numbers

If we add the additive inverse of a rational number and other rational number then this is called subtraction of two rational numbers. So the subtraction is inverse process of addition and the term add the negative of use for subtraction.

Subtraction of Rational Numbers Problems with Solutions

1. Find value of $\frac{2}{3} - \frac{4}{5}$.

Solution:

$$\frac{2}{3} + \text{additive inverse of } \frac{4}{5}.$$

$$= \frac{2}{3} + \left(\frac{-4}{5}\right) = \frac{2\times 5}{3\times 5} + \left(\frac{-4\times 3}{5\times 3}\right)$$

$$= \frac{10}{15} + \frac{-12}{15} = \frac{10+(-12)}{15}$$

$$= \frac{-2}{5}.$$

2. Find value of $\frac{2}{7} - \left(\frac{-5}{3}\right)$.

Solution:

$$\frac{2}{7} - \left(\frac{-5}{3}\right) = \frac{2}{7} + \text{ additive inverse of } \left(\frac{-5}{3}\right)$$

$$= \frac{2}{7} + \frac{5}{3}$$

$$= \frac{2\times 3}{7\times 3} + \frac{5\times 7}{3\times 7}$$

$$= \frac{6}{21} + \frac{35}{21} = \frac{6+35}{21} = \frac{41}{21}.$$

3. Subtract $\frac{-5}{8}$ from $\frac{-3}{7}$.

Solution:

The given rational numbers are $\frac{-5}{8}$ and $\frac{-3}{7}$.

Therefore,

$$\frac{-3}{7}-\left(\frac{-5}{8}\right)=\frac{-3}{7}+\frac{5}{8}$$

$$=\frac{-24}{56}+\frac{35}{56}=\frac{-24+35}{56}=\frac{11}{56}$$

4. Simplify : $\frac{1}{6}+\frac{-2}{5}-\frac{-2}{15}$.

Solution:

We have,

$$\frac{1}{6}+\frac{-2}{5}-\frac{-2}{15}=\frac{1}{6}-\frac{2}{5}+\frac{2}{15}$$

$$\left[\text{Since } -\left(\frac{-2}{15}\right)=\frac{2}{15}\right]$$

$$=\frac{1\times 5-2\times 6+2\times 2}{30} \quad [\text{LCM of 6, 5, 15 is 30}]$$

$$=\frac{5-12+4}{30}=\frac{9-12}{30}=\frac{-3}{30}=\frac{-1}{10}$$

5. What number should be added to $\frac{-5}{8}$ so that the sum is $\frac{5}{9}$?

Solution:

The number will be obtained by subtracting $\frac{-5}{8}$ from $\frac{5}{9}$.

So, $\frac{5}{9}-\left(\frac{-5}{8}\right)=\frac{5}{9}+\frac{5}{8}=\frac{5\times 8+5\times 9}{72}$

$$=\frac{40+45}{72}=\frac{85}{72}$$

Therefore, required number is $\frac{85}{72}$.

6. What number should be subtracted from $\frac{27}{11}$ so as to get $\frac{-5}{33}$?

Solution:

We have, difference of the given number and the required number = $\frac{-5}{33}$

Given number = $\frac{27}{11}$

Therefore, $\frac{27}{11} - \frac{-5}{33} = \frac{27}{11} + \frac{5}{33}$

$= \frac{27 \times 3 + 5 \times 1}{33} = \frac{81+5}{33} = \frac{86}{33}$

Therefore, required number is $\frac{86}{33}$.

7. The sum of two rational numbers is $\frac{-3}{5}$ If one of them is $\frac{-9}{10}$ Find the other.

Solution:

Given, sum of the numbers = $\frac{-3}{5}$

One of the numbers = $\frac{-9}{10}$

The other number

= Sum of the numbers – One of the numbers

$= \frac{-3}{5} - \frac{-9}{10} = \frac{-3}{5} + \frac{9}{10}$

$= \frac{-3 \times 2 + 9 \times 1}{10}$

$= \frac{-6+9}{10} = \frac{3}{10}$

Therefore, required number is $\frac{3}{10}$.

How do you Multiply and Divide Rational Numbers?

Multiplication of Rational Numbers

(a) Let $\frac{a}{b}$ and c are two rational numbers, then $\frac{a}{b} \times c = \frac{ac}{b}$

Eg : Find product of $\frac{-5}{7}$ and 9.

$$\frac{-5}{7} \times 9 = \frac{-5}{7} \times \frac{9}{1} = \frac{-5 \times 9}{7 \times 1} = \frac{-45}{7}$$

(b) When we multiply two rational numbers :

$$\text{i.e., } \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

$$\text{e.g., } \frac{1}{2} \times \frac{-3}{4} = \frac{1 \times (-3)}{2 \times 4} = \frac{-3}{8}.$$

1. On multiplying two rational numbers, we get result as a rational number.

   $$\text{e.g., } \frac{-3}{4} \times \frac{5}{7} = \frac{-3 \times 5}{4 \times 7} = \frac{-15}{28}$$

   (closure property)

2. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{3}$ (commutative i.e., on changing the order the result remains same)
3. $\frac{1}{3} \times \left(\frac{1}{4} \times \frac{1}{5}\right) = \left(\frac{1}{3} \times \frac{1}{4}\right) \times \frac{1}{5}$ (associative)
4. If 0 is multiplied to any rational number, the result is always zero.

   e.g.,

   $$\text{(a) } 0 \times \frac{4}{5} = \frac{0 \times 4}{5} = \frac{0}{5} = 0$$

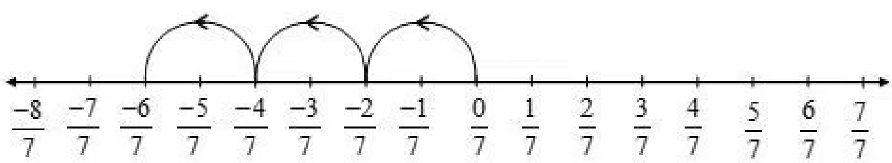
   $$\text{(b) } \frac{-2}{3} \times 0 = \frac{-2 \times 0}{3} = \frac{0}{3} = 0.$$

Multiplication of Rational Numbers on a Number Line

The product of two rational numbers on the number line can be calculated in the following way.

When we multiply $\frac{-2}{7}$ by 3 on a number line, it means 3 jumps of $\frac{-2}{7}$ to the left from zero. Now we reach at $\frac{-6}{7}$. Thus we find $\frac{-2}{7} \times 3 = \frac{-6}{7}$, i.e.,

$$\frac{-2}{7} \times 3 = \frac{-2}{7} \times \frac{3}{1} = \frac{-2 \times 3}{7 \times 1} = \frac{-6}{7}.$$



This result reconfirms that the product of two rational numbers is rational number whose numerator is

the product of the numerators of the given rational numbers and the denominator is the product of the denominators of the given numbers.
∴ Multiplication is closure (product is rational), commutative (ab = ba) and associative (a(bc) = (ab)c) for rational number.

Division of Rational Numbers

(a) Let $\frac{a}{b}$ be a rational number then its reciprocal will be $\frac{b}{a}$

1. The product of a rational number with its reciprocal is always 1.

   For example,

   (a) $\frac{5}{7} \times \frac{7}{5} = 1$ (b) $\frac{-3}{8} \times \frac{-8}{3} = 1$

2. Zero has no reciprocal as reciprocal of 0 = $\left(\frac{0}{1}\right)$ is $\frac{1}{0}$ (which is not defined).
3. The reciprocal of a rational number is called the multiplicative inverse of rational number.
4. 1 and −1 are the only rational numbers which are their own reciprocal.

   Reciprocal of 1 = $\frac{1}{1} = 1.$

   Reciprocal of −1 = $\frac{1}{-1} = -1.$

5. Reciprocal of a (+ve) rational number is (+ve) and reciprocal of (−ve) rational number is (−ve).
   To divide one rational number by other rational numbers we multiply the rational number by the reciprocal of the other

   $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \text{reciprocal of } \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

   For example, $\frac{5}{-6} \div \frac{-3}{+2}$

   $$\frac{5}{-6} \div \frac{-3}{2} = \frac{5}{-6} \times \text{reciprocal of} \left(\frac{-3}{2}\right)$$

   $$= \frac{5}{-6} \times \frac{+2}{-3}$$

   $$= \frac{5 \times 2}{-6 \times (-3)} = \frac{10}{18} = \frac{5}{9}.$$

6. Zero divided by any rational number is always equal to zero.

   For example, $0 \div \frac{2}{5} = 0$; $0 \div \frac{-3}{8} = 0$

Note:

1. When a rational number (except zero) is divided by another rational number (except 0) the quotient is always a rational number. (closed under division)

   i.e., $\frac{a}{b} \div \frac{c}{d} = \left(\frac{ad}{bc}\right)$ is also rational number.

2. Division of any rational number by itself gives the quotient 1.

For example, $\frac{4}{5} \div \frac{4}{5} = 1.$

3. When a rational number is divided by 1, the quotient is a rational number itself.

For example, $\frac{3}{4} \div 1 = \frac{3}{4}.$

Multiplication and Division of Rational Numbers Problems with Solutions

1. Find the product:

(i) $\frac{7}{2} \times \left(\frac{-5}{4}\right)$

(ii) $\frac{7}{10} \times (-9)$

(iii) $\frac{4}{-5} \times \frac{-5}{4}$

(iv) $\frac{4}{7} \times \left(\frac{-2}{5}\right)$

Solution:

(i) $\frac{7}{2} \times \left(\frac{-5}{4}\right) = \frac{7 \times (-5)}{2 \times 4} = \frac{-35}{8}$

(ii) $\frac{7}{10} \times (-9) = \frac{7}{10} \times \frac{(-9)}{1} = \frac{7 \times (-9)}{10 \times 1} = \frac{-63}{10}$

(iii) $\frac{4}{-5} \times \frac{-5}{4} = \frac{4 \times (-5)}{(-5) \times 4} = \frac{-20}{-20} = 1$

(iv) $\frac{4}{7} \times \left(\frac{-2}{5}\right) = \frac{4}{7} \times \left(\frac{-2}{5}\right) = \frac{4 \times (-2)}{7 \times 5} = \frac{-8}{35}$

2. Find the value of:

(i) $(-6) \div \frac{2}{3}$

(ii) $\frac{-4}{5} \div 2$

(iii) $\frac{3}{13} \div \left(\frac{-4}{65}\right)$

(iv) $\frac{-2}{8} \div \frac{-2}{8}$

(v) $\left(\frac{-6}{9}\right) \div 1$

Solution:

(i) $(-6) \div \frac{2}{3} = \frac{-6}{1} \times \frac{3}{2} = \frac{-6 \times 3}{1 \times 2} = \frac{-18}{2} = -9$

So, $(-6) \div \frac{2}{3} = -9.$

(ii) $\frac{-4}{5} \div 2 = \frac{-4}{5} \times \frac{1}{2} = \frac{-4 \times 1}{5 \times 2}$

$= \frac{-2 \times 1}{5} = \frac{-2}{5} = \frac{-2}{5}$

So, $\frac{-4}{5} \div 2 = \frac{-2}{5}.$

(iii) $\frac{3}{13} \div \left(\frac{-4}{65}\right) = \frac{3}{13} \times \frac{65}{-4} = \frac{3 \times 65}{13 \times (-4)}$

$= \frac{15}{-4} = \frac{-15}{4}$

So, $\frac{3}{13} \div \left(\frac{-4}{65}\right) = \frac{-15}{4}.$

(iv) $\frac{-2}{8} \div \frac{-2}{8} = \frac{-2}{8} \times \frac{8}{-2} = \frac{(-2) \times 8}{8 \times (-2)} = \frac{1 \times 1}{1 \times 1} = 1$

So, $\frac{-2}{8} \div \frac{-2}{8} = 1.$

(v) $\left(\frac{-6}{9}\right) \div 1 = \left(\frac{-6}{9}\right) \div \frac{1}{1} = \frac{-6}{9} \times \frac{1}{1}$

$= \frac{-6 \times 1}{9 \times 1} = \frac{-2 \times 1}{3 \times 1} = \frac{-2}{3}.$

So, $\left(\frac{-6}{9}\right) \div 1 = \frac{-2}{3}.$

3. Multiply:

(i) $\left(\frac{-8}{25}\right)$ by $\left(\frac{-5}{16}\right)$

(ii) $\left(\frac{9}{-11}\right)$ by $\left(\frac{22}{-27}\right)$

Solution:

(i) Multiplication of $\left(\frac{-8}{25}\right)$ by $\left(\frac{-5}{16}\right)$

$$= \frac{-8}{25} \times \frac{-5}{16} = \frac{-8 \times -5}{25 \times 16} = \frac{40}{400}$$

Dividing both the numerator and denominator by the greatest common divisor of 40 and 400 which is 40.

$$= \frac{40 \div 40}{400 \div 40} = \frac{1}{10}$$

(ii) Multiplication of $\left(\frac{9}{-11}\right)$ by $\left(\frac{22}{-27}\right)$

$$= \frac{9}{-11} \times \frac{22}{-27}$$

$$= \frac{9 \times 22}{-11 \times -27} = \frac{198}{297}$$

$$= \frac{198 \div 99}{297 \div 99} = \frac{2}{3}$$

4. Simplify:

$$\left(\frac{-2}{3} \times \frac{9}{5}\right) + \left(\frac{2}{3} \times \frac{-6}{7}\right).$$

Solution:

We have, $\left(\frac{-2}{3} \times \frac{9}{5}\right) + \left(\frac{2}{3} \times \frac{-6}{7}\right)$

$$= \frac{-18}{15} + \left(\frac{-12}{21}\right) = \frac{-18 \times 7 + (-12 \times 5)}{105}$$

$$= \frac{-126 + (-60)}{105} = \frac{-126 - 60}{105} = \frac{-186}{105} = \frac{-62}{35}$$