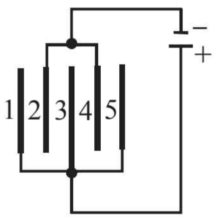
CHAPTER 12

Electrostatics

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

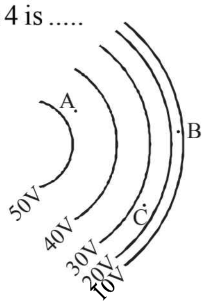
1. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance d apart, the plates are connected to a source of emf V as shown in the figure



(1984- 2 Marks)

The charge on plate 1 is ..... and on plate 4 is .....

2. Figure shows line of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point ...

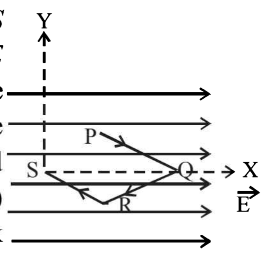


(1984- 2 Marks)

3. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is .... and the tension in each string is .... newtons. *(1986 - 2 Marks)*

4. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential differences across the capacitors now becomes............. *(1988 - 2 Marks)*

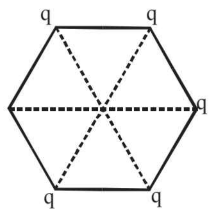
5. A point charge q moves from point P to point S along the path $PQRS$ (fig.) in a uniform electric field E pointing parallel to the positive direction of the X-axis. The cooridnates of the points P, Q, R and S are (a, b, O), $(2a, O, O)$ $(a, -b, O)$ and (O, O, O) respectively. The work done by the field in the above process is given by the expression



.... *(1989 - 2 Marks)*

6. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volts. The electric field at the point (1m, 0, 2 m) is .... V/m. *(1992 - 1 Mark)*

7. Five point charges, each of value $+ q$ coul, are placed on five vertices of a regular hexagon of side L metres. The magnitude of the force on the point charge of value $- q$ coul. placed at the centre of the hexagen is .... newton. *(1992 - 1 Mark)*



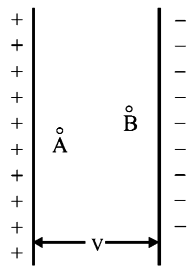
B True/False

1. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. *(1981- 2 Marks)*

2. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulombs and the other an equal negative charge. Their masses after charging are different. *(1983 - 2 Marks)*

3. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. *(1983 - 2 Marks)*

4. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical.



(1986 - 3 Marks)

5. A ring of radius R carries a uniformly distributed charge $+ Q$. A point charge $- q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. *(1988 - 2 Marks)*

6. An electric line of forces in the $x - y$ plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1, y = 0$ in the $x - y$ plane, will move along the circular line of force. *(1988 - 2 Marks)*

C MCQs with One Correct Answer

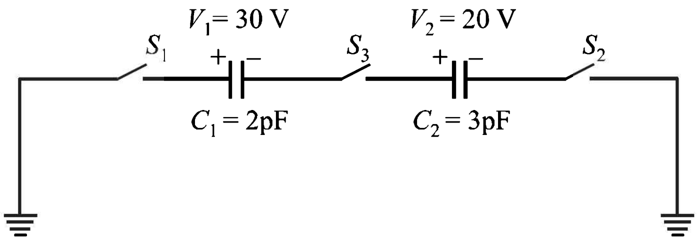
1. A hollow metal sphere of radius 5 cms is charged such that the potential on its surface is 10 volts. The potential at the centre of the sphere is *(1983 - 1 Mark)*
   (a) zero
   (b) 10 volts
   (c) same as at a point 5 cms away from the surface
   (d) same as at a point 25 cms away from the surface

2. Two point charges $+q$ and $-q$ are held fixed at $(-d, o)$ and (d, o) respectively of a x-y coordinate system. Then *(1995S)*
   (a) The electric field E at all points on the x-axis has the same direction
   (b) Electric field at all points on y-axis is along x-axis
   (c) Work has to be done in bringing a test charge from ∞ to the origin
   (d) The dipole moment is $2qd$ along the x-axis

3. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is *(1995S)*
   (a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

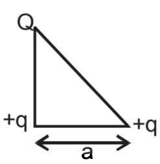
4. Two identical metal plates are given positive charges Q_1 and Q_2 $(<Q_1)$ respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is *(1999 - 2 Marks)*
   (a) $(Q_1+Q_2)/(2C)$ (b) $(Q_1+Q_2)/C$
   (c) $(Q_1-Q_2)/C$ (d) $(Q_1-Q_2)/(2C)$

5. For the circuit shown in Figure, which of the following statements is true? *(1999 - 2 Marks)*

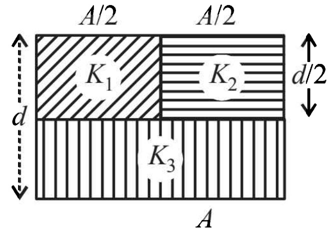


   (a) With S_1 closed $V_1 = 15$ V, $V_2 = 20$ V
   (b) With S_3 closed, $V_1 = V_2 = 25$ V
   (c) With S_1 and S_2 closed, $V_1 = V_2 = 0$
   (d) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

6. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to *(2000S)*
   (a) $\frac{-q}{1+\sqrt{2}}$
   (b) $\frac{-2q}{2+\sqrt{2}}$
   (c) $-2q$
   (d) $+q$

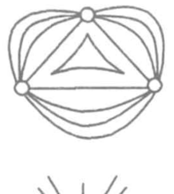
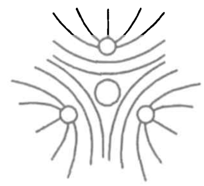


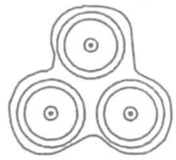
7. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by *(2000S)*



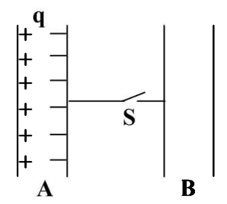
   (a) $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{2K_3}$ (b) $\frac{1}{K} = \frac{1}{K_1+K_2} + \frac{1}{2K_3}$
   (c) $K = \frac{K_1K_2}{K_1+K_2} + 2K_3$ (d) $K = K_1 + K_2 + 2K_3$

8. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in *(2001S)*

(a)  (b) 

(c)  (d) 

9. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is *(2001S)*



   (a) zero (b) $q/2$
   (c) q (d) $2q$

10. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy: *(2001S)*
   (a) $V_A < V_B$ (b) $V_A > V_B$
   (c) $V_A < V_C$ (d) $V_A > V_C$

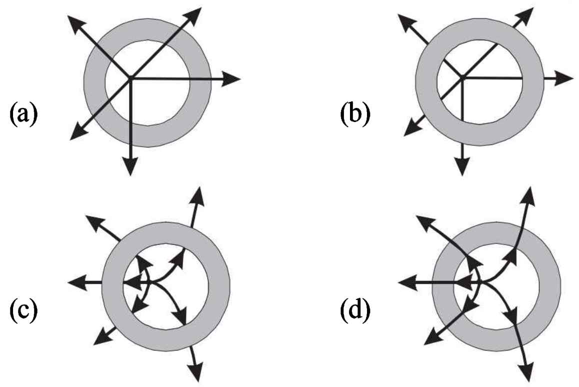
11. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to *(2002S)*
   (a) x (b) x^2
   (c) x^3 (d) $1/x$

12. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is *(2002S)*

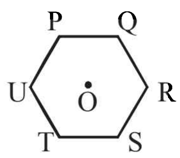
(a) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$ (b) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$

(c) $\frac{1}{4}C\left(V_1 - V_2\right)^2$ (d) $\frac{1}{4}C\left(V_1 + V_2\right)^2$

13. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces? *(2003S)*



14. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on *PQRSTU* corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at *R*. Which of the following arrangement of charge is possible for P, Q, R, S, T and U respectively. *(2004S)*



(a) +, +, +, –, –, – (b) –, +, +, +, –, –

(c) –, +, +, –, +, – (d) +, –, +, –, +, –

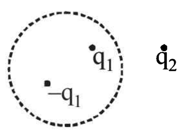
15. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be *(2004S)*

(a) due to q_1 and q_2 only

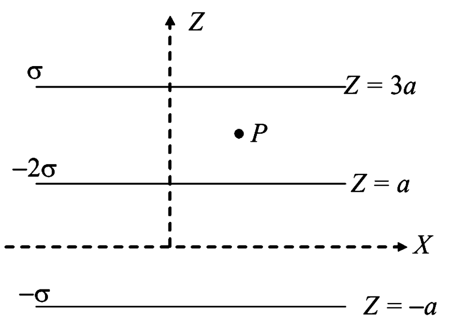
(b) due to q_2 only

(c) zero

(d) due to all



16. Three infinitely long charge sheets are placed as shown in figure. The electric field at point *P* is *(2005S)*



(a) $\frac{2\sigma}{\varepsilon_0}\hat{k}$ (b) $\frac{4\sigma}{\varepsilon_0}\hat{k}$ (c) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$ (d) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

17. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. *(2007)*

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.

(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.

(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders

(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

18. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then *(2007)*

(a) negative and distributed uniformly over the surface of the sphere

(b) negative and appears only at the point on the sphere closest to the point charge

(c) negative and distributed non-uniformly over the entire surface of the sphere

(d) zero

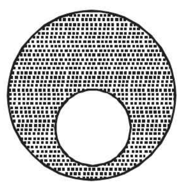
19. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is *(2007)*

(a) zero everywhere

(b) non-zero and uniform

(c) non-uniform

(d) zero only at its center



20. Positive and negative point charges of equal magnitude are kept at $\left(0,0,\frac{a}{2}\right)$ and $\left(0,0,\frac{-a}{2}\right)$ respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is *(2007)*

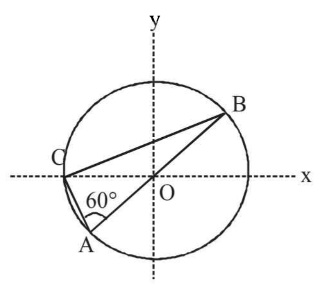
(a) positive

(b) negative

(c) zero

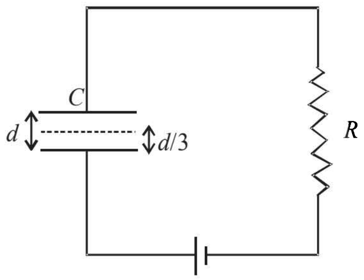
(d) depends on the path connecting the initial and final positions

21. Consider a system of three charges $q/3$, $q/3$ and $-2q/3$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB = 60°$ *(2008)*



(a) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis
(b) The potential energy of the system is zero
(c) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$
(d) The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

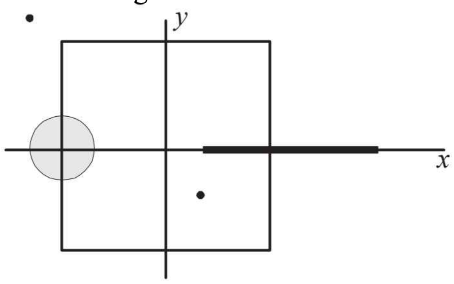
22. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is – ***(2008)***



(a) $\frac{6\varepsilon_0 R}{5d + 3vt}$
(b) $\frac{(15d + 9vt)\,\varepsilon_0 R}{2d^2 - 3dvt - 9v^2t^2}$
(c) $\frac{6\varepsilon_0 R}{5d - 3vt}$
(d) $\frac{(15d - 9vt)\,\varepsilon_0 R}{2d^2 - 3dvt - 9v^2t^2}$

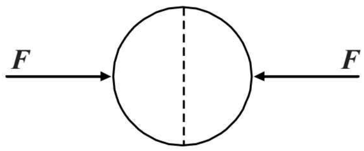
23. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is ***(2009)***
(a) $1:2:3$ (b) $1:3:5$
(c) $1:4:9$ (d) $1:8:18$

24. A disc of radius $a/4$ having a uniformly distributed charge 6C is placed in the x - y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x - axis from $x = a/4$ to $x = 5a/4$. Two point charges – 7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is ***(2009)***



(a) $\frac{-2C}{\varepsilon_0}$ (b) $\frac{2C}{\varepsilon_0}$ (c) $\frac{10C}{\varepsilon_0}$ (d) $\frac{12C}{\varepsilon_0}$

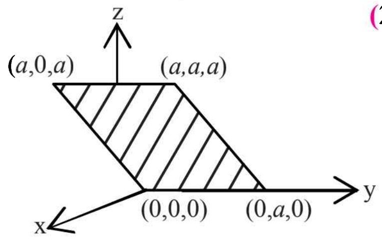
25. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to ***(2010)***



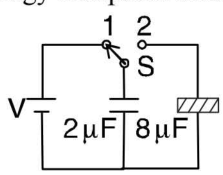
(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$ (c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

26. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5\ \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ \text{ms}^{-1}$. Given $g = 9.8\ \text{m s}^{-2}$, viscosity of the air $= 1.8 \times 10^{-5}\ \text{Ns m}^{-2}$ and the density of oil $= 900\ \text{kg m}^{-3}$, the magnitude of q is ***(2010)***
(a) 1.6×10^{-19}C (b) 3.2×10^{-19}C
(c) 4.8×10^{-19}C (d) 8.0×10^{-19}C

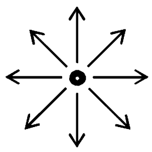
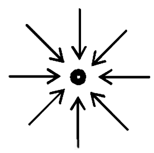
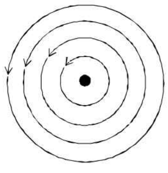
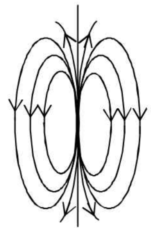
27. Consider an electric field $\vec{E} = E_0\hat{x}$ where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **(2011)**
(a) $2E_0a_2$
(b) $\sqrt{2}E_0a^2$
(c) $E_0 a^2$
(d) $\frac{E_0a^2}{\sqrt{2}}$



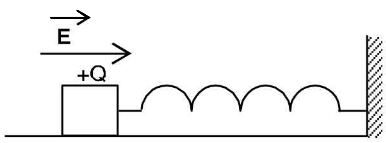
28. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is ***(2011)***
(a) 0%
(b) 20%
(c) 75%
(d) 80%



29. Which of the field patterns given below is valid for electric field as well as for magnetic field? ***(2011)***

(a) 
(b) 
(c) 
(d) 

30. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be ***(2011)***

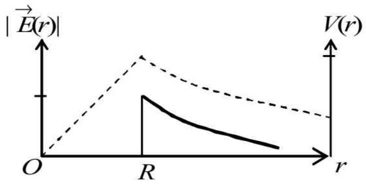


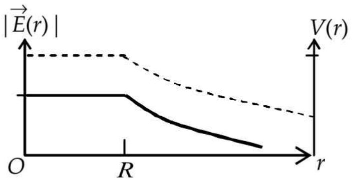
(a) of the same frequency and with shifted mean position.
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position.
(d) of changed frequency and with the same mean position.

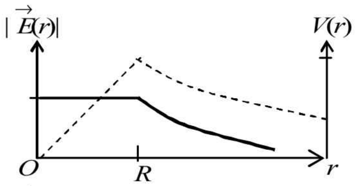
31. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

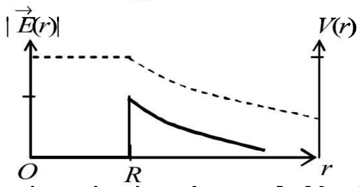
(a) 1×10^{-5} V (b) 1×10^{-7} V ***(2012)***
(c) 1×10^{-9} V (d) 1×10^{-10} V

32. Consider a thin spherical shell of radius R with centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph? ***(2012)***

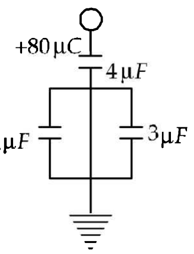
(a)


(b)


(c)


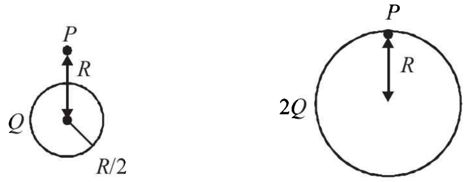
(d)


33. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is ***(2012)***

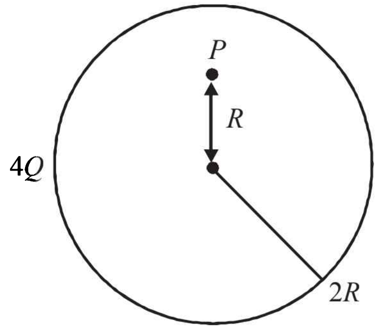


(a) +32 μC
(b) +40 μC
(c) +48 μC
(d) +80 μC

34. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitude of the electric fields at point P at a distance R from the centre of sphere 1, 2 and 3 are E_1, E_2 and E_3 respectively, then ***(JEE Adv. 2014)***



Sphere 1 Sphere 2



Sphere 3

(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$
(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

D MCQs with One or More than One Correct

1. Two equal negative charges $-q$ are fixed at points $(0, -a)$ and $(0, a)$ on y – axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x - axis. The charge Q will ***(1984- 2 Marks)***

(a) execute simple harmonic motion about the origin
(b) move to the origin remain at rest
(c) move to infinity
(d) execute oscillatory but not simple harmonic motion

2. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as ***(1985 - 2 Marks)***

(a) $Q > Q_0$ (b) $V > V_0$
(c) $E > E_0$ (d) $U > U_0$

3. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to : ***(1987 - 2 Marks)***

(a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

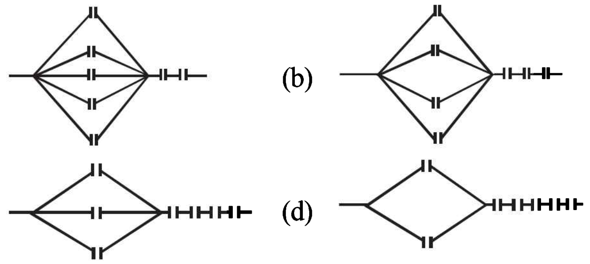
4. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles : ***(1987 - 2 Marks)***

(a) the charge on the capacitor increases.
(b) the voltage across the plates increases.
(c) the capacitance increases.
(d) the electrostatic energy stored in the capacitor increases

5. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is : ***(1989 - 2 Marks)***

(a) V (b) $2V$
(c) $4V$ (d) $-2V$

6. Seven capacitors each of capacitance $2\mu F$ are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{10}{11}\right)\mu F$. Which of the combination (s) shown in figure will achieve the desired result ? ***(1990 - 2 Marks)***



7. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in question, in the process of inserting the slab, then ***(1991 - 2 Marks)***

(a) $Q = \frac{\varepsilon_0 AV}{d}$ (b) $Q = \frac{\varepsilon_0 KAV}{d}$

(c) $E = \frac{V}{Kd}$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left[1 - \frac{1}{K}\right]$

8. Two identical thin rings, each of radius R metres, are coaxially placed a distance R metres apart. If Q_1 coulomb, and Q_2 coulomb, are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is ***(1992 - 2 Marks)***

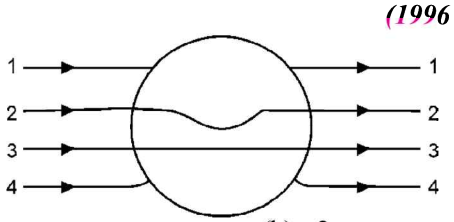
(a) zero (b) $\frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

(c) $\frac{q\sqrt{2}(Q_1+Q_2)}{(4\pi\varepsilon_0 R)}$ (d) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

9. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor. ***(1996 - 2 Marks)***

(a) is same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as $1/r$, where r is the distance from axis
(d) varies as $1/r^2$ where r is the distance from the axis.

10. A metallic solid sphere is placed in a uniform electric fied. The lines of force follow the path(s) shown in Figure as ***(1996 - 2 Marks)***



(a) 1 (b) 2
(c) 3 (d) 4

11. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$,

(a) the magnitude of the electric field remains the same.
(b) the direction of the electric field remains the same.
(c) the electric potential increases continuously.
(d) the electric potential increases at first, then decreases and again increases. ***(1998S - 2 Marks)***

12. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$,..... $x = \infty$ on the x axis, and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$,.... $x = \infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is ***(1998S - 2 Marks)***

(a) 0 (b) $\frac{q}{8\pi\varepsilon_0 x_0 \ln 2}$

(c) ∞ (d) $\frac{q \ln 2}{4\pi\varepsilon_0 x_0}$

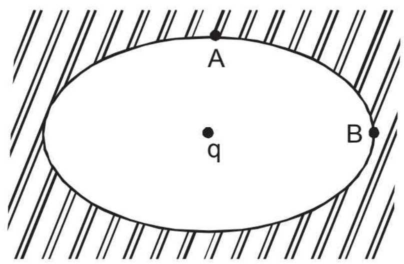
13. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is ***(1998S - 2 Marks)***

(a) periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
(b) simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$
(c) approximately simple harmonic, provided $z_0 << R$
(d) such that P crosses O and continues to move along the negative z axis towards $z = -\infty$

14. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre ***(1998S - 2 Marks)***

(a) increases as r increases, for $r < R$.
(b) decreases as r increases, for $0 < r < \infty$.
(c) decreases as r increases, for $R < r < \infty$.
(d) is discontinuous at $r = R$.

15. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then ***(1999S - 3 Marks)***



(a) electric field near A in the cavity = electric field near B in the cavity
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0

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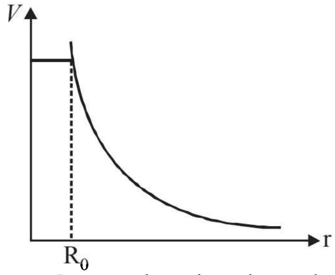
16. A spherical symmetric charge system is centered at origin. Given, Electric potential *(2006S - 5 Marks)*

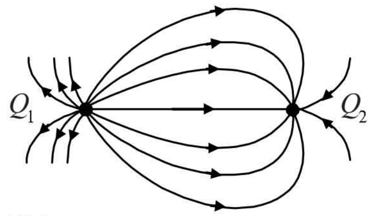
$$V=\frac{Q}{4\pi\varepsilon_0 R_0}\ (r\le R_0),\ V=\frac{Q}{4\pi\varepsilon_0 r}\ (r>R_0)$$



(a) Within $r=2R_0$ total enclosed net charge is Q
(b) Electric field is discontinued at $r=R_0$
(c) Charge is only present at $r=R_0$
(d) Electrostatic energy is zero for $r<R_0$

17. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s). *(2009)*
(a) The angular momentum of the charge $-q$ is constant
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

18. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that *(2010)*



(a) $|Q_1|>|Q_2|$
(b) $|Q_1|<|Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

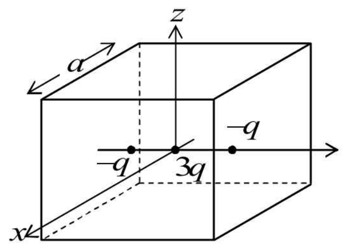
19. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B(<R_A)$ are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then *(2011)*

(a) $E_A^{inside}=0$ (b) $Q_A>Q_B$

(c) $\frac{\sigma_A}{\sigma_B}=\frac{R_B}{R_A}$ (d) $E_A^{on\ surface}<E_B^{on\ surface}$

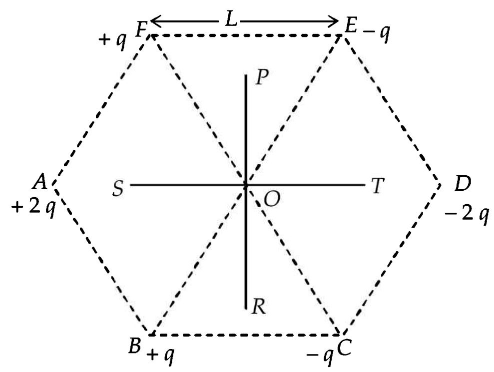
20. Which of the following statement(s) is/are correct? *(2011)*
(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
(b) The Gauss law can be used to calculate the field distribution around an electric dipole.
(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is (V_B-V_A).

21. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0,-a/4,0)$, $+3q$ at $(0,0,0)$ and $-q$ at $(0,+a/4,0)$. Choose the correct options(s) *(2012)*



(a) The net electric flux crossing the plane $x=+a/2$ is equal to the net electric flux crossing the plane $x=-a/2$
(b) The net electric flux crossing the plane $y=+a/2$ is more than the net electric flux crossing the plane $y=-a/2$.
(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$
(d) The net electric flux crossing the plane $z=+a/2$ is equal to the net electric flux crossing the plane $x=+a/2$.

22. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K=\frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct? *(2012)*



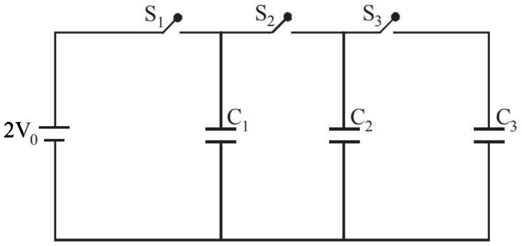
(a) The electric field at O is 6K along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

23. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be *(JEE Adv. 2013)*

(a) -4 (b) $-\frac{32}{25}$

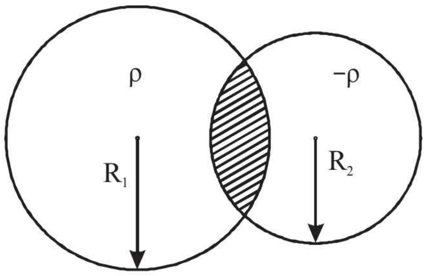
(c) $\frac{32}{25}$ (d) 4

24. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time *(JEE Adv. 2013)*



(a) The charge on the upper plate of C_1 is $2CV_0$
(b) The charge on the upper plate of C_1 is CV_0
(c) The charge on the upper plate of C_2 is 0
(d) The charge on the upper plate of C_2 is $-CV_0$

25. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region



(JEE Adv. 2013)

(a) The electrostatic field is zero
(b) The electrostatic potential is constant
(c) The electrostatic field is constant in magnitude
(d) The electrostatic field has same direction

26. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric field at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then *(JEE Adv. 2014)*

(a) $Q = 4\sigma\pi r_0^2$

(b) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(c) $E_1(r_0/2) = 2E_2(r_0/2)$

(d) $E_2(r_0/2) = 4E_3(r_0/2)$

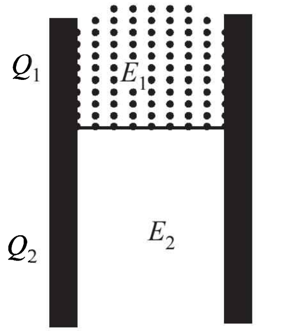
27. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. *(JEE Adv. 2014)*

(a) $\dfrac{E_1}{E_2} = 1$

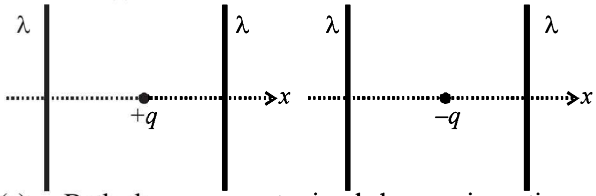
(b) $\dfrac{E_1}{E_2} = \dfrac{1}{K}$

(c) $\dfrac{Q_1}{Q_2} = \dfrac{3}{K}$

(d) $\dfrac{C}{C_1} = \dfrac{2+K}{K}$

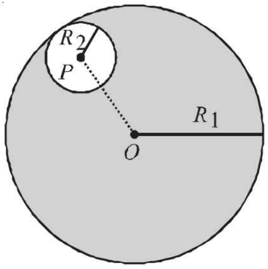


28. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are) *(JEE Adv. 2015)*



(a) Both charges execute simple harmonic motion
(b) Both charges will continue moving in the direction of their displacement
(c) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement
(d) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement

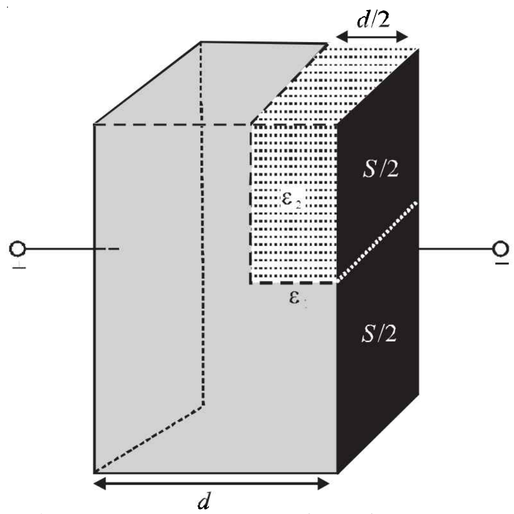
29. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is (are) *(JEE Adv. 2015)*



(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$
(b) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$
(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$
(d) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

30. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative primitivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is

(JEE Adv. 2015)

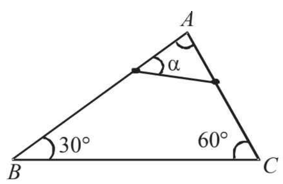


(a) 6/5 (b) 5/3
(c) 7/5 (d) 7/3

E Subjective Problems

1. Three charges each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle. *(1978)*
(i) If $Q = -q$, will the charges at the corners move towards centre or fly away from it.
(ii) For what value of Q will the charges remain stationary? In this situation how much work is done in removing the charges to infinity?

2. A rigid insulated wire frame, in the form of right triangle ABC is set in a vertical plane. Two beads of equal masses m each carrying charges q_1 and q_2 are connected by a chord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine : *(1978)*

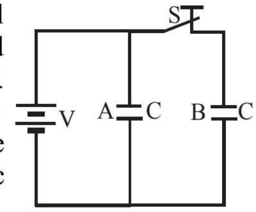


(i) the angle α,
(ii) the tension in the chord, and
(iii) the normal reactions on the beads.
If the chord is now cut, what are the values of the charges for which the beads continue to remain stationary?

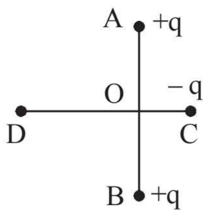
3. A charge 'Q' is distributed over two concentric hollow spheres of radii 'r' and 'R' ($>r$) such that the surface densities are equal. Find the potential at the common centre.
(1981- 3 Marks)

4. A thin fixed ring of radius 1 metre has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0. 9 gm and having a negative charge of 1×10^{-6} coulomb is placed on the axis at a distance of 1 cm from the centre of the ring. show that the motion of the negatively charged particle is approaximately simple harmonic. Calculate the time period of oscillations. *(1982 - 5marks)*

5. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. *(1983 - 6 Marks)*



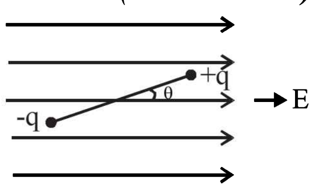
6. Two fixed, equal, positive charges, each of magnitude 5×10^{-5} coul are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. *(1985 - 6 Marks)*



The moving charge, when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.

7. Three particles, each of mass 1 gm and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10$ m/s^2). *(1988 - 5 Marks)*

8. A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-$ q respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5 degree) with the field direction, fig. Find an expression for the minimum time needed for the rod to become parallel to the field after it is set free. *(1989 - 8mark)*



9. Three concentric spherical metallic shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities σ, $-\sigma$ and σ respectively. *(1990 -7 Marks)*
(i) Find the potential of the three shells A, B and C.
(ii) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.

10. Two fixed charges $-2Q$ and Q are located at the points with coordinates ($-3a$, 0) and ($+ 3a$, 0) respectively in the x-y plane. *(1991 - 4 + 2 + 2 Marks)*
(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.
(b) Give the expression $V(x)$ at a general point on the x - axis and sketch the function $V(x)$ on the whole x-axis.
(c) If a particle of charge $+ q$ starts form rest at the centre of the circle, show by a short quantative argument that the particle eventually crosses the circle. Find its speed when it does so.

GP_3481

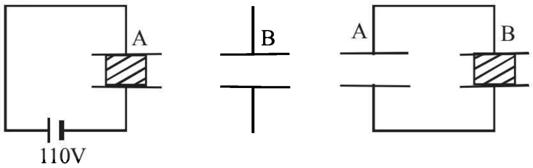
11. (a) A charge of Q coulomb is uniformly distributed over a spherical volume of radius R metres. Obtain an expression for the energy of the system.

(b) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles ?

Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg. m.

(c) If the same charge of Q coulomb as in part (a) above is given to a spherical conductor of the same radius R, what will be energy of the system ? ***(1992 - 10 Marks)***

12. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate area of A and B are 0.04 m^2 and 0.02m^2 respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B. ***(1993 - 2 + 3 + 2 Marks)***



(i) The dielectric slab is placed inside A as shown in figure (a). A is then charged to a potential difference of 110V. Calculate the capacitance of A and the energy stored in it.

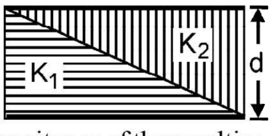
(ii) The battery is disconnected and then the dielectric slab is moved from A. Find the work done by the external agency in removing the slab from A.

(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (c). Calculate the energy stored in the system.

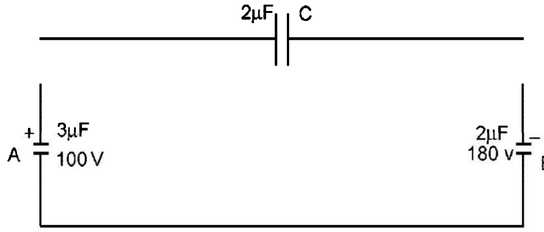
13. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $(R\sqrt{3}, 0, 0)$ on the positive x-axis directly towards O, with an initial speed v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P. ***(1993-4 Marks)***

14. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12}$C^{2}N^{-1}m^{-1}) ***(1994 - 6 Marks)***

15. The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2, respectively. Find the capacitance of the resulting capacitor. ***(1996 - 2 Marks)***



16. Two capacitors A and B with capacities 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate ***(1997 - 5 Marks)***



(i) the final charge on the three capacitors. and

(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit.

17. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given a charge Q, brought into contact with S_2, and removed. S_1 is recharged such that the charge on it is again Q; and it is again brought into contact with S_2 and removed. This procedure is repeated n times. ***(1998 - 8 Marks)***

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

18. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $q/m = 4 \epsilon_0 \, g/\sigma$ ***(1999 - 10 Marks)***

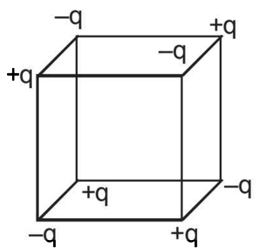
(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

19. Four point charges $+8mC, -1mC, -1mC$, and $+8mC$ are fixed at the points $-\sqrt{\frac{27}{2}}$m, $-\sqrt{\frac{3}{2}}$m, $+\sqrt{\frac{3}{2}}$m and $+\sqrt{\frac{27}{2}}$m respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the -x direction. Its speed at $x = +\infty$ is V_0. Find the least value of V_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free.

Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm}^2/\text{C}^2$. ***(2000 - 10 Marks)***

20. Charges $+q$ and $-q$ are located at the corners of a cube of side as show in the figure. Find the work done to separate the charges to infinite distance. ***(2003 - 2 Marks)***



21. A charge $+Q$ is fixed at the origin of the co-ordinate system while a small electric dipole of dipole moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin. ***(2003 - 4 Marks)***

(a) Calculate the K.E. of the dipole when it reaches to a point $(d, 0)$.

(b) Calculate the force on the charge $+Q$ at this moment.

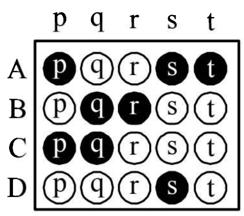
22. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and σ_2 $(\sigma_1 > \sigma_2)$ are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length $a(a < d)$ at an angle 45° with the normal to S_1. Calculate the work done by the electric field ***(2004)***

23. A conducting liquid bubble of radius a and thickness t $(t << a)$ is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. ***(2005 - 2 Marks)***

F Match the Following

DIRECTIONS (Q. No. 1) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column-II**. In each case, a point M and line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to the equivalent to a steady current. ***(2009)***

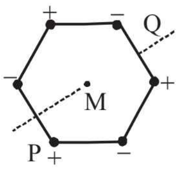
Column-I **Column-II**

A) $E = 0$

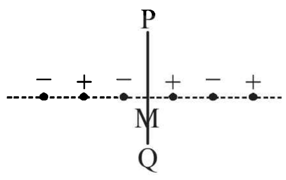
(B) $V \neq 0$

(C) $B = 0$

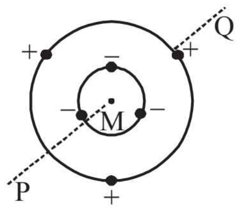
(D) $\mu \neq 0$

(p) 

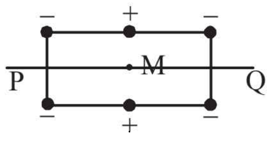
Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon .

(q) 

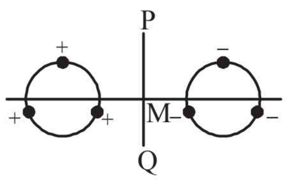
Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.

(r) 

Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

(s) 

Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.

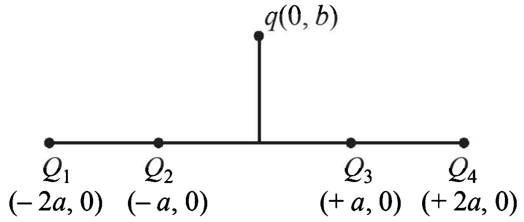
(t) 

Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

DIRECTIONS (Q. No. 2) : *Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

2. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II. Match List-I with List-II and select the correct answer using the code given below the lists. *(JEE Adv. 2014)*

	List - I		List - II
P.	Q_1, Q_2, Q_3, Q_4 all positive	1.	$+x$
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	2.	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	3.	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	4.	$-y$



Codes:

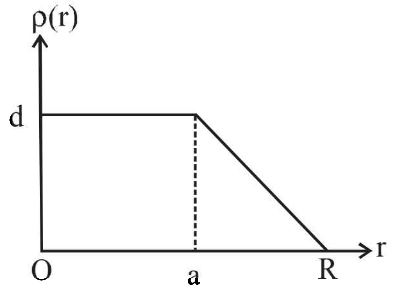
(a) P-3, Q-1, R-4, S-2 (b) P-4, Q-2, R-3, S-1

(c) P-3, Q-1, R-2, S-4 (d) P-4, Q-2, R-1, S-3

G Comprehension Based Questions

PASSAGE-I

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density ρ (r) [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure The electric field is only along the radial direction. *(2008)*



1. The electric field at r = R is

(a) independent of a

(b) directly proportional to a

(c) directly proportional to a^2

(d) inversely proportional to a

2. For a = 0, the value of d (maximum value of ρ as shown in the figure) is –

(a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$

(c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

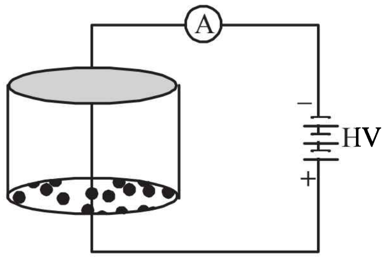
3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies.

(a) a = 0 (b) a = R/2

(c) a = R (d) a = 2R/3

PASSAGE-II

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r <<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



4. Which one of the following statements is correct? *(JEE Adv. 2016)*

(a) The balls will stick to the top plate and remain there

(b) The balls will bounce back to the bottom plate carrying the same charge they went up with

(c) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(d) The balls will execute simple harmonic motion between the two plates

5. The average current in the steady state registered by the ammeter in the circuit will be *(JEE Adv. 2016)*

(a) zero

(b) proportional to the potential V_0

(c) proportional to $V_0^{1/2}$

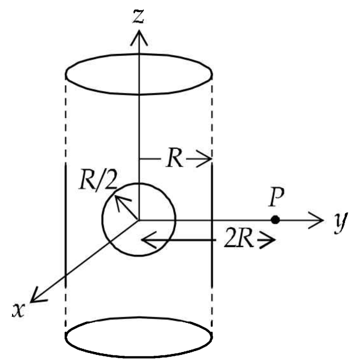
(d) proportional to V_0^2

H Assertion & Reason Type Questions

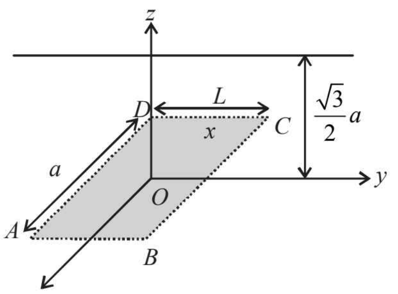
1. **STATEMENT-1 :** For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
and
STATEMENT-2 : The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$. ***(2008)***
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

I Integer Value Correct Type

1. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre.
If the electric field at $r = \frac{R}{2}$ *is* $\frac{1}{8}$ times that at $r = R$, find the value of a. ***(2009)***
2. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is ***(2011)***
3. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16K\varepsilon_0}$. The value of k is ***(2012)***



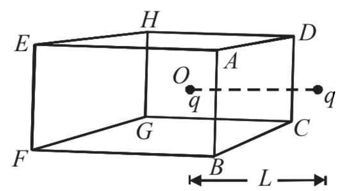
4. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the $y-z$ plane at $z = \frac{\sqrt{3}}{2}$ a (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the $x-y$ plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



(JEE Adv. 2015)

Section-B JEE Main / AIEEE

1. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is **[2002]**
(a) 0.1 V (b) 8V
(c) 2V (d) 0.5V.
2. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to **[2002]**
(a) CV (b) $\frac{1}{2}nCV^2$
(c) CV^2 (d) $\frac{1}{2n}CV^2$
3. A charged particle q is placed at the centre O of cube of length L ($A\,B\,C\,D\,E\,F\,G\,H$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is **[2002]**
(a) $q/4\pi\epsilon_0 L$
(b) zero
(c) $q/2\pi\epsilon_0 L$
(d) $q/3\pi\epsilon_0 L$



4. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is **[2002]**
(a) $Q/2$ (b) $-Q/2$
(c) $Q/4$ (d) $-Q/4$
5. Capacitance (in F) of a spherical conductor with radius 1 m is **[2002]**
(a) 1.1×10^{-10} (b) 10^{-6}
(c) 9×10^{-9} (d) 10^{-3}

6. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **[2003]**

(a) $(\phi_2 - \phi_1)\varepsilon_o$ (b) $(\phi_1 + \phi_2)/\varepsilon_o$

(c) $(\phi_2 - \phi_1)/\varepsilon_o$ (d) $(\phi_1 + \phi_2)\varepsilon_o$

7. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor **[2003]**

(a) decreases (b) remains unchanged

(c) becomes infinite (d) increases

8. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P a distance $\frac{R}{2}$ from the centre of the shell is **[2003]**

(a) $\frac{2Q}{4\pi\varepsilon_o R}$ (b) $\frac{2Q}{4\pi\varepsilon_o R} - \frac{2q}{4\pi\varepsilon_o R}$

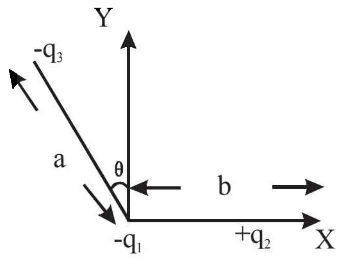
(c) $\frac{2Q}{4\pi\varepsilon_o R} + \frac{q}{4\pi\varepsilon_o R}$ (d) $\frac{(q+Q)2}{4\pi\varepsilon_o R}$

9. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is **[2003]**

(a) 16×10^{-32} joule (b) 3.1×10^{-26} joule

(c) 4×10^{-10} joule (d) 32×10^{-32} joule

10. Three charges $-q_1$, $+q_2$ and $-q_3$ are place as shown in the figure. The x - component of the force on $-q_1$ is proportional to **[2003]**



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

11. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be **[2003]**

(a) 200% (b) 100%

(c) 50% (d) 300%

12. Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that B but **uncharged** is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **[2004]**

(a) $F/8$ (b) $3F/4$

(c) $F/4$ (d) $3F/8$

13. A charge particle 'q' is shot towards another charged particle 'Q' which is fixed, with a speed 'v'. It approaches 'Q' upto a closest distance r and then returns. If q were given a speed of '$2v$' the closest distances of approach would be **[2004]**

(a) $r/2$ (b) $2r$

(c) r (d) $r/4$

14. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is **[2004]**

(a) $-\frac{Q}{2}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{4}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

15. A charged oil drop is suspended in a uniform field of 3×10^4 v/m so that it neither falls nor rises. The charge on the drop will be (Take the mass of the charge = 9.9×10^{-15} kg and $g = 10$ m/s^2) **[2004]**

(a) 1.6×10^{-18} C (b) 3.2×10^{-18} C

(c) 3.3×10^{-18} C (d) 4.8×10^{-18} C

16. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is **[2005]**

(a) $\frac{L}{4}$ (b) $2L$

(c) $4L$ (d) $8L$

17. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is **[2005]**

(a) $\frac{q}{2\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

(b) $\frac{qR}{4\pi\epsilon_0 d^2}$

(c) $\frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

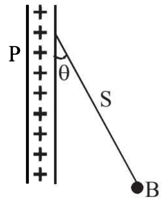
(d) zero

18. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is 'C' then the resultant capacitance is **[2005]**

(a) $(n+1)C$ (b) $(n-1)C$

(c) nC (d) C

19. A charged ball *B* hangs from a silk thread *S*, which makes an angle θ with a large charged conducting sheet *P*, as shown in the figure. The surface charge density σ of the sheet is proportional to **[2005]**



(a) $\cot\theta$
(b) $\cos\theta$
(c) $\tan\theta$
(d) $\sin\theta$

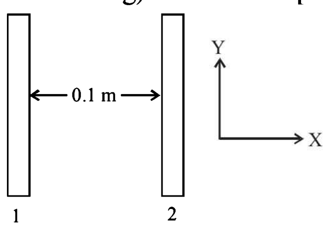
20. A fully charged capacitor has a capacitance '*C*'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity '*s*' and mass '*m*'. If the temperature of the block is raised by 'ΔT', the potential difference '*V*' across the capacitance is **[2005]**

(a) $\dfrac{mC\Delta T}{s}$ (b) $\sqrt{\dfrac{2mC\Delta T}{s}}$

(c) $\sqrt{\dfrac{2ms\Delta T}{C}}$ (d) $\dfrac{ms\Delta T}{C}$

21. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **[2006]**
(a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as a translational force
(d) a torque only

22. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (i.e., plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg) **[2006]**

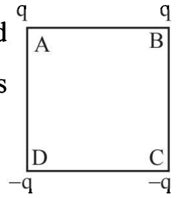


(a) 2.65×10^6 m/s
(b) 7.02×10^{12} m/s
(c) 1.87×10^6 m/s
(d) 32×10^{-19} m/s

23. Two spherical conductors *A* and *B* of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres *A* and *B* is
(a) 4 : 1 (b) 1 : 2 **[2006]**
(c) 2 : 1 (d) 1 : 4

24. An electric charge $10^{-3}\,\mu$C is placed at the origin (0, 0) of *X* – *Y* co-ordinate system. Two points *A* and *B* are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points *A* and *B* will be **[2007]**
(a) 4.5 volts (b) 9 volts
(c) Zero (d) 2 volt

25. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and *V* the potential at the centre. If the charges on *A* and *B* are interchanged with those on *D* and *C* respectively, then **[2007]**



(a) $\vec{E}$ changes, *V* remains unchanged
(b) $\vec{E}$ remains unchanged, *V* changes
(c) both $\vec{E}$ and *V* change
(d) $\vec{E}$ and *V* remain unchanged

26. The potential at a point *x* (measured in μm) due to some charges situated on the *x*-axis is given by
$V(x) = 20/(x^2 - 4)$ volt
The electric field *E* at $x = 4\ \mu$m is given by **[2007]**
(a) (10/9) volt/μm and in the +ve *x* direction
(b) (5/3) volt/μm and in the –ve *x* direction
(c) (5/3) volt/μm and in the +ve *x* direction
(d) (10/9) volt/μm and in the –ve x direction

27. A parallel plate condenser with a dielectric of dielectric constant *K* between the plates has a capacity *C* and is charged to a potential *V* volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[2007]**

(a) zero (b) $\dfrac{1}{2}(K-1)\,CV^2$

(c) $\dfrac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

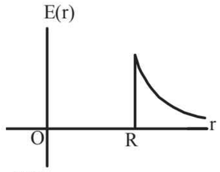
28. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio **[2007]**

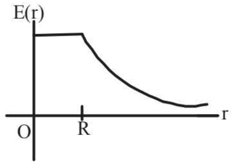
$\dfrac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be

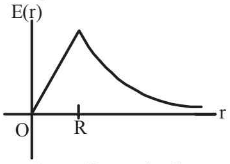
(a) g_M / g_E (b) 1
(c) 0 (d) g_E / g_M

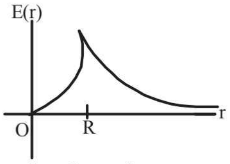
29. A parallel plate capacitor with air between the plates has capacitance of 9 *pF*. The separation between its plates is '*d*'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $\dfrac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\dfrac{2d}{3}$. Capacitance of the capacitor is now **[2008]**
(a) 1.8 *pF* (b) 45 *pF*
(c) 40.5 *pF* (d) 20.25 *pF*

30. A thin spherical shell of radus *R* has charge *Q* spread uniformly over its surface. Which of the following graphs most closely represents the electric field *E*(*r*) produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[2008]**

(a) 

(b) 

(c) 

(d) 

31. Two points P and Q are maintained at the potentials of 10 V and – 4 V, respectively. The work done in moving 100 electrons from P to Q is : **[2009]**

(a) 9.60×10^{-17} J (b) -2.24×10^{-16} J
(c) 2.24×10^{-16} J (d) -9.60×10^{-17} J

32. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals: **[2009]**

(a) –1 (b) 1 (c) $-\frac{1}{\sqrt{2}}$ (d) $-2\sqrt{2}$

33. *This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.* **[2009]**

Statement-1 : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

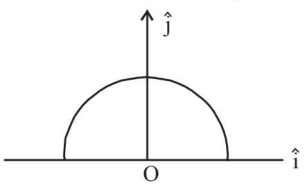
Statement-2 : The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

34. Let $P(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is : **[2009]**

(a) $\frac{Q}{4\pi \in_0 r_1^2}$ (b) $\frac{Q r_1^2}{4\pi \in_0 R^4}$
(c) $\frac{Q r_1^2}{3\pi \in_0 R^4}$ (d) 0

35. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is **[2010]**



(a) $\frac{q}{4\pi^2 \varepsilon_0 r^2} \hat{j}$
(b) $-\frac{q}{4\pi^2 \varepsilon_0 r^2} \hat{j}$
(c) $-\frac{q}{2\pi^2 \varepsilon_0 r^2} \hat{j}$
(d) $\frac{q}{2\pi^2 \varepsilon_0 r^2} \hat{j}$

36. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by **[2010]**

(a) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$
(c) $\frac{4\rho_0 r}{4\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

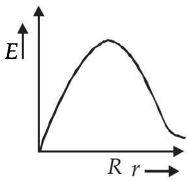
37. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as a function of distance x between them, **[2011]**

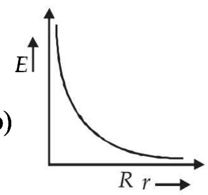
(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

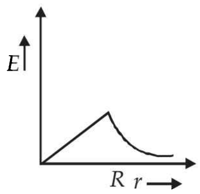
38. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre and a, b are constants. Then the charge density inside the ball is : **[2011]**

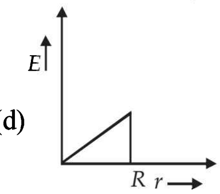
(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

39. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre, The graph which would correspond to the above will be: **[2012]**

(a) 

(b) 

(c) 

(d) 

40. This questions has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describe the two statements. **[2012]**

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinite is zero.

Statement -1 : When a charge q is take from the centre of the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement -2 : The electric field at a distance r ($r < R$) from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of statement 1.
(b) Statement 1 is true Statement 2 is false.
(c) Statement 1 is false Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1

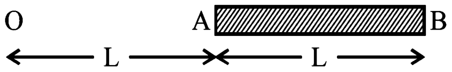
41. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that connecting them together the potential on each one can be made zero. Then **[JEE Main 2013]**

(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

42. Two charges, each equal to q, are kept at x = – a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y <<a) along the y-axis, the net force acting on the particle is proportional to **[JEE Main 2013]**

(a) y (b) –y
(c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

43. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is **[JEE Main 2013]**



(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$
(c) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$ s

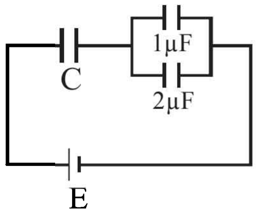
44. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is: **[JEE Main 2014]**

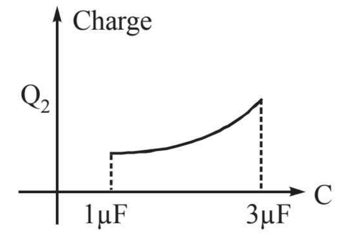
(a) 120 J/C (b) -120 J/C
(c) -80 J/C (d) 80 J/C

45. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[JEE Main 2014]**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2
(c) 3×10^4 C/m^2 (d) 6×10^4 C/m^2

46. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by: (*figures are drawn schematically and are not to scale*) **[JEE Main 2015]**



(a)


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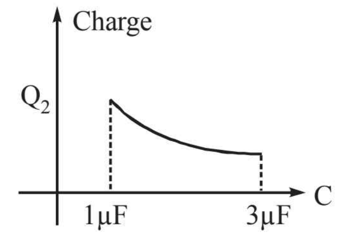
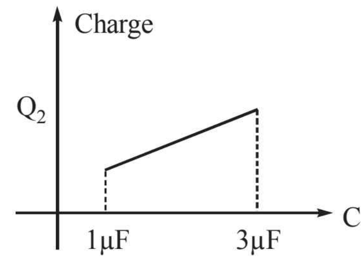
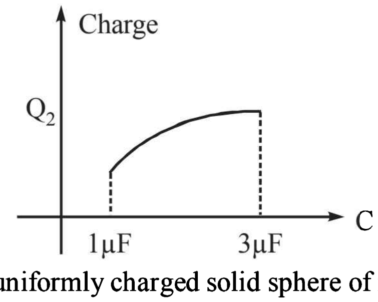


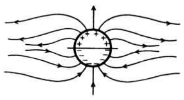
(c)


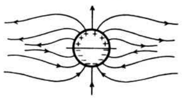
(d)


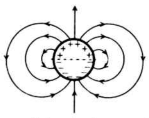
47. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[JEE Main 2015]**

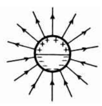
(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(b) $2R < R_4$
(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

48. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge - σ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*) **[JEE Main 2015]**

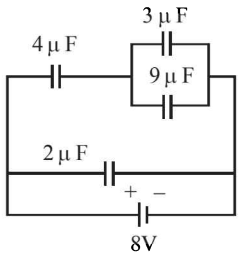
(a) 

(b) 

(c) 

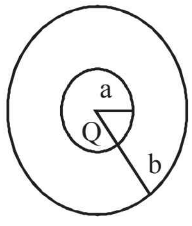
(d) 

49. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μ F and 9 μ F capacitors), at a point distance 30 m from it, would equal : **[JEE Main 2016]**



(a) 420 N/C (b) 480 N/C
(c) 240 N/C (d) 360 N/C

50. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is : **[JEE Main 2016[**



(a) $\frac{2Q}{\pi(a^2 - b^2)}$ (b) $\frac{2Q}{\pi a^2}$

(c) $\frac{Q}{2\pi a^2}$ (d) $\frac{Q}{2\pi(b^2 - a^2)}$

12 Electrostatics

Section-A : JEE Advanced/ IIT-JEE

A

1. $\frac{\varepsilon_0 A}{d}\times V\,;\ \frac{2\varepsilon_0 A}{d}\times V$ **2.** B **3.** $180°,\ \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}$ **4.** $\frac{3V}{k+2}$

5. $-qEa$ **6.** -8 **7.** $\frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2}$

B

1. F	**2.** T	**3.** T	**4.** T	**5.** F	**6.** F

C

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (b)
8. (c)	**9.** (a)	**10.** (b)	**11.** (b)	**12.** (c)	**13.** (c)	**14.** (c)
15. (d)	**16.** (c)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (c)	**21.** (c)
22. (a)	**23.** (b)	**24.** (a)	**25.** (a)	**26.** (d)	**27.** (c)	**28.** (d)
29. (c)	**30.** (a)	**31.** (c)	**32.** (d)	**33.** (c)	**34.** (c)	

D

1. (d)	**2.** (a, d)	**3.** (b)	**4.** (b, d)	**5.** (a)	**6.** (a)	**7.** (a, c, d)
8. (b)	**9.** (c)	**10.** (d)	**11.** (b, c)	**12.** (d)	**13.** (a, c)	**14.** (a, c)
15. (c, d)	**16.** (a,b,d)	**17.** (a)	**18.** (a, d)	**19.** (a, b, c, d)	**20.** (c, d)	**21.** (a, c, d)
22. (a, b, c)	**23.** (d)	**24.** (b, d)	**25.** (c, d)	**26.** (c)	**27.** (a, d)	**28.** (c)
29. (d)	**30.** (d)					

E

1. **(i)** Move towards centre; **(ii)** $Q=\frac{4\sqrt{3}\,q}{9}$, $3(2+\sqrt{3})K\frac{q^2}{a^2}$

2. **(i)** 60°; **(ii)** $mg=\pm k\frac{q_1q_2}{l^2}$; **(iii)** $N_1=\sqrt{3}\,mg$; $N_2=mg$ **3.** $\frac{KQ(R+r)}{R^2+r^2}$ **4.** 0.628 sec. **5.** $\frac{3}{5}$

6. 8.48m **7.** 3.16×10^{-9}C **8.** $\frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

9. **(i)** $\frac{\sigma}{\epsilon_0}(a-b+c)$, $\frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b}-b+c\right)$, $\frac{\sigma}{\epsilon_0}\left(\frac{a^2-b^2+c^2}{c}\right)$ **(ii)** $c=a+b$

10. **(a)** $4a, (5a,0)$ **(b)** $KQ\left[\frac{1}{3a-x}-\frac{2}{3a+x}\right]$ **(c)** $\sqrt{\frac{1}{4\pi\epsilon_0}\left(\frac{Qq}{2ma}\right)}$

11 **(a)** $\frac{3Q^2}{20\pi\epsilon_0 R}$ **(b)** $\frac{3GM^2}{5R}$, $1.5\times10^{32}J$ **(c)** $\frac{Q^2}{8\pi\epsilon_0 R}$

12. **(i)** 2×10^{-9} F, 1.21×10^{-5} J **(ii)** 4.84×10^{-5} J **(iii)** 1.1×10^{-5} J **13.** $\sqrt{\frac{\lambda q}{2\epsilon_0 m}}$

14. 4.425×10^{-9}A. **15.** $\frac{K_1K_2A\epsilon_0}{d(K_1-K_2)}\log\frac{K_1}{K_2}$

16. **(i)** 90×10^{-6}C, 210×10^{-6}C, 150×10^{-6}C **(ii)** 4.74×10^{-2}J, 1.8×10^{-2}J

17. **(a)** $\frac{1}{2}\times\frac{1}{4\pi\epsilon_0 R}\times\left\{\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\right\}^2$ **(b)** $\frac{Q^2R}{2(4\pi\varepsilon_0)r^2}$ **18.** **(a)** $\frac{4a}{3}$ **(b)** $\frac{a}{\sqrt{3}}$

19. 3 m/s, 3×10^{-4} J **20.** $\frac{1}{4\pi\epsilon_0}\frac{q^2}{a}\cdot\frac{4}{\sqrt{6}}(3\sqrt{3}-3\sqrt{6}-\sqrt{2})$

21. **(a)** $\frac{1}{4\pi\epsilon_0}\frac{pQ}{d^2}$ **(b)** $\frac{1}{4\pi\epsilon_0}\frac{2pQ}{d^3}\hat{i}$ **22.** $\frac{q_0(\sigma_1-\sigma_2)a}{\sqrt{2}\epsilon_0}$ **23.** $V\left(\frac{a}{3t}\right)^{1/3}$

F **1.** (A)-(p, r, s): (B)-(r, s); (C)-(p, q, t); (D)-(r, s) **2.** (a)
G **1.** (a) **2.** (b) **3.** (c) **4.** (c) **5.** (d)
H **1.** (a)
I **1.** 2 **2.** 3 **3.** 6 **4.** 6

Section-B : JEE Main/ AIEEE

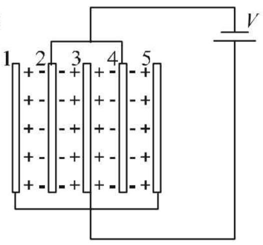
1. (a)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)
10. (b)	**11.** (d)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)
19. (c)	**20.** (c)	**21.** (c)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (a)
28. (b)	**29.** (c)	**30.** (a)	**31.** (c)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (a)
37. (d)	**38.** (c)	**39.** (c)	**40.** (c)	**41.** (b)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (a)
46. (d)	**47.** (a,b)	**48.** (c)	**49.** (a)	**50.** (c)				

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. On the plate 1 there is +ve charge

$$q = CV = \frac{\varepsilon_0 A}{d}\times V$$

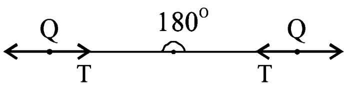


On the plate 4 the charge is

$$-2q = \frac{-2\varepsilon_0 A}{d}\times V$$

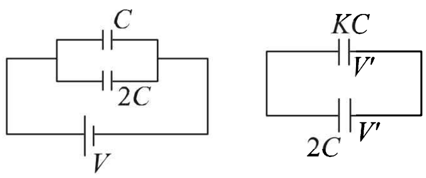
2. It is greatest at point B because at B the equipotential surfaces are closest.

3. There is no gravitational force acting. Only electrostatic force of repulsion is acting which will take the two balls as far as possible. The angle between the two strings will be 180°. The tension in the string will be equal to the electrostatic force of repulsion



$$T = \frac{1}{4\pi\varepsilon_0}\times\frac{Q\times Q}{(2L)^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}$$

4. Initially charge on capacitance $C = q_1 = CV$
Charge on capacitance $C = q_2 = 2CV$



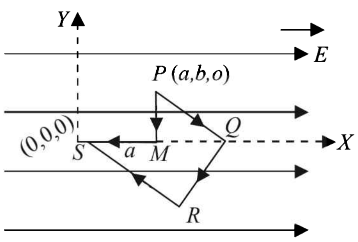
Finally charge on capacitance $C = q_1' = KCV'$
Charge on capacitance $2C = q_2' = 2CV'$
Total charge will remain conserved

$$\therefore\quad CV + 2CV = KCV' + 2CV' \quad \text{or, } V' = \frac{3V}{K+2}$$

5. **NOTE :** Since electric field is conservative in nature, the work done by the field along $PQRS$ will be same as along PMS

Work done from P to $M = \vec{F}\cdot\overrightarrow{PM}$

$= F(PM)\cos 90° = 0$



Work done from M to $S = \vec{F}.\vec{MS}$

$= F(MS)\cos 180°$ $[\because F = qE]$

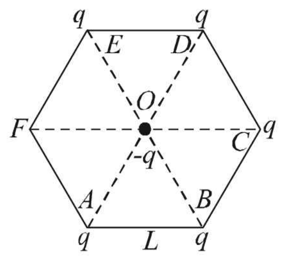
$= -qEa$

6. Electric potential $V = 4x^2$ volts
The electric potential changes only along x-axis,
We know that

$$E_x = \frac{-dV}{dx} \Rightarrow E_x = -\frac{d(4x^2)}{dx} = -8x$$

The electric field at point (1, 0, 2) will be (here $x = 1$)
$E_x = -8$ volt/m.

7.



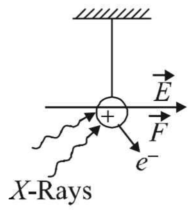
Force on (–q) due to charge at D will get cancelled out by force on (–q) due to change on A. Similarly force on –q due to charge at E will get cancelled out due to charge on B. So the net force will be because of charge on C $F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}$ directed from O to C.

B. True/ False

1. Electrostatic force is conservative in nature, therefore work done is path independent.

2. The metallic sphere which gets negatively charged gains electrons and hence its mass increases.
The metallic sphere which gets positively charged loses electrons and hence its mass decreases.

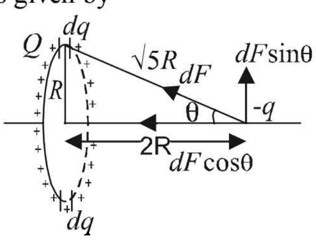
3. When high energy X-ray beam falls, it will knock out electrons from the small metal ball making it positively charged. Therefore the ball will be deflected in the direction of electric field.



4. The electric field produced between the parallel plate capacitor is uniform. The force acting on charged particle placed in an electric field is given by $F = qE$.
In the case of two protons, q and E are equal and hence force will be equal.

5. **KEY CONCEPT :** Force on charge $(-q)$ due to small charge dq situated at length $d\ell$ is given by

$$dF = k\frac{qdq}{5R^2}$$



Resolving this force into two parts $dF\cos\theta$ and $dF\sin\theta$ as shown in figure.

If we take another diametrically opposite length $d\ell$, the charge on it being dq. Then the force on charge $(-q)$ by this small charge dq will be

$$dF = k\frac{qdq}{5R^2}$$

Again resolving this force, we find $dF\sin\theta$ components of the two forces cancel out and $dF\cos\theta$ component adds up.

$\therefore$ The total force

$$F = \int_0^{2\pi R} dF\cos\theta = \int_0^{2\pi R}\frac{kqdq}{5R^2}\times\frac{2R}{\sqrt{5}R}$$

Charge on length $2\pi R = Q$

$\therefore$ Charge on length $d\ell = \frac{Qd\ell}{2\pi R} = dq$

$$\therefore \quad F = \int_0^{2\pi R}\frac{2kq}{5\sqrt{5}R^2}\times\frac{Qd\ell}{2\pi R}$$

$$= \frac{2kQq}{5\sqrt{5}\times 2\pi R^3}\times 2\pi R = \frac{2kQq}{5\sqrt{5}R^2}$$

This is not an equation of simple harmonic motion.

6. For a particle to move in circular motion, we need a centripetal force which is not available.
The statement is false.

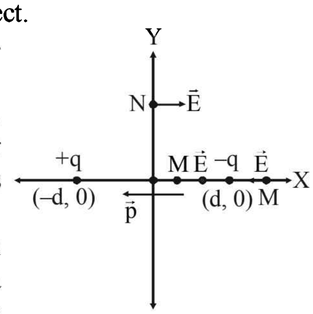
C. MCQs with ONE Correct Answer

1. **(b)** The potential at the surface of a hollow or conducting sphere is same as the potential at the centre of the sphere and any point inside the sphere.

2. **(b)** The two charges form an electric dipole. If we take a point M on the X-axis as shown in the figure, then the net electric field is in $-X$-direction.

$\therefore$ Option (a) is incorrect.

If we take a point N on Y-axis, we find net electric field along $+X$ direction. The same will be true for any point on Y-axis. (b) is a correct option.



NOTE : For any point on the equatorial line of a dipole, the electric field is opposite to the direction of dipole moment.

(b) $W_{\infty - 0} = q(V_\infty - V_0) = q(0-0) = 0$

$\therefore$ (c) is incorrect. The direction of dipole moment is from –ve to + ve. Therefore (d) is incorrect.

3. **(b)** C and $2C$ are in parallel to each other.

$\therefore$ Resultant capacity $= (2C + C)$

$C_R = 3C$

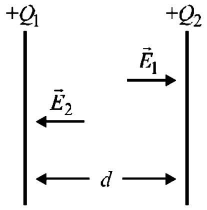
Net potential $= 2V - V$

$V_R = V$

$\therefore$ Final energy $= \frac{1}{2}C_R(V_R)^2 = \frac{1}{2}(3C)(V)^2 = \frac{3}{2}CV^2$

4. **(d)** Within the capacitor,

$$E_1 = \frac{Q_1}{2\varepsilon_0 A}; E_2 = \frac{Q_2}{2\varepsilon_0 A};$$



where A = area of each plate
d = separation between two plate

$$E = E_1 - E_2 = \frac{1}{2\varepsilon_0 A}(Q_1 - Q_2)$$

Hence, $V = Ed$

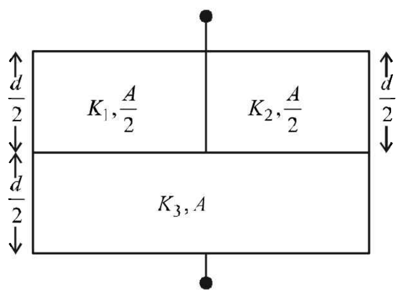
$$= \frac{1}{2}\frac{d}{\varepsilon_0 A}(Q_1 - Q_2) = \frac{Q_1 - Q_2}{2C}$$

5. **(d)** With the closing of switch S_3 and S_1 the negative charge on C_2 will attract the positive charge on C_2 thereby maintaining the negative charge on C_1. The negative charge on C_1 will attract the positive charge on C_1. No transfer of charge will take place. Therefore p.d across C_1 and C_2 will be 30 V and 20 V.

6. **(b)** Here we have $\frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$

$$\therefore \quad Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$$

7. **(b)**



Let C_1 = Capacity of capacitor with K_1
C_2 = Capacity of capacitor with K_2
C_3 = Capacity of capacitor with K_3

$$\therefore \quad C_1 = K_1\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_1}{d}$$

$$\therefore \quad C_2 = K_2\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_2}{d}$$

$$\therefore \quad C_3 = K_3(A)\frac{\varepsilon_0 \times 2}{d} = \frac{2A\varepsilon_0 K_3}{d}$$

C_1 and C_2 are in parallel

$$\therefore \quad C_{eq} = \frac{A\varepsilon_0}{d}(K_1 + K_2)$$

C_{eq} and C_3 are in series

$$\therefore \quad \frac{1}{C} = \frac{d}{A\varepsilon_0(K_1+K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

But $C = \frac{KA\varepsilon_0}{d}$ for single equivalent capacitor

$$\therefore \quad \frac{d}{KA\varepsilon_0} = \frac{d}{A\varepsilon_0(K_1+K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

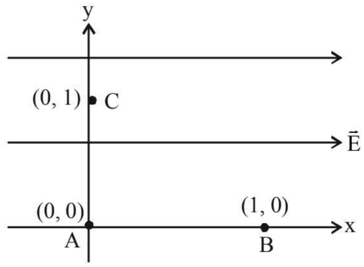
or $\frac{1}{K} = \frac{1}{K_1+K_2} + \frac{1}{2K_3}$.

8. **(c)** Electric field lines do not form closed loops. Therefore options (a), (b) and (d) are wrong.
Option (c) is correct. There is repulsion between similar charges.

9. **(a)** When S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$. Neither there will be any shifting of charge from B to A.

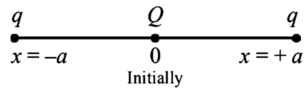
10. **(b)** **NOTE :** As we move along the direction of electric field the potential decreases.

$\therefore \quad V_A > V_B$

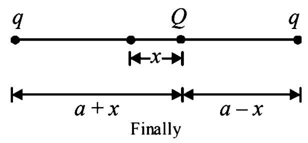


11. **(b)** Initial energy

$$U_i = \frac{2Qq}{4\pi\varepsilon_0 a}$$



Final energy



$$U_f = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{a+x} + \frac{1}{a-x}\right] = \frac{2Qqa}{4\pi\varepsilon_0(a^2 - x^2)}$$

$$U_i - U_f = \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{a}{(a^2 - x^2)}\right]$$

$$= \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{a^2 - x^2 - a^2}{a(a^2 - x^2)}\right] = \frac{-2Qqx^2}{4\pi\varepsilon_0 a^3}$$

when $x << a$ then x^2 is neglected in denominator.

$$U_i - U_f = \left(\frac{-Qq}{2\pi\varepsilon_0 a^3}\right)x^2$$

12. **(c)** Initially we know that

$$\Delta U = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$\Delta U = \frac{1}{2} \times \frac{C \times C}{2C}(V_1 - V_2)^2$$

$$\Delta U = \frac{C}{4}(V_1 - V_2)^2$$

13. **(c)** Electric field everywhere inside the metallic portion of shell is zero.

Hence options (a) and (d) are incorrect.

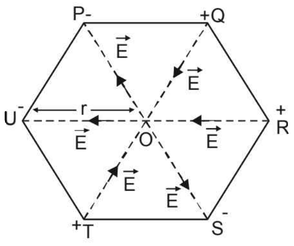
Electric field lines are always normal to a surface. Hence option (b) is incorrect. Only option (c) represents the correct answer.

14. **(c)** $|\vec{E}| = \frac{Kq}{r^2}$

Electric field due to P on O is cancelled by electric field due to S on O.

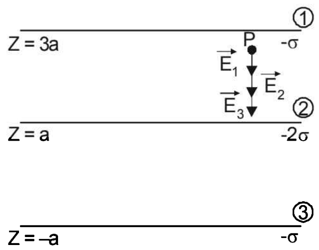
Similarly Electric field due to Q on O is cancelled by electric field due to T and O.

The electric field due to R on O is in the same direction as that of U on O. Therefore the net electric field is $2\vec{E}$.



15. **(d)** The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface will be due to the charges present in side the Gaussian surface and outside it. It will be due to all the charges.

16. **(c)** Figure shows the electric fields due to the sheets 1, 2 and 3 at point P. The direction of electric fields are according to the charge on the sheets (away from positively charge sheet and towards the negatively charged sheet and perpendicular).



The total electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

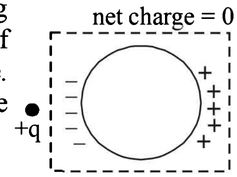
$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$

$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) \quad = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

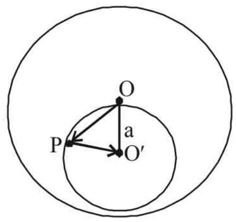
17. **(a)** When a charge density is given to the inner cylinder, the potential developed at its surface is different from that on the outer cylinder. This is because the potential decreases with distance for a charged conducting cylinder when the point of consideration is outside the cylinder.

But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Therefore no potential difference will be produced between the cylinders in this case.

18. **(d)** When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface. But the total charge on the sphere is zero as no charge has left or entered the sphere.



19. **(b)** Let us consider a uniformly charged solid sphere without any cavity. Let the charge per unit volume be σ and O be the centre of the sphere. Let us consider a uniformly charged sphere of negative charged density σ having its centre at O'. Also let OO' be equal to a.



Let us consider an arbitrary point P in the small sphere. The electric field due to charge on big sphere

$$\vec{E}_1 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OP}$$

Also the electric field due to small sphere

$$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{PO'}$$ $\therefore$ The total electric field

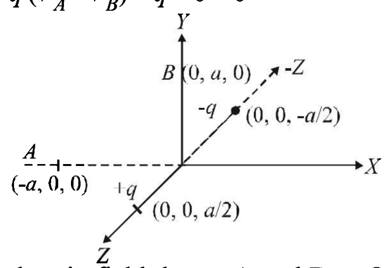
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}[\overrightarrow{OP} + \overrightarrow{PO'}] = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OO'}$$

Thus electric field will have a finite value which will be uniform.

20. **(c)** The charges make an electric dipole. A and B points lie on the equatorial plane of the dipole.

Therefore, potential at A = potential at $B = 0$

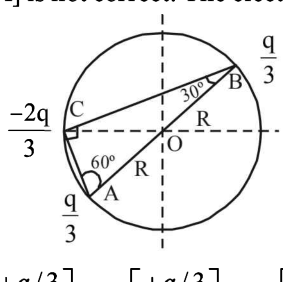
$W = q(V_A - V_B) = q \times 0 = 0$



21. **(c)** The electric field due to A and B at O are equal and opposite producing a resultant which is zero. The electric field at O due to C is

$$E = \frac{1}{4\pi\epsilon_0}\frac{2q/3}{R^2} = \frac{q}{6\pi\epsilon_0 R^2}.$$

$\therefore$ Option [A] is not correct. The electric potential at O is



$$V_O = K\left[\frac{+q/3}{R}\right] + K\left[\frac{+q/3}{R}\right] + K\left[\frac{-2q/3}{R}\right] = 0$$

$\therefore$ Option [D] is wrong

In $\Delta ABC \frac{AC}{AB} = \sin 30^\circ \Rightarrow AC = \frac{AB}{2} = R$

Also $\frac{BC}{AB} = \sin 60^\circ \Rightarrow \quad BC = \frac{\sqrt{3}AB}{2} = \sqrt{3}R$

Potential energy of the system

$$K\left[\frac{(q/3)(2/3)}{2R}\right] + K\left[\frac{(q/3)(-2q/3)}{R}\right] + K\left[\frac{(q/3)(-2q/3)}{\sqrt{3}R}\right]$$

$$= \frac{kq^2}{9R}\left[\frac{1}{2} - 2 - \frac{2}{\sqrt{3}}\right] \neq 0$$

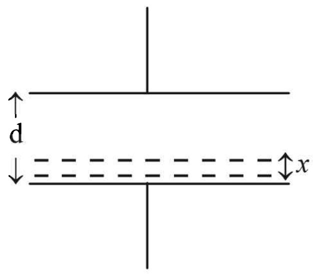
$\therefore$ Option [B] is wrong

Magnitude of force between B and C is

$$F = \frac{1}{4\pi \epsilon_0} \frac{(2q/3)(q/3)}{(\sqrt{3}R)^2} = \frac{q^2}{54\pi \epsilon_0 R^2}$$

22. (a) Let the level of liquid at an instant of time 't' be x. Then

$$v = -\frac{dx}{dt} \Rightarrow dx = -v dt$$



$$\Rightarrow \int_{d/3}^{x} dx = -v \int_{0}^{t} dt$$

$$\Rightarrow x - \frac{d}{3} = -vt$$

$$\Rightarrow x = \frac{d}{3} - vt$$

Also the capacitance can be considered as an equivalent of two capacitances in series such that

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{\frac{\epsilon_0 A}{d-x}} + \frac{1}{\frac{\epsilon_0 AK}{x}} = \frac{d-x}{\epsilon_0 A} + \frac{x}{\epsilon_0 AK}$$

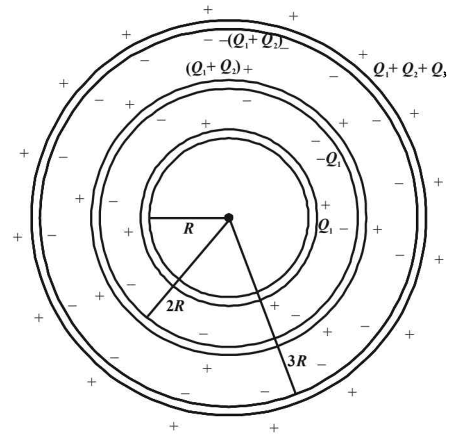
$$\therefore C_{eq} = \frac{\epsilon_0 AK}{Kd + x(1-K)}$$

But $A = 1$, $K = 2$ and $x = \frac{d}{3} - vt$

$$\therefore C_{eq} = \frac{\epsilon_0 \times 1 \times 2}{2d + \left[\frac{d}{3} - vt\right](1-2)} = \frac{6\epsilon_0}{5d + 3vt}$$

$$\therefore \text{Time constant } \tau = RC_{eq} = \frac{6R\epsilon_0}{5d + 3vt}$$

23. (b) The charges on the surfaces of the metallic spheres are shown in the diagram. It is given that the surface charge densities on the outer surfaces of the shells are equal. Therefore



$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2} = x(\text{say})$$

$$\therefore \quad Q_1 = 4\pi R^2 x$$

$$Q_1 + Q_2 = 4\pi(2R)^2 k = 4[4\pi R^2 x]$$

$$\Rightarrow Q_2 = 4[4\pi R^2 x] - Q_1$$

$$= 4[4\pi R^2 x] - 4\pi R^2 x = 3[4\pi R^2 x]$$

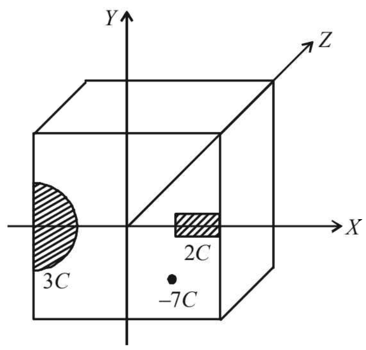
Also $Q_1 + Q_2 + Q_3 = 4\pi(3R)^2 x = 9[4\pi R^2 x]$

$$\therefore \quad Q_3 = 9[4\pi R^2 x] - Q_1 - Q_2 = 9[4\pi R^2 x] - [4\pi R^2 x] - 3[4\pi R^2 x] = 5[4\pi R^2 x]$$

$$\Rightarrow Q_1 : Q_2 : Q_3 = 1 : 3 : 5$$

24. (a) From the figure it is clear that the charge enclosed in the cubical surface is $3C + 2C - 7C = -2C$. Therefore the electric flux through the cube is

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$$



25. (a) The electrostatic pressure at a point on the surface of a uniformly charged sphere $= \frac{\sigma^2}{2\epsilon_0}$

$\therefore$ The force on a hemispherical shell $= \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$

26. (d) **When the electric field is on**

Force due to electric field = weight

$qE = mg$

$qE = \frac{4}{3}\pi R^3 \rho g \quad \therefore q = \frac{4\pi R^3 \rho g}{3E}$ ...(i)

When the electric field is switched off

Weight = viscous drag force

$mg = 6\pi\eta R v_t$

$\frac{4}{3}\pi R^3 \rho g = 6\pi\eta R v_t \quad \therefore R = \sqrt{\frac{9\eta v_t}{2\rho g}}$ ...(ii)

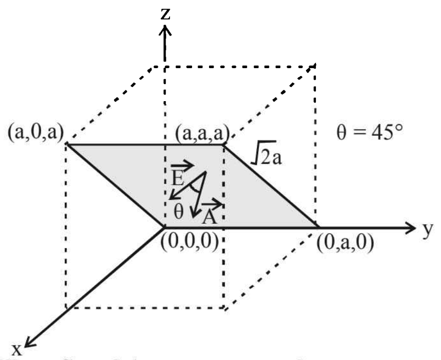
From (i) & (ii) $q = \frac{4}{3}\pi\left[\frac{9\eta v_t}{2\rho g}\right]^{\frac{3}{2}} \times \frac{\rho g}{E}$

$= \frac{4}{3}\times\pi\left[\frac{9\times1.8\times10^{-5}\times2\times10^{-3}}{2\times900\times9.8}\right]^{\frac{3}{2}}\times\frac{900\times9.8\times7}{81\pi\times10^5}$

$= 7.8\times10^{-19}$ C

27. **(c)** Given $\vec{E} = E_o\hat{x}$

This shows that the electric field acts along $+x$ direction and is a constant. The area vector makes an angle of 45° with the electric field. Therefore the electric flux through the shaded portion whose area is $a\times\sqrt{2}a = \sqrt{2}a^2$ is $\phi = \vec{E}.\vec{A} = EA\cos\theta = E_0(\sqrt{2}a^2)$ cos $45° = E_0(\sqrt{2}a^2)\times\frac{1}{\sqrt{2}} = E_0a^2$



28. **(d)** **When S and 1 are connected**

The 2μF capacitor gets charged. The potential difference across its plates will be V.

The potential energy stored in 2 μF capacitor

$U_i = \frac{1}{2}CV^2 = \frac{1}{2}\times2\times V^2 = V^2$

When S and 2 are connected

The 8μF capacitor also gets charged. During this charging process current flows in the wire and some amount of energy is dissipated as heat. The energy loss is

$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

Here, $C_1 = 2\mu F, C_2\, 8\,\mu F, V_1 = V, V_2 = 0$

$\therefore \Delta U = \frac{1}{2}\times\frac{2\times8}{2+8}(V-0)^2 = \frac{4}{5}V^2$

The percentage of the energy dissipated $= \frac{\Delta U}{Ui}\times100$

$= \frac{\frac{4}{5}V^2}{V^2}\times100 = 80\%$

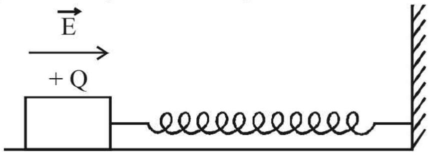
29. **(c)** The pattern of field lines shown in option (c) is correct because

(a) a current carrying toroid produces magnetic field lines of such pattern

(b) a changing magnetic field with respect to time in a region perpendicular to the paper produces induced electric field lines of such pattern.

30. **(a)** The frequency of SHM performed by wooden block is

$v_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

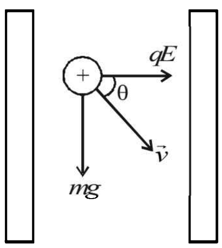


when electric field is switched on, the value of k and m is not affected and therefore the frequency of SHM remains the same. But as an external force QE starts acting on the block towards right, the mean position of SHM shifts towards right by $\frac{QE}{k}$

correct option is (a).

Note : In SHM if a constant additional force is applied then it only shift the quilibrium position and does not change the frequency fo SHM.

31. **(c)** The two forces acting on the proton just after the release are shown in the figure. In this situation



$qE = mg \qquad [\therefore \theta = 45°]$

$\therefore \quad q\left(\frac{V}{d}\right) = mg$

$\therefore \quad V = \frac{mgd}{q} = \frac{1.67\times10^{-27}\times10\times10^{-2}}{1.6\times10^{-19}} = 10^{-9}V$

32. **(d)** For a thin uniformly positive charged spherical shell

(i) Inside the shell at any point

$E = O$ and $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$ = constt.

where q = charge on sphere

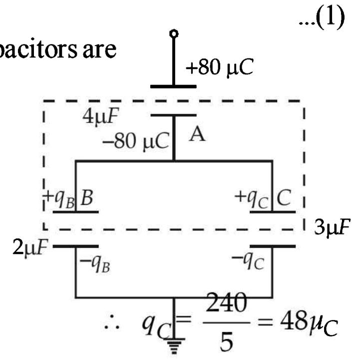
R = Radius of sphere

(ii) Outside the shell at any point at any distance r from the centre $E \propto \frac{1}{r^2}$ and $V \propto \frac{1}{r}$

33. **(c)** The total charge on plate A will be $-80\ \mu C$. If q_B and q_C be the charges on plate B and C then

$q_B + q_C = 80\ \mu C$ ...(1)

Also $2\mu F$ and $3\mu F$ capacitors are in parallel. Therefore,



$$\frac{q_B}{2} = \frac{q_C}{3}$$

$$\therefore \quad \frac{80 - q_C}{2} = \frac{q_C}{3}$$

$$\therefore \quad 240 - 3q_C = 2q_C \qquad \therefore \ q_C = \frac{240}{5} = 48\mu C$$

This charge will obviously be positive.

34. **(c)** $E_1 = \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{Q}{R^2}$;

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{R^2};\ E_3 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q/2}{R^2}$$

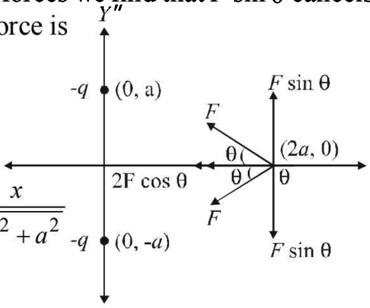
Clearly $E_2 > E_1 > E_3$

where $Q/2$ is the charge enclosed in a sphere of radius R concentric with the given sphere.

$$\left[\frac{4Q}{\frac{4}{3}\pi(2R)^3} = \frac{Q'}{\frac{4}{3}\pi R^3}\right]$$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(d)** Let us consider the positive charge Q at any instant of time t at a distance x from the origin. It is under the influence of two forces $\vec{F_1}$ (= F) and $\vec{F_2}$ (= F). On resolving these two forces we find that $F \sin\theta$ cancels out. The resultant force is



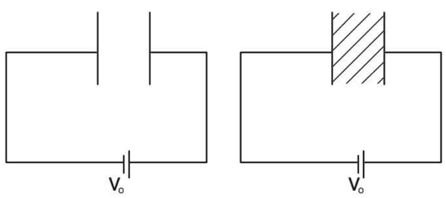
$$F_R = 2F\cos\theta$$

$$= 2 \times \frac{kQq}{(x^2 + a^2)} \times \frac{x}{\sqrt{x^2 + a^2}}$$

$$= \frac{2kQqx}{(x^2 + a^2)^{3/2}}$$

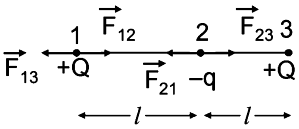
Since F_R is not proportional to x, the motion is NOT simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin the net force is zero but due to momentum it will cross the origin and more towards left. As it comes on negative x-axis, the force is again towards the origin.

2. **(a, d)**



(i) P.d. = V_0 Capacitance = C	P.d. = V_0 Capacitance = KC [K is the dielectric constant of slab $K > 1$]
(ii) $Q_0 = CV_0$	New charge = KCV_0
(iii) Potential Energy $= \frac{1}{2}CV_0^2$	New potential energy $= \frac{1}{2}KCV_0^2$
(iv) $E = \frac{V_0}{d}$	$E = \frac{V_0}{d}$

3. **(b)** q has to be negative for equilibrium.



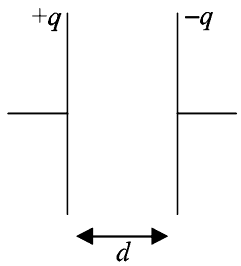
Considering equilibrium of 1

$$F_{13} = F_{12}$$

$$\frac{KQ \times Q}{(2\ell)^2} = \frac{KQ(-q)}{\ell^2} \quad \therefore q = -\frac{Q}{4}$$

4. **(b, d)**

Charge on plate is q

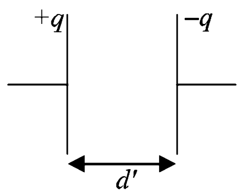


$$C = \frac{\varepsilon_0 A}{d}$$

$$q = CV \Rightarrow V = \frac{q}{C},$$

$$U = \frac{1}{2}q \times V$$

Charge on plate is q



$$C' = \frac{\varepsilon_0 A}{d'} \Rightarrow C' < C,$$

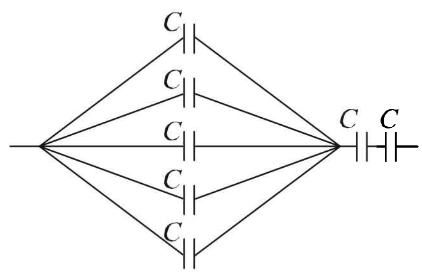
$$V' = \frac{q}{C'} \Rightarrow V' > V$$

$$U' = \frac{1}{2}qV' \Rightarrow U' = U$$

5. **(a)** The potential inside the shell will be the same everywhere as on its surface. As we add $-3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore the potential difference remains the same.

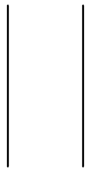
6. **(a)** The equivalent capacitance

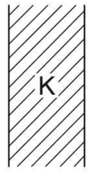
$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2 \times 5} = \frac{11}{10} \Rightarrow C_{eq} = \frac{10}{11}\mu F$$



7. **(a, c, d)**

As $C = \frac{\varepsilon_0}{d}A$ $\qquad$ $C' = \frac{K\varepsilon_0 A}{d}$





$V' = \frac{V}{K}$

$Q = CV$

$= \frac{\varepsilon_0 A}{d} \times V$ $\qquad$ $Q = \frac{\varepsilon_0 AV}{d} = C'V'$

Q will remain same as no charge is leaving or entering the plates during the process of slab insertion

Now, $Q = C'V' = C'E'd$

$$E' = \frac{Q}{C'd} = \frac{\frac{\varepsilon_0 AV}{d}}{\frac{K\varepsilon_0 A}{d}} \times \frac{1}{d} = \frac{V}{Kd}$$

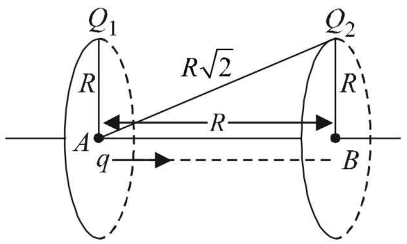
Work done is the change in energy stored

$$W = \frac{1}{2}CV^2 - \frac{1}{2}C'V'^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 AV^2}{d} - \frac{1}{2}\frac{K\varepsilon_0 A}{d} \times \left(\frac{V}{K}\right)^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$W = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left[1 - \frac{1}{K}\right]$$

8. **(b)** The work done in moving a charge from A to B



$W = (T.P.E.)_A - (T.P.E.)_B$ where $T.P.E.$ = Total Potential Energy

$$(T.P.E.)_A = \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right) \times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

$$(T.P.E.)_B = \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right)q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

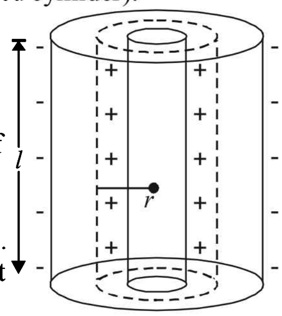
$$\therefore \quad W = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$$

9. **(c)** Let λ be the charge per unit length. Let us consider a Gaussian surface (dotted cylinder).

Applying Gauss's law

$$\phi = \oint \vec{E}.\vec{ds} = \frac{\lambda \ell}{\varepsilon_0}$$



For the flat portions of Gaussian surface, the angle between electric field and surface is 90°. Hence flux through flat portions is zero.

NOTE : By symmetry, the electric field on the curved surface is same throughout.

The angle between $\vec{E}$ and $\vec{ds}$ is 0° (for curved surface)

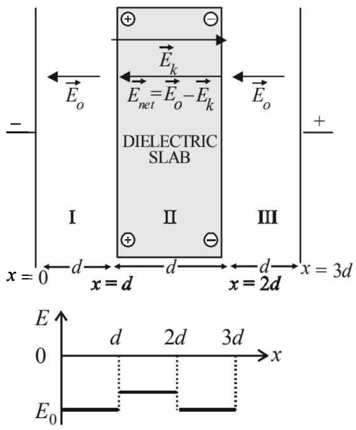
$$\Rightarrow \quad E\int ds = \frac{\lambda \ell}{\varepsilon_0} \quad \Rightarrow \quad E \times 2\pi r\ell = \frac{\lambda \ell}{\varepsilon_0}$$

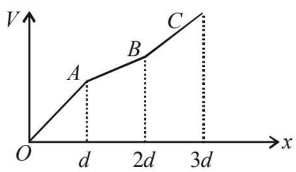
$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \Rightarrow \quad E \propto \frac{1}{r}$$

10. **(d)** The electric lines of force cannot enter the metallic sphere as electric field inside the solid metallic sphere is zero. Also, the origination and termination of the electric lines of force from the metallic surface is normally (directed towards the centre).

11. **(b, c)**

In region I and III, there will be electric field $\vec{E}_0$ directed from positive to negative. In region II, due to orientation of dipoles, there is an electric field $\vec{E}_k$ present in opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $\vec{E}_0 - \vec{E}_k$ in the direction of $\vec{E}_0$ as shown in the diagram. ($\because E_0 > E_k$)





NOTE : When one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the potential increase. Since in region II, the electric field is less as compared to I and III therefore the increase in potential will be less but there has to be increase in potential in all the regions from $x = 0$ to $x = 3d$. Also where E is uniform, $\frac{dV}{dx} = $ constt.

12. **(d)** Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + ...\right]$$

$$V = \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

13. **(a, c)**
Let Q be the charge on the ring, the negative charge $-q$ is released from point $P(0, 0, Z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{QZ_0}{(R^2 + Z_0^2)^{3/2}}$$

Therefore, force on charge P will be towards centre as shown, and its magnitude is

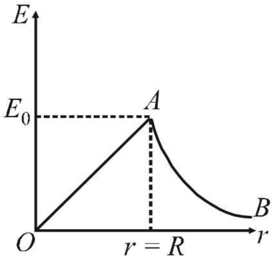
$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(R^2 + Z_0^2)^{3/2}}\cdot Z_0 \qquad ...(1)$$

Similarly, when it crosses the origin, the force is again towards centre O.
Thus the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.
Secondly if $Z_0 << R$, $(R^2 + Z_0^2)^{3/2} \approx R^3$

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}\cdot Z_0 \qquad \text{[From equation 1]}$$

i.e. the restoring force $F_e \propto -Z_0$. Hence the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position).

14. **(a, c)**
KEY CONCEPT : The expressions of the electric field inside the sphere $(r < R)$ $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}r$; outside the sphere $(R < r < \infty)$ $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$



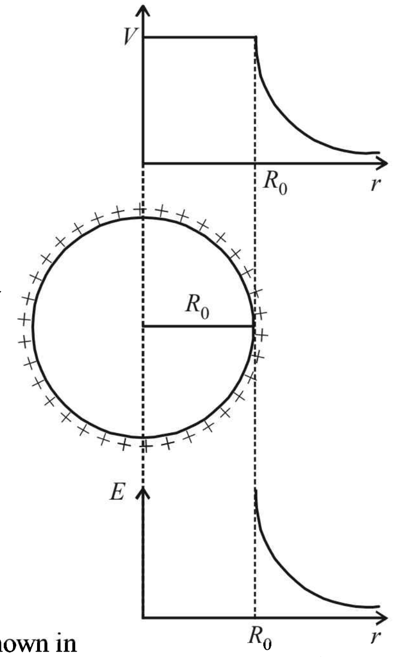
Hence, E increases for $r < R$ and decreases for $R < r < \infty$.

15. **(c, d)**
When two points are connected with a conducting path in electrostatic condition, then the potential of the two points is equal. Thus potential at A = Potential at B
(c) is the correct option.
Option (d) is a result of Gauss's law

Total electric flux through cavity $= \frac{q}{\epsilon_0}$

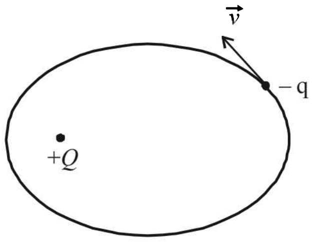
Option (a) and (b) are dependent on the curvature which is different at points A and B.

16. **(a,b,d)**



(a) The whole charge Q will be enclosed in a sphere of diameter $2R_0$.
(b) Electric field E = zero inside the sphere. Hence electric field is discontinued at $r = R_0$.
(c) Changes in V and E are continuously present for $r > R_0$. Option (c) is incorrect.
(d) For $r < R_0$, the potential V is constant and the electric intensity is zero. Obviously, the electrostatic energy is zero for $r < R_0$.

17. **(a)** The situation is shown in the figure which is similar to a planet revolving around sun. The distance of $-q$ from $+Q$ is changing, therefore, force between the charges will change.



The speed of the charge $-q$ will be greater when the charge is nearer to $+Q$ as compared to when it is far. Therefore, the angular velocity of charge $-q$ is also variable. The direction of the velocity changes continously, therefore, linear momentum is also variable. The angular momentum of $(-q)$ about Q is constant because the torque about $+Q$ is zero.

18. **(a,d)**
The electric field lines are orginating from Q_1 and terminating on Q_2. Therefore Q_1 is positive and Q_2 is negative.
As the number of lines associated with Q_1 is greater than that associated with Q_2, therefore $|Q_1| > |Q_2|$.
Option (a) is correct.
At a finite distance on the left of Q_1 , the electric field intensity cannot be zero because the electric field created by Q_1 will be greater than Q_2. This is because the magnitude of Q_1 is greater and the distance smaller $\left[E \propto \frac{Q}{r^2}\right]$

At a finite distance to the right of Q_2, the electric field is zero. Here, the electric field created by Q_2 at a particular point will cancel out the electric field created by Q_1.

19. **(a, b, c, d)**
Electric field inside a spherical metallic shell with charge on the surface is always zero. Therefore option [a] is correct.
When the shells are connected with a thin metal wire then electric potentials will be equal, say V.

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi\epsilon_0}\frac{Q_B}{R_B} = V$$

As $R_A > R_B$ therefore $Q_A > A_B$. option [b] is also correct.

As $\dfrac{\sigma_A}{\sigma_B} = \dfrac{\frac{Q_A}{4\pi R_A^2}}{\frac{Q_B}{4\pi R_B^2}} = \dfrac{R_B^2}{R_A^2} \times \dfrac{Q_A}{Q_B} = \dfrac{R_B^2}{R_A^2} \times \dfrac{4\pi\epsilon_0 R_A V}{4\pi\epsilon_0 R_B V}$

$\therefore \dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ Option (c) is also correct

Also $E_A = \dfrac{\sigma_A}{\epsilon_0}$ & $E_B = \dfrac{\sigma_B}{\epsilon_0}$

$\dfrac{E_A}{E_B} = \dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A} < 1 \quad \therefore E_A < E_B$

Option (d) is also correct

20. (c, d)

(a) is not correct because it is valid only when $E \propto r^{-2}$

(b) is not correct

(c) is correct as between two point charges we will get a point where the electric field due to the two point charges cancel out each other.

(d) is correct when the work done is without accelerating the charge.

21. (a, c, d)

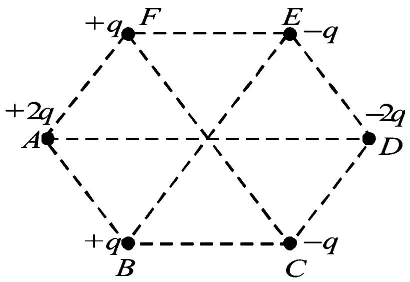
The electric flux passing through $x = +\dfrac{a}{2}$,

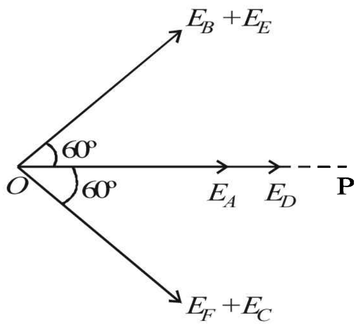
$x = -\dfrac{a}{2}, z = +\dfrac{a}{2}$ is same due to symmetry.

The net electric flux through the cubical region is

$\dfrac{-q+3q-q}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$.

22. (a, b, c)





Here $\dfrac{|\vec{E_A}|}{2} = |\vec{E_B}| = |\vec{E_C}| = \dfrac{|\vec{E_D}|}{2} = |\vec{E_E}| = |\vec{E_F}| = K$

$\therefore \quad E_O = E_A + E_D + (E_F + E_C)\cos 60° + (E_B + E_C)\cos 60°$

$= 2K + 2K + (K + K) \times \dfrac{1}{2} + (K + K) \times \dfrac{1}{2} = 6K$

The electric potential at O is

$$V_O = \frac{1}{4\pi\varepsilon_0 L}[2q + q + q - q - q - 2q] = 0$$

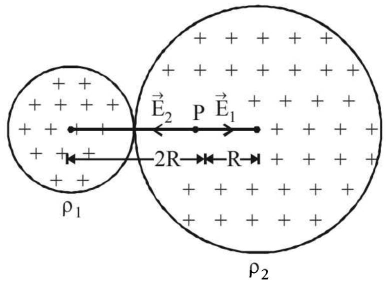
PR is perpendicular bisector (the equatorial line) for the electric dipoles AB, FE and BC. Therefore the electric potential will be zero at any point on PR.

At any point ST, the electric field will be directed from S to T. The potential decreases along the electric field line.

23. (b, d)

Electric field E_1 due to smaller sphere at P is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\rho_1 \times \frac{4}{3}\pi R^3}{(2R)^2}$$



$$E_1 = \frac{1}{4\pi\epsilon_0} \times \frac{\rho_1 \pi R}{3} = \frac{\rho_1 R}{4\epsilon_0 \times 3}$$

Electric field E_2 due to bigger sphere at P is

$$E_2 = \frac{\rho_2 R}{3\epsilon_0}$$

As $E_1 = E_2 \quad \therefore \dfrac{\rho_1 R}{4\epsilon_0 \times 3} = \dfrac{\rho_2 R}{3\epsilon_0} \Rightarrow \dfrac{\rho_1}{\rho_2} = 4$

Option (d) is correct.

24. (b, d)

Step 1 : When S_1 is pressed. The capacitor C_1 gets charged such that its upper plate acquires a positive charge $+2CV_0$ and lower plate $-2CV_0$.

Step 2 : When S_2 is pressed (S_1 open). As $C_1 = C_2$ the charge gets distributed equal. The upper plates of C_1 and C_2 now take charge $+CV_0$ each and lower plate $-CV_0$ each.

(b) and (d) are correct option.

25. (c, d)

Let us consider a point P on the overlapping region. The electric field intensity at P due to positively charged sphere

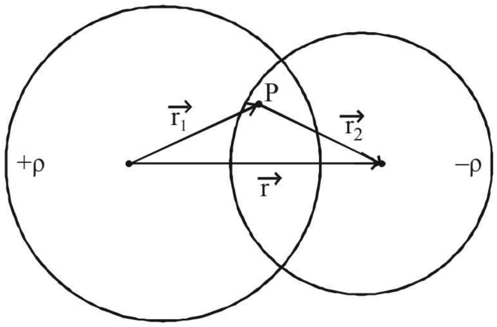
$= \dfrac{\rho\vec{r}_1}{3\epsilon_0}$

The electric field intensity at P due to negatively charged sphere $= \dfrac{\rho\vec{r}_2}{3\epsilon_0}$. The total electric field,

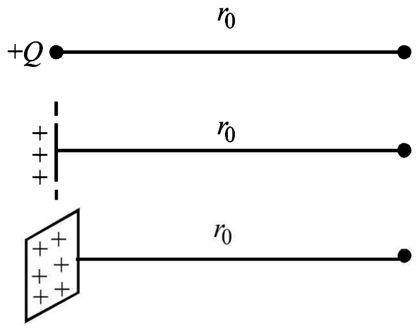
$$\vec{E} = \frac{\rho\vec{r}_1}{3\epsilon_0} + \frac{\rho\vec{r}_2}{3\epsilon_0} = \frac{\rho}{3\epsilon_0}\left[\vec{r}_1 + \vec{r}_2\right]$$

$$\vec{E} = \frac{\rho}{3\epsilon_0}\vec{r}$$

Therefore the electric field is same in magnitude and direction option (c) and (d) are correct.



26. (c)



$E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0}$

$E_3 = \frac{\sigma}{2\epsilon_0}$

$E_1 = E_2$ (Given)

$$\frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0}$$

$\therefore Q = 2\lambda r_0$ ...(1)

$E_2 = E_3$ (Given)

$$\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0} = \frac{\sigma}{2\epsilon_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$$

$\therefore$ (b) is incorrect

$E_1 = E_3$ (Given)

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2} = \frac{\sigma}{2\epsilon_0} \Rightarrow Q = 2\pi\sigma r_0^2$$

$\therefore$ (a) is incorrect

$$\text{Now } E_1(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{4Q}{r_0^2}$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{4 \times 2\lambda r_0}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$$

$$\text{and } 2E_2(r_0/2) = 2\left[\frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0}\right] = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$$

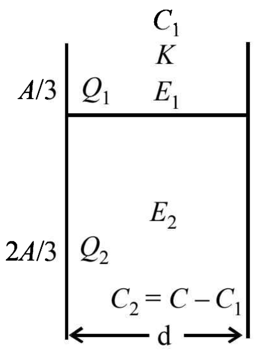
$\therefore$ (c) is correct

$$E_2(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0/2} = \frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0} = \frac{\lambda}{\pi\epsilon_0 r_0}$$

$$4E_3(r_0/2) = \frac{4\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0} = \frac{2}{\epsilon_0} \times \frac{\lambda}{\pi r_0}$$

$\therefore$ (d) is incorrect.

27. (a, d)



This is a combination of two capacitors in parallel. Therefore

$C = C_1 + C_2$ $\quad\therefore C_2 = C - C_1$

where $C_1 = \frac{kA}{3\epsilon_0 d}$ and $C - C_1 = \frac{2A}{3\epsilon_0 d}$

$$\therefore \frac{C - C_1}{C_1} = \frac{2}{k}$$

$$\therefore \frac{C}{C_1} - 1 = \frac{2}{k}$$

$$\therefore \frac{C}{C_1} = \frac{2}{k} + 1$$

$$\frac{C}{C_1} = \frac{2}{k} + 1$$

$\therefore$ (d) is a correct option.

$$\text{Now, } Q_1 = C_1 V = \frac{kA}{3\epsilon_0 d} \times V$$

$$\text{and } Q_2 = (C - C_1)V = \frac{2A}{3\epsilon_0 d} \times V$$

$\therefore \frac{Q_1}{Q_2} = \frac{k}{2}$ $\quad\therefore$ (c) is incorrect

Also $V = E \times d$

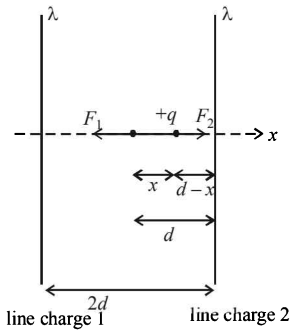
$\therefore E = \frac{V}{d} = E_1 = E_2$ $\quad\therefore$ (a) is a correct option

28 (c) Force on change q when it is given a small displacement x is $F_{net} = F_1 - F_2$

$$F_{net} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d - x} - \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d + x}$$

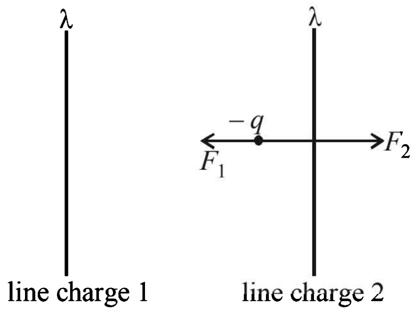
$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{d + x - d + x}{d^2 - x^2}\right]$$

$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\frac{2x}{d^2 - x^2}$$



When $x << d$ then

$F_{net} = \frac{\lambda}{\pi\varepsilon_0}x$ and is directed towards the mean position therefore the charge +q will execute SHM.

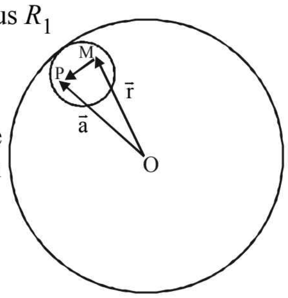


In case of charge $(-q)$

$F_2 > F_1$ therefore the charge $-q$ continues to move in the direction of its displacement.

[C] is the correct option.

29. **(d)** Assume the cavity to contain similar charge distribution of positive and negative charge as the rest of sphere. Electric field at M due to uniformly distributed charge of the whole sphere of radius R_1



$$\vec{E} = \frac{\rho}{3\varepsilon}\vec{r}$$

Electric field at M due the negative charge distribution in the cavity

$$\vec{E}_2 = \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

∴ The total electric field at M is

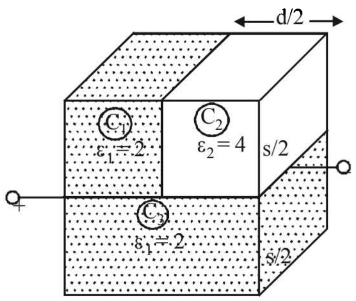
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\left(\vec{a} - \vec{r}\right)\left[\because\ \vec{r} + \overrightarrow{MP} = \vec{a}\right]$$

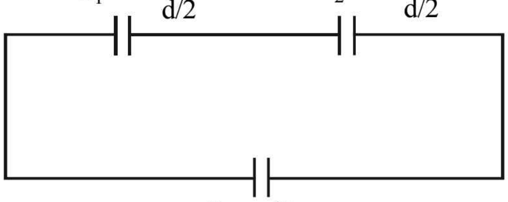
$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{a}$$

(d) is the correct option

30. **(d)**



$$C_1 = \frac{2\,\varepsilon_0\,s/2}{d/2} \qquad C_2 = \frac{4\,\varepsilon_0\,s/2}{d/2}$$



$$C_3 = \frac{2\,\varepsilon_0\,s/2}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2} + C_3 = \frac{\frac{2\varepsilon_0 s}{d} \times \frac{4\,\varepsilon_0 s}{d}}{\frac{6\varepsilon_0 s}{d}} + \frac{\varepsilon_0 s}{d}$$

$$= \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d}$$

$$\therefore \quad C_{eq} = \frac{7}{3}\frac{\varepsilon_0 s}{d} = \frac{7}{3}C_1 \qquad \left[\because C_1 = \frac{\varepsilon_0 s}{d}\right]$$

E. Subjective Problems

1. **(i)** The force on charge q kept at A due to charges kept at B and C

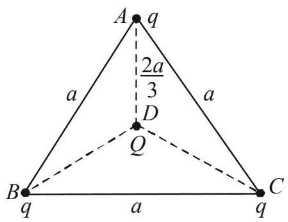
$F_1 = 2F\cos 30°$

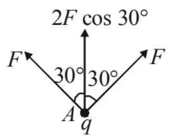
$$F_1 = \sqrt{3} \times \left(9 \times 10^9 \frac{q^2}{a^2}\right)$$

The force on q due to charge $(-q)$ kept at D

$$F_2 = 9 \times 10^9 \frac{q^2}{(2a/3)^2} = \frac{9}{4} \times \left(9 \times 10^9 \times \frac{q^2}{a^2}\right)$$

Clearly the two forces are not equal. Also as $F_2 > F_1$ the charges will move towards the centre.





(ii) For charges to remain stationary

$$2 \times K\frac{q^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{9}{4} \times K \times \frac{q^2 Q}{a^2} \Rightarrow \frac{4\sqrt{3}\,q}{9} = Q$$

The charge Q should be negative.

The potential energy of the system is

$$= 3\left[K\frac{q^2}{a^2} + K\frac{q^2}{a^2}\right] + 3\left[K \times \frac{4\sqrt{3}}{9}\frac{q \times q}{(2a/3)^2}\right]$$

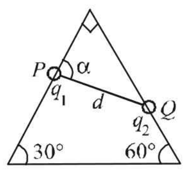
$$= 6K \times \frac{q^2}{a^2} + 3\sqrt{3}\,K\frac{q^2}{a^2} = 3(2 + \sqrt{3})K\frac{q^2}{a^2}$$

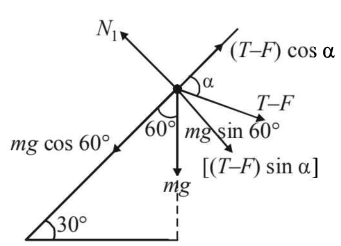
This is the amount of work needed to move the charges to infinity.

2. Because of equilibrium of charge q_1

$N_1 = mg\sin 60° + (T - F)\sin\alpha$ ... (i)

and $(T - F)\cos\alpha = mg\cos 60°$ ... (ii)

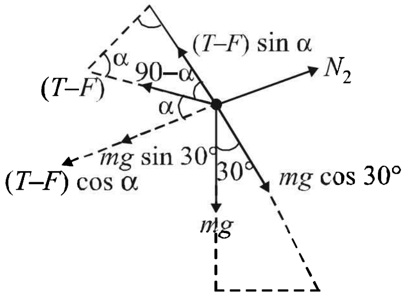




Because of equilibrium of charge q_2

$(T-F)\sin\alpha = mg\cos 30°$ ... (iii)

and $N_2 = (T-F)\cos\alpha + mg\sin 30°$ ... (iv)



From (i) and (iii)

$N_1 = mg\sin 60° + mg\cos 30°$

$$= mg\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,mg$$

From (ii) and (iv)

$$N_2 = mg\cos 60° + mg\sin 30° = mg\left(\frac{1}{2}+\frac{1}{2}\right) = mg$$

Also, $F = k\frac{q_1q_2}{\ell^2}$

Now from eqn. (ii) and (iii), we get

$(T-F)^2\cos^2\alpha + (T-F)^2\sin^2\alpha$

$= m^2g^2\cos^2 60° + m^2g^2\cos^2 30°$

$$\Rightarrow (T-F)^2 = m^2g^2\left[\frac{1}{4}+\frac{3}{4}\right] = m^2g^2$$

$\Rightarrow T - F = \pm mg$ ... (v)

$\Rightarrow T = mg + F = mg + k\frac{q_1q_2}{\ell^2}$ ... (vi)

[Taking positive sign]

From (ii) and (v)

$mg\cos\alpha = mg\cos 60° \Rightarrow \cos\alpha = \cos 60°$

$\therefore$ $\alpha = 60°$

when the string is cut, $T = 0$

$\therefore$ From (vi)

$$mg = \pm k\frac{q_1q_2}{\ell^2} \Rightarrow q_1q_2 = \pm\frac{mg\ell^2}{k}$$

Now the charges should be unlike for equilibrium.

3. Let q be the charge on the inner sphere and $(Q-q)$ be the charge on outer sphere.

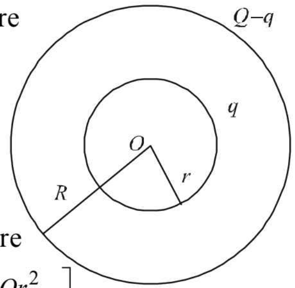
Given that surface charge densities are equal.

$$\therefore \frac{q}{4\pi r^2} = \frac{Q-q}{4\pi R^2} \quad \left(\text{Surface charge density}, \sigma = \frac{q}{A}\right)$$

or, $qR^2 = (Q-q)r^2$ or, $qR^2 = Qr^2 - qr^2$

$$\therefore q = \frac{Qr^2}{R^2+r^2}$$

Potential at O due to inner sphere



$$V_i = K\frac{q}{r} = \frac{K}{r}\left(\frac{Qr^2}{R^2+r^2}\right)$$

$$V_i = K\frac{Qr}{R^2+r^2}$$

Potential at O due to outer sphere

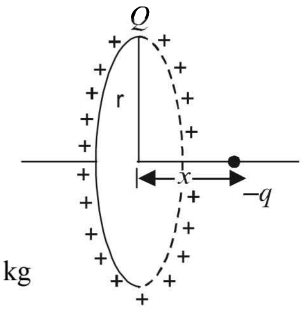
$$V_0 = K\frac{(Q-q)}{R} = \frac{K}{R}\left[Q - \frac{Qr^2}{R^2+r^2}\right]$$

$$= \frac{K}{R}\frac{\left[QR^2+Qr^2-Qr^2\right]}{(R^2+r^2)} = \frac{K(QR)^2}{R\left(R^2+r^2\right)} = \frac{KQR}{\left(R^2+r^2\right)}$$

The total potential at the common centre

$$V = V_i + V_0 = \frac{KQr}{R^2+r^2} + \frac{KQR}{R^2+r^2} = \frac{KQ(R+r)}{R^2+r^2}$$

4. **KEY CONCEPT :** The electric field due a uniformly charged ring of radius r at a point distant x from its center on its axis is given by



Fixed Ring

$$E = k\frac{Qx}{(r^2+x^2)^{3/2}}$$

$r = 1\text{m}$

$Q = 10^{-5}\,\text{C}$

mass of particle $m = 0.9\times 10^{-3}$ kg

charge on particle $q = -10^{-6}$ C

$\therefore$ Force on the negative charge q will be $F = qE$

$$\therefore F = \frac{-kQq}{(r^2+x^2)^{3/2}}\times x \quad \text{or,} \quad mA = \frac{-kQq}{(r^2+x^2)^{3/2}}\times x$$

$$\text{or,} \quad A = -k\frac{Qq}{m(r^2+x^2)^{3/2}}\times x$$

For $x \ll r$ $A = -\frac{kQq}{r^3}\times x$

$\Rightarrow$ The motion is simple harmonic in nature.

Comparing the above equation with $A = -\omega^2 x$ we get

$$\therefore \omega^2 = \frac{kQq}{mr^3} \quad \text{or} \quad \omega = \sqrt{\frac{kQq}{mr^3}}$$

$$\therefore \frac{2\pi}{T} = \sqrt{\frac{kQq}{mr^3}} \Rightarrow T = 2\pi\sqrt{\frac{mr^3}{kQq}}$$

$$T = 2\times 3.14\left[\frac{0.9\times10^{-3}\times1^3}{9\times10^9\times10^{-5}\times10^{-6}}\right]^{1/2}$$

$= 6.28\,[0.01]^{1/2} = 6.28\,[0.1]$

$T = 0.628$ sec

5. The potential difference across each capacitor is V.
Total Energy = Energy in A + Energy in B

$$= \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

When the switch opened and a dielectric is inserted between the plates of capacitors, the new capacitance is 3C.

$$\text{Energy in } A = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2 \quad (V \text{ is the same})$$

$$\text{Energy in } B = \frac{1}{2}\frac{q^2}{KC} = \frac{1}{2} \times \frac{(CV)^2}{3C}$$

$$= \frac{CV^2}{6}$$ (charge on capacitor B remains same when switch is opened)

Total Energy = Energy in A + Energy in B

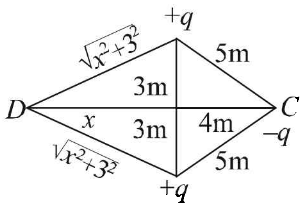
$$\therefore \quad \text{Total Energy} = \frac{3}{2}CV^2 + \frac{1}{6}CV^2 = \frac{5}{3}CV^2 \quad \text{...(ii)}$$

$$\frac{\text{Total Energy initially}}{\text{Total energy finally}} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5}$$

6. Total energy of the system of three charges when the charge $-q$ is at C

= P.E. + K.E.

$$= \left[\frac{Kq \times q}{6} + \frac{K(q)(-q)}{5} + \frac{Kq(-q)}{5}\right] + 4 \quad \text{...(i)}$$



Final energy of the system of three charges when $-q$ is at D and momentarily at rest

= P.E. + K.E.

$$= \left[\frac{Kq \times q}{6} + \frac{Kq(-q)}{\sqrt{x^2+3^2}} + \frac{Kq(-q)}{\sqrt{x^2+3^2}}\right] + 0$$

$$= \frac{Kq \times q}{6} + \frac{2Kq(-q)}{\sqrt{x^2+3^2}} \quad \text{...(ii)}$$

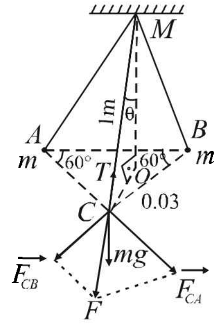
By the principle of conservation of energy from (i) and (ii), we get

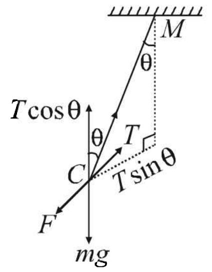
$$\frac{kq \times q}{6} + \frac{2kq(-q)}{5} + 4 = \frac{kq \times q}{6} + \frac{2kq(-q)}{\sqrt{x^2+3^2}}$$

$$2 = kq^2\left[\frac{1}{5} - \frac{1}{\sqrt{x^2+3^2}}\right]$$

$\therefore \quad x^2 + 9 = 81 \quad \therefore x = 8.48$ m

7. Each mass will be in equilibrium under the act of three force namely tension of string, weight, resultant electrostatic force of the two other charges out of these three forces F and mg are perpendicular.





Let T make an angle θ with the vertical

$$OC = \frac{2}{3}\sqrt{(0.03)^2 - (0.015)^2} = 0.0173 \text{ m}$$

$\therefore \quad OM = 0.9997$

NOTE THIS STEP : Resolving T in the direction of mg and F and applying the condition of equilibrium, we get

$T\cos\theta = mg$; $\qquad T\sin\theta = F$

$$\therefore \quad \tan\theta = \frac{F}{mg} \quad \text{...(i)}$$

$$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA}F_{CB}\cos\alpha}$$

$$\therefore \quad F = \sqrt{F_{CA}^2 + F_{CA}^2 + 2F_{CA}^2 \times \frac{1}{2}}$$

$$F = \sqrt{3}F_{CA} = \sqrt{3} \times \frac{kq^2}{(CA)^2} \quad \text{...(ii)}$$

[where F_{CB} = Force on C due to B
F_{CA} = Force on C due to A
$|\vec{F}_{CB}| = |\vec{F}_{CA}|$ and $\alpha = 60°$]

$$\text{Also, } \tan\theta = \frac{OC}{OM} = \frac{0.0173}{0.9997} \quad \text{....(iii)}$$

From (i), (ii) and (iii)

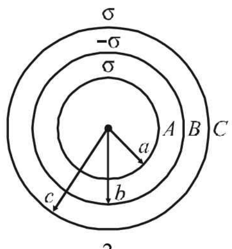
$$\frac{0.0173}{0.9997} = \frac{\sqrt{3} \times 9 \times 10^9 \times q^2}{(0.03)^2 \times 10^{-3} \times 9.8}$$

On solving, we get $q = 3.16 \times 10^{-9}$ C.

8. Time for the dipole to align along the direction of electric field will be

$$t = \frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{ML}{2qE}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}.$$

9. Charge on Shell $A = q_A = \sigma(4\pi a^2)$
Charge on Shell $B = q_B = \sigma(4\pi b^2)$
Charge on Shell $C = q_C = \sigma(4\pi c^2)$
The potential of shell A



$$V_A = \frac{kq_A}{a} + \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$= \frac{k\sigma(4\pi a^2)}{a} + \frac{k(-\sigma)(4\pi b^2)}{b} + \frac{k\sigma(4\pi c^2)}{c}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \sigma \times \frac{4\pi a^2}{a} - \frac{1}{4\pi\varepsilon_0}\sigma\frac{(4\pi b^2)}{b} + \frac{1}{4\pi\varepsilon_0} \times \sigma \frac{(4\pi c^2)}{c}$$

$= \frac{\sigma}{\varepsilon_0}[a-b+c]$ Similarly, $V_B = \frac{kq_A}{b} + \frac{kq_B}{b} + \frac{kq_C}{c}$

and $V_C = \frac{kq_A}{c} + \frac{kq_B}{c} + \frac{kq_C}{c}$

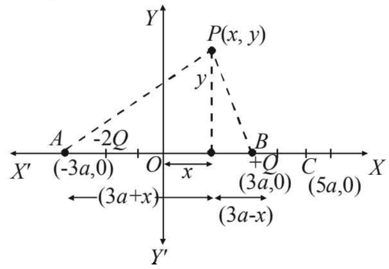
$$V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right] \text{ and } V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

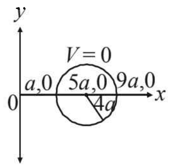
Given that $V_A = V_C$

$$\frac{\sigma}{\varepsilon_0}(a-b+c) = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2+c^2}{c}\right]$$

or $ac - bc + c^2 = a^2 - b^2 + c^2$ or $c = a + b$

10. **(a)** Let P be a point in the X-Y plane with coordinates (x, y) at which the potential due to charges $-2Q$ and $+Q$ placed at A and B respectively be zero.





$$\therefore \quad \frac{K(2Q)}{\sqrt{(3a+x)^2 + y^2}} = \frac{K(+Q)}{\sqrt{(3a-x)^2+y^2}}$$

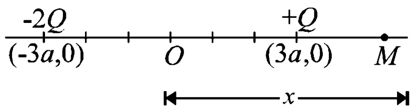
$$\Rightarrow \quad 2\sqrt{(3a-x)^2+y^2} = \sqrt{(3a+x)^2+y^2}$$

$$\Rightarrow \quad (x-5a)^2 + (y-0)^2 = (4a)^2$$

This is the equation of a circle with centre at $(5a, 0)$ and radius $4a$. Thus C $(5a, 0)$ is the centre of the circle.

(b) **For $x > 3a$**

To find $V(x)$ at any point on X-axis, let us consider a point (arbitrary) M at a distance x from the origin.



The potential at M will be

$$V(x) = \frac{K(-2Q)}{x+3a} + \frac{K(+Q)}{(x-3a)} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore \quad V(x) = KQ\left[\frac{1}{x-3a} - \frac{2}{x+3a}\right] \text{ for } |x| > 3a$$

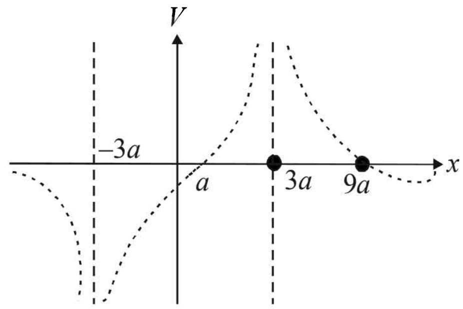
Similarly, for $0 < |x| < 3a$

$$V(x) = KQ\left[\frac{1}{3a-x} - \frac{2}{3a+x}\right]$$

Since circle of zero potential cuts the x-axis at $(a,0)$ and $(9a,0)$

Hence, $V(x) = 0$ at $x = a$, at $x = 9a$

- From the above expressions
  $V(x) \to \infty$ at $x \to 3a$ and $V(x) \to -\infty$ at $x \to -3a$
- $V(x) \to 0$ as $x \to \pm\infty$
- $V(x)$ varies $\frac{1}{x}$ in general.



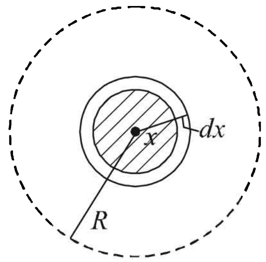
(c) Applying Energy Conservation

$(\text{K.E.} + \text{P.E.})_{\text{centre}} = (\text{K.E.} + \text{P.E.})_{\text{circumference}}$

$$0 + K\left[\frac{Qq}{2a} - \frac{2Qq}{8a}\right] = \frac{1}{2}mv^2 + K\left[\frac{Qq}{6a} - \frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2 = \frac{KQq}{4a}, \quad v = \sqrt{\frac{KQq}{2ma}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

11. **(a)** Let us consider a shell of the thickness dx at a distance x from the centre of a sphere



The vol. of the shell $= \frac{4}{3}\pi\left[(x+dx)^3 - \frac{4}{3}\pi x^3\right]$

$$= \frac{4}{3}\pi\left[(x+dx)^3 - x^3\right]$$

$$= \frac{4}{3}\pi x^3\left[\left(1+\frac{dx}{x}\right)^3 - 1\right]$$

$$= \frac{4}{3}\pi x^3\left[1 + \frac{3dx}{x} - 1\right]$$

$$= \frac{4}{3}\pi x^3 \times \frac{3dx}{x} = 4\pi x^2 dx$$

Let ρ be the charge per unit volume of the sphere

$\therefore$ Charge of the shell $= dq = 4\pi x^2 \rho dx$ ... (i)

Potential at the surface of the sphere of radius x

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \times \frac{4}{3}\pi x^3}{x} \qquad \left[\because V = k\frac{q}{r}\right]$$

$\therefore$ Potential at the surface of the sphere of radius x =

$$\frac{\rho x^2}{3\varepsilon_0}$$

Work done in bringing the charge dq on the sphere of radius x

$$dW = \frac{\rho x^2}{3\varepsilon_0} \times dq \Rightarrow dW = \frac{\rho x^2}{3\varepsilon_0} \times 4\pi x^2 \rho\, dx$$

Therefore the work done in accumulating the charge Q over a spherical volume of radius R meters

$$W = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} x^4 dx = \frac{4\pi\rho^2}{3\varepsilon_0}\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5}$$

$$= \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{4/3\pi R^3}\right)^2 \frac{R^5}{5} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This is also the energy stored in the system.

(b) The above energy calculated is

$$E = \frac{3Q^2}{5\times(4\pi\varepsilon_0)R} = \frac{3KQ^2}{5R} \text{ where } K = \frac{1}{4\pi\varepsilon_0}$$

NOTE : In case of earth and gravitational pull, K may be replaced by G. Therefore the energy required to disassemble the planet earth against the gravitational pull amongst its constituent particle is the work required to make earth from its constituent particles.

$$\therefore \quad E = \frac{3GM^2}{5R} \qquad [\because Q \text{ is replaced by } M]$$

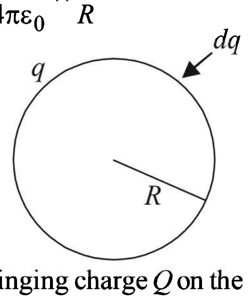
$$\text{But } g = \frac{GM}{R^2} \Rightarrow gMR = \frac{GM^2}{R}$$

$$F = \frac{Kq_1q_2}{r^2};\ F = \frac{Gm_1m_2}{r^2}$$

$$\therefore \quad E = \frac{3}{5}gMR = \frac{3}{5}\times 10\times 2.5\times 10^{31} = 1.5\times 10^{32}\text{ J}$$

(c) During the charging process, let at any instant the spherical conductor has a charge q on its surface.

The potential at the surface $= \frac{1}{4\pi\varepsilon_0}\times\frac{q}{R}$



Small amount of work done in increasing charge dq more on the surface will be

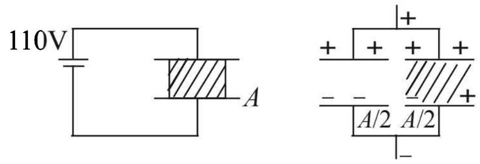
$$dW = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{R}\times dq$$

$\therefore$ Total amount of work done in bringing charge Q on the surface of spherical conductor.

$$W = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\,dq = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{(8\pi\varepsilon_0 R)}$$

12. **(i) NOTE :** The capacitor A with dielectric slab can be considered as two capacitors in parallel, one having dielectric slab and one not having dielectric slab. Each capacitor has an area of $\frac{A}{2}$.

The combined capacitance is



$$C = C_1 + C_2$$

$$= \frac{(A/2)\varepsilon_0}{d} + \frac{(A/2)\varepsilon_0\varepsilon_r}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}[1+\varepsilon_r]$$

$$= \frac{0.4\times 8.85\times 10^{-12}}{2\times 8.85\times 10^{-4}}[1+9] = 2\times 10^{-9}F$$

$$\therefore \quad \text{Energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}\times 2\times 10^{-9}\times(110)^2$$

$$= 1.21\times 10^{-5}\text{ J}$$

(ii) Work done in removing the dielectric slab = (Energy stored in capacitor without dielectric) – (Energy stored in capacitor with dielectric).

NOTE : While taking out the dielectric, the charge on the capacitor plate remains the same.

$$\therefore \quad W = \frac{q^2}{2C'} - \frac{q^2}{2C} \quad \text{Here,} \quad C = 2\times 10^{-9}F,$$

$$C' = \frac{A\varepsilon_0}{d} = \frac{0.04\times 8.85\times 10^{-14}}{8.85\times 10^{-4}} = 0.4\times 10^{-9}F$$

$$q = CV = 2\times 10^{-9}\times 110 = 2.2\times 10^{-7}C$$

$$\therefore \quad W = \frac{(2.2\times 10^{-7})^2}{2}\left[\frac{1}{0.4\times 10^{-9}} - \frac{1}{2\times 10^{-9}}\right]$$

$$= 4.84\times 10^{-5}\text{ J}$$

(iii) The capacitance of $B = \frac{\varepsilon_0\varepsilon_r A_B}{d}$

$$= \frac{8.85\times 10^{-12}\times 9\times 0.02}{8.85\times 10^{-4}}$$

$$C_B = 1.8\times 10^{-9}F$$

The charge on A, $q_A = 2.2\times 10^{-7}C$ gets distributed into two parts.

$\therefore \quad q_1 + q_2 = 2.2\times 10^{-7}C$

also the potential difference across A = p.d. across B

$$\frac{q_1}{C_A} = \frac{q_2}{C_B}$$

$$\Rightarrow q_1 = \frac{C_A}{C_B}q_2 = \frac{0.4\times 10^{-9}}{1.8\times 10^{-9}}q_2 = 0.22\,q_2$$

$$\therefore \quad 0.22\,q_2 + q_2 = 2.2\times 10^{-7}$$

$$\Rightarrow q_2 = \frac{2.2}{1.22}\times 10^{-7} = 1.8\times 10^{-7}C$$

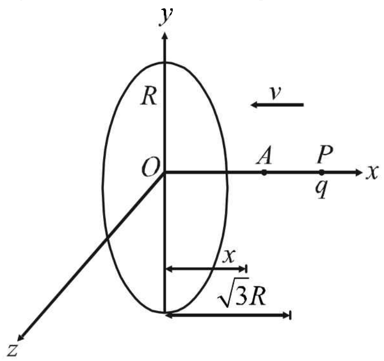
$$\Rightarrow q_1 = 0.4\times 10^{-7}C$$

Total energy stored $= \dfrac{q_1^2}{2C_A} + \dfrac{q_2^2}{2C_B}$

$$= \frac{0.4 \times 0.4 \times 10^{-14}}{2 \times 0.4 \times 10^{-9}} + \frac{1.8 \times 1.8 \times 10^{-14}}{2 \times 1.8 \times 10^{-8}}$$

$= 0.2 \times 10^{-5} + 0.9 \times 10^{-5} = 1.1 \times 10^{-5}$ J

13. Potential energy can be found at the initial point A and final point O. The difference in potential energy has to be provided by the K.E. of the charge at A.



$$V(x) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\sqrt{R^2 + x^2}}, \text{ at } A.$$

$$V_O = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi R\lambda}{R}, \text{ at } O. \text{ or } V_O = \frac{\lambda}{2\varepsilon_0}$$

$$V_P = \frac{1}{4\pi\varepsilon_0} \frac{2\pi R\lambda}{\sqrt{R^2 + (\sqrt{3}R)^2}} = \frac{\lambda}{4\varepsilon_0}$$

Potential difference between points O and $P = V$

$\therefore \quad V = V_O - V_P$

or $\quad V = \dfrac{\lambda}{2\varepsilon_0} - \dfrac{\lambda}{4\varepsilon_0}$ or $V = \dfrac{\lambda}{4\varepsilon_0}$

The kinetic energy of the charged particle is converted into its potential energy at O.

$\therefore$ Potential energy of charge $(q) = qV$

Kinetic energy of charged particle $= \dfrac{1}{2}mv^2$

For minimum speed of particle so that it does not return to P,

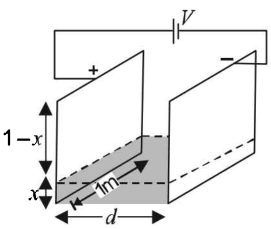
$$\frac{1}{2}mv^2 = qV \text{ or } v^2 = \frac{2qV}{m} = \frac{2q \times \lambda}{m \times 4\varepsilon_0}$$

or $\quad v = \sqrt{\dfrac{q\lambda}{2\varepsilon_0 m}}$

14. The adjacent figure is a case of parallel plate capacitor. The combined capacitance will be

$C = C_1 + C_2$

$$= \frac{k\varepsilon_0(x \times 1)}{d} + \frac{\varepsilon_0[(1-x) \times 1)}{d}$$



$$C = \frac{\varepsilon_0}{d}[kx + 1 - x] \qquad \text{... (i)}$$

Differentiating the above quation w.r.t. time

$$\frac{dC}{dt} = \frac{\varepsilon_0}{d}(k-1)\frac{dx}{dt} = \frac{\varepsilon_0}{d}(k-1)v$$

where $\quad v = \dfrac{dx}{dt}$

We know that $q = CV$, $\dfrac{dq}{dt} = V\dfrac{dC}{dt}$

From (iii) and (iv)

$$I = V\frac{\varepsilon_0}{d}(k-1)v$$

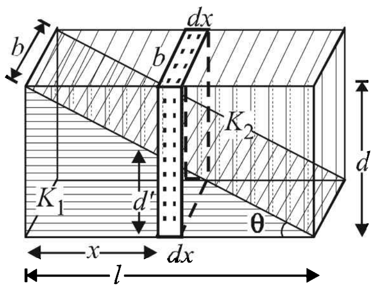
$$I = \frac{500 \times 8.85 \times 10^{12}}{0.01}(11-1) \times 0.001$$

$= 4.425 \times 10^{-9}$ Amp.

15. **Case (i)** When no dielectric :

Given $\quad C = \dfrac{\varepsilon_0 A}{d}$

Case (ii) When dielectric is filled : A small dotted element of thickness dx is considered as shown in the figure.



The small capacitance of the dotted portion

$\dfrac{1}{dC} = \dfrac{1}{dC_1} + \dfrac{1}{dC_2}$ where dC_1 = capacitance of capacitor with dielectric K_1

dC_2 = capacitance of capacitor with dielectric K_2.

Let ℓ, b the length and breadth of the capacitor plate. Therefore $\ell \times b = A$.

$$dC_1 = \frac{K_1(bdx)\varepsilon_0}{d'}$$

$$d' = d - x\frac{d}{\ell} = d\left[1 - \frac{x}{\ell}\right]$$

$$\therefore \quad dC_1 = \frac{K_1 b(dx)\,\varepsilon_0}{d\left[1 - \dfrac{x}{\ell}\right]} - \frac{K_1 b\ell(dx)\,\varepsilon_0}{d(\ell - x)} = \frac{K_1 A\,\varepsilon_0(dx)}{d(\ell - x)}$$

Similarly, $dC_2 = \dfrac{K_2\,\varepsilon_0(bdx)}{d - d'} = \dfrac{K_2\,\varepsilon_0 bdx}{d - d + \dfrac{xd}{\ell}}$

$$\frac{K_2\varepsilon_0 b.\ell.dx}{xd} = \frac{K_2\,\varepsilon_0 A\,dx}{xd}$$

$\therefore \quad \dfrac{1}{dC}=\dfrac{d(\ell-x)}{K_1A\varepsilon_0(dx)}+\dfrac{xd}{K_2A\varepsilon_0(dx)}$

$\Rightarrow \quad \dfrac{K_1K_2A\varepsilon_0dx}{K_2\ell d+d(K_1-K_2)x}=dC$

To find the capacitance of the whole capacitor, we integrate the above equation.

$$C=\int_0^{\ell}\frac{K_1K_2A\varepsilon_0dx}{K_2\ell d+d(K_1-K_2)x}$$

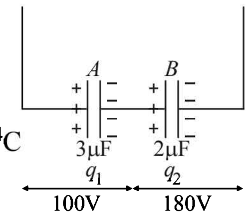
$$=K_1K_2A\varepsilon_0\int_0^{\ell}\frac{dx}{K_2\ell d+d(K_1-K_2)x}$$

$$=K_1K_2A\varepsilon_0\left[\frac{\log[K_2\ell d+d(K_1-K_2)x]}{d(K_1-K_2)}\right]_0^{\ell}$$

$$C=\frac{K_1K_2A\varepsilon_0}{d(K_1-K_2)}\log\frac{K_1}{K_2}$$

16. **(i)** **KEY CONCEPT :** Use charge conservation to solve this problem.

INITIALLY:



Charge on capacitor A

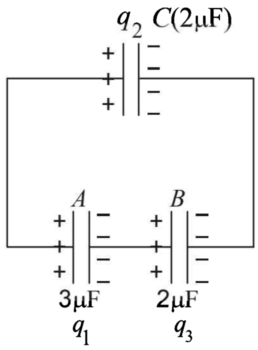
$q_A=3\times10^{-6}\times100=3\times10^{-4}$C

Charge on capacitor B

$q_B=2\times10^{-6}\times180=3.6\times10^{-4}$C

FINALLY:

Let the charge on capacitor A, C and B be q_1, q_2 and q_3 respectively.



By charge conservation.

The sum of charge on +ve plate of capacitor A and C should be equal to q_A

$\therefore \quad q_1+q_2=3\times10^{-4}$C... (i)

Similarly the sum of charge on –ve plates of capacitor C and B will be equal to q_B

$\therefore \quad -q_2-q_3=-3.6\times10^{-4}$C

$\Rightarrow \quad q_2+q_3=3.6\times10^{-4}$C ... (ii)

Applying Kirchoff's law in the closed loop, we get

$$\frac{q_1}{3\times10^{-6}}-\frac{q_2}{2\times10^{-6}}+\frac{q_3}{2\times10^{-6}}=0$$

$\Rightarrow \quad 2q_1-3q_2+3q_3=0$ ....(iii)

On solving (i), (ii) and (iii), we get

$q_1=90\times10^{-6}$ C, $q_2=210\times10^{-6}$ C,

and $q_3=150\times10^{-6}$ C,

(ii) Amount of electrostatic energy in the system initially

$$U_i=U_A+U_B=\frac{1}{2}C_A(V_A)^2+\frac{1}{2}C_B(V_B)^2$$

$$=\frac{1}{2}\times3\times10^{-6}(100)^2+\frac{1}{2}\times2\times10^{-6}(180)^2$$

$=4.74\times10^{-2}$J

Amount of electrostatic energy stored finally

$$U_f=\frac{1}{2}\frac{q_1^2}{C_A}+\frac{1}{2}\frac{q_2^2}{C_B}+\frac{1}{2}\frac{q_3^2}{C_C}$$

$$=\frac{1}{2}\frac{(90\times10^{-6})^2}{3\times10^{-6}}+\frac{1}{2}\frac{(210\times10^{-6})^2}{2\times10^{-6}}+\frac{1}{2}\frac{(150\times10^{-6})^2}{2\times10^{-6}}$$

$$+\frac{1}{2}\frac{(150\times10^{-6})^2}{2\times10^{-6}}=1.8\times10^{-2}\text{ J}$$

17. Limiting value of energy as $n\to\infty$.

Let us calculate q_n when n tends to ∞.

For G.P., $S_\infty=\dfrac{a}{1-r_1}$ where r_1 = common ratio

$\therefore \quad q_\infty=\dfrac{QR}{R+r}\left[\dfrac{1}{1-\dfrac{R}{R+r}}\right]$ or $q_\infty=\dfrac{QR}{r}$

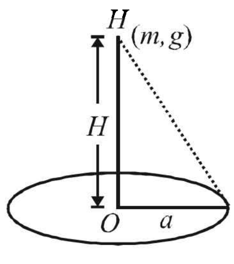
$\therefore \quad U_\infty=\dfrac{q_\infty^2}{2C}=\left(\dfrac{QR}{r}\right)^2\times\dfrac{1}{2\times(4\pi\varepsilon_0)\times(R)}$

or $\quad U_\infty=\dfrac{Q^2R^2}{r^2\times2\times4\pi\varepsilon_0R}$ or $U_\infty=\dfrac{Q^2R}{2(4\pi\varepsilon_0)r^2}$

18. **(a)** **KEY CONCEPT :** The K.E. of the particle, when it reaches the disc is zero.

Given that a = radius of disc, σ = surface charge density, $q/m=4\varepsilon_0g/\sigma$

Potential due to a charged disc at any axial point situated at a distance x from O is,



$$V(x)=\frac{\sigma}{2\varepsilon_0}[\sqrt{a^2+x^2}-x]$$

Hence, $V(H)=\dfrac{\sigma}{2\varepsilon_0}[\sqrt{a^2+H^2}-H]$

and $V(O)=\dfrac{\sigma a}{2\varepsilon_0}$

NOTE : According to law of conservation of energy, loss of gravitational potential energy = gain in electric potential energy

$mgH=q\Delta V$

$=q\,[V(0)-V(H)]$

$mgH=q\,\dfrac{\sigma}{2\varepsilon_0}\,[a-\{\sqrt{(a^2+H^2)}-H\}]$ ...(1)

From the given relation : $\dfrac{\sigma q}{2\varepsilon_0}=2mg$

Putting this in equation (1), we get,

$mgH=2mg\,[a-\{\sqrt{(a^2+H^2)}-H\}]$

or $\quad H=\dfrac{4a}{3}$ $\quad$ [$\because H=O$ is not valid]

(b) Total potential energy of the particle at height H

$U(x) = mgx + qV(x)$

$$= mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2+x^2} - x)$$

$$= mgx + 2mg\,[\sqrt{(a^2+x^2)} - x] \quad ....(2)$$

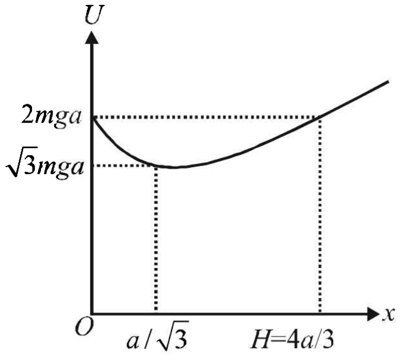
$$U_{(A)} = mgH + 2mg\left[\sqrt{a^2+H^2} - H\right]$$

$$= mg\left[2\sqrt{a^2+H^2} - H^2\right] \quad ....(3)$$

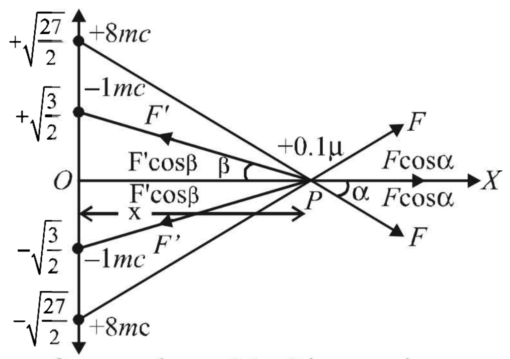
For equilibrium : $\frac{dU}{dH} = 0$

This gives : $H = \frac{a}{\sqrt{3}}$ $\therefore$ $U_{min} = \sqrt{3}$ mga

From equation (2), graph between $U(x)$ and x is as shown above.



19. Let the particle at some instant be at a point P distant x from the origin. As shown in the figure, there are two forces of repulsion acting due to two charges of + 8 mC. The net force is $2F\cos\alpha$ towards right.
Similarly there are two forces of attraction due to two charges of – 1 mC. The net force due to these force is $2F'\cos\beta$ towards left.



The net force on charge 0.1 μC is zero when
$2F\cos\alpha = 2F'\cos\beta$

$$\frac{K\times 8\times10^{-6}\times0.1\times10^{-6}}{\left(\sqrt{x^2+\frac{27}{2}}\right)^2}\times\frac{x}{\sqrt{x^2+\frac{27}{2}}}$$

$$= \frac{K\times 1\times10^{-6}\times0.1\times10^{-6}}{\left(\sqrt{x^2+\frac{3}{2}}\right)^2}\times\frac{x}{\sqrt{x^2+\frac{3}{2}}}$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{5}{2}}$$

This means that we need to move the charge from – ∞ to $\sqrt{\frac{5}{2}}$. Thereafter the attractive forces will make the charge move to origin.

The electric potential of the four charges at $x = \sqrt{\frac{5}{2}}$ is

$$V = \frac{2\times9\times10^9\times8\times10^{-6}}{\sqrt{\frac{5}{2}+\frac{27}{2}}} - \frac{2\times9\times10^9\times10^{-6}}{\sqrt{\frac{5}{2}+\frac{3}{2}}}$$

$$= 2\times9\times10^9\times10^{-6}\left[\frac{8}{4}-\frac{1}{2}\right] = 2.7\times10^4\,\text{V}$$

Kinetic energy is required to overcome the force of repulsion from ∝ to $x = \sqrt{\frac{5}{2}}$.

The work done in this process is $W = q\,(V)$

where V = p.d between ∞ and $x = \sqrt{\frac{5}{2}}$.

$\therefore$ $W = 0.1\times10^{-6}\times2.7\times10^4 = 2.7\times10^{-3}$ J

By energy conservation $\frac{1}{2}mV_0^2 = 2.7\times10^{-3}$

$$\Rightarrow \quad \frac{1}{2}\times6\times10^{-4}V_0^2 = 2.7\times10^{-3}$$

$\Rightarrow$ $V_0 = 3$ m/s

K.E. at the origin

Potential at origin

$$V_{x=0} = \frac{2\times9\times10^9\times8\times10^{-6}}{\sqrt{\frac{27}{2}}} - \frac{2\times9\times10^9\times10^{-6}}{\sqrt{\frac{3}{2}}}$$

$= 2.4\times10^4$

Again by energy conservation

$$\text{K.E.} = q\left[V_{x=\frac{\sqrt{5}}{2}} - V_{x=0}\right]$$

$\therefore$ K.E. $= 0.1\times10^{-6}\,[2.7\times10^4 - 2.4\times10^4]$
$= 0.1\times10^{-6}\times0.3\times10^4$
$= 3\times10^{-4}$ J

20. $$W_{\text{external}} = \Delta PE = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\frac{-3}{1}+\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right]\times4$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}.\frac{4}{\sqrt{6}}[3\sqrt{3} - 3\sqrt{6} - \sqrt{2}]$$

21. **(a)** Potential energy of the dipole-charge system
$U_i = 0$ (since the charge is far away)

$$U_f = -Q \times \frac{1}{4\pi\varepsilon_0}\frac{p}{d^2} \quad \text{[at a point } (d, \omega)]$$

$\therefore$ K.E. $= |U_f - U_i| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{pQ}{d^2}$

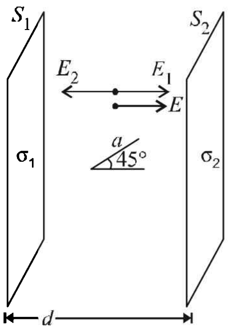
(b) Electric field at origin due to dipole

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i}$$

Thus, force on charge Q is given by

$$\vec{F} = Q\vec{E} = \frac{2pQ}{4\pi\varepsilon_0 d^3}\hat{i}$$

22. Electric field due to S_1, $E_1 = \dfrac{\sigma_1}{\varepsilon_0}$



Electric field due to S_2, $E_2 = \dfrac{\sigma_2}{\varepsilon_0}$

$\therefore$ $E = E_1 - E_2$

$= \dfrac{\sigma_1 - \sigma_2}{\varepsilon_0} \quad (\because \sigma_1 > \sigma_2)$

Work done by electric field

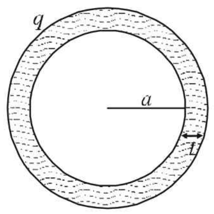
$$W = (q_0 E) a \cos 45^\circ = q_0 E \times \frac{a}{\sqrt{2}}$$

$$\therefore \quad W = \frac{q_0(\sigma_1 - \sigma_2)a}{\sqrt{2}\,\varepsilon_0}$$

23. *LIQUID BUBBLE :* The potential of the liquid bubble is V.

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} \quad \text{...(1)}$$

where q is the charge on the liquid bubble.



LIQUID DROPLET

The volume of liquid droplet = Volume (of the liquid) in liquid bubble.

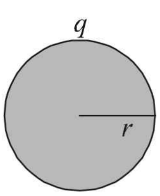
$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(a+t)^3 - \frac{4}{3}\pi a^3$$

or, $r^3 = a^3 + t^3 + 3a^2t + 3at^2 - a^3$

or, $r^3 = 3a^2t$

($\because$ t is very small as compared to a)

or, $r = [3a^2t]^{1/3}$ ... (iii)



NOTE : By charge conservation we can conclude that charge on liquid bubble is equal to charge on liquid droplet Let charge on liquid droplet is q.

$\therefore$ Potential on liquid droplet

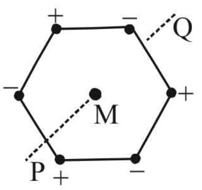
$$V_{\text{droplet}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

or, $V_{\text{droplet}} = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{4\pi\varepsilon_0 V \times a}{[3a^2t]^{1/3}}$ [From (i) and (ii)]

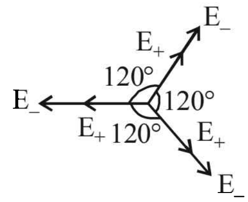
or, $V_{\text{droplet}} = V\left[\dfrac{a}{3t}\right]^{1/3}$

F. Match the Following

1.



The electric field at M due to the charges at the corners of regular hexagon is as shown



Here $|E_+| = |E_-|$. The symmetry of the situation shows that $E = 0$ at M.

Therefore (A) is the correct option.

The electric potential due to all the charges at M is zero.

Therefore (B) is incorrect option.

When the system of charges is rotated about line PM, the net current will be zero.

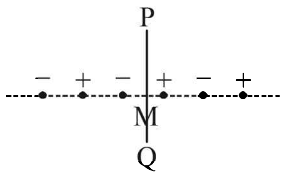
Therefore the magnetic field at M is zero.

(C) is the correct option.

When magnetic field is zero, then $\mu = 0$

(D) is incorrect option.

(q)



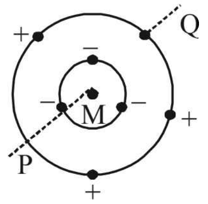
The electric field due to the inner most positive and negative charges at M is $E_1 = 2\left[k\dfrac{q}{r^2}\right]$ towards left. The electic field due to the next positive and negative charges at M is $E_2 = 2\left[k\dfrac{q}{(2r)^2}\right]$ towards right. The electric field due to the outermost positive and negative charges at M is $E_3 = 2\left[k\dfrac{q}{(3r)^2}\right]$ towards left. Clearly the vector sum of these three electric field is not zero.

(A) is incorrect option.

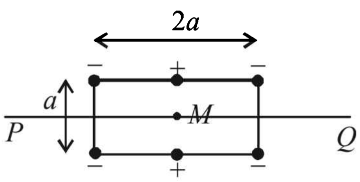
The electric potential due to the charges at M

$$= k\left[\frac{+q}{r} \frac{-q}{r} + \frac{q}{2r} - \frac{q}{2r} + \frac{q}{3r} - \frac{q}{3r}\right] = 0$$

(B) is incorrect option.
The net current due to the innermost positive and negative charges is zero. Similarly the net current due to other charges in pairs is zero. Therefore the magnetic field at M is zero. Also the magnetic moment is zero.
(C) is the correct option
(D) is incorrect option.
(r)
The net electric field due to negative charges in the inner circle is zero. Similarly the net electric field due to positive charges in the outercircle is zero.



(A) is the correct option.
The electric potential due to negative charges at M is different from the electric potential due to positive charges at M. Therefore the electric potential at M is not equal to zero.
(B) is the correct option.
When the system of charges rotate, we get a current I_1 due to negative charges and another current I due to positive charges. The magnitude of the magnetic at M due to the currents is different. Therefore $B \neq 0$ and $\mu \neq 0$.
(C) is incorrect option
(D) is the correct option.
(s)

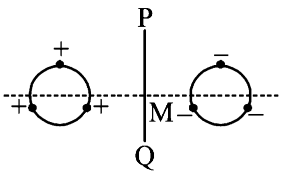


The electric field at M due to all the charges is zero because the electric field due to different charges cancel out in pairs.
(A) is the correct option.
The potential at M due to the charges is

$$V = k\left[\frac{+q}{a/2} + \frac{q}{a/2} - 4\left(\frac{q}{\frac{\sqrt{5}a}{4}}\right)\right] \neq 0$$

(B) is the correct option.
When the whole system is set into rotation with a constant angular velocity about the line PQ we get three loops in which current is flowing.
The magnetic field due to these currents produce a resultant magneic field at M which is not equal to zero. Therefore a net magnetic dipole moment will be produced.
(C) is an incorrect option.
(D) is correct option.

(t)



There will be a net electric field due to the arrangement of charges at M towards the right side.
(A) is an incorrect option.
The electic potential at M will can out in pairs by positive and negative charges, due to symmetrical arrangement of charges.
(B) is an incorrect option.

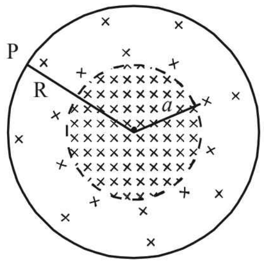
When the system of charges rotates about PQ, the net current is zero due to symmetrical arrangement of charges. Therefore $B = 0$ and $\mu = 0$
(C) is the correct option.
(D) is the incorrect option.

18. (a) If Q_1, Q_2, Q_3 and Q_4 are all positive, then the force will be along $+y$-direction.
If Q_1, Q_2 are positive and Q_3, Q_4 are negative the force will act along $+x$-direction.
If Q_1, Q_4 are positive and Q_2, Q_3 are negative then attractive force will dominate repulsive force and the force will be along $-y$ direction.

G. Comprehension Based Questions

1. (a) When the point of observation is on the surface of sphere then the whole charge inside the sphere (when distributed symmetrically about the centre) behaves as a point charge on the centre. Therefore until the charge distribution is symmetrical about the centre it does not matter what is the ratio a/R. The electric field remains constant and is equal to

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{Ze}{R}.$$



2. (b) For a = 0, the graph is as shown. The equation for the graph line is

$$\rho = d - \frac{r}{R}dr$$

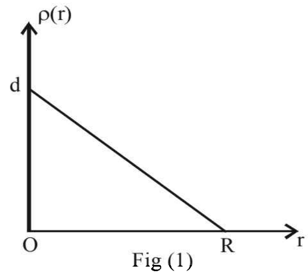


Fig (1)

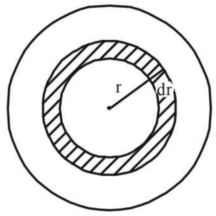


Fig (2)

The charge in the dotted element shown in Fig (2) is
$dq = \rho \times 4\pi r^2 dr$

$$\therefore dq = \left(d - \frac{d}{R}r\right)4\pi r^2 dr \Rightarrow Ze = \int_0^R 4\pi d r^2 dr - \int_0^R \frac{4\pi d}{R} r^3 dr$$

$$Ze = 4\pi d\frac{R^3}{3} - \frac{4\pi d}{R}\frac{R^4}{4}$$

$$\therefore \frac{Ze.}{4\pi d R^3} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \qquad \therefore d = \frac{3Ze}{\pi R^3}$$

3. (c) If the volume charge density is constant then $E \propto r$.

4. **(c)** After colliding the top plate, the ball will gain negative charge and get repelled by the top plate and bounce back to the bottom plate.

5. **(d)** $I_{av} \propto \frac{Q}{t}$ ...(i)

Here $Q \propto V_0$ ...(ii)

Also $S = ut + \frac{1}{2}at^2$

$$h = \frac{1}{2}\frac{QE}{m}t^2 = \frac{1}{2}\left(\frac{Q\times 2V_0}{mh}\right)\times t^2$$

$\therefore$ $t \propto \frac{1}{V_0}$ – (iii) $[\because Q \propto V_0]$

From (i), (ii) and (iii)

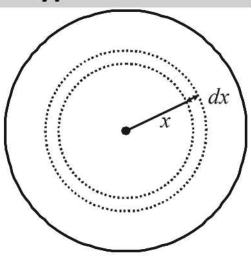
$$I_{av} \propto \frac{V_0}{1/V_0} = I_{av} \propto V_0^2$$

H. Assertion & Reason Type Questions

1. **(a)** Both the statements are true and statement-2 is the correct explanation of statement-1

I. Integer Value Correct Type

1. **2** Let us consider a spherical shell of radius x and thickness dx. The volume of this shell is $4\pi x^2(dx)$. The charge enclosed in this spherical shell is



$$dq = (4\pi x^2)dx \times kx^a$$

$\therefore$ $dq = 4\pi k x^{2+a} dx$.

For $r = R$:

The total charge enclosed in the sphere of radius R is

$$Q = \int_0^R 4\pi k\, x^{2+a}\, dx = 4\pi k \frac{R^{3+a}}{3+a}.$$

$\therefore$ The electric field at $r = R$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k R^{3+a}}{(3+a)R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

For $r = R/2$:

The total charge enclosed in the sphere of radius $R/2$ is

$$Q' = \int_0^{R/2} 4\pi k\, x^{2+a} dx = \frac{4\pi k(R/2)^{3+a}}{3+a}$$

$\therefore$ The electric field at $r = R/2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\frac{(R/2)^{3+a}}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\left(\frac{R}{2}\right)^{1+a}$$

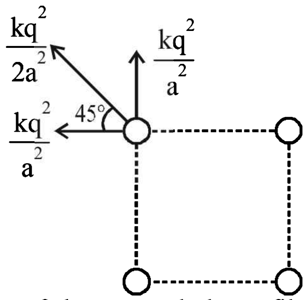
Given, $E_2 = \frac{1}{8}E_1$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}\left(\frac{R}{2}\right)^{1+a} = \frac{1}{2^3}\times\frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

$\Rightarrow$ $1 + a = 3 \Rightarrow a = 2$

2. **3**

$$F_{electric} = \frac{kq^2}{2a^2} + 2\left[\frac{kq^2}{a^2}\times\frac{1}{\sqrt{2}}\right] = \frac{q^2}{a^2}\times \text{constant}$$



As the system of charges and planar film is in equlibrium, therefore

$$\frac{q^2}{a^2}\times \text{constant} = \gamma a \times \text{constant}$$

$$\therefore \quad a = k\left(\frac{q^2}{\gamma}\right)^{1/3} \quad \therefore \quad N = 3$$

3. **6** We suppose that the cavity is filled up by a positive as well as negative volume charge of ρ. So the electric field now produced at P is the superposition of two electric fields.

(a) The electric field created due to the infinitely long solid cylinder is

$E_1 = \frac{\rho R}{4\varepsilon_0}$ directed towards the $+Y$ direction

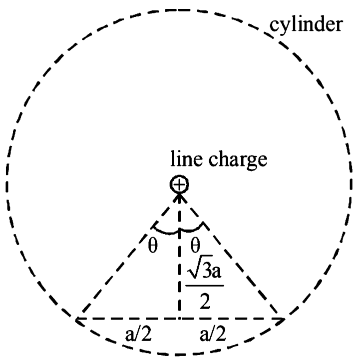
(b) The electric field created due to the spherical negative charge density

$E_2 = \frac{\rho R}{96\varepsilon_0}$ directed towards the $-Y$ direction.

$\therefore$ The net electric field is

$$E = E_1 - E_2 = \frac{1}{6}\left[\frac{23\rho R}{16\varepsilon_0}\right]$$

4. **(6)**



$$\tan\theta = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}$$

$\therefore$ $\theta = 30°$

The flux through the dotted cylinder by Gauss's law is

$$\phi\ \text{cylinder} = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$\therefore$ For 360° angle the flux is $\frac{\lambda L}{\varepsilon_0}$

$\therefore$ For 60° angle the flux will be $\frac{\lambda L}{6\varepsilon_0}$

Therefore n = 6

Section-B JEE Main/ AIEEE

1. (a) We know that $\frac{W_{AB}}{q} = V_B - V_A$

$\therefore V_B - V_A = \frac{2J}{20C} = 0.1\,\text{J/C} = 0.1\text{V}$

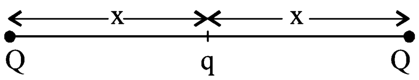
2. (b) The equivalent capacitance of n identical capacitors of capacitance C is equal to nC. Energy stored in this capacitor

$E = \frac{1}{2}(nC)V^2 = \frac{1}{2}nCV^2$

3. (b) Both the charges are identical and placed symmetrically about $ABCD$. The flux crossing $ABCD$ due to each charge is $\frac{1}{6}\left[\frac{q}{\epsilon_0}\right]$ but in opposite directions. Therefore the resultant is zero.

4. (d) For equilibrium of charge Q

$K\frac{Q \times Q}{(2x)^2} + K\frac{Qq}{x^2} = 0 \Rightarrow q = -\frac{Q}{4}$



5. (a) For an isolated sphere, the capacitance is given by

$C = 4\pi \epsilon_0 r = \frac{1}{9 \times 10^9} \times 1 = 1.1 \times 10^{-10} F$

6. (a) The flux entering an enclosed surface is taken as negative and the flux leaving the surface is taken as positive, by convention. Therefore the net flux leaving the enclosed surface $= \phi_2 - \phi_1$

$\therefore$ the charge enclosed in the surface by Gauss's law is

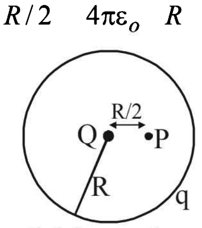
$q = \epsilon_0 (\phi_2 - \phi_1)$

7. (b) The capacitance of a parallel plate capacitor in which a metal plate of thickness t is inserted is given by

$C = \frac{\varepsilon_o A}{d - t}$. Here $t \to 0 \therefore C = \frac{\varepsilon_o A}{d}$

8. (c) Electric potential due to charge Q placed at the centre of the spherical shell at point P is

$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R/2} = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}$



Electric potential due to charge q on the surface of the spherical shell at any point inside the shell is

$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$

$\therefore$ The net electric potential at point P is

$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R} + \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$

9. (d) The work done is stored as the potential energy. The potential energy stored in a capacitor is given by

$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{\left(8 \times 10^{-18}\right)^2}{100 \times 10^{-6}} = 32 \times 10^{-32}\,\text{J}$

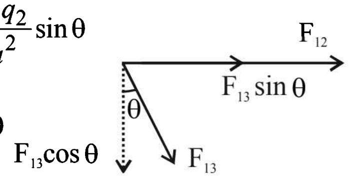
10. (b) Force on charge q_1 due to q_2 is $F_{12} = k\frac{q_1 q_2}{b^2}$

Force on charge q_1 due to q_3 is $F_{13} = k\frac{q_1 q_3}{a^2}$

The X - component of the force (F_x) on q_1 *is* $F_{12} + F_{13}\sin\theta$

$\therefore F_x = k\frac{q_1 q_2}{b^2} + k\frac{q_1 q_2}{a^2}\sin\theta$

$\therefore F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$



11. (d) $R_f = n^2 R_i$

Here n = 2 (length becomes twice)

$\therefore R_f = 4R_i$

New reresistance = 400 of R_i

$\therefore$ Increase = 300%

12. (d) $F \propto \frac{Q_A Q_C}{x^2}$

x is distance between the spheres. After first operation charge on B is halved i.e $\frac{Q}{2}$ and charge on third sphere becomes $\frac{Q}{2}$. Now it is touched to C, charge then equally distributes them selves to make potential same, hence charge on C becomes $\left(Q + \frac{Q}{2}\right)\frac{1}{2} = \frac{3Q}{4}$.

$\therefore F_{new} \propto \frac{Q_C' Q_B'}{x^2} = \frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{2}\right)}{x^2} = \frac{3}{8}\frac{Q^2}{x^2}$

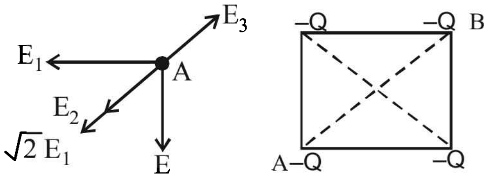
or $F_{new} = \frac{3}{8}F$

13. (d) $\frac{1}{2}mv^2 = \frac{kQq}{r} \Rightarrow \frac{1}{2}m(2v)^2 = \frac{kqQ}{r'} \Rightarrow r' = \frac{r}{4}$

14. (b) Net field at A should be zero

$\sqrt{2}\,E_1 + E_2 = E_3$

$$\therefore \frac{kQ\times\sqrt{2}}{a^2}+\frac{kQ}{(\sqrt{2}\,a)^2}=\frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$$



$$\Rightarrow \frac{Q\sqrt{2}}{1}+\frac{Q}{2}=2q \Rightarrow q=\frac{Q}{4}(2\sqrt{2}+1)$$

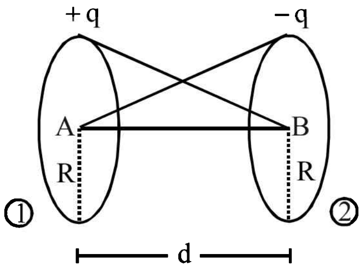
15. (c) At equilibrium, electric force on drop balances weight of drop.

$$qE=mg \Rightarrow q=\frac{mg}{E}=\frac{9.9\times10^{-15}\times10}{3\times10^4}=3.3\times10^{-18}\,\mathrm{C}$$

16. (b) $\frac{-K2q}{(x-L)^2}+\frac{K8q}{x^2}=0 \Rightarrow \frac{1}{(x-L)^2}=\frac{4}{x^2}$

or $\frac{1}{x-L}=\frac{2}{x} \Rightarrow x=2x-2L$ or $x=2L$

17. (a)



$V_A=V_{\text{self}}+V_{\text{due to (2)}}$

$$\Rightarrow V_A=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

$V_B=V_{\text{self}}+V_{\text{due to (1)}}$

$$\Rightarrow V_B=\frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{R}+\frac{q}{\sqrt{R^2+d^2}}\right]$$

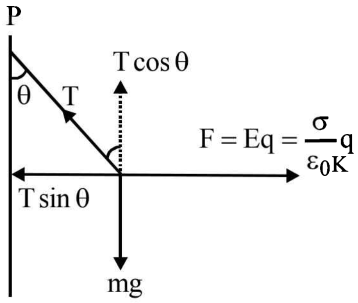
$\Delta V=V_A-V_B$

$$=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R}+\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

$$=\frac{1}{2\pi\varepsilon_0}\left[\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

18. (b) As n plates are joined, it means $(n-1)$ capacitor joined in parallel.
$\therefore$ resultant capacitance = $(n-1)\,C$

19. (c)



$$T\sin\theta=\frac{\sigma}{\varepsilon_0 K}.q \qquad \text{.... (i)}$$

$$T\cos\theta=mg \qquad \text{.... (ii)}$$

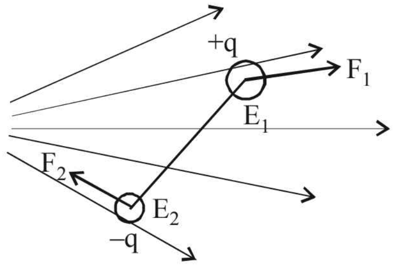
Dividing (i) by (ii),

$$\tan\theta=\frac{\sigma q}{\varepsilon_0 K.mg} \quad \therefore\ \sigma \propto \tan\theta$$

20. (c) Applying conservation of energy,

$$\frac{1}{2}CV^2=m.s\Delta T; \quad V=\sqrt{\frac{2m.s.\Delta T}{C}}$$

21. (c)



The electric field will be different at the location of the two charges. Therefore the two forces will be unequal. This will result in a force as well as torque.

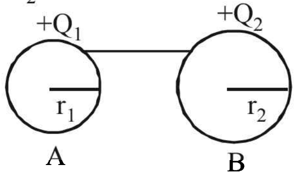
22. (a) $eV=\frac{1}{2}mv^2$

$$\Rightarrow v=\sqrt{\frac{2eV}{m}}=\sqrt{\frac{2\times1.6\times10^{-19}\times20}{9.1\times10^{-31}}}$$

$$=2.65\times10^6\ \mathrm{m/s}$$

23. (c) After connection, $V_1=V_2$

$$\Rightarrow K\frac{Q_1}{r_1}=K\frac{Q_2}{r_2}$$



$$\Rightarrow \frac{Q_1}{r_1}=\frac{Q_2}{r_2}$$

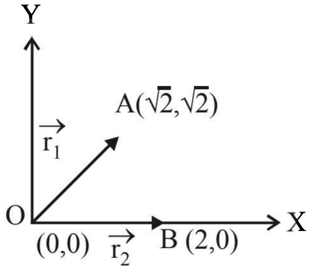
The ratio of electric fields

$$\frac{E_1}{E_2}=\frac{K\frac{Q_1}{r_1^2}}{K\frac{Q_2}{r_2^2}}=\frac{Q_1}{r_1^2}\times\frac{r_2^2}{Q_2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{r_1 \times r_2^2}{r_1^2 \times r_2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{2}{1}$$

Since the distance between the spheres is large as compared to their diameters, the induced effects may be ignored.

24. (c)



The distance of point $A\left(\sqrt{2}, \sqrt{2}\right)$ from the origin,

$$OA = |\vec{r_1}| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2 \text{ units.}$$

The distance of point $B(2, 0)$ from the origin,

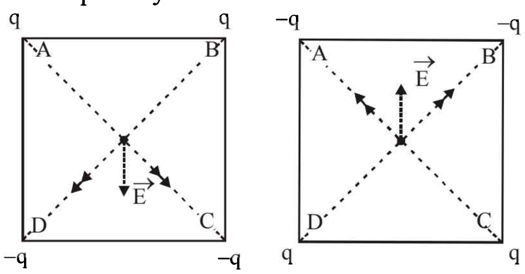
$$OB = |\vec{r_2}| = \sqrt{(2)^2 + (0)^2} = 2 \text{ units.}$$

Now, potential at A, $V_A = \frac{1}{4\pi \in_0} \cdot \frac{Q}{(OA)}$

Potential at B, $V_B = \frac{1}{4\pi \in_0} \cdot \frac{Q}{(OB)}$

$\therefore$ Potential difference between the points A and B is zero.

25. (a) As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude, but opposite directions.
Also, the potential will be same in both cases as it is a scalar quantity.



26. (a) Here, $V(x) = \frac{20}{x^2 - 4}$ volt

We know that $E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{20}{x^2 - 4}\right)$

or, $E = +\frac{40x}{(x^2 - 4)^2}$

At $x = 4\ \mu\text{m}$,

$$E = +\frac{40 \times 4}{(4^2 - 4)^2} = +\frac{160}{144} = +\frac{10}{9} \text{ volt}/\mu\text{m}.$$

Positive sign indicates that $\vec{E}$ is in +ve x-direction.

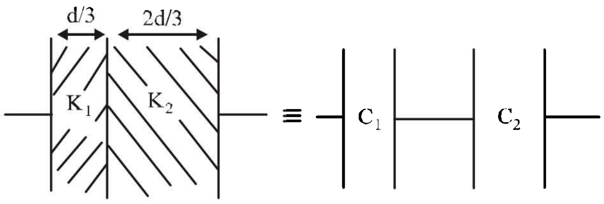
27. (a) The potential energy of a charged capacitor before removing the dielectric slat is $U = \frac{Q^2}{2C}$.

The potential energy of the capacitor when the dielectric slat is first removed and the reinserted in the gap between the plates is $U = \frac{Q^2}{2C}$

There is no change in potential energy, therefore work done is zero.

28. (b) Electronic charge does not depend on acceleration due to gravity as it is a universal constant.
So, electronic charge on earth
= electronic charge on moon
$\therefore$ Required ratio = 1.

29. (c)



The given capacitance is equal to two capacitances connected in series where

$$C_1 = \frac{k_1 \in_0 A}{d/3} = \frac{3k_1 \in_0 A}{d} = \frac{3 \times 3 \in_0 A}{d} = \frac{9 \in_0 A}{d}$$

and

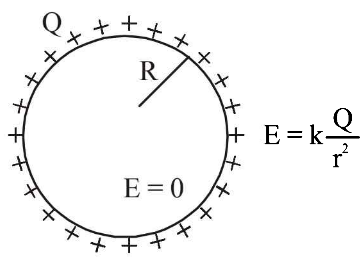
$$C_2 = \frac{k_2 \in_0 A}{2d/3} = \frac{3k_2 \in_0 A}{2d} = \frac{3 \times 6 \in_0 A}{2d} = \frac{9 \in_0 A}{d}$$

The equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{9 \in_0 A} + \frac{d}{9 \in_0 A} = \frac{2d}{9 \in_0 A}$$

$$\therefore C_{eq} = \frac{9}{2} \frac{\in_0 A}{d} = \frac{9}{2} \times 9\ pF = 40.5\,pF$$

30. (a) The electric field inside a thin spherical shell of radius R has charge Q spread uniformly over its surface is zero.



Outside the shell the electric field is $E = k\frac{Q}{r^2}$. These characteristics are represented by graph (a).

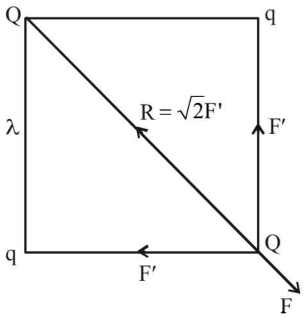
31. (c) $\frac{W_{PQ}}{q} = (V_Q - V_P)$

$\Rightarrow W_{PQ} = q(V_Q - V_P)$

$= (-100 \times 1.6 \times 10^{-19})(-4-10)$

$= +2.24 \times 10^{-16}\text{J}$

32. (d) Let F be the force between Q and Q. The force between q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. For equilibrium



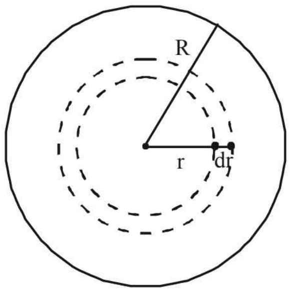
$\sqrt{2}F' = -F$

$\sqrt{2} \times k\frac{Qq}{\ell^2} = -k\frac{Q^2}{(\sqrt{2}\ell)^2}$

$\Rightarrow \frac{Q}{q} = -2\sqrt{2}$

33. (a) Statement 1 is true.
Statement 2 is true and is the correct explanation of (1)

34. (b)



Let us consider a spherical shell of thickness dx and radius x. The volume of this spherical shell $= 4\pi r^2 dr$.
The charge enclosed within shell

$= \frac{Qr}{\pi R^4}[4\pi r^2 dr]$

The charge enclosed in a sphere of radius r_1 is

$= \frac{4Q}{R^4}\int_0^{r_1} r^3 dr = \frac{4Q}{R^4}\left[\frac{r^4}{4}\right]_0^{r_1} = \frac{Q}{R^4}r_1^4$

$\therefore$ The electric field at point p inside the sphere at a distance r_1 from the centre of the sphere is

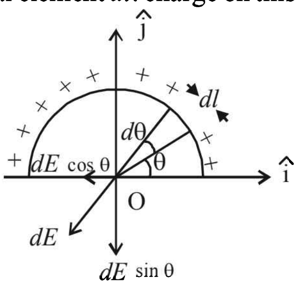
$$E = \frac{1}{4\pi\epsilon_0}\frac{\left[\frac{Q}{R^4}r_1^4\right]}{r_1^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^4}r_1^2$$

35. (c) Let us consider a differential element dl. charge on this element.

$dq = \left(\frac{q}{\pi r}\right)dl$

$= \frac{q}{\pi r}(rd\theta)$ $(\because dl = rd\theta)$

$= \left(\frac{q}{\pi}\right)d\theta$



Electric field at O due to dq is

$$dE = \frac{1}{4\pi\epsilon_0}\cdot\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{\pi r^2}d\theta$$

The component $dE\cos\theta$ will be counter balanced by another element on left portion. Hence resultant field at O is the resultant of the component $dE\sin\theta$ only.

$$\therefore E = \int dE\sin\theta = \int_0^{\pi}\frac{q}{4\pi^2 r^2\epsilon_0}\sin\theta d\theta$$

$$= \frac{q}{4\pi^2 r^2\epsilon_0}[-\cos\theta]_0^{\pi} = \frac{q}{4\pi^2 r^2\epsilon_0}(+1+1)$$

$$= \frac{q}{2\pi^2 r^2\epsilon_0}$$

The direction of E is towards negative y-axis.

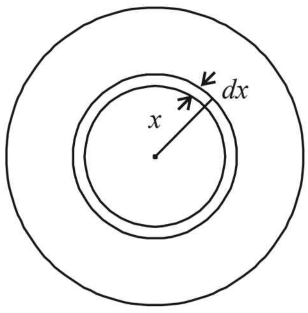
$$\therefore \vec{E} = -\frac{q}{2\pi^2 r^2\epsilon_0}\hat{j}$$

36. (a) Let us consider a spherical shell of radius x and thickness dx.
Charge on this shell

$$dq = \rho.4\pi x^2 dx = \rho_0\left(\frac{5}{4}-\frac{x}{R}\right).4\pi x^2 dx$$

$\therefore$ Total charge in the spherical region from centre to r $(r < R)$ is

$$q = \int dq = 4\pi\rho_0\int_0^r\left(\frac{5}{4}-\frac{x}{R}\right)x^2 dx$$



$$= 4\pi\rho_0\left[\frac{5}{4}\cdot\frac{r^3}{3}-\frac{1}{R}\cdot\frac{r^4}{4}\right] = \pi\rho_0 r^3\left(\frac{5}{3}-\frac{r}{R}\right)$$

$\therefore$ Electric field at r, $E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{\pi\rho_0 r^3}{r^2}\left(\frac{5}{3}-\frac{r}{R}\right) = \frac{\rho_0 r}{4\epsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$$

37. (d) At any instant

$T\cos\theta = \text{mg}$ ....(i)

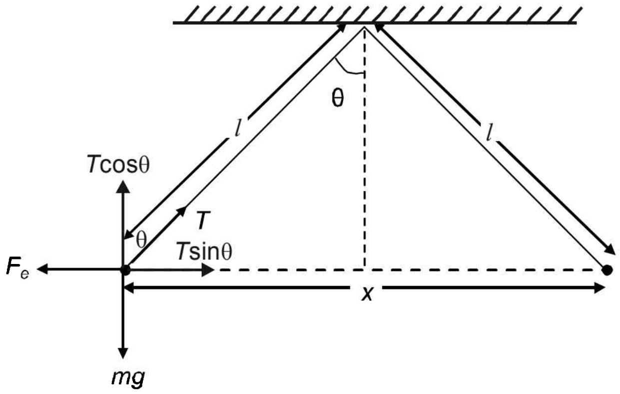
$T\sin\theta = \text{F}_e$ ....(ii)

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{F_e}{mg} \Rightarrow \text{F}_e = \text{mg}\tan\theta$

$\Rightarrow \quad \frac{kq^2}{x^2} = mg\tan\theta \Rightarrow q^2 \propto x^2\tan\theta$

$\sin\theta = \frac{x}{2l}$

For small θ, $\sin\theta \approx \tan\theta \quad \therefore\ q^2 \propto x^3$



$\Rightarrow \quad q\frac{dq}{dt} \propto x^2\frac{dx}{dt}$

$\therefore \quad \frac{dq}{dt} = \text{const.}$

$\therefore \quad q \propto x^2.v \Rightarrow x^{3/2}\alpha x^2.v \quad [\because q^2 \propto x^3]$

$\Rightarrow \quad v \propto x^{-1/2}$

38. (c) Electric field

$$E = -\frac{d\phi}{dr} = -2ar \qquad ....(i)$$

By Gauss's theorem

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad ....(ii)$$

From (i) and (ii),

$q = -8\pi\varepsilon_0 ar^3$

$\Rightarrow \quad dq = -24\pi\varepsilon_0 ar^2\, dr$

Charge density, $\rho = \frac{dq}{4\pi r^2 dr} = -6\varepsilon_0 a$

39. (c) $E_{in} \propto r$

$E_{out} \propto \frac{1}{r^2}$

40. (c) The electric field inside a uniformly charged sphere is

$\frac{\rho.r}{3\epsilon_0}$

The electric potential inside a uniformly charged sphere

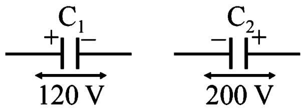
$= \frac{\rho R^2}{6\epsilon_0}\left[3 - \frac{r^2}{R^2}\right]$

∴ Potential difference between centre and surface

$= \frac{\rho R^2}{6\epsilon_0}[3-2] = \frac{\rho R^2}{6\epsilon_0}$

$\Delta U = \frac{q\rho R^2}{6\epsilon_0}$

41. (b)

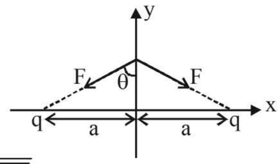


For potential to be made zero, after connection

$120\,C_1 = 200\,C_2 \quad \left[\because C = \frac{q}{v}\right]$

$\Rightarrow \quad 3C_1 = 5C_2$

42. (a) $\Rightarrow F_{net} = 2F\cos\theta$

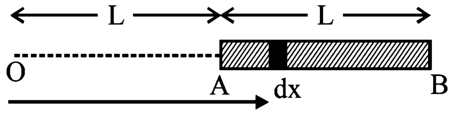


$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \Rightarrow \frac{kq^2y}{a^3}$$

So, $F \propto y$

43. (d)



Electric potential is given by,

$$V = \int_L^{2L}\frac{kdq}{x} = \int_L^{2L}\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{L}\right)dx}{x} = \frac{q}{4\pi\varepsilon_0 L}\ln(2)$$

44. (c) Potential difference between any two points in an electric field is given by,

$dV = -\vec{E}.\vec{dx}$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80J/C$$

45. (a) Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

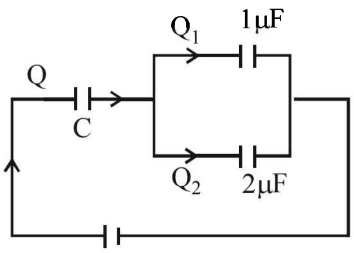
$$E = \frac{\sigma}{K\varepsilon_0}$$

Then, charge density

$\sigma = K\varepsilon_0 E$

$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}\ C/m^2$

46. (d)

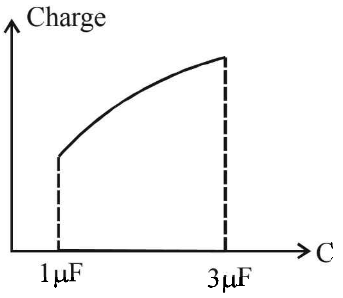


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$$Q = E\left(\frac{C\times 3}{C+3}\right)$$

$$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$$

Therefore graph d correctly dipicts.



47. (a,b) We know, $V_0 = \frac{Kq}{R} = V\text{surface}$

Now, $V_i = \frac{Kq}{2R^3}(3R^2 - r^2)$ [For r < R]

At the centre of sphere r = 0. Here

$$V = \frac{3}{2}V_0$$

$$\text{Now, } \frac{5}{4}\frac{Kq}{R} = \frac{Kq}{2R^3}(3R^2 - r^2)$$

$$R_2 = \frac{R}{\sqrt{2}}$$

$$\frac{3}{4}\frac{Kq}{R} = \frac{Kq}{R^3}$$

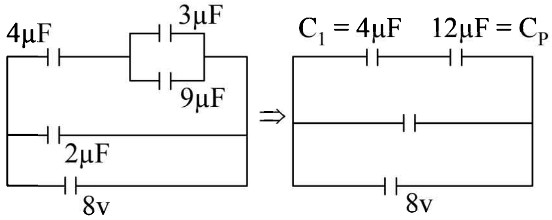
$$\frac{1}{4}\frac{Kq}{R} = \frac{Kq}{R_4}$$

$R_4 = 4R$

Also, $R_1 = 0$ and $R_2 < (R_4 - R_3)$

48. (c) Field lines originate perpendicular from positive charge and terminate perpendicular at negative charge. Further this system can be treated as an electric dipole.

49. (a)



Charge on C_1 is $q_1 = \left[\left(\frac{12}{4+12}\right)\times 8\right]\times 4 = 24\mu c$

The voltage across C_P is $V_P = \frac{4}{4+12}\times 8 = 2v$

$\therefore$ Voltage across 9μF is also 2V

$\therefore$ Charge on 9μF capacitor = 9 × 2 = 18μC

$\therefore$ Total charge on 4 μF and 9μF = 42μc

$$\therefore E = \frac{KQ}{r^2} = 9\times 10^9 \times \frac{42\times 10^{-6}}{30\times 30} = 420\,Nc^{-1}$$

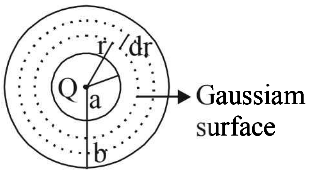
50. (c) Applying Gauss's law

$$\oint_S \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0}$$

$$\therefore E\times 4\pi r^2 = \frac{Q + 4\pi a r^2 - 4\pi A a^2}{\epsilon_0}$$

$$\rho = \frac{dr}{dv}$$

$$Q = \rho 4\pi r^2$$



$$Q = \int_a^r \frac{A}{r}4\pi r^2 dr = 4\pi A[r^2 - a^2]$$

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{Q - 4\pi A a^2}{r^2} + 4\pi A\right]$$

For E to be independent of 'r'

$Q - 2\pi A a^2 = 0$

$$\therefore A = \frac{Q}{2\pi a^2}$$