Natural numbers & Whole numbers

Concept of Whole Numbers

Can you say what counting numbers are?

The numbers that we use for counting are called counting numbers. They start with 1. They are 1, 2, 3, 4 …. . These counting numbers are also called **Natural Numbers**. Therefore, we can define the natural numbers as follows:

> "**The collection of all counting numbers is known as Natural Numbers.**"

What will we obtain, if we subtract 1 from 1?

If we subtract 1 from 1, then we obtain

1 − 1 = 0

The number 0 (zero) with all the natural numbers form a system of numbers, which is called **Whole Numbers**. This means whole numbers are a set of numbers starting from 0 i.e., 0, 1, 2 … and this can be defined as follows:

> "**If zero is added to the collection of natural numbers, then we obtain the collection of whole numbers, or in other words, we can say that all natural numbers along with zero are called whole numbers.**"

Remember: All natural numbers are whole numbers, but all whole numbers are not natural numbers.

Think of any big number, say 20958340. We can write this number using symbols 0, 2, 3, 4, 5, 8 and 9.

Similarly, we can write a natural number using 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each of such symbols is called a **digi**t or a **figure**.

On observing the natural and whole numbers it is found that:

- The value of numbers increase as we move from left to right.
- On moving further to the right, we keep on finding more numbers. Thus, these numbers are endless and it is not possible to tell the highest natural or whole number.

- 1 is the smallest natural number.
- 0 is the smallest whole number.

The number 0 follow certain rules:

- $a + 0 = 0 + a$, for all natural numbers a
- $a. 0 = 0. a$, for all natural numbers a
- 0 + 0 = 0
- 0.0 = 0

Well ordering property of natural numbers:

Well ordering property of natural numbers states that every non-empty subset of natural numbers of **N** (or **W**) has the smallest element.

For example, let us consider the set of all even natural numbers i.e., {2, 4, 6, ...}. This set is the subset of natural number. 2 is the smallest element of the set of even natural number.

Let us now try and solve the following puzzle to check whether we have understood this concept.

Successors and Predecessors of Numbers

Suppose you have a bag full of candies. Now, suppose you want to count the number of candies in the bag. **How will you start counting?**

You will start counting as 1, 2, 3, 4, 5 and so on.

Therefore, we can say that the above numbers are counting numbers and are known as **natural numbers**.

Now, consider the natural number 5. **Can you tell which number is the predecessor of 5 and which number is the successor of 5?**

The predecessor of 5 is 4 and the successor of 5 is 6.

How? Let us first define these terms.

Successor

"If 1 is added to any natural number, then we obtain the successor of that number."

For example, the successor of 11 will be 11 + 1 = 12.

Predecessor

"If 1 is subtracted from any natural number, then we obtain the predecessor of that number."

For example, the predecessor of 11 will be 11 − 1 = 10.

To understand this better take a look at the given video.

Note:

(1) ***Every natural number has a successor because there is no largest natural number.***

(2) ***Successor and predecessor are reverse of each other.***

For example, 20 is the successor of 19 and 19 is the predecessor of 20.

(3) ***Every number except 1 has a predecessor in case of natural numbers.*** This is because 1 − 1 = 0 and 0 is not a natural number. It is a whole number.

Let us now see a few examples to understand the concept better.

Example 1:

Write two successors and two predecessors of 245600.

Solution:

First Successor = 245600 + 1 = 245601

Second Successor = 245601 + 1 = 245602

First Predecessor = 245600 − 1 = 245599

Second Predecessor = 245599 − 1 = 245598

Example 2:

Write the number right before and after 893.

Solution:

Number before 893 = 893 − 1 = 892

Number after 893 = 893 + 1 = 894

Example 3:

Write four numbers next to 91.

Solution:

This means that we have to find four successors of 91.

Successor of 91 = 91 + 1 = 92

Successor of 92 = 92 + 1 = 93

Successor of 93 = 93 + 1 = 94

Successor of 94 = 94 + 1 = 95

Thus, four numbers next to 91 are 92, 93, 94, and 95.

Closure Property of Whole Numbers over Addition and Multiplication

Let us consider the whole numbers 6, 17, 4, and 15. Now, we add these numbers two at a time. This can be done as follows.

6 + 17 = 23, 6 + 4 = 10, 6 + 15 = 21

17 + 4 = 21, 17 + 15 = 32, 15 + 4 = 19

We can observe that in each case, the new number obtained is also a whole number.

Can we say that this is true for any pair of whole numbers?

Yes, if we add any two whole numbers, then we will get a whole number again. Thus, we can say that **the whole numbers are closed under addition.**

Closure property for addition of whole numbers can be stated as follows.

"The sum of any two whole numbers is again a whole number".

Now, let us find if the closure property is also valid for multiplication.

Consider the whole numbers 8, 10, 5, and 0. We now multiply them taking two at a time. The possible combinations of numbers are

8 × 10 = 80, 8 × 5 = 40, 8 × 0 = 0

10 × 5 = 50, 10 × 0 = 0, 5 × 0 = 0

Here, we can observe that the product of any two whole numbers is again a whole number. Thus, the numbers 8, 10, 5, and 0 are closed under multiplication of whole numbers.

Therefore, when we multiply any two whole numbers, their product will always be a whole number. Thus, we can say that **whole numbers are closed under multiplication.**

Closure property for multiplication of whole numbers can be stated as follows.

> **"The product of any two whole numbers is again a whole number".**

Let us now see if the closure property of whole numbers is valid for **Subtraction** and Division also.

If we subtract 3 from 1, we will obtain −2 which is not a whole number.

1 − 3 = −2 (The result is not a whole number)

Similarly, in the following examples, −6 and −4 are not whole numbers.

1 − 7 = −6 (The result is not a whole number)

3 − 7 = −4 (The result is not a whole number)

Thus, we can say that **whole numbers are not closed under subtraction**.

Now, consider the whole numbers 12, 1, and 4. Let us divide one number by another number.

12 ÷ 4 = 3 (whole number)

12 ÷ 1 = 12 (whole number)

1 ÷ 12, the result will not be a whole number**.**

1 ÷ 4, the result will not be a whole number.

4 ÷ 1 = 4 (whole number)

4 ÷ 12, the result will not be a whole number.

Here, we can observe that the division of two whole numbers does not always give a whole number. Thus, we can say that **whole numbers are not closed under division.**

From our discussion, we can conclude

Addition and multiplication of whole numbers hold closure property.
Subtraction and division of whole numbers do not hold closure property.

These properties hold true for natural numbers as well.

Have a look at the given video to understand this concept better.

Commutative and Associative Properties of Whole Numbers over Addition and Multiplication

We will now learn two important properties of whole numbers. Let us start by taking two numbers and adding them.

What do we obtain on adding 3 and 4?

We obtain, 3 + 4 = 7

Now, **what would be the result, if we interchange the places of the numbers and then add them?**

We obtain, 4 + 3 = 7

Observe that the result is the same i.e., 3 + 4 = 4 + 3 = 7. This means that we can add 4 and 3 in any order.

If we try the same with other whole numbers, then we will find that their sum always remains the same, regardless of the order in which they are added. This property of whole numbers is called the **commutative property of addition.**

Commutative property of addition can be stated as follows.

"The addition of two whole numbers always gives the same result, irrespective of the order in which they are added".

Now, similar to addition, in multiplication also, can we multiply two numbers in any order and still obtain the same result?

Let us take an example and find this out.

Consider the multiplication of two numbers 5 and 2.

Now, $2 \times 5 = 10$ and also, $5 \times 2 = 10$

This means $2 \times 5 = 5 \times 2$

Thus, we obtain the same result when we interchange the places of the numbers in case of multiplication as well.

This means that similar to addition, the **commutative property of multiplication** also holds for all whole numbers.

Commutative property of multiplication can be stated as follows.

"Multiplication of two whole numbers always gives the same result, irrespective of the order in which they are multiplied".

Let us now find out whether the commutative property of whole numbers holds true for **subtraction** and **division** also or not.

Consider two whole numbers 2 and 4.

First, we subtract 2 from 4.

Now, $4 - 2 = 2$ (Whole number)

Now, we subtract 4 from 2.

$2 - 4 = -2$ (The result is not a whole number)

Here, we can observe that $2 - 4 \neq 4 - 2$

Thus, we can conclude that the **commutative property of whole numbers is not valid for subtraction.**

Now, consider the same whole numbers 2 and 4 again and check the commutative property for division.

4 ÷ 2 = 2 (Whole number)

2 ÷ 4 = $\frac{1}{2}$ (The result is not a whole number)

∴ 2 ÷ 4 ≠ 4 ÷ 2

Thus, the **commutative property is not valid for division also.**

We can summarize our discussion as follows.

Addition and multiplication are commutative for whole numbers.
Subtraction and division are not commutative for whole numbers.

Let us now consider one example. **Suppose we have three bags; Bag A has 3 candies, Bag B has 5 candies, and Bag C has 8 candies. How many candies do we have in total?**

We can find the answer in two ways. Let us see how.

Thus, the **associative property of addition of whole numbers** states that

For any three whole numbers *a, b,* and *c,*

$$a + (b + c) = (a + b) + c$$

Similarly, associative property for multiplication of whole numbers can be stated as follows.

For any three whole numbers *a, b,* and *c,*

$$a \times (b \times c) = (a \times b) \times c$$

Thus, we can summarize our discussion as follows.

(1) Addition is associative for whole numbers.

(2) Multiplication is associative for whole numbers.

Now,

For any three whole numbers *a, b,* and *c,*

$a - (b - c) = a - b + c$

Subtraction is not associative for whole numbers.

Thus, whole numbers are not associated under subtraction.

The associative properties of addition and multiplication can help in making our calculations much simpler. Let us see how.

Consider the case where we have to add the numbers 1234, 7, and 993. One way of doing this is by first finding the sum of 1234 and 7 and then adding it to 993. This can be done as follows.

$1234 + 7 + 993 = (1234 + 7) + 993 = 1241 + 993 = 2234$

As seen here, the calculations were quite long and time consuming. A simpler method would be to use the associative property of addition.

$1234 + 7 + 993 = 1234 + (7 + 993) = 1234 + 1000 = 2234$

Did you notice that the latter calculations were simpler?

Let us take another example where we have to multiply the numbers 658, 25, and 4.

One way of doing this is as follows.

$658 \times 25 \times 4 = (658 \times 25) \times 4$

Now, multiplication of 658 and 25 will take a long time.

Instead of this, using the associative property of multiplication, we can easily find the product of the three numbers as follows.

658 × 25 × 4 = 658 × (25 × 4) = 658 × 100 = 65800

Therefore, before making any lengthy addition or multiplication calculations, we should check whether we can use the associative property and make the task much easier.

These properties hold true for natural numbers also.

Let us now solve some more examples.

Example 1:

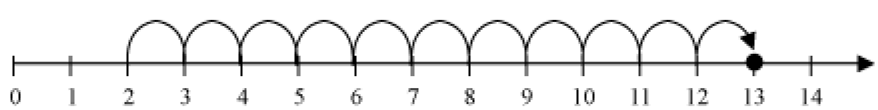
Verify the commutative property of addition for two numbers 2 and 11 using the number line.

Solution:

The commutative property of addition states that the addition of two whole numbers always gives the same result, irrespective of the order in which they are added.

We can add the numbers 2 and 11 as 2 + 11.

On the number line, we start from 2 and move 11 units to the right. The final position obtained on the number line is 13. This implies that the result of the expression (2 + 11) is 13.



Now, let us find the value of the expression (11 + 2).

On the number line, we start from 11 and move 2 units to the right. The final position obtained on the number line is 13. This implies that the result of the expression (11 + 2) is 13.

The result obtained is same in both cases. Hence, the commutative property of addition for the numbers 2 and 11 is verified.

Example 2:

Find the value of the expression (27 + 300 + 73) using any two ways.

Solution:

The sum of 27 + 300 + 73 can be calculated as follows.

27 + 300 + 73 = 327 + 73 = 400

The other way is as follows.

(27 + 300 + 73)

= 27 + (300 + 73)

= 27 + (73 + 300) (Using commutative property)

= (27 + 73) + 300 (arranging the numbers, using associative property of addition)

= 100 + 300

= 400

Example 3:

Verify the associative property of addition for the expression (7 + 14 + 13).

Solution:

The associative property of addition states that we can add three whole numbers by grouping them in different orders and still obtain the same result.

Now, 7 + 14 + 13 = (7 + 14) + 13 = 21 + 13 = 34

Also, 7 + 14 + 13 = 7 + (14 + 13) = 7 + (27) =34

∴ (7 + 14) + 13 = 7 + (14 + 13)

This verifies the associative property for addition.

Example 4:

Find the value of the expression (125 × 19 × 4) by using properties of whole numbers.

Solution:

125 × 19 × 4 = 125 × 4 × 19 (19 × 4 = 4 × 19, by commutativity)

= (125 **×** 4) **×** 19 (By associative property of multiplication)

= 500 **×** 19

= 9500

Example 5.

Find the value of *a* if (12 + 17) + *a* = 12 + (17 + 9).

Answer:

(12 + 17) + *a* = 12 + (17 + 9)

⇒12 + 17 + *a* = 12 + 17 + 9 (Addition is associative for whole numbers)

⇒29 + *a* = 29 + 9

⇒ *a* = 9

Distributive Property of Whole Numbers for Multiplication over Addition

Let us consider two whole numbers 4 and 5. The product of 4 and 5 is 20.

Now, if we break 5 as (2 + 3), then we can write 4 × 5 as 4 × (2 + 3) = 20

Observe that (4 × 2) + (4 × 3) = 8 + 12 = 20

Thus, 4 × (2 + 3) = (4 × 2) + (4 × 3)

This is called the distributive property of whole numbers for multiplication over addition. We can generalize this property as follows.

"Suppose we have three whole numbers *a*, *b*, and *c*. It does not matter whether we add *b* and *c* first, and then multiply the product with *a* or whether we multiply *a* with *b* and *a* with *c* and then add the two products".

This can be written as $a \times (b + c) = a \times b + a \times c$

This property also holds for multiplication over subtraction i.e., $a \times (b - c) = a \times b - a \times c$

This property can be very useful in multiplying bigger numbers. Suppose we have to multiply 19 and 45. It would take us a long time to do this if we follow the usual approach.

Therefore, let us try to find the product of 19 and 45 by using the distributive property.

In this way, we can reduce our calculation work by making use of distributive property of multiplication over addition or subtraction.

Let us now solve a few more examples to understand the concept better.

Example 1:

Find the value of the expression (22 × 103) using distributive property.

Solution:

We can write 103 as (100 + 3). Therefore,

$22 \times 103 = 22 \times (100 + 3)$

$= 22 \times 100 + 22 \times 3$ (By distributive property of multiplication over addition)

$= 2200 + 66$

$= 2266$

Example 2:

Rohit buys two packets (of different sizes) of same candies. The bigger packet has 25 candies while the smaller packet has 15 candies. The cost of each candy is 50 p. What amount does Rohit require to pay for the two packets?

Solution:

Cost of each candy = 50 p

Number of candies in the bigger packet = 25

∴ Cost of the bigger packet = (50 × 25) p

Number of candies in the smaller packet = 15

∴ Cost of the smaller packet = (50 × 15) p

Cost of the two packets = (50 × 25 + 50 × 15) p

= [50 × (25 + 15)] p (By using distributive property of multiplication over addition)

= (50 × 40) p

= 2000 p

= Rs 20

Thus, Rohit requires Rs 20 to pay for the two packets of candies.

Additive and Multiplicative Identities for Whole Numbers

When you add 5 and 0, what do you obtain?

We obtain 5 + 0 = 5

Now, we consider the addition of the following numbers.

1 + 0	1
9 + 0	9
3 + 0	3
7 + 0	7

What did you observe?

Yes, you are right. The addition of a number with zero gives the same number again. Therefore, we can conclude that the addition of any whole number with zero gives the same number. This property of whole numbers is known as **the identity property of addition.**

Identity property of addition of whole numbers can be stated as follows:

“If we add zero to any whole number, then we obtain the same whole number again and the number zero is called the identity for addition”.

Now, let us consider the multiplication of 5 with 1.

5 × 1 = 5

Here, we can see that we obtained the same number i.e., 5 on multiplying it with 1.

Now, let us consider the multiplication of the following numbers with 1.

1 × 1	1
7 × 1	7
4 × 1	4
9 × 1	9

What do you observe?

The multiplication of any number with one gives the same number again. Therefore, we can conclude that multiplying any whole number with one gives the same number. This is known as the **identity property of multiplication.**

Identity property of multiplication of whole numbers can be stated as follows:

“If we multiply any whole number with 1, then we will obtain the same number again and the number 1 is called the identity for multiplication”.

The given video will explain the above concept better.

Important properties of the additive identity (i.e., 0)

What will be the result when we multiply 5 with 0?

Now, 5 × 0 = 0

Similarly, if we multiply the following numbers with zero, then we will obtain zero as a result.

6 × 0	0
7 × 0	0
12 × 0	0
8 × 0	0
35 × 0	0

Therefore, we can conclude that multiplying any whole number with zero gives zero as the result.

But division by zero cannot be defined. We cannot divide any number by zero.

Let us now consider some more examples.

Example 1:

Find the values of *a*, *b*, and *c* in the following expressions.

1. **$a + 9 = 9$**
2. **$6 \times b = 0$**
3. **$3 \times c = 3$**

Solution:

i. $a + 9 = 9$

$a = 0$ (By identity property of addition)

ii. $6 \times b = 0$

$b = 0$ (Multiplication of any number with zero gives zero)

iii. $3 \times c = 3$

$c = 1$ (By identity property of multiplication)

Example 2:

Which of the following equations are correct?

(i) 6 × 0 = 0

(ii) 19 ÷ 0 = 0

(iii) 21 × 1 = 1

(iv) 9 + 0 = 0

Solution:

(i) Multiplication of any number with zero gives zero. Therefore, the given equation is correct.

(ii) We cannot divide a number by zero. Therefore, the given relation, 19 ÷ 0 = 0, is incorrect.

(iii) We know that any number multiplied by 1 gives the number itself. By using the identity property of multiplication, the given equation is incorrect.

(iv) Addition of zero to any number gives the number itself. Therefore, the given equation is incorrect.