Trigonometry & Co-ordinate Geometry

Introduction

Trigonometry is the study of the relationship between the angles and the sides of a triangle.

a. Units of Angle

Angles are measured in many units, viz. degree, minute, seconds and radians.

Hence, we have:

1 degree = 60 minutes, 1 minute = 60 seconds, π radians = 180°

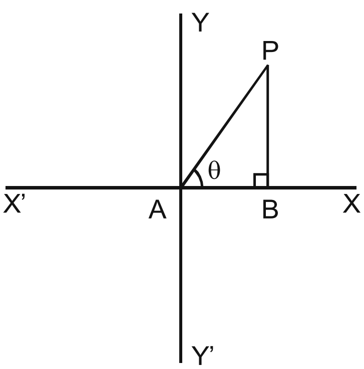
b. Trigonometrical Ratios

In a right angled triangle ABP, if θ be the angle between AP and AB, then we define

1. $\sin\theta = \dfrac{\text{Height}}{\text{Hypotenuse}} = \dfrac{PB}{AP}$

2. $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AP}$

3. $\tan\theta = \dfrac{\text{Height}}{\text{Base}} = \dfrac{PB}{AB}$



4. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\text{Base}}{\text{Height}} = \dfrac{AB}{PB}$

5. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AP}{AB}$

6. $\textbf{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{Hypotenuse}}{\text{Height}} = \dfrac{AP}{PB}$

Important Formulae

1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

2. $\sin^2\theta + \cos^2\theta = 1$

3. $1 + \tan^2\theta = \sec^2\theta$

4. $1 + \cot^2\theta = \text{cosec}^2\theta$

Trigonometric measures of certain angles

Angle	**sinθ**	**cosθ**	**tanθ**
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞

Signs of trigonometric ratios

<u>IInd quadrant</u> Here, only sin and cosec are positive.	**<u>Ist quadrant</u>** Here all ratios (sin, cos, tan, sec, cosec and cot) are positive.
<u>IIIrd quadrant</u> Here, only tan and cot are positive.	**<u>IVth quadrant</u>** Here, only cos and sec are positive.

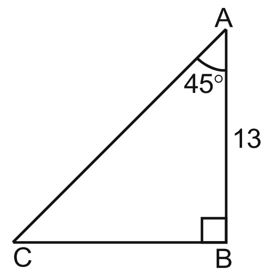
You can remember above table as:

School	**After**
To	**College**

Solved Examples

1. In ΔABC, $\angle A = 45°$, $\angle B = 90°$ and AB = 13 cm. Find BC.

Solution :



$$\tan A = \frac{BC}{AB}$$

$$\tan 45° = \frac{BC}{13}$$

$\Rightarrow$ BC = 13 × tan45° = 13 × 1 = 13

so BC = 13 cm

2. Prove that $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

Solution :

$$\frac{1-\cos\theta}{\sin\theta} = \frac{1-\cos\theta}{\sin\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

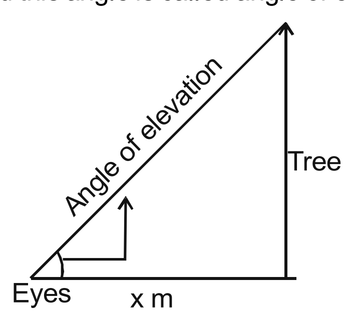
$$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$

c. Angle of Elevation and Depression

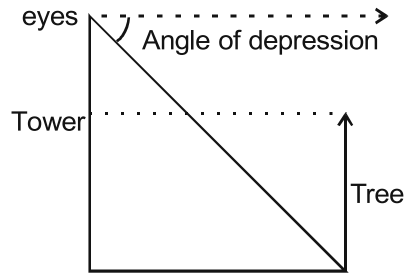
Angle of elevation

Suppose there is a big tree and our eyes and the bottom of the tree is in one horizontal. We want to see the top of the tree, so what will happen in that process if we will lift up our head to view the top of the tree. In this process our eyes move up by a certain angle and this angle is called angle of elevation.



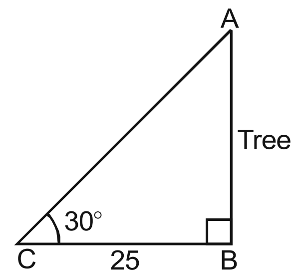
Angle of depression

Suppose we are standing on a tower and there is a tree whose height is equal to that of tower. We are facing straight and want to see the bottom of the tree, so what will happen in that process if we will look down to view the bottom of the tree. In this process our eyes cover certain angles and this angle is called angle of depression.



3. A man is standing at a distance of 25 m from the bottom of the tree, and he finds that the angle of elevation of the top of the tree is 30°. Find the height of the tree. (Consider height of man to be negligible)

Solution :



The man's eyes are at point C.

AB is the height of the tree and BC is given as 25 m and $\angle ACB = 30°$.

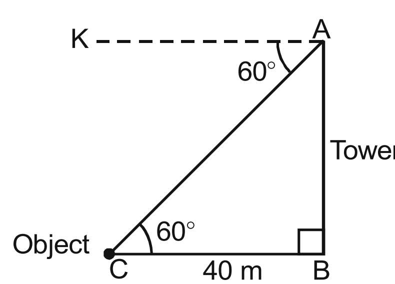
$$\tan 30° = \frac{AB}{BC} = \frac{AB}{25}$$

So $\frac{1}{\sqrt{3}} = \frac{AB}{25}$ and $AB = \frac{25}{\sqrt{3}}$

so the height of the tree is $\frac{25}{\sqrt{3}}$ m

4. A tower is at a distance of 40 m from an object. From the top of the tower the angle of depression of the object is 60°. Find the height of the tower.

Solution :



AB is a tower and C is the object.

If we draw a line AK parallel to BC, then

$\angle KAC = \angle ACB$ (Alternate angles)

$\tan 60^\circ = \frac{AB}{BC} = \frac{AB}{40}$

$\Rightarrow \sqrt{3} = \frac{AB}{40}$; so AB = $40\sqrt{3}$

So the height of the tower is $40\sqrt{3}$ m

(ii) Co-ordinate Geometry

Introduction

Co-ordinate Geometry is based on your basic knowledge of Geometry and that of graphing techniques that you have learnt in Algebra earlier.

It is nothing special except that you use algebraic equations in two variable to represent various geometric shapes.

We shall be dealing with two-dimensional problems, where there are two variables to be handled.

The variables are normally denoted by the ordered pair (x, y).

The horizontal axis is the X-axis and the vertical axis is the Y-axis. If the coordinates of a point on the X-Y plane is (x, y), it implies that it is at a perpendicular distance of x from the Y-axis and at a perpendicular distance y from the X-axis. The point of intersection of the X and Y-axes is called the origin and the coordinates of this point is (0, 0).

a. Some fundamental formulae:

1. Distance between the points (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. The points that divides the line joining two given points (x_1, y_1) and (x_2, y_2) in the ratio m : n internally and externally are

$$\left(\frac{mx_2 \pm nx_1}{m \pm n}, \frac{my_2 \pm ny_1}{m \pm n}\right)$$

Note:

It would be '+' in the case of internal division and '–' in the case of external division.

3. The coordinates of the mid-point of the line joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
4. The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Note:

If three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, then area of triangle ABC is zero.

5. Centroid of a triangle.
   The centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is
   $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
6. Slope of the line joining the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$; $(x_1 \neq x_2)$. The slope is also indicated by m.
7. If the slopes of two lines be m_1 and m_2, then the lines will be
   i. Parallel if $m_1 = m_2$
   ii. Perpendicular if $m_1 m_2 = -1$

b. Straight line

Standard forms

1. All straight lines can be written as $y = mx + c$, where m is the slope of the straight line, c is the Y intercept or the Y coordinate of the point at which the straight line cuts the Y-axis.
2. The equation of the straight line passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.
3. The equation of the straight line passing through two points (x_1, y_1) and (x_2, y_2) is
   $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
4. The point of intersection of any two lines of the form y = ax + b and y = cx + d is same as the solution arrived at when these two equations are solved.
5. The length of perpendicular from a given point (x_1, y_1) to a given line ax + by + c = 0 is $\left|\frac{ax_1 + by_1 + c}{\sqrt{(a^2 + b^2)}}\right| = p$, where p is the length of perpendicular. In particular, the length of perpendicular from origin (0, 0) to the line ax + by + c = 0 is $\frac{c}{\sqrt{a^2 + b^2}}$.

5. A line passes through the mid-point of the line joining the points (–3, –4) and (–5, 6) and has a slope of $\frac{3}{4}$. Find the equation.

Solution :

Slope $m = \frac{3}{4}$, mid-point of the line joining (–3, –4) and (–5, 6)

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3-5}{2}, \frac{-4+6}{2}\right) = (-4, 1)$

$\therefore$ The equation of the line is $y - y_1 = m(x - x_1)$,

$y - 1 = \frac{3}{4}(x + 4)$, i.e. $3x - 4y + 16 = 0$

6. Find the equation of the line through (2, –4) and parallel to the line joining the points (2, 3) and (–4, 5).

Solution :

Slope of the line joining (2, 3) and (–4, 5) is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-4-2} = \frac{2}{-6} = -\frac{1}{3}$

$\therefore$ Slope of any line parallel to it $= -\frac{1}{3}$,

point $(x_1, y_1) = (2, -4)$ and $m = -\frac{1}{3}$

$\therefore$ The equation is $y - y_1 = m(x - x_1)$

$y + 4 = -\frac{1}{3}(x - 2)$,

i.e. $3y + 12 = -x + 2$, $x + 3y + 10 = 0$

7. The vertices of a triangle are (1, 3), (–2, 4) and (3, –5). Find the equation of the altitude from (1, 3) to the opposite side.

Solution :

Let the vertices be A(1, 3), B (–2, 4) and C(3, –5), slope of

$BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-4}{3+2} = \frac{-9}{5}$

$\therefore$ Slope of altitude through A $= \frac{5}{9}$, since the altitude passes through A (1, 3).

Its equation is $y - y_1 = m(x - x_1)$,

i.e. $y - 3 = \frac{5}{9}(x - 1)$, i.e. $9y - 27 = 5x - 5$, i.e. $5x - 9y + 22 = 0$

8. Find the equation of the perpendicular bisector of the line joining (5, 6) and (2, –2).

Solution :

Let the points be A(5, 6) and B (2, –2),

mid-point $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\therefore$ Mid-point of AB $= \left(\frac{5+2}{2}, \frac{6-2}{2}\right)$

$= \left(\frac{7}{2}, \frac{4}{2}\right) = \left(\frac{7}{2}, 2\right)$

Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{2-5} = \frac{-8}{-3} = \frac{8}{3}$

$\therefore$ Slope of perpendicular $= -\frac{3}{8}$

Now equation is $y - y_1 = m(x - x_1)$,

i.e. $y - 2 = -\frac{3}{8}\left(x - \frac{7}{2}\right)$,

$8y - 16 = -3x + \frac{21}{2}$, $3x + 8y - 16 - \frac{21}{2} = 0$

$6x + 16y - 32 - 21 = 0$, $6x + 16y - 53 = 0$.

9. What is the equation of the line which joins the points A (–1, 3) and B (4, –2)?

Solution :

Equation is given by $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$,

i.e. $\frac{y-3}{-2-3} = \frac{x+1}{4+1}$, $\frac{y-3}{-5} = \frac{x+1}{5}$,

$y - 3 = -(x + 1)$, $y - 3 = -x - 1$,

$x + y - 2 = 0$.

10. Find the equation of the line passing through (2, –1) and whose intercepts on the axes are equal in magnitude but opposite in sign.

Solution :

Let the intercepts on the axes be a and – a. The intercept form of the equation is

$\frac{x}{a} + \frac{y}{b} = 1$, $\frac{x}{a} + \frac{y}{-a} = 1$, i.e. $x - y = a$

The line passes through (2, –1)

$\therefore$ Substituting (2, –1) in the equation

$2 - (-1) = a$, $2 + 1 = a$, i.e. $a = 3$

The required equation is $x - y = 3$ or $x - y - 3 = 0$

11. Find the equation of the line passing through the point of intersection of $4x - y - 3 = 0$ and $x + y - 2 = 0$ and perpendicular to $2x - 5y + 3 = 0$

Solution :

The two lines are $4x - y - 3 = 0$ and $x + y - 2 = 0$. Solving the two equations, $5x = 5$, $x = 1$.

Put $x = 1$ in $4x - y - 3 = 0$, i.e. $4(1) - y - 3 = 0$, $-y + 1 = 0$, i.e. $y = 1$

$\therefore$ The point of intersection is (1, 1). The required line passes through (1, 1) and is perpendicular to $2x - 5y + 3 = 0$. Slope of the line $2x - 5y + 3 = 0$ is $\frac{2}{5}$.

$\therefore$ Slope of perpendicular is $-\frac{5}{2}$. Equation of the required line is $y - y_1 = m(x - x_1)$, i.e. $(y - 1) = -\frac{5}{2}(x - 1)$, $2y - 2 = -5x + 5$, i.e. $2y + 5x = 7$

12. The distance of the point (7, 5) from the point of intersection of the lines $x + y = 4$ and $2x - y = 2$ is

(a) 6 (b) $\sqrt{34}$

(c) 5 (d) 7

Solution :

The two equations are $x + y = 4$ and $2x - y = 2$, solving these two equations, we get $x = 2$ and $y = 2$

$\therefore$ The point of intersection is (2, 2)

so, the distance between (2, 2) and (7, 5) is

$$\sqrt{(7-2)^2 + (5-2)^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$$

Exercise

1. The value of $\frac{\sin 45^\circ \cos 60^\circ}{\tan 30^\circ}$ is

   (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{2}}{\sqrt{3}}$

   (c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{\sqrt{2}}$

2. $4\cos^2 60^\circ - \sin^4 30^\circ$ is equal to

   (a) $\frac{15}{16}$ (b) $\sqrt{3}+\sqrt{2}$

   (c) 1 (d) $\frac{2}{\sqrt{3}}$

3. The value of sin45° × cos45° is

   (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

   (c) $\sqrt{2}$ (d) $\sqrt{3}$

4. The angles of elevation of an aeroplane flying vertically above the ground as observed from the two consecutive stones 2 km apart are 45° and 60°. The height of the aeroplane above the ground is

   (a) $\frac{\sqrt{3}+2}{2\sqrt{2}}$ km (b) $\frac{2\sqrt{3}}{\sqrt{3}-1}$ km

   (c) $\frac{2\sqrt{3}}{\sqrt{3}}$ km (d) $\frac{2\sqrt{3}}{2}$ km

5. The height of a tower is 200 m. When the angle of elevation of the sun changes from 45° to 60°, the shadow of the tower becomes shorter by x m. Find the value of x.

   (a) $\frac{600-\sqrt{3}}{\sqrt{3}}$ (b) $\frac{600+\sqrt{3}}{3}$

   (c) $\frac{600+\sqrt{3}}{\sqrt{3}}$ (d) $\frac{200\left(3-\sqrt{3}\right)}{3}$

6. The value of the expression

   $\sin^2 45^\circ + \sin^2 30^\circ + \sin^2 60^\circ + \sin^2 90^\circ$ is

   (a) 4.5 (b) 3

   (c) 2.5 (d) 2

7. If $\sin\theta = \frac{5}{8}$, then find the value of $\cos\theta$.

   (a) $\frac{\sqrt{14}}{3}$ (b) $\frac{\sqrt{39}}{8}$

   (c) 5 (d) 6

8. tan 860° = ?

   (a) tan 40° (b) – tan 40°

   (c) cos 40° (d) cot 40°

9. 485° lies in quadrant

   (a) 1st (b) 2nd

   (c) 3rd (d) 4th

10. From a tower 100 m high, the angle of depression of a car is 30°. Find how far the car is from the tower.

    (a) $100\sqrt{3}$ m (b) 100 m

    (c) $100\sqrt{2}$ m (d) $50\sqrt{2}$ m

11. A kite is flying with a thread 150 m long. If the thread makes an angle of 30° with horizontal, find the height of kite from ground.

    (a) 100 m (b) 75 m

    (c) 50 m (d) 80 m

12. A tree is broken by wind and the top of it struck the ground at angle of 30° and at a distance of 50 m from the foot, find the initial height of tree.

    (a) $\frac{75}{\sqrt{3}}$ m (b) $\frac{150}{\sqrt{3}}$ m

    (c) 100 m (d) $\frac{100}{\sqrt{3}}$ m

13. The shadow of a tree, when the angle of elevation of sun is 45° is found to be 20 m longer than when it is 60°. Find the height of tree.

    (a) 50 m (b) 27.39 m

    (c) 47.38 m (d) 40 m

14. A tower stands vertically on the ground. At a point on ground, 20 m away from the foot of tower, the angle of elevation of the top of tower is 30°. Find the height of tower.

    (a) $\frac{20}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m

    (c) 20 m (d) 30 m

15. From the top of a 200 m high pole, the angle of depression of two points is 30° and 45°, if one point is directly behind the other, find the distance between the two points.

    (a) 150 m (b) 100 m

    (c) 143 m (d) 146 m

16. Find the horizontal distance between the two trees whose heights are 10 m and 18 m, and the distance between their tops is 17 m.

    (a) 15 m (b) 20 m

    (c) 18 m (d) 12 m

17. If the point (xy) is equidistant from (a + b, b – a) and (a – b, a + b) then

    (a) ax = by (b) bx = ay

    (c) ab = xy (d) None of these

18. The value of x, if the distance between (x, 6) and (3, 0) is 10, is

    (a) 11 (b) – 5

    (c) Both 11 and – 5 (d) 13

19. The ratio in which the x-axis divide the line segment joining the points (4, – 6) and (1, 3) is

    (a) 2 : 1 (b) 1 : 2

    (c) 1 : 4 (d) 4 : 1

20. If the coordinates of two vertices of a triangle are (–2, 5) and (–4 , 4) and centroid of the triangle lies on origin then coordinate of third vertex is.

    (a) (–6, –9) (b) (6, –9)

    (c) (–6, 9) (d) (6, 9)

21. The slope of a line which is perpendicular to the line joining the points (2, 6) and (–3, 1) is.

    (a) 1 (b) $-\frac{1}{2}$

    (c) –1 (d) 2

22. If the points (2, –k), (0, –5) and $\left(\frac{5}{2},0\right)$ are collinear, then value of 'k' is

    (a) –1 (b) 1

    (c) 0 (d) $\frac{3}{2}$

Answer Key

1. (c)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (a)
11. (b)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (b)
21. (c)	**22.** (b)								

Explanations

1. c $\frac{\sin 45°\cdot\cos 60°}{\tan 30°} = \frac{\frac{1}{\sqrt{2}}\cdot\frac{1}{2}}{\frac{1}{\sqrt{3}}} = \frac{\frac{1}{2\sqrt{2}}}{\frac{1}{\sqrt{3}}} = \frac{1}{2\sqrt{2}}\times\frac{\sqrt{3}}{1} = \frac{\sqrt{3}}{2\sqrt{2}}$

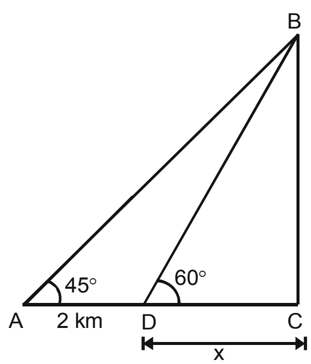
2. a $4\cos^2 60° - \sin^4 30° = 4\times\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^4$

$= 4\times\frac{1}{4} - \frac{1}{16} = 1 - \frac{1}{16} = \frac{15}{16}$

3. a $\sin 45° \times \cos 45° = \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

4. b Let A and D are two stones so that AD = 2 km and BC is the height of the aeroplane from the ground.

Let DC = x km. So, in ΔBAC



$\tan 45° = \frac{BC}{AC}$ … (i)

$\Rightarrow 1 = \frac{BC}{2+x}$ and 2 + x = BC

In ΔBDC, $\tan 60° = \frac{BC}{CD}$

So, $\sqrt{3} = \frac{BC}{x}$ and $BC = \sqrt{3}x$ … (ii)

Equating (i) and (ii)

2 + x = $\sqrt{3}x$ and $\sqrt{3}x - x = 2$

So, $x\left(\sqrt{3}-1\right) = 2$ and $x = \frac{2}{\sqrt{3}-1}$

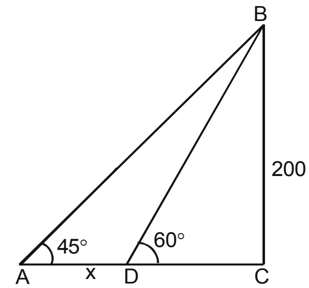
and BC = $\sqrt{3}x = \sqrt{3}\left(\frac{2}{\sqrt{3}-1}\right) = \frac{2\sqrt{3}}{\sqrt{3}-1}$

5. d Let BC is the height of the tower and AD = x metres

In ΔBAC

$\tan 45° = \frac{BC}{AC} \Rightarrow 1 = \frac{200}{AC} \Rightarrow AC = 200$

In ΔBDC, $\tan 60° = \frac{BC}{DC} = \frac{BC}{AC-AD}$



$\Rightarrow \sqrt{3} = \frac{200}{200-x} \Rightarrow 200\sqrt{3} - \sqrt{3}x = 200$

$\Rightarrow \sqrt{3}x = 200\left(\sqrt{3}-1\right)$

$\Rightarrow x = \frac{200\left(\sqrt{3}-1\right)}{\sqrt{3}} = \frac{200\left(3-\sqrt{3}\right)}{3}$

6. c $\sin^2 45° + \sin^2 30° + \sin^2 60° + \sin^2 90°$

7. b $\sin\theta = \frac{5}{8} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

So base^2 = (Hypotenuse)^2 – (Perpendicular)^2

= $(8)^2 - (5)^2 = 64 - 25 = 39$

base = $\sqrt{39}$

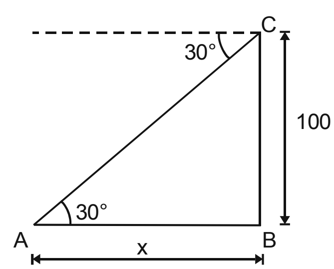
$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{39}}{8}$

8. b $\tan 860° = \tan\left(180°\times n \pm \theta\right)$

$= \tan\left(180°\times 5 - 40°\right) = -\tan 40°$

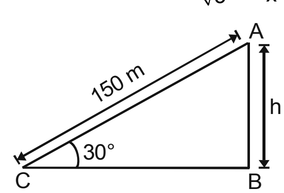
9. b $485° = 360 + 90 + 35$. So it lies in 2nd quadrant

10. a



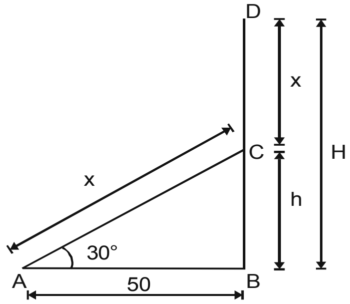
tan30 = $\frac{100}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x} \Rightarrow x = 100\sqrt{3}$ m

11. b



$\frac{h}{150} = \sin 30°$; $h = \frac{1}{2}\times 150 = 75$ m.

12. b



Distance AC = DC = x

$\frac{h}{50} = \tan 30° \Rightarrow h = \frac{50}{\sqrt{3}}$

$\Rightarrow \frac{50}{x} = \cos 30°$

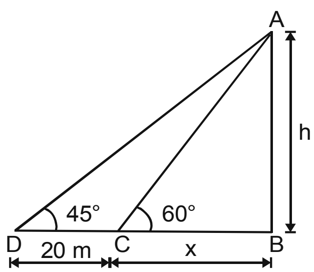
$x = \frac{50}{\cos 30°} = \frac{50 \times 2}{\sqrt{3}} = \frac{100}{\sqrt{3}}$

$\therefore$ Height of tree = x + h

$= \frac{50}{\sqrt{3}} + \frac{100}{\sqrt{3}} = \frac{150}{\sqrt{3}}$ m.

13. c $\frac{h}{x} = \tan 60° \Rightarrow h = x \tan 60°$

$\frac{h}{x+20} = \tan 45°$



$\Rightarrow \frac{x \tan 60°}{x+20} = \tan 45°$

$\sqrt{3}x = 1 \times (x+20)$

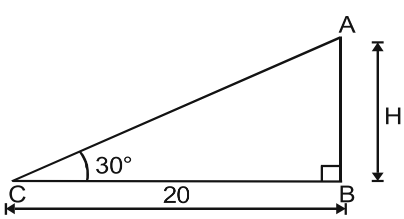
$\sqrt{3}x = x + 20$

1.73x – x = 20

$x = \frac{20}{0.73} = 27.39$

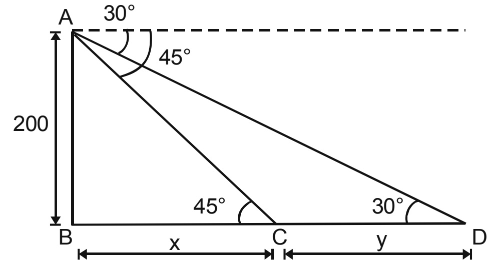
Height of tree = h = $27.39 \times \sqrt{3} = 47.39$ m.

14. a



$\frac{H}{20} = \tan 30°$; $H = \frac{20}{\sqrt{3}}$ m.

15. d



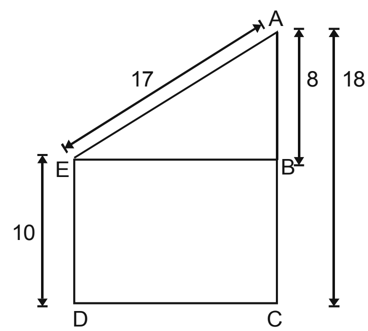
y = Distance between two points

$\frac{200}{x} = \tan 45° \Rightarrow x = 200$

$\frac{200}{x+y} = \tan 30° \Rightarrow x + y = 200\sqrt{3}$

$y = 200\sqrt{3} - 200 = 200 \times 0.73 = 146$ m.

16. a



BE = $\sqrt{17^2 - 8^2} = \sqrt{289 - 64}$

17. b Let the points be $A(x, y)$, $B(a+b, b-a)$ and $C(a-b, a+b)$

Given: AB = AC

or $AB^2 = AC^2$

or $[x-(a+b)]^2 + [y-(b-a)]^2$

$= [x-(a-b)]^2 + [y-(a+b)]^2$

or $x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a)$

$= x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$

or $-ax - bx - by + ay = -ax + bx - ay - by$

(cancelling 2)

or $2bx = 2ay$

or $bx = ay$.

18. c Given the distance between (x, 6) and (3, 0) is 10.

Hence $\sqrt{(x-3)^2 + (6-0)^2} = 10$.

or $(x-3)^2 + 6^2 = 10^2$

or $(x-3)^2 = 10^2 - 6^2 = 8^2$

or $(x-3) = \pm 8$.

or $x = -5$ and 11.

19. a Let the required ratio be x : 1

Hence the co-ordinates of the point at which the x-axis cuts the line segment joining (4, –6) and (1, 3) is $\left(\frac{x+4}{x+1}, \frac{3x-6}{x+1}\right)$.

Now obviously the y – co-ordinate of this point has to be 0. So $\frac{3x-6}{x+1} = 0$ or x = 2

Hence, the ratio is 2 :1.

20. b Let the third vertex of the triangle is (x, y)

Centroid of the triangle is (0, 0)

$\therefore 0 = \frac{-2-4+x}{3} \Rightarrow x = 6$

Hence, the third vertex is (6, –9)

21. c Let the slope of the required line = m_1

Slope (m_2) of the line joining (2, 6) and (–3, 1)

$= \frac{1-6}{-3-2} = \frac{-5}{-5} = 1$

As required line is perpendicular to the line joining the points (2, 6) and (–3, 1)

$\therefore m_1 m_2 = -1$

$\Rightarrow m_1 = \frac{-1}{m_2} = -1$

Hence, slope of the required line = –1

22. b Let the point (2, –k), (0, –5) and $\left(\frac{5}{2}, 0\right)$ are represented by the points A, B and C respectively.

As we know if the points A, B and C are collinear then Slope of AB = slope of BC

$\frac{-5-(-k)}{0-2} = \frac{0-(-5)}{\frac{5}{2}-0} \Rightarrow \frac{k-5}{-2} = \frac{5}{\frac{5}{2}}$

$\Rightarrow \frac{k-5}{-2} = 2$

$\therefore k = 1$

Alternate method:

The value of ‘k’ can be found by putting area of the triangle ABC = 0