Fundamental Concepts

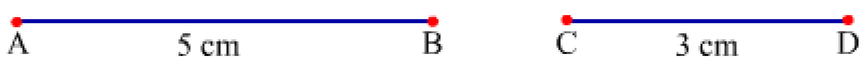
Points, Line Segments, Lines, Rays, Planes and Space

Points, line segments, lines, and rays are the basic ideas on which the world of geometry is based. In order to understand geometry, we should be very clear about these basic ideas.

Let us start with the concept of a point. In order to understand the idea of a point, let us consider the sharp tip of a pencil or the pointed end of a pin.

These sharp tips represent dots. These small dots can be described as points. Let us go through the video to understand the concept of points and other related geometrical concepts such as lines, rays, and line segments.

Now, look at the line segments AB and CD.



Length of segment AB is 5 cm and that of segment CD is 3 cm.

Mathematically, their respective lengths can be written as follows:

***l* (AB) = 5 cm** and ***l* (CD) = 3 cm**

We have seen what lines, rays, and line segments are, so let us now study another concept, which is the concept of planes.

A plane is a flat surface having length and width, but no thickness. We can say that a plane is a flat surface, which extends indefinitely in all directions.

For example, surface of a wall, floor of a ground, etc.

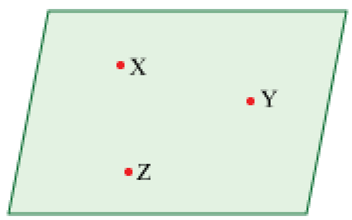
A plane can be denoted by writing small letters inside it such as letters *p, q*, etc.

For example:

This plane is read as “plane *p*”.

Also, a plane can be denoted by taking 3 different points, say X, Y, Z in the plane, but not on same line.

For example:



This plane is read as “plane XYZ”.

Do you think that there is any relation between points and lines in a plane?

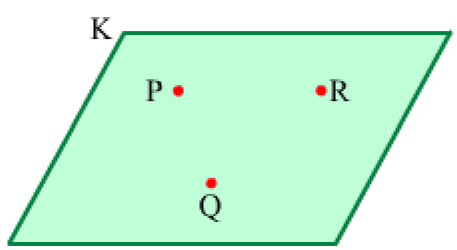
The answer to this question is ‘yes’ and the relations between them are called incidence properties.

To understand the incidence properties, look at the following video.

Some axioms:

(1) There is exactly one plane passing through three non-collinear points.

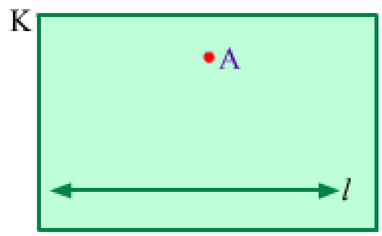
Observe the given figure.



Here, points P, Q and R are three non-collinear points. K is the plane passing through these points.

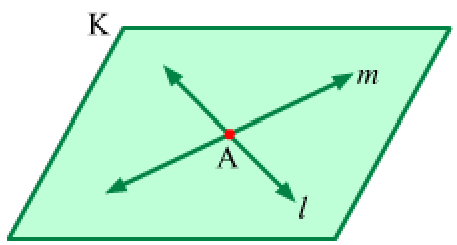
(2) There is exactly one plane which passes through a line and a point not lying on the line.

Look at the following figure.



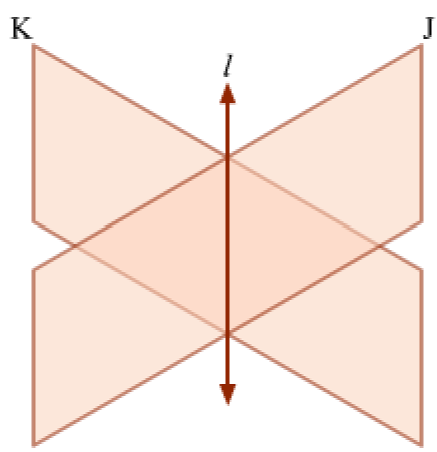
Here, plane K passes through the line *l* and the point A which does not lie on the line *l*.

(3) Only one plane can pass through two distinct intersecting lines.



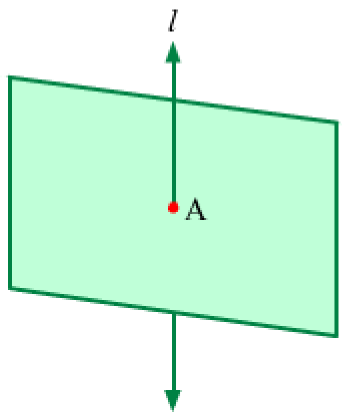
In the figure given above, lines *l* and *m* intersect each other at point A. Plane K passes through these lines.

(4) A line is obtained by intersection of two planes.



In the figure given above, planes J and K intersect each other. Line *l* is obtained by their intersection.

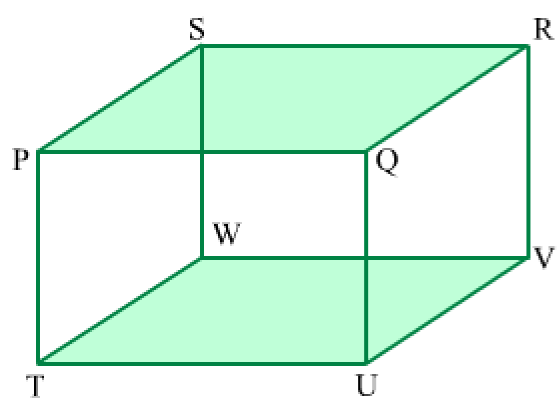
(5) When a plane is intersected by a line, which does not lie in the plane, then a point is obtained.



In the figure given above, line *l* does not lie in the plane K and it intersects the plane at point A.

Parallel planes:

Planes which do not intersect each other are said to be parallel.



In the above given figure, planes PQRS and TUVW do not intersect each other and hence, these are parallel planes.

Similarly, PTWS and QUVR is a pair of parallel planes while PTUQ and SWVR is another pair of parallel planes.

Length of a line segment:

The length of a line segment is the distance between the end points of the line segment.

Length of a line segment PQ is denoted as *l*(PQ) and distance between two points is denoted as *d*(P, Q).

Therefore, *l*(PQ) = *d*(P, Q).

Here after *l*(PQ) is denoted as PQ and thus, PQ = *l*(PQ) = *d*(P, Q).

Congruent segment:

If two line segments are of equal length, then they are said to be congruent.

In the above figure, *l*(PQ) = *l*(RS). So, line segments PQ and RS are congruent.

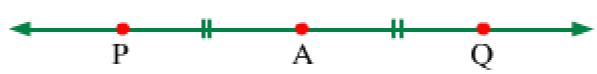
Mathematically, if *l*(PQ) = *l*(RS), then seg PQ $\cong$ seg RS.

Note: While considering the length of segment PQ, we write only PQ but while considering the set of points between P and Q (segment as a whole), we write seg PQ or side PQ.

Mid-point of a segment:

If A is a point such that P – A – Q and *d*(P, A) = *d*(A, Q) then A is said to be the mid-point of the segment PQ.

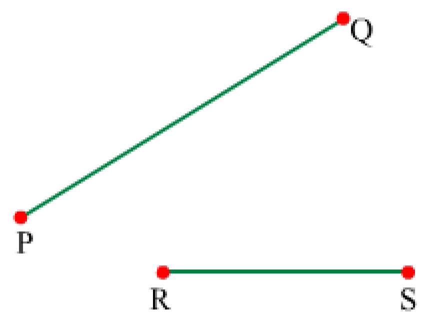
Every line segment has one and only one mid-point.



In the above figure, seg PA $\cong$ seg AQ.

Comparison of segments:

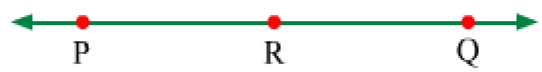
Observe the below given figure.



Here, RS < PQ so, it can be said seg RS is smaller than seg PQ. This information is denoted as seg RS < seg PQ.

Opposite rays:

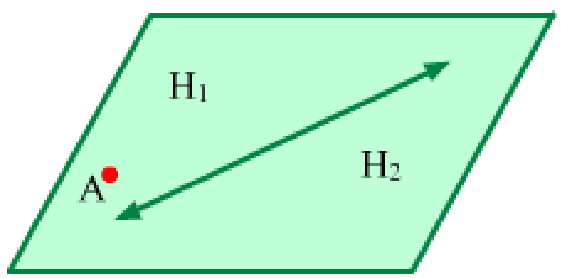
If two rays have same origin and are contained in the same line, then the rays are known as opposite rays.



In the above figure, ray RP and ray RQ are opposite rays.

Plane separation axiom:

In a plane, a given line and points which do not lie on the line form two disjoint sets say H_1 and H_2.



Each of the sets H_1 and H_2 is known as the half plane and the given line is called the edge of each half plane.

If A is any point in any of the half planes then then that half plane is known as A side of the half plane.

Let us discuss some examples based on these basic geometrical ideas.

Example 1:

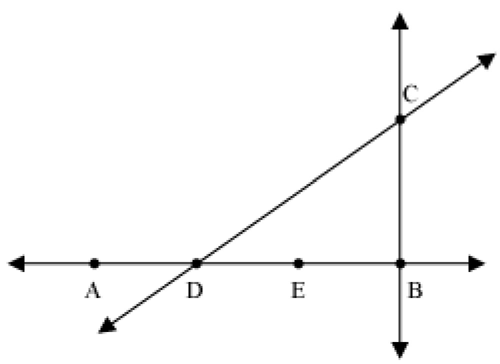
With respect to the given figure, name

(a) any six line segments

(b) a line

(c) three rays

(d) two pairs of intersecting lines



Solution:

(a) Six line segments shown in the figure are $\overline{AD}, \overline{AE}, \overline{DE}, \overline{DB}, \overline{BE}$, and $\overline{BC}$.

(b) $\overleftrightarrow{AB}$ is a line in the given figure.

(c) Three rays in the given figure are $\overrightarrow{DA}, \overrightarrow{DB}$, and $\overrightarrow{DC}$.

(d) Two pairs of intersecting lines in the given figure are $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$, and $\overleftrightarrow{DC}$ and $\overleftrightarrow{BC}$.

Example 2:

Draw rough figures of the following.

(a) A line $\overleftrightarrow{AB}$

(b) Points P and Q that lie on line $\overleftrightarrow{AB}$

(c) A line $\overleftrightarrow{XY}$ that intersects line $\overleftrightarrow{AB}$ at point Q

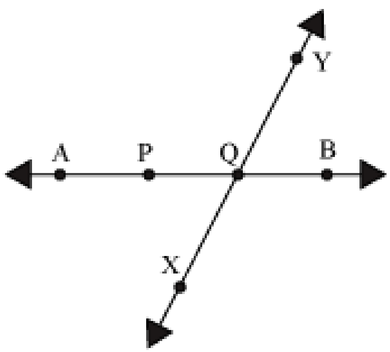
(d) A line $\overleftrightarrow{DE}$ that is parallel to line $\overleftrightarrow{XY}$ and passes through point P

Solution:

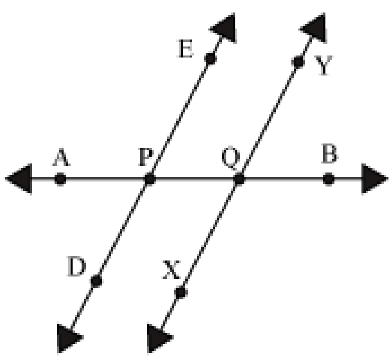
(a)

(b)

(c)

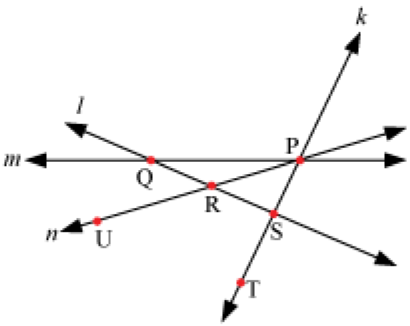


(d)



Example 3:

With respect to the given figure, state whether the following statements are correct or incorrect.



(a) The points P, S, and T are collinear points.

(b) The lines *k*, *l*, and n are concurrent lines.

(c) The point U, R, and S are collinear points.

(d) The point P is the point of concurrence of lines *k*, *m*, and *n*.

Solution:

(a) Correct

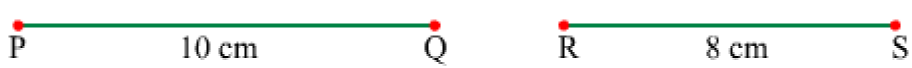
(b) Incorrect; since the lines *k, l*, and *n* do not pass through the same point

(c) Incorrect; since the points U, R, and S do not lie on the same line

(d) Correct

Example 4:

Write the lengths of given line segments.



Solution:

It can be seen that the length of segment PQ is 10 cm and that of segment RS is 8 cm.

Mathematically, their respective lengths can be written as follows:

l (PQ) = 10 cm and *l* (RS) = 8 cm

Perpendicular Lines and Perpendicular Bisectors

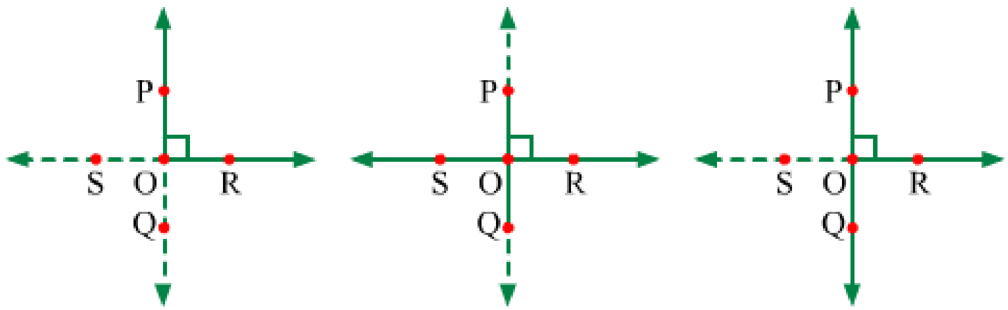
Two intersecting lines can have any angle between them, but perpendicular lines are special lines, which intersect each other at an angle of ninety degrees.

You can understand the concept of perpendicular lines and perpendicular bisectors by going through the video.

Perpendicularity of segments and rays:

Two rays or two segments or a segment and a ray are said to be perpendicular to each other if the lines containing them are perpendicular.

Observe the following figure.



In figure (i), ray OP is perpendicular to ray OR i.e., ray OP ⊥ ray OR.

In figure (ii), segment PQ is perpendicular to segment RS i.e., seg PQ ⊥ seg RS.

In figure (iii), line PQ is perpendicular to ray OR i.e., line PQ ⊥ ray OR.

For each case, we have $m\angle POR = 90°$.

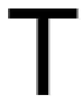
Foot of the perpendicular:

In the figure (ii), line OP intersects line SR at point O at right angle. Thus, line OP ⊥ line SR. Here, the point of intersection O is known as the foot of the perpendicular.

Let us now look at some examples to understand the concept better.

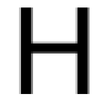
Example 1:

Identify the perpendicular lines and show them by dotted extended lines in the following figures.



(i)

(ii)

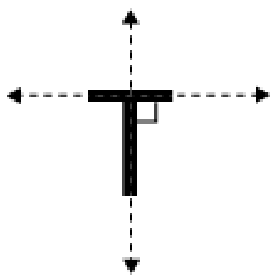


(iii)

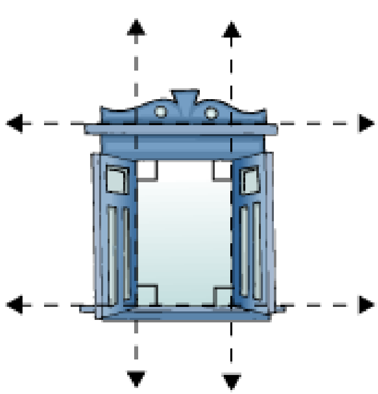
Solution:

The dotted extended lines shown below are perpendicular lines.

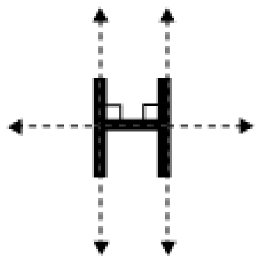
(i)



(ii)

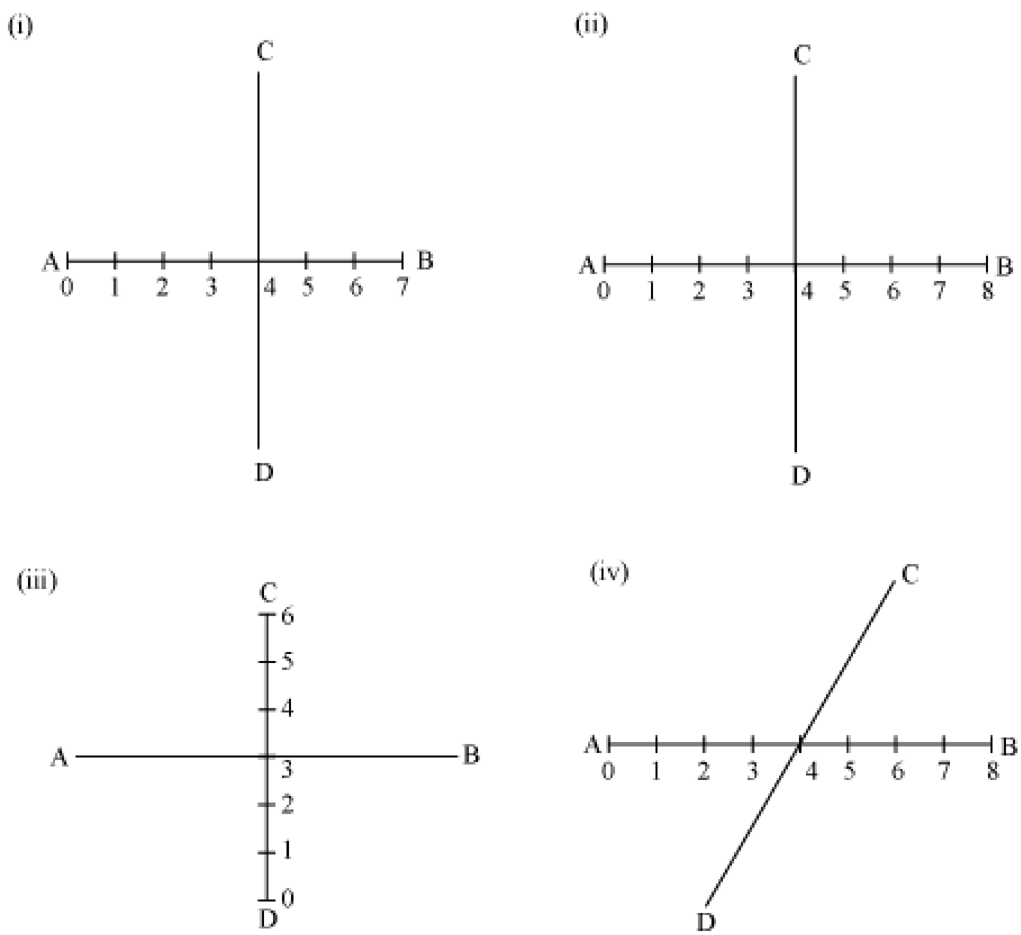


(iii)



Example 2:

Which two among the following figures represent a perpendicular bisector?



Solution:

(i) CD is perpendicular to AB. However, it does not bisect AB.

(ii) CD acts as a perpendicular bisector to AB.

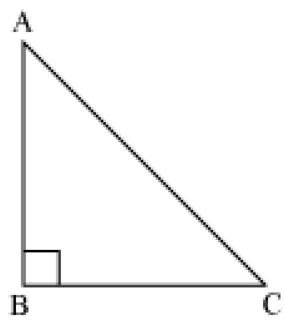
(iii) AB acts as a perpendicular bisector to CD.

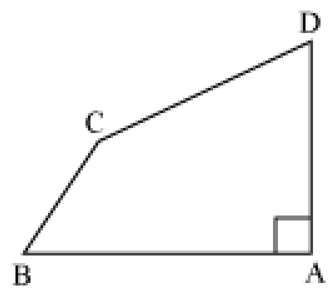
(iv) AB and CD are not perpendicular to each other. Therefore, neither AB nor CD is a perpendicular bisector.

Hence, figures **(ii)** and **(iii)** represent a perpendicular bisector.

Example 3:

Identify perpendicular lines in the following figures.





(i) **(ii)**

Solution:

1. Sides AB and BC form a right angle.

   Therefore, AB ⊥ BC

2. Sides AB and DA form a right angle.

   Therefore, AB ⊥ DA

Classification of Curves as Open and Closed

Look at the following figures.

(i) (ii) (iii)

Each of these shapes is an example of a **curve.** In fact, any shape that we draw is a curve. We can define a curve as follows.

Any figure drawn on a paper is known as a curve. A curve may or may not be straight.

Note: In real life, we do not consider straight lines as curves. However, in mathematics, straight lines are also considered as curves.

Can we find any difference among the three curves that we discussed in the beginning?

Curves (i) and (iii) do not intersect themselves, while curve (ii) does. Also, curves (i) and (iii) are not closed figures, while curve (ii) is a closed figure. On the basis of these observations, we classify curves as follows.

1. Simple curves
2. Closed Curves
3. Open curves

Let us discuss each of these with the help of the following video.

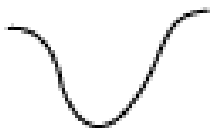
Let us discuss some more examples based on classification of curves.

Example 1:

Classify each of the following curves as open or closed.

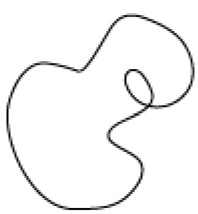
(a)

(b)



(c)

(d)



Solution:

(a) Since no end points can be seen in the curve, it is an example of a closed curve.

(b) Since the two end points of the curve can be seen, it is an example of an open curve.

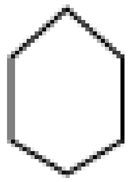
(c) Since the two end points of the curve can be seen, it is an example of an open curve.

(d) Since no end points can be seen in the curve, it is an example of a closed curve.

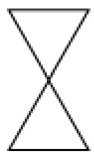
Example 2:

State whether each of the following curves is simple or not.

(a)



(b)



Solution:

(a) Since the curve does not cross itself, it is a simple curve.

(b) Since the curve crosses itself at one point, it is not a simple curve.