CHAPTER 05

Compression Members

5.1 Introduction

- A compression member is the one which carries predominantly the opposite compressive forces on the two ends.
- Ideal Compression Member: An ideal compression member is the one which is perfectly straight, has no crookedness or the imperfection; the loads are applied uniformly across the section with center of gravity of the loads coinciding with the center of gravity of the compression member. However such an ideal compression member never exists in practice.

5.2 Different Types of Compression Member

- Column, stanchion or post: This type of compression member supports floors or girders of a building and usually carries very heavy loads.
- Strut: It is lightly loaded compression member used in a truss and is usually of small span. A typical strut may be continuous or discontinuous. A continuous strut passes over more than one joint (apart from end joints) like top chord member of a truss bridge, principal rafter of roof truss etc. A discontinuous strut spans between two end joints only like the vertical or the inclined compression member of a truss.
- Boom: It the compression member of a crane.

5.3 Design Aspects of Compression Member

- In the design of a compression member, stability plays a very indispensable role.
- By the term stability in a compression member, we usually mean buckling.
- Among the three types of equilibrium viz. stable equilibrium, neutral equilibrium and unstable equilibrium, the unstable equilibrium is the most unfavorable since here in this equilibrium, a slight deflection produces further large deflections leading to complete collapse of the structure.
- Compression members are the case of unstable equilibrium since even a slightly bent compression member when placed in a structure may undergo significant bending which is not the case with tension members where the bent member tries to straighten on application of tensile force.
- Column design also depends on the length of the member while for designing tension member; length does not play any significant role.

- Based on length, column may be long/slender, intermediate or short. Whether a column is long, intermediate or short is defined by its **slenderness ratio.**
- As the length of the column increases, the tendency of buckling increases provided compressive load acting on the column and cross-sectional area remains constant.

Table 5.1 Salient features of failure in columns

S.No.	Type of column	Failure feature
1.	Long	• Due to elastic buckling. • Stress in the column will not exceed proportional limit and is much lower than this limit.
2.	Intermediate	• Due to inelastic buckling (yielding+ buckling) and flexural rigidity EI changes continuously. • Extreme fiber reaches the yield point while stress in all other fibres remains elastic.
3.	Short	• Due to yielding (crushing) • Axial shortening of column occurs till it gets crushed.

5.4 Effective Length

- Compression member on buckling deflects in a type of curve the shape of which depends on the type of end conditions of the column.
- In each of the compression member there is a part of compression member which deflects as if it is pin-jointed i.e. hinged.
- **Effective length:** The ends of this portion of compression member are called as **points of contra flexure** and distance between these points is called as **effective length** of compression member.
- This effective length is arrived at from the actual length of the compression member and its end conditions.

NOTE

The end conditions of a compression member are accounted for by the use of **effective length factors** (K) which when multiplied with the **actual length** (L) gives **effective length** (l) of the compression member.

5.4.1 Factors Affecting the Magnitude of Effective Length Factor (K)

(a) The amount of rotational restraint available at the ends of the compression member.

(b) The resistance or restraint available to lateral movement at the ends of the compression member.

Table: 5.2 Effective lengths of prismatic compression members

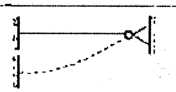
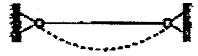
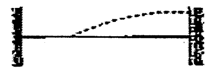
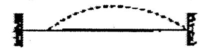
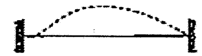
Boundary Conditions				Schematic Representation	Effective length (KL)
At One End		At the Other End			
Translation	Rotation	Translation	Rotation		
Restrained	Restrained	Free	Free		2.0 L
Free	Restrained	Restrained	Free		2.0 L
Restrained	Free	Restrained	Free		1.0 L
Restrained	Restrained	Free	Restrained		1.2 L
Restrained	Restrained	Restrained	Free		0.8 L
Restrained	Restrained	Restrained	Restrained		0.65 L

Table: 5.3 Effective lengths of struts

S.No.	Type	Sections	Effective length	
			In the plane of gusset	Perpendicular to gusset
1.	Continuous	Single or double angle	0.7 L to 1.00 L	1.00 L
2.	Discontinuous	*Single-angle connected with one bolt	1.00 L	1.00 L
3.	Discontinuous	*Single-angle connected with more than one bolt or welded	0.85 L	1.00 L
4.	Discontinuous	Double-angles placed back to back on opposite side of gusset plate	0.70 L to 0.85 L	1.00 L
5.	Discontinuous	Double-angle placed back to back on same side of gusset plate	0.70 L to 0.85 L	1.00 L

Remember: If a compression member is supported differently with respect to its principal axes then its effective lengths in the two directions will be different.

- The values of effective length factors given in **Table 5.2** are applicable for isolated columns only.
- As per IS 800, where there is no exact analysis available, there the effective length of columns in a framed structure may be obtained by the product of actual length of the column between the centers of laterally supporting members (beams) with the effective length factor (K) where the factor K is computed using the expressions (based on Wood's curves) as given below provided the connection between the beam and column is rigid.
- **Braced (non-sway) frames:** The effective length factor (K) is given by

$$K = \left[\frac{1+0.145(\beta_1+\beta_2)-0.265\beta_1\beta_2}{2-0.364(\beta_1+\beta_2)-0.247\beta_1\beta_2}\right] \quad ...(5.1)$$

- **Moment resisting (sway) frames:** The effective length factor (l) is given by,

$$K = \left[\frac{1-0.2(\beta_1+\beta_2)-0.12\beta_1\beta_2}{1-0.8(\beta_1+\beta_2)-0.6\beta_1\beta_2}\right]^{1/2} \quad ...(5.2)$$

where β_1 and β_2 are the coefficients at the top and bottom end of the column respectively which are given by,

$$\beta = \frac{\sum K_c}{\sum K_c + \sum K_b} \quad ...(5.3)$$

K_b, K_c = Effective flexural stiffness of the beams and the columns respectively meeting at a joint at the ends of a column and rigidly connected at the joints and are computed as:

$$K_b = \frac{I}{L}, \quad K_c = C\frac{I}{L}$$

I = Moment of inertia of the member about an axis normal to the plane of the frame

L = Length of the member = Center to center distance of the intersecting members

C = Connection factor as given in Table 5.4

Remember

1. For columns fixed at the base, $\Sigma K_b \rightarrow \infty$ and hence $\beta_2 \rightarrow 0$.
2. For columns hinged at the base, $\Sigma K_b = 0$ and hence $\beta_2 = 1$.
3. Stiff columns connected to flexible beams can rotate much freely and approach the hinged end condition but K will relatively be a large value.
4. Very long columns connected to girders of large cross section are not free to rotate at the ends and hence approach fixed end conditions and K will relatively be a small value.

where

$\bar{n} = \frac{P}{P_e}$

P_e = Euler's elastic buckling load

$= \frac{\pi^2 EI}{L^2}$

P = Applied axial load

L = Unbraced length of the column

Table: 5.4 Values of connection factor (C)

S.No.	Far End Condition	Connection factor (C)	
		Braced Frame	Unbraced Frame
1.	Pinned	$1.5(1-\bar{n})$	$1.5(1-\bar{n})$
2.	Rigidly connected to column	$1.0(1-\bar{n})$	$1.0(1-0.2\bar{n})$
3.	Fixed	$2.0(1-0.4\bar{n})$	$0.67(1-0.4\bar{n})$

5.5 Slenderness Ratio (λ)

- For the same area of cross section, as the length of column increases, the tendency of the column to buckle increases and its load carrying capacity decreases.
- This tendency of a column to buckle is measured in terms of slenderness ratio (λ) which is defined as the ratio of effective length of the member to its appropriate radius of gyration i.e.

$$\text{Slenderness ratio } (\lambda) = \frac{l}{r} = \frac{KL}{r} \quad ...(5.4)$$

- Least radii of gyration occurs when the column has equal unbraced lengths in both the axes with similar end conditions about both the axes for a particular column section.

$$\lambda_{max} = \frac{KL}{r_{min}}$$

where $$r_{min} = \sqrt{\frac{I_{min}}{A}} \quad ...(5.5)\ (\because I = Ar^2)$$

- The slenderness ratio of a column should be kept as small as possible from economic point of view.
- For the same area of cross-section, the moment of inertia is the maximum when the material is farthest away from the centroid of the section. Thus the section gets thinner as the length of the column increases and ultimately local buckling puts a check on the column size.
- Lateral support in a column decreases the unsupported length and thus permitting the use of smaller sections for the same axial load. Sometimes two slenderness ratios in the two different directions are defined as:

$$\lambda_z = \frac{l_z}{r_z} \quad \text{where, } r_z = \sqrt{\frac{I_z}{A}}$$

$$\lambda_y = \frac{l_y}{r_y} \quad \text{where, } r_y = \sqrt{\frac{I_y}{A}}$$

- Those steel sections which give two different values of slenderness ratio in the two directions should be so selected and fixed in a structure that difference the two slenderness ratios λ_y and λ_z is the minimum.

Table 5.5 : Maximum slenderness ratio for compression members

S.No.	Type of Member	λ
1.	Tension member prone to reversal of stresses due to the loads other than wind or earthquake.	180
2.	Member carrying compressive loads due to dead and imposed (live) loads.	180
3.	Member carrying compressive force due to the combination of wind and earthquake only provided deformation of such members does not adversely affect the stress in any part of the structure.	250
4.	Compression flange of a beam restrained against lateral torsional buckling.	300
5.	A member normally acting as a tie in a roof truss or a bracing system not considered effective when subjected to possible reversal of stresses due to wind or earthquake forces.	350

Do You Know?

Why slenderness ratio of a compression member is limited?

1. Limiting the slenderness ratio takes care of accidental and construction (fabrication, transportation, erection etc.) loads.
2. The bracing members can be used as walkway for workmen or to provide temporary support for the equipment.
3. To take care of the stresses induced due to possible unexpected vibrations.

5.6 Types of Sections for Compression Members

- The section should be so proportioned to have maximum moment of inertia for a given cross sectional area.
- This is achieved by putting the maximum proportion of the area away from the center of gravity of the section.
- It is preferable to have equal moment of inertias about all the in plane axes.

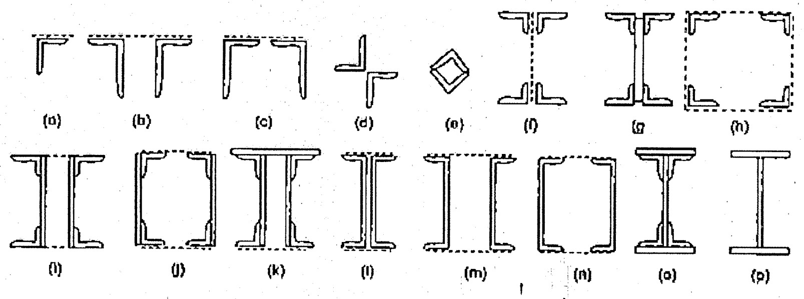


Fig. 5.1 Various shapes of compression members

5.7 Various Types of Buckling

- It is quite possible that an individual element of a column i.e. flanges and web may buckle locally giving rise to wrinkles. This type of buckling leading to column failure is called as local buckling. This local buckling can be avoided by limiting the width to thickness ratios as given in Table 1.1.
- But when a column buckles wholly (and not locally) then this buckling occurs in any one of the following three ways viz.:

  (a) Flexural buckling (b) Torsional buckling (c) Flexural torsional buckling

5.7.1 Flexural Buckling

- It is also called as **Euler** buckling when elastic behavior predominates.
- In this, deflection occurs due to bending about an axis corresponding to the largest slenderness ratio. This axis is the minor principal axis i.e. the one with the smallest radius of gyration.
- Compression members of any type of cross-sectional shape can fail in this type of buckling.

5.7.2 Torsional Buckling

- This type of buckling occurs due to bending wherein sections get displaced from their original position by translation without rotation.
- Thin walled members with open cross-sectional shapes are weak in torsion and hence undergo buckling by twisting rather than bending.
- Torsional buckling occurs when torsional rigidity of the member is very small as compared to flexural rigidity.
- Failure occurs due to rotation about longitudinal axis (xx-axis) of the member.
- This occurs only with doubly symmetric sections with very slender cross sectional elements.
- Standard hot rolled sections are not prone to torsional buckling but a member made of thin plate elements must be checked for possible torsional buckling.
- Torsional buckling is quite complex and thus it is always avoided. This is done by proper arrangement of the members and by providing bracing to prevent lateral movement and twisting.

5.7.3 Flexural Torsional Buckling

- This type of buckling failure is caused by combination of flexural buckling and torsional buckling.
- Herein the member bends and twists simultaneously.
- This type of failure occurs with unsymmetrical sections including both with one axis of symmetry like channels, T-sections, double angle sections etc. and with no axis of symmetry at all.
- Usually an analysis of torsional or flexural-torsional buckling only is made as and when it seems to be appropriate.
- This type of buckling can either be elastic or inelastic in nature.

5.8 Elastic and Inelastic Buckling of Columns

- Fig. 5.2 shows an idealized elasto-plastic stress strain curve and from the curve it can be established that maximum load a short column can carry is $P_y = f_y A$ and after that plastic yielding of the whole section occurs.
- Stress at failure of a column can in no case exceed yield stress f_y.
- But much before a column reaches its full strength, buckling due to instability occurs. This Euler critical load (P_{cr}) for column with both ends pinned (i.e. hinged) is given by,

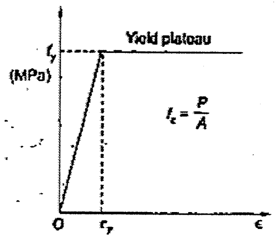


Fig. 5.2 Idealised elasto-plastic stress strain curve for mild steel

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad ...(5.6)$$

- From Eq. (5.6), this critical buckling load P_{cr} is independent of the strength of steel, increases with flexural rigidity and obviously decreases with length (L).
- The limitation of Euler's buckling load expression is that it does not give correct results for stocky or less slender compression members.
- **Buckling:** Euler's critical load P_{cr} causes elastic buckling of the column and represents the maximum load that the column can carry safely. With any additional increase of load beyond this, the column will not be able to support it and initial deformation grows significantly as column becomes unstable. This unstable state is called as **buckling, flexural buckling or lateral buckling.**
- Lateral buckling implies bending or warping of steel element out of its longitudinal axis or plane under the action of longitudinal force. This instability is shown in Fig. 5.3.

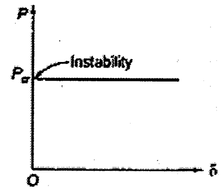


Fig. 5.3 Instability in an ideal column

- The mean compressive stress at buckling is given by,

$$\text{Buckling stress,} \quad f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 EAr^2}{AL^2} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} = \frac{\pi^2 E}{\lambda^2} \qquad ...(5.7)$$

where $\lambda = L/r$ = Slenderness ratio

- Thus Euler's critical stress f_{cr} is inversely proportional to square of slenderness ratio λ.

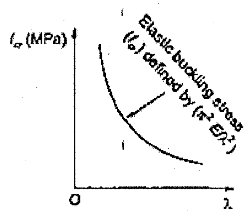


Fig. 5.4 Euler's buckling relationship between f_{cr} and λ

Remember: The Euler's elastic critical load P_{cr} gives an idea about the slenderness of a compression member while the maximum load P_y gives an idea about the resistance of the column to yielding.

- Thus for an axially loaded column, the average stress under the applied axial load must remain less than the critical stress f_{cr}.
- **Fig. 5.5 (a)** shows the strength curve of an axially loaded and initially straight hinged column. This plot is sometimes represented in non-dimensional form (plot of f_c/f_y versus $\sqrt{f_y/f_{cr}}$) as shown in **Fig. 5.5 (b)**.

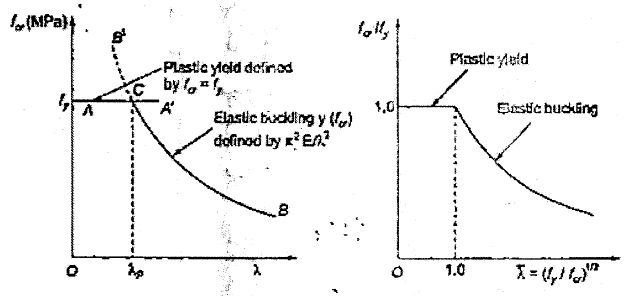


Fig. 5.5 Column strength curve

- **Validity of Euler's stress :** The upper limit of validity of Euler's stress is the proportional limit (f_y) i.e.

$$f_{cr} = f_y$$

$$\Rightarrow \quad f_y = \frac{\pi^2 E}{\lambda_p^2}$$

$$\Rightarrow \quad \lambda_p = \pi\sqrt{\frac{E}{f_y}} = 88.858 \simeq 88.86 \quad (\text{for } f_y = 250\text{N/mm}^2) \qquad ...(5.8)$$

(Upper limit of slenderness ratio for Euler's stress)

where λ_p = Slenderness ratio which explains the change point from yielding to buckling failure.

- The load carrying capacity of compression member with slenderness ratio exceeding 88.86 depends on various factors in addition to slenderness ratio and material characteristics. Some of these factors are:
  - (a) Shape of the column cross section
  - (b) Presence of residual stresses
  - (c) Presence of initial curvature in the member
  - (d) Eccentricity of the applied load

- In case the stress at which buckling occurs exceeds the proportional limit of the material, then the relationship between the stress and strain will not be linear and modulus of elasticity E can no longer be used.
- Thus better estimate of critical buckling load can be obtained by using variable tangent modulus E_t instead of E as shown in Fig. 5.6.

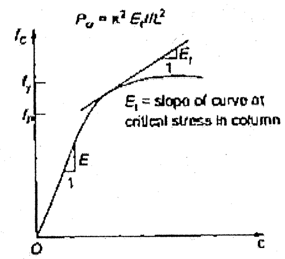


Fig. 5.6 Tangent modulus (variable)

Remember: For critical stress lesser than proportional limit of the material, the slope of stress strain curve is constant being equal to Young's modulus of elasticity E and the buckling is elastic. For critical stress value larger than proportional limit of the material, the slope of stress strain curve is E_t and the buckling is inelastic.

- **Inelastic buckling:** It is quite difficult to assess correctly the inelastic critical load. But in the elastic range, the modulus of elasticity E is constant and critical load can be assessed correctly.
- The maximum residual compressive stresses f_r are assumed to be $f_y/2$ to take into account unknown residual stresses.
- Very slender columns fail by buckling i.e. $f_{cr} \ll f_y$
- Inclusion of all possible variables is quite difficult and thus a transitional relationship is developed for a column as it transits from elastic behaviour to its strength limit state ($f_y A$)

Thus, $$f_p = f_y - \frac{f_y}{2} = \frac{f_y}{2} \qquad ...(5.9)$$

$$\therefore \quad f_p = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{A(kL)^2} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} \qquad ...(5.10)$$

- In the inelastic range, a simple parabolic expression is used to fit the average performance of steel columns as:

$$f_{cr} = f_y - \frac{f_p}{\pi^2 E}(f_y - f_p)\left(\frac{kL}{r}\right)^2 \qquad ...(5.11)$$

$$f_{cr} = f_y - \frac{f_y}{2\pi^2 E}\left(f_y - \frac{f_y}{2}\right)\left(\frac{kL}{r}\right)^2 \quad \left(\because f_p = \frac{f_y}{2}\right)$$

$$f_{cr} = f_y - \frac{f_y^2}{4\pi^2 E}\left(\frac{kL}{r}\right)^2$$

- The curve given by Eq. (5.11) becomes tangent to Euler's curve at the assumed proportional limit of the material.
- Fig. 5.7 shows a composite curve that describes completely the strength of any column of any given material. This curve is called as column strength curve which is also used in IS specifications for the design of columns.

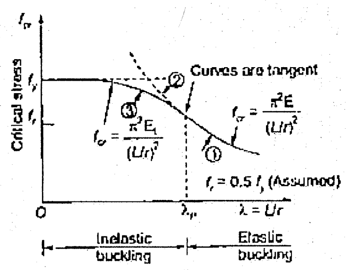


Fig. 5.7 Column strength curve

NOTE: Tangent modulus E_t is smaller than E and thus for the same slenderness ratio (kL/r), E_t results in smaller critical buckling load.

5.9 Column Formula

- Only the short columns can resist maximum compressive load $P_y = f_y A$ and the stress at failure of the column can never exceed f_y. The failure of column (neglecting the possibility of torsion) is due to bending.
- Earlier IS 800 recommended the use of secant formula taking into account the eccentricity of load, initial crookedness and different end conditions and using a factor of safety of 1.68. The allowable mean stress f_c was given by,

$$f_c = \frac{f_y/1.68}{1+0.20\sec\frac{l}{2r}\sqrt{\frac{1.68F_c}{E}}}\left(1.2-\frac{l}{800r}\right) \qquad \text{...(5.12)}$$

  Here r is the appropriate radius of gyration of the section.
- IS 800:1984 recommendeds the use of **Merchant Rankine** formula which is expressed as:

$$\frac{1}{f^n} = \frac{1}{f_e^n}+\frac{1}{f_y^n}$$

$$f = \frac{f_e f_y}{\left[f_e^n+f_y^n\right]^{1/n}} \qquad \text{...(5.13)}$$

  where f_e = Elastic critical stress which is same as f_{cc} as used by IS code.

  f_y = Failure stress

  The direct stress in compression of an axially loaded column is limited to $0.6f_y$. Thus the formula for permissible compressive stress derived from Merchant Rankine formula gets modified as:

$$f = 0.6\frac{f_{cc} f_y}{\left[f_{cc}^n+f_y^n\right]^{1/n}} \qquad \text{...(5.14)}$$

  where f_y = Yield stress in N/mm²

  f_{cc} = Elastic critical stress in compression = π^2E/λ^2

  λ = Slenderness ratio

  n = A factor which ranges from 1 to 3 and usually taken as 1.4
- From Merchant Rankine formula, it follows that for small values of slenderness ratio (λ), the failure stress tends to become yield stress (f_y) and for large values of slenderness ratio, it becomes the elastic critical stress (f_c).
- IS 800 : 2007 recommends different column curves viz.a, b, c and d in non-dimensional form based on Perry Robertson approach based on cross-section classification.
- Due to various imperfections that exist in a real column, this yield stress need to be reduced but it is quite difficult to quantify the amount (or fraction) of reduction in this yield stress.
- Thus based on statistical test results, curves a, b, c and d were proposed which takes into account all these imperfections.
- The buckling curves (a, b, c and d) give the value of reduction factor χ of the resistance of the column as a function of non-dimensional effective slenderness ratio called as reference slenderness for various types of cross sectional shapes.

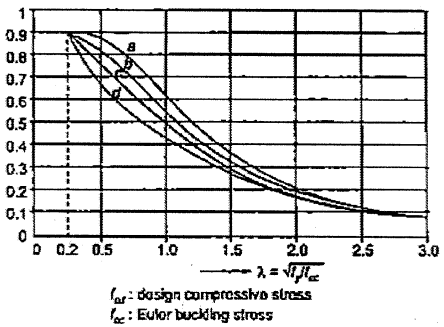


Fig. 5.8 Column buckling curves

- The imperfection factor α depends on:
  (i) Shape of the column cross section
  (ii) Direction in which buckling occurs
  (iii) Fabrication process (i.e. hot rolled, cold rolled, welded etc.).

  The value of α increases with imperfection as given in **Table 5.6** and also in **Table 7** of **IS 800:2007**.

Table 5.6 Imperfection factor (α)

Buckling curve	a	b	c	d
α	0.21	0.34	0.49	0.76

Do You Know ?

1. For stub column ($\lambda \le 0.2$), full plastic resistance develops on the cross section and buckling need not to be checked.
2. For $\lambda > 0.2$, reduction of load resistance must be taken into account because of buckling.

Curve a: It represents quasi perfect shapes like hot rolled I-section (with $h/b_f > 1.2$ and $t_f \le 40$ mm) if buckling is normal to the major axis. It also represents hot rolled hollow sections.

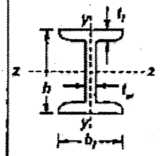
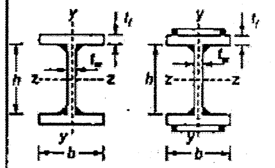
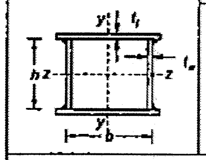
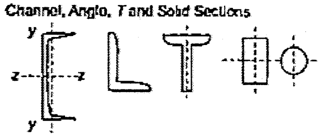
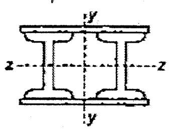
Curve b: It represents shapes with medium imperfections. It defines the behaviour of rolled I-section buckling about the minor axis. Welded I-sections of thin flanges ($t_f \le 40$ mm) and of rolled I-sections with medium flanges (40 mm < t_f < 100 mm) if buckling is about the major axis and most welded box sections.

Curve c: It represents cross sectional shapes with lot of imperfections like channels, angles, T-sections, thick welded box sections, hot rolled I-sections ($h/b_f \le 1.2$ and $t_f \le 100$ mm) buckling about the minor axis, I-sections ($t_f > 40$ mm) buckling about the major axis.

Curve d: It represents shapes with maximum imperfections. It is used for hot rolled I-sections with very thick flanges ($t_f > 100$ mm) and thick welded I-sections ($t_f > 40$ mm) if buckling is about the minor axis.

Table 5.7 aids in selection of appropriate buckling curve as a function of type of cross section and axis about which the buckling is taking place.

Table 5.7 Buckling curves for cross sections

Cross Section	Limits		Buckling about Axis	Buckling Curve
Rolled *I*-Sections 	$h/b_f > 1.2$:	$t_f \le 40$ mm	z-z	*a*
			y-y	*b*
		40 mm < $t_f \le$ 100 mm	z-z	*b*
			y-y	*c*
	$h/b_f \le 1.2$:	$t_f \le$ 100 mm	z-z	*b*
			y-y	*c*
		$t_f >$ 100 mm	z-z	*d*
			y-y	*d*
Welded *I*-Sections 		$t_f \le 40$ mm	z-z	*b*
			y-y	*c*
		$t_f > 40$ mm	z-z	*c*
			y-y	*d*
Hollow Sections	Hot rolled		Any	*a*
	Cold formed		Any	*b*
	Generally (Except as below)			
Welded Box Section 	Generally (Except as below)		Any	*b*
	Thick welds and			
		$b/t_f < 30$	z-z	*c*
		$b/t_w < 30$	y-y	*c*
Channel, Angle, *T* and Solid Sections 			Any	*c*
Built-up Member 			Any	*c*

- **Cl. 7.1.2.1** of **IS 800:2007** recommends the following formula for estimating the design compressive stress f_{cd} of an axially loaded compression member as:

$$f_{cd} = \frac{f_y/\gamma_{m0}}{\phi + \sqrt{\phi^2 - \lambda^2}} = \chi \frac{f_y}{\gamma_{m0}} \le \frac{f_y}{\gamma_{m0}} \qquad \text{...(5.15)}$$

where $\phi = 0.5[1 + \alpha(\lambda - 0.2) + \lambda^2]$, λ = Non dimensional effective slenderness ratio

$$\lambda = \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{f_y\left(\frac{KL}{r}\right)^2}{\pi^2 E}} \qquad \text{...(5.16)}$$

f_{cc} = Euler's buckling stress, α = Imperfection factor,

γ_{m0} = Partial factor of safety for material (steel) = 1.1

Stress reduction factor $$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \qquad \text{...(5.17)}$$

Stress reduction factor (χ) is given in Table 5.8.

Table 5.8 Stress reduction factor (χ) and design compressive stress (f_{cd}) for different buckling curves of steel of grade f_y = 250 N/mm².

S.No.	Curve a		Curve b		Curve c		Curve d	
	χ	f_{cd}	χ	f_{cd}	χ	f_{cd}	χ	f_{cd}
10.	1.000	227	1.000	227	1.000	227	1.000	227
20.	0.994	226	0.991	225	0.987	224	0.980	223
30.	0.969	220	0.950	216	0.930	211	0.896	204
40.	0.939	213	0.906	206	0.870	198	0.815	185
50.	0.904	205	0.855	194	0.807	183	0.736	137
60.	0.859	195	0.798	181	0.740	168	0.659	150
70.	0.803	182	0.732	166	0.670	152	0.587	133
80	0.734	167	0.661	150	0.600	136	0.521	118
90.	0.657	149	0.589	134	0.533	121	0.461	105
100.	0.579	132	0.520	118	0.471	107	0.408	92.6
110.	0.507	115	0.458	104	0.416	94.6	0.361	82.1
120.	0.443	101	0.403	91.7	0.368	83.7	0.321	73.0
130	0.388	88.3	0.356	81.0	0.327	74.3	0.287	65.2
140.	0.342	77.8	0.316	71.8	0.291	66.2	0.257	58.4
150.	0.303	68.9	0.281	64.0	0.261	59.2	0.231	52.6
160.	0.270	61.4	0.252	57.3	0.234	53.3	0.209	47.5
170.	0.242	55.0	0.227	46.5	0.212	48.1	0.190	43.1
180.	0.218	49.5	0.205	46.57	0.192	43.67	0.173	39.3
190.	0.197	44.7	0.186	42.2	0.175	39.7	0.158	35.9
200.	0.179	40.7	0.169	38.5	0.160	36.3	0.145	33.0
210.	0.163	37.1	0.155	35.2	0.146	33.3	0.134	30.0
220.	0.149	34.0	0.142	32.2	0.135	30.6	0.123	28.0
230.	0.137	31.2	0.131	29.8	0.124	28.3	0.114	26.0
240.	0.127	28.8	0.121	27.5	0.115	26.2	0.106	24.1
250.	0.117	26.6	0.112	27.5	0.107	24.3	0.099	22.5

- The design compressive strength (P_d) of the compression member is then given by,

$$P_d = A_e f_{cd}$$

where A_e = Effective sectional area = Gross sectional area if bolt holes are filled with bolts

5.10 Design Strength of Axially Loaded Compression Member

- Torsional buckling does not produce a limit state since either elastic or inelastic axial buckling limit state will occur at much lower loads.
- But sections like angle, T and channel which do not have any appreciable torsional buckling stability buckle torsionally before elastic or inelastic limit states are reached.
- For steel with yield stress other than 250 N/mm², Table 8 and Table 9 of IS 800:2007 can be referred for stress reduction factor χ and design compressive stess f_{cd}

5.11 Procedure for the Design of Axially Loaded Compression Member

Assumptions:

(a) The ideal column is assumed to be absolutely straight with no initial crookedness which is in fact a purely hypothetical situation and never occurs in practice.

(b) The modulus of elasticity is assumed to be constant in a built up column.

(c) Certain types of secondary stresses which may even be of the order of 25% to 40% of primary stresses are completely ignored i.e. not taken into account.

- There are two unknowns and it is required to assume one and calculate the other. Obviously the section has further to be checked for safety.

Design procedure:

Step-1. For an average column of height 3 m to 5 m, the slenderness ratio lies between 40 and 60. For slender column, a little higher value than 60 is assumed. For column with heavy factored load, a smaller value of slenderness ratio is assumed.

For the assumed value of slenderness ratio, the design compressive stress for that particular value is determined from Table 5.8 for the curve that is relevant to the buckling class of cross section as given in Table 5.7.

Alternatively, assume a design compressive stress in the compression member.

Assume slenderness ratio and design compressive stress as:

(i)	Rolled steel beam sections	70 - 90	135 N/mm²
(ii)	Angle struts	110 - 130	90 N/mm²

Step-2. The cross sectional area required to carry the factored axial compressive load is given by,

$$A_g\,(\text{in mm}^2) = \frac{P_u\,(\text{in N})}{\text{Assumed compressive stress}\,(\text{in N/mm}^2)}$$

Step-3. Select a suitable section from IS Handboook No.1 having area about 20% more than the area required in Step-2. The section is so chosen that the minimum radius of gyration selected is as large as possible. This appropriate least radius of gyration is recorded.

Step-4. Knowing the type of end condition from Table 5.2 or Table 5.3 and the type of connection, determine the effective length of the member.

Step-5. Determine slenderness ratio ($\lambda = l/r$) which must be less than the permissible slenderness ratio as given in Table 5.5 and hence the design compressive stress f_{cd} is computed from Eq. 5.17 or determined from Table 5.8 for both columns and struts.

Step-6. For single angle section loaded concentrically, the design compressive strength is determined from Eq. $P_d = A_c \cdot f_{cd}$ and the design compressive stress using Eq. 5.17 and the class c curve.

- The three limit states that may occur in single angle compression member loaded through one of its legs are flexural buckling, local buckling of leg of angle and flexural-torsional buckling.
- As per Cl. 7.5.[illegible] of IS 800:2007, the equivalent slenderness ratio in such a case is given by,

$$\lambda_e = \sqrt{k_1 + k_2\lambda_{vv}^2 + k_3\lambda_\phi^2} \qquad \text{...(5.18)}$$

where k_1, k_2, k_3 are the constants depending on end conditions as given in Table 5.9.

$$\lambda_{vv} = \frac{\left(\frac{l}{r_w}\right)}{\varepsilon\sqrt{\frac{\pi^2 E}{250}}} \qquad \text{...(5.19)}$$

$$\lambda_\phi = \frac{(b_1+b_2)}{2t\varepsilon\sqrt{\frac{\pi^2 E}{250}}} \qquad \text{...(5.20)}$$

Table 5.9 Constants k_1, k_2, k_3

No. of bolts at each end connection	Gusset/connecting member fixity	k_1	k_2	k_3
≥ 2	Fixed	0.20	0.35	20
	Hinged	0.70	0.60	5
1	Fixed	0.75	0.35	20
	Hinged	1.25	0.50	60

l = Center to center length of the supporting member

r_v = Radius of gyration about the minor axis

b_1, b_2 = Width of the two legs of the angle section

t = Thickness of the leg

ε = Yield stress ratio = $\sqrt{250/f_y}$

Step-7. The design compressive strength (P_d) of the member is computed by multiplying the design compressive stress (f_{cd}) with the effective cross sectional area (A_e). This design compressive strength must be more than the factored compressive load i.e.

$$P_d > P_u$$

Revise the section if P_d calculated differs considerably from the factored axial compressive load P_u.

5.12 Built-up Columns

- When already available rolled steel sections cannot furnish the required sectional properties like cross sectional area, radius of gyration etc. then built up section is the possible recourse.
- For economy in designing heavily loaded long columns, the least radius of gyration of the column section is increased to the maximum. This is achieved by placing the rolled sections away from the centroidal axis of the column and is connected by some system like lacings or battens.
- This system is referred to as lattice system. Such types of columns are called as latticed columns or open columns.

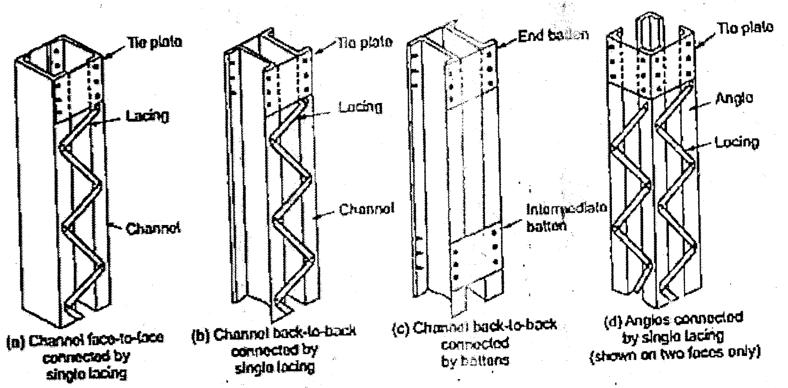


Fig. 5.9 Some possible lattice systems of built-up column

NOTE

The capacity of latticed column is always less than the capacity of solid wall column. For the same moment of inertia, they are more efficient since effective distribution of area enable the moment of inertia about z-axis to be equal to about y-axis.

- While calculating the strength of latticed column, neglect lacings and battens as primary load carrying elements.
- The buckling strength of a latticed column is less than that of solid column having the same cross sectional area and same slenderness ratio, since shearing component of the axial load produces deformation in the lattice which reduces the overall stiffness of the column and thus decreases the buckling strength of the column.

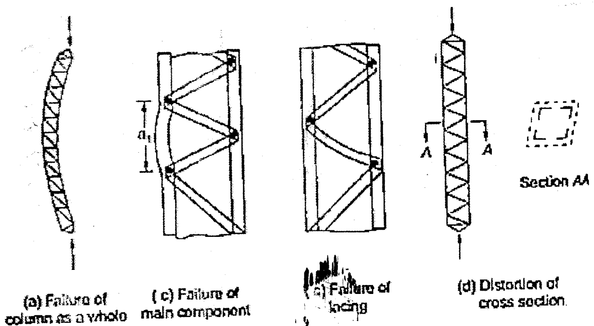


Fig. 5.10 Modes of failure in latticed columns

- Design of built up columns considers the following four conditions:
  - (a) Column buckling as a whole under axial load. [Fig. 5.10 (a)]
  - (b) Buckling of component column. [Fig. 5.10 (b)]
  - (c) Failure of lattice member. [Fig. 5.10 (c)]
  - (d) Distortion of the cross section. [Fig. 5.10 (d)]

5.13 Lacing

- Lacing hold the various members of a compression member straight, parallel and at correct distance apart in order to distribute the stress uniformly between the various parts.
- Lacing generally consists of flat or angle sections.

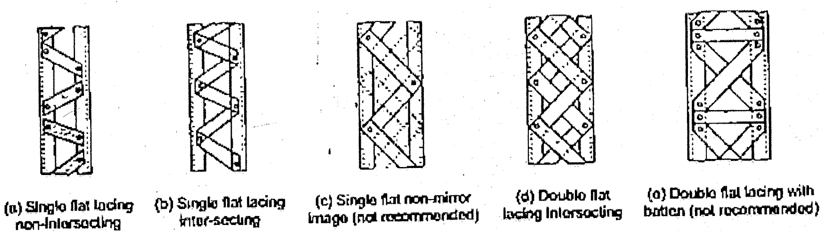


Fig. 5.11 Lacing system

- Lacings can also be used with battens as shown in Fig. 5.11(e) but this is not preferred since it gives undesirable effects.
- Lacing bars must not project beyond the column section. They are connected with single rivet or bolt at the end but two rivets or bolts are desirable assuming that one rivet or bolt may be defective in spite of the fact that stresses are usually small and only one rivet or bolt is enough.

NOTE: Lacings are designed as compression members and checked for tension (i.e., they are idealised as truss members).

- Single lacing system is quite sufficient for the transverse shear force for which the lacing is designed, if it is designed properly but often designers consider double lacing to be more safe and superior (in spite that it is uneconomical) and recommend it also.

Remember

1. For laced columns, the radius of gyration of the column cross section about an axis normal to the plane of lacing shall not be less than the radius of gyration about an axis parallel to the plane of the lacing.
2. Lacing system must not be accompanied by tie plates except at the column ends.
3. Lacing on two parallel planes should be the shadow of each other and not the crossed one. This provision prevents torsion about the column axis.
4. Uniform lacing system should be provided throughout the length of the column i.e. the inclination of lacings with the column should be equal on either side and a uniform section should be provided for the lacings.
5. The effective slenderness ratio of laced column is taken as 5% more than the actual maximum slenderness ratio. This provision takes care of shear deformations.
6. As stated, lacings are designed for a transverse shear equal to 2.5% of the axial load. But if bending moment is also there on the column then addition shear has to be considered.

5.13.1 Procedure for the Design of Laced Column

Step-1. A design compressive stress is assumed. For steel with f_y = 250 N/mm^2, a trial design stress can be assumed as 125 to 175 N/mm^2. For other types of steel, this design compressive stress range may have to be suitable modified.

Step-2. The cross sectional area required to carry the factored compressive load P_u at the assumed design compressive stress is calculated as,

$$A\,(\text{in mm}^2) = \frac{P_u\,(\text{in N})}{\text{Assumed design compressive stress (in N/mm}^2)}$$

Step-3. After having the cross sectional area required, a suitable section consisting of two channels or four angles or two I-sections with or without extra plates is selected from IS Handbook No. 1.The area given for the section is noted down which should preferably be about 10-20% more than the area calculated in Step-2 above.

Step-4. The sections are so spaced that the radius of gyration of the section about an axis normal to the plane of lacing is not less than the radius of gyration about an axis in the plane of lacing. This can be obtained by making the radius of gyration about *yy* axis equal to or greater than about *zz* axis i.e. $r_y \geq r_z$.

Step-5. An assessment of the effective length of the column is made ($l = KL$) and slenderness ratio (KL/r) is determined. The effective slenderness ratio of the laced column is taken as 1.05 times (KL/r) i.e. 5% more than the actual maximum slenderness ratio in order to take into account the shear deformations.

Step-6. For the estimated value of slenderness ratio, the design compressive stress f_{cd} is computed from Table 5.8.

Step-7. The design capacity of the member is calculated and it must be more than the factored load on the column i.e.

$$\text{Design capacity} = A_g f_{cd} > P_u$$

Step-8. The angle of inclination of the lacing bar with the longitudinal axis of the column should be kept between 40° to 70° as given in Cl. 7.6.4 of IS 800:2007. Generally as a trial value initially, this is assumed as 45°. The spacing of lacing bars a_1 is calculated.

NOTE: Lacings are most effective between 35° to 45° inclination and become less efficient beyond this range.

- The critical buckling load for the column in inelastic range is given by,

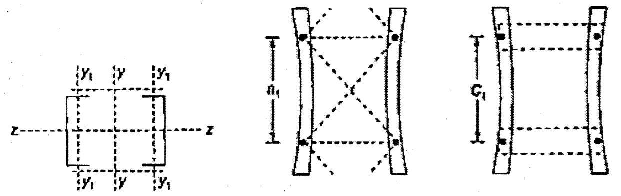
$$P_{cr} = \frac{P}{1 + \dfrac{P_t}{A_d E \sin\theta \cos^2\theta}} \qquad \text{...(5.21)}$$

where P_t = Tangent modulus load

A_d = Total cross sectional area of diagonals at any point along the span

The maximum value of $\sin\theta \cos^2\theta$ lies between 35° to 45° and beyond this limit, it becomes smaller and lacings become inefficient.

Step-9. The maximum spacing of lacing bars a_1 should be such that the minimum slenderness ratio of the component member (a_1/r_y) does not exceed 50 (in order to prevent local buckling) or 0.7 times the slenderness ratio of the member as a whole where a_1 is the length of the component member and r_y is the radius of gyration about *yy* axis of the component member. In laced and battened columns, there exists a possibility that the collapse of the entire column may be triggered by buckling of individual rib (or ribs) about y_1 axis (Fig. 5.12) at a lower load than that is required to cause buckling of the composite section. Thus there arises the need to limit the slenderness ratio of the component member by a factor (< 1) times the slenderness ratio of the member. IS Code recommends this factor as 0.7.



(a) Cross section of latticed column (b) Laced column (c) Battened column

Fig. 5.12 Buckling of open column ribs

Remember

1. The angle θ in single lacing system can be increased from 40° up to 70° only. Even if the component column is found to be unsafe then a double lacing system remains the only recourse.
2. Cl. 7.6.1.5 of IS 800:2007 states that the effective slenderness ratio of laced column should be 5% more than the actual maximum slenderness ratio so that shear deformation effects are accounted for.

Step-10. Cl. 7.6.6.1 of IS 800:2007 recommends that the lacings for compression members are designed for a transverse shear force V_t equal to 2.5% of the axial load on the column. This transverse shear V_t is distributed equally in all parallel planes N in which there is shear resisting elements like lacings or continuous plates. Thus V_t/N is the transverse force to which the lacing is subjected.

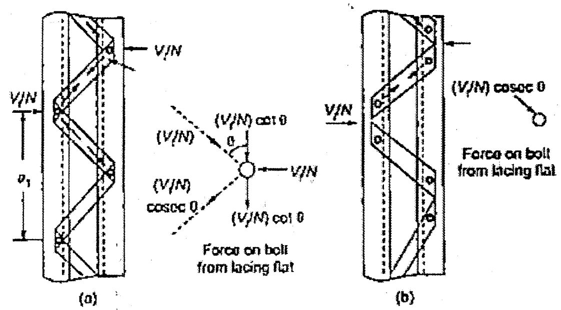


Fig. 5.13 Forces in a lacing bar

Remember

1. N = 2 for two channels laced on both faces and N = 4 for four angles laced on all the faces.
2. The shear V_t so determined is required to be enhanced for shear due to weight of the member or due to other forces and lacing is proportioned for the combined shear.

Step-11. The compressive force in the lacing bar is calculated which is equal to (V_t/N) cosecθ for single lacing system and $(V_t/2N)$ cosecθ for double lacing system.

Step-12. The lacing bars are so proportioned that they will not yield or buckle under the effect of transverse shear.

Width: At the very first instance, assume a bolt or rivet diameter. Cl. 7.6.2 of IS 800 : 2007 recommends that the minimum width of the flat must not be less than three times the nominal diameter of the end connector.

Thickness: In single lacing system, the thickness t of the lacing flat $\nless$ 1/40 of its effective length and in double lacing system, this thickness t of the lacing flat $\nless$ 1/60 of its effective length as per Cl. 7.6.3 of IS 800:2007.

Step-13. The minimum radius of gyration of the lacing flat is computed as:

$$r_{min} = t/\sqrt{12}$$

Step-14. As per Cl. 7.6.6.3 of IS 800:2007 the slenderness ratio of the lacing bar should be less than 145. For riveted or bolted connection, the effective length of lacing bar is taken as the length between the inner end rivets or bolts for single lacing system and 0.7 times of this distance for double lacing system. In welded connections, the effective length of lacing bar is taken as 0.7 times the distance between the inner ends of the welds connecting the lacing bars to the member.

Step-15. For the calculated value of slenderness ratio, the design compressive strength of the lacing bar is computed which must be more than the force in the lacing bar.

Step-16. The tensile strength of the lacing bar is computed and it must be more than the force in the lacing bar.

Remember: The tensile force in lacing bar is quite small and thus the possibility of block shear failure is very low and accordingly the lacing is checked for net section fracture only.

Step-17. For riveted or bolted connection, the strength of rivet or bolt is computed and it must be more than the load coming on rivet or bolt. When two lacing bars are connected at the same rivet or bolt, the load on the rivet or bolt will be $2(V_t/N)\cot\theta$ and for lacing bars connected at different points, the load on rivet or bolt will be (V_t/N) cosec θ.

For welded connections, the welding of lacing bar to the main member should be sufficient enough to transmit the load in the lacing bars. Welding should be done along each side of the lacing bar for full lap length. The overlap of lacing bar should not be less than four times the thickness of the bar or the member, whichever is less.

Step-18. In order to prevent the distortion of column section at the column ends, tie plates (i.e. battens) are provided at the ends of the lacing system and these must be able to resist the forces to which the lacing bar is subjected.

5.14 Batten

- Battens are plates or any other rolled sections that are used to connect main components of the compression member.
- Battens are placed opposite to each other on the two parallel faces of the compression member and must be spaced and proportioned uniformly as shown in Fig. 5.14.
- Number of battens should be such that the compression member is divided into at least three bays within its actual length.
- Batten may be either **end batten** or **intermediate batten**. End battens are provided at the end of battens and intermediate battens are provided in between the end battens.
- As far as strength of battened columns is considered then, it is same as that of laced columns but are not economically beneficial and are thus used sparsely.

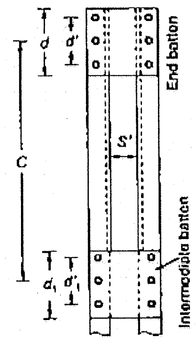


Fig. 5.14 Battened Columns

NOTE: Battens are not suitable for columns subjected to eccentric loading in the plane of the connecting system.

5.14.1 Procedure for the Design of Battened Column

Design of battened column is almost similar to that of laced column with the only difference that in case of battened column, the **effective slenderness ratio is enhanced by 10%** in order to account for shear deformations occurring in battened column. Thus effective slenderness ratio (λ_e) of battened column is,

$$\lambda_e = 1.1\left(\frac{KL}{r}\right)$$

Once the column section has been arrived at, then battens are designed as per the following procedure:

Step-1. As per Cl. 7.7.3 of IS 800:2007, the maximum spacing of battens (C) should be such that the minimum slenderness ratio of the component member (C/r_y) does not exceed 50 or 0.7 times the slenderness ratio of the column as a whole i.e. $0.7(KL/r)$.

NOTE As per IS 800:2007, the radius of gyration of battened column about an axis normal to the plane of the batten shall not be less than the radius of gyration of that battened column about an axis parallel to the plane of the batten.

Step-2. The batten section is selected as per the recommendations given below and later on the batten section is checked for the forces acting on it.

Depth: The effective depth of end batten must not be less than the distance between the CGs of the component members and must be greater than twice the width of one component member.

The effective depth of intermediate batten is taken as 75% (i.e. 3/4[th]) of the depth of end batten and must be more than twice the width of component member as given in Cl. 7.7.2.3 of IS 800:2007.

As shown in Fig. 5.14, the two channels are placed back-to-back spaced at a distance S' apart and are connected by battens.

Thus effective depth of end batten $(d') = S' + 2C_{yy}$ or $2b$ (whichever is maximum)

Overall depth of end batten $(d) = d' + 2$(Edge distance)

Effective depth of intermediate batten $(d_1') = \left(\frac{3}{4}\right)d'$ or $2b$ (whichever is maximum)

Overall depth of intermediate batten $(d_1) = d_1' + 2$(Edge distance)

where distance C_{yy} is as given in IS Handbook No. 1 for the section.

Thickness: The thickness t of the batten must not be less than 1/50th of the distance between the innermost connecting lines of the rivets or bolts or welds normal to the main member i.e.

$$t \le \frac{1}{50}(S' + 2g)$$

where g = Gauge distance for the section as given in IS Handbook No. 1

Remember: The above requirements of depth and thickness of batten do not apply in case rolled sections like angle section, I-section, and channel sections etc. which are used as battens.

Step-3. Battens are designed to carry the flexural moment and shear force that arise due to transverse shear force V_t which is taken as 2.5% of the total axial load on the compression member as a whole. The transverse shear force is divided equally in all the parallel planes N in which there is shear resisting elements like battens or continuous plates. As far as buckling of battened columns is considered in the plane of battens then unlike in laced columns, there are no diagonal elements to carry the transverse shear. The transverse shear thus must be resisted by the main ribs.

Step-4. The transverse shear in ribs in battened columns gives rise to longitudinal shear that must be resisted by battens. This shear force is obtained by considering points of contra flexure in each longitudinal rib midway between the battens and at its mid-point in each batten. Battens must be able to resist longitudinal shear and moment due to transverse shear V_t as shown in Fig. 5.15.

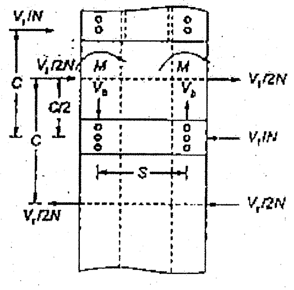


Fig. 5.15 Forces acting on batten

$$V_b = \frac{V_t C}{NS} \quad ...(5.22)$$

$$M = \frac{V_t C}{2N} \quad ...(5.23)$$

where V_b = Longitudinal shear force

V_t = Transverse shear force

C = Center to center distance between the battens

S = Minimum transverse distance between the CGs of rivet or bolt or weld

Step-5. The shear stress (V_t/A_t) is computed in the batten section and it must not exceed $\left(f_y/\sqrt{3}\,\gamma_{m0}\right)$

where A_t = Cross sectional area of the batten = dt

γ_{m0} = Partial factor of safety = 1.1

d = Overall depth of the batten

t = Thickness of the batten

Step-6. The flexural moment in the batten section is calculated and must not exceed f_y/γ_{m0}

$$\sigma_{bc,cal} = \frac{M}{Z} = \frac{M}{\frac{1}{6}td^2} = \frac{6M}{td^2} < \frac{f_y}{\gamma_{m0}}$$

The connection (riveted or bolted or welded) of battens to main members is designed to resist the longitudinal shear and flexural moment.

(i) For riveted or bolted connection, not less than two rivets or bolts sould be used.

(ii) For welded connection, the aggregate length of the weld on each edge parallel to the depth must not be less than half the depth of batten plate. At least one third of the weld must be placed at each end of this edge.

5.15 Encased Column

- In tall buildings with steel as skeletal structure, columns are encased in concrete in order to have flush surfaces from architectural point of view as shown in Fig. 5.16.
- Encasing a column with concrete increases the fire resistance as well of the column and it prevents corrosion.
- IS 800:2007 does not cover the design aspect of encased column but IS 800:1984 leys down the design procedure of such columns by working stress method of design.
  - (a) The member should be symmetrical I-section or two channel sections placed back to back with or without cover plates.
  - (b) The overall size of the steel section should not exceed 750 mm x 450 mm with larger dimension being measured parallel to the web.
  - (c) The column must be unpainted and solidly encased in dense concrete with 20 mm nominal size aggregates with a minimum concrete grade of M15.
  - (d) The minimum width of the solid casing should be $(b_o + 100$ mm$)$ where b_o is the width of steel flange of the column.
  - (e) The surface and edges of the steel column should have a concrete cover of at least 50 mm.
  - (f) The concrete should be reinforced with steel wires in the form of stirrups of diameter at least 5 mm @ 150 mm c/c spacing. These stirrups are supported by 10 mm diameter bars at the four corners.
  - (g) Steel core encased columns must be machined precisely at splices.

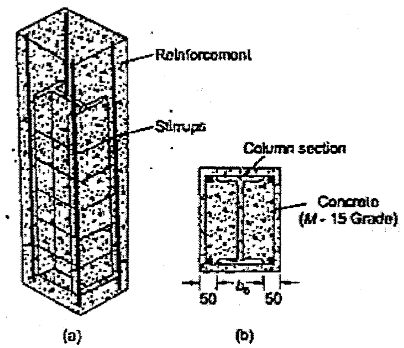


Fig. 5.16 Encased column

5.16 Column Splices

- A splice is a joint provided in the length of the member.
- In case of column splice, if the load is truly concentric then theoretically no splice is required since compression will be directly transferred through bearing. But truly axial load on column never occurs.

Also columns are most of the times also subjected to bending. This raises the necessity of column splices. Column sections are required to be spliced for the following cases:

(a) When the overall length of the column is more than the length of the column section available.

(b) In case of high rise structures, the columnsection required in upper stories is small than that required for lower stories and thus column needs to be spliced.

- For splicing compression members, care must be taken to see whether the ends of the connecting compression members are faced, milled or machined for complete bearing or not.
- If bearing in the two sections is achieved completely then a major portion of the load passes through the sections only and splice needs to be designed only for the remaining force.
- Often it is not possible to machine the surfaces perfectly and thus load to be transmitted through the splice plate cannot be assessed perfectly. Thus, it is desirable to design the splice for strength of the member.
- Tension splices are designed to transmit the design action or the 30% of member design capacity, whichever is more.

5.16.1 Specification for the Design of Splices

(a) Where the ends of the compression members are faced for complete bearing over the whole area there the splices are designed to hold the members accurately in position and to resist any tension where bending is also there.

(b) In case the connecting members are not faced for complete bearing then splices are designed to transmit all the forces to which they are subjected to.

(c) Splices are designed as short columns.

- Ideally a splice plate should be located at a place where flexural moment in the column is zero i.e. at the locations of point of contra flexure.
- Due to direct load, there are two points of contra flexure varying from middle of the column to the points above or below the middle.

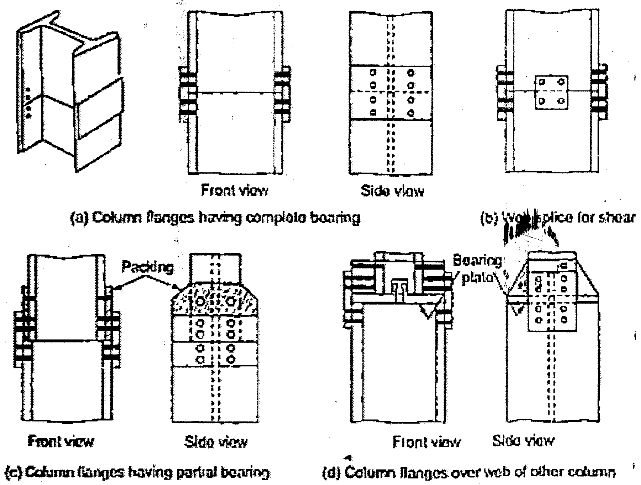


Fig. 5.17 Various types of column splices

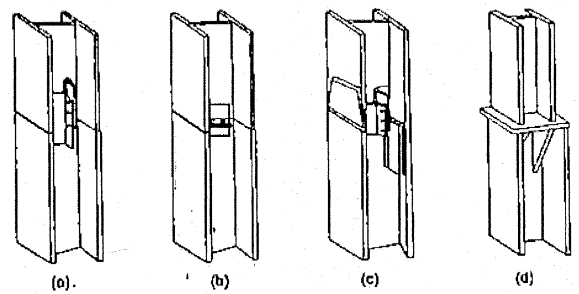


Fig. 5.18 Welded column splices

5.16.2 Procedure for the Design of Column Splices

Step-1. For the case of axial compressive load, the splice plates are provided on the flanges of the two column sections which are to be jointed.

(a) For machined end columns, the splice is designed only to keep the columns in position and to carry the tension due to bending moment to which the splice plate may be subjected. The splice plate and its connection must be designed to carry 50% of axial load and tension (due to bending moment if any).

(b) If the column ends are not machined then splice and its connections are designed to resist the total axial load and tension (due to bending moment if any).

Let P_u = Factored load on the column

P_{u1} = Design load for splice due to axial column load

P_{u2} = Design load for splice due to bending moment

For machined end columns,

$$P_{u1} = \frac{P_u}{4}$$

For non-machined end columns,

$$P_{u1} = \frac{P_u}{2}$$

$$P_{u2} = \frac{M_u}{\text{Lever arm}}$$

Here 'lever arm' is the center to center distance between the two splice plates.

Step-2. Splice plates are regarded as short columns with zero slenderness ratio. Thus these plates will be prone to yield stress.

Step-3. The cross sectional area of splice plate is given by,

Cross sectional area of splice plate required = Factored load on splice plate / Yield stress

Step-4. The width of splice plate is generally kept equal to with of the column flange. Thus thickness of splice plate is given by,

Thickness of splice plate = Cross sectional area of the splice plate required (from STEP 3) / Width of the splice plate

Step-5. Assume some nominal diameter of the bolts/rivets and determine the strength of rivet/bolt.

No. of bolts/rivets required = Factored load on splice / Strength of rivet/bolt

Step-6. When bearing plate is also required than length and width of the bearing plate is kept equal to the column depth and flange width of the upper column and thickness is computed by equating the ultimate bending moment due to the factored load to the moment of resistance of the plate section.

Illustrative Examples

Example 5.1 Determine the design axial load carrying capacity of a column made of ISHB 300 @ 577 N/m if length of the column is 3 m which is pinned at both the ends. Use Fe410 steel.

Solution:

For Fe410. $f_y = 250\ \text{N/mm}^2$, $f_u = 410\ \text{N/mm}^2$

$E = 2 \times 10^5\ \text{N/mm}^2$

∵ Column is pinned at both the ends

∴ $kL = L = 3\text{ m}$

For ISHB 300 @ 577 N/m

$h = 300\text{ mm}$ $b_f = 250\text{ mm}$

$t_f = 10.6\text{ mm}$ $A_g = A = 7484\ \text{mm}^2$

∴ $$\frac{h}{b_f} = \frac{300}{250} = 1.2 \text{ and } t_f < 100\text{ mm}$$

From Table 10 of IS 800 : 2007 or Table 5.7 of this chapter the section fails under buckling class '*b*' about *z-z* axis and is under class '*c*' for buckling about *y-y* axis.

$r_{min} = r_{yy} = 54.1\text{ mm}$

∴ $$f_{cc} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2 \times 2 \times 10^5}{\left(\frac{3000}{54.1}\right)^2} = 641.92\ \text{N/mm}^2$$

Non-dimensional effective slenderness ratio (λ)

$$= \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{250}{641.92}} = 0.6241$$

For buckling class '*b*', imperfection factor (α) = 0.34 (From Table 5.6)

∴ $$\phi = 0.5[1 + \alpha(\lambda - 0.2)] + \alpha^2]$$

$$= 0.5[1 + 0.34(0.6241 - 0.2) + (0.6241)^2] = 0.7668$$

∴ Design compressive stress (f_{cd})

$$= \frac{f_y/\gamma_{m0}}{\phi + \sqrt{\phi^2 - \lambda^2}} \le \frac{f_y}{\gamma_{m0}} = \frac{250/1.1}{0.7668 + \sqrt{0.7668^2 - 0.6241^2}}$$

$$= 187.47\ \text{N/mm}^2 < \frac{f_y}{\gamma_{m0}} = \frac{250}{1.1} = 227.27\ \text{N/mm}^2 \quad \text{(OK)}$$

∴ Design compressive strength of column (P_d) is

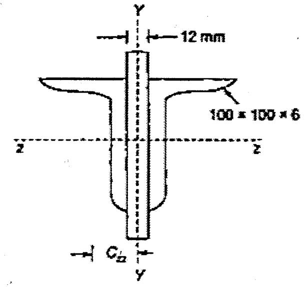
$$P_d = f_{cd} A_g$$

$$= 187.47 \times 7484\text{ N} = 1403\text{ kN}$$

∴ Working compressive load $= \frac{1403}{1.5} = 935.33\text{ kN} = 935\text{ kN (say)}$

Example 5.2 A strut in a roof truss is 3 m long and consists of ISA × 100 × 100 × 6. Determine the factored strength of the strut if it is connected on both sides by 12 mm gusset plate by

(a) one bolt (b) two bolts (c) welding



Solution:

For ISA 100 × 100 × 6

$A = 1167\ \text{mm}^2$

$C_{zz} = C_{yy} = 26.7\text{ mm}$

$r_{zz} = r_{yy} = 30.9\text{ mm}$

Now r_{zz} of single angle ISA 100 × 100 × 6 is same for two ISA 100 × 100 × 6 since *z-z* axis is same for both single and two ISAs.

∴ $r_{zz} = 30.9\text{ mm}$

$I_{yy} = 2[I_{yy}$ of one ISA + Area of one ISA$(C_{zz} + 6)^2]$

But I_{yy} of one ISA $= 111.3 \times 10^4\ \text{mm}^4$

∴ $I_{yy} = 2[111.3 \times 10^4 + 1167(26.7 + 6)^2]$

$= 472.17 \times 10^4\ \text{mm}^4$

∴ $$r_{yy} = \sqrt{\frac{I_{yy}}{A_g}} = \sqrt{\frac{472.17 \times 10^4}{2 \times 1167}} = 44.98\text{ mm} < r_{zz} = 30.9\text{ mm}$$

∴ r_{zz} governs the strength of member

(a) When one bolt is used

$r = r_{zz} = 30.9\text{ mm}$

$kL = L = 3000\text{ mm}$

∴ $$\frac{kL}{r} = \frac{3000}{30.9} = 97.09$$

The member belongs to buckling class '*c*'.

∴ For $\frac{kL}{r} = 97.09$ and $f_y = 250\ \text{N/mm}^2$

$f_{cd} = 111.2\ \text{N/mm}^2$ **(From Table 5.8)**

∴ Design strength of column (P_d)

$= A_c \cdot f_{cd} = 2 \times 1167 \times 111.2\text{ N} = 259.54\text{ kN}$

(b) When two bolts are used

with two bolts, effective length gets reduced which can be taken as 0.85 times actual length.

∴ $kL = 0.85 \times 3000 = 2550\text{ mm}$

∴ $$\frac{kL}{r} = \frac{2550}{30.9} = 82.52$$

$\therefore$ f_{cd} = 132.25 N/mm² (From Table 5.8)

$\therefore$ P_d = 2 × 1167 × 132.25 N = 308.67 kN

(c) When welding is used

Welding makes the joint rigid

$\therefore$ $kL = 0.7 \times 3000 = 2100$ mm

$\therefore$ $$\frac{kL}{r} = \frac{2100}{30.9} = 67.96$$

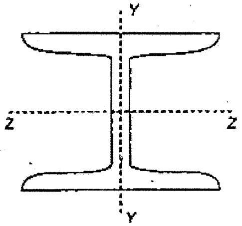
$\therefore$ f_{cd} = 155.3 N/mm² (From Table 5.8)

$\therefore$ P_d = 2 × 1167 × 155.3 N = 362.47 kN

Example 5.3 Compute the design compressive load for a column 3.2 m long and its section is ISHB 350 @ 710.2 N/m. The column is fixed at both the ends. Use steel of grade Fe410.

Solution:

For steel of grade Fe410, f_y = 250 N/mm², f_u = 410 N/mm²



Partial factor of safety for the material, $\gamma_{m0} = 1.1$

Effective length factor for column fixed at both the ends, $K = 0.65$

For ISHB 350 @ 710.2 N/m

Depth of section, h = 350 mm

Width of flange, b_f = 250 mm

Thickness of flange, t_f = 11.6 mm

Thickness of web, t_w = 10.1 mm

Gross area, A_g = 9221 mm²

Radius of gyration about z - z axis, r_z = 146.5 mm

Radius of gyration about y - y axis, r_y = 52.2 mm

$$\frac{h}{b_f} = \frac{350}{250} = 1.4 > 1.2$$

Now, t_f = 11.6 mm < 40 mm

Thus buckling curve to be used about z-z axis is 'a' and about y - y axis is 'b'. Now

$\therefore$ $r_y < r_z$

Thus column is prone to buckle about y - y axis and governing design compressive stress will depend on λ_y. In order to check this, the design compressive strength will be computed about both y - y and z - z axis.

Another point to note is that, the design compressive stress (f_{cd}) can be computed either by using the Eq. (5.17) or from Table 5.8.

Approach-I

Design compressive stress using Eq. (5.17)

(i) About y - y axis

$\because$ About y - y axis, curve 'b' is to be used

Thus imperfection factor for curve 'b' from Table 5.6, $\alpha = 0.34$

Non-dimensional slenderness ratio,

$$\lambda_y = \sqrt{\frac{f_y\left(\frac{kL}{r_y}\right)^2}{\pi^2 E}} = \sqrt{\frac{250\left(\frac{0.65 \times 3200}{52.2}\right)^2}{\pi^2 \times 2 \times 10^5}} = 0.448$$

$$\phi_y = 0.5[1 + \alpha(\lambda_y - 0.2) + \lambda_y^2]$$

$$= 0.5[1 + 0.34(0.448 - 0.2) + 0.448^2] = 0.643$$

$\therefore$ Design compressive stress (f_{cd})

$$= \frac{f_y/\gamma_{m0}}{\phi_y + \left[\phi_y^2 - \lambda_y^2\right]^{1/2}} = \frac{250/1.1}{0.643 + \left[0.643^2 - 0.448^2\right]^{1/2}}$$

$$= 227.27 \text{ N/mm}^2$$

$\therefore$ Design compressive strength (p_d) = $f_{cd} A_g$

$$= 205.82 \times 9221 \text{ N} = 1897.9 \text{ kN}$$

(ii) About z-z axis

$\because$ About z-z axis, curve 'a' is to be used

Thus imperfection factor for curve 'a' from Table 5.6, $\alpha = 0.21$

Non-dimensional slenderness ratio,

$$\lambda_y = \sqrt{\frac{f_y\left(\frac{kL}{r_z}\right)^2}{\pi^2 E}} = \sqrt{\frac{250\left(\frac{0.65 \times 3200}{146.5}\right)^2}{\pi^2 \times 2 \times 10^5}} = 0.16 < 0.2$$

Thus $\lambda_z = 0.2$

$$\phi_z = 0.5[1 + \alpha(\lambda_z - 0.2) + \lambda_z^2]$$

$$= 0.5[1 + 0.21(0.2 - 0.2) + 0.2^2] = 0.52$$

$\therefore$ Design compressive stress (f_{cd})

$$= \frac{f_y/\gamma_{m0}}{\phi_z + \left[\phi_z^2 - \lambda_z^2\right]^{1/2}} = \frac{250/1.1}{0.513 + \left[0.52^2 - 0.2^2\right]^{1/2}} = 227.27 \text{ N/mm}^2$$

$\therefore$ Design compressive strength (P_d) = $f_{cd} \times A_g$

$$= 227.27 \times 9221 \text{ N}$$

$$= 2095.7 \text{ kN} > 1897.9 \text{ kN}$$

Hence design compressive strength (P_d)

= 1897.9 kN (about y-y axis as stated above)

Approach-II

Design compressive stress using Table 5.8.

Strength reduction factor $\chi = \dfrac{1}{\phi + \left(\phi^2 - \lambda^2\right)^{1/2}}$ which can be seen from Table 5.8 corresponding to effective slenderness ratio (kL/r)

(i) About *y-y* axis

Effective slenderness ratio about $y-y$ axis $= \frac{kL}{\lambda_y} = \frac{0.65 \times 3200}{52.2} = 39.847$

$\because$ About $y-y$ axis, curve '*b*' is to be used

$\therefore$ From table - $\chi = 0.907$

$\therefore$ Design compressive stress $(f_{cd}) = \left(\frac{f_y}{\gamma_{m0}}\right)\chi = \frac{250}{1.1} \times 0.907 = 206.14$ N/mm^2

$\therefore$ Design compressive strength $(P_d) = f_{cd} \cdot A_g = 206.14 \times 9221$ N $= 1900.8$ kN

(which is near to value calculated using Eq.(5.17) which 1897.9 kN)

(ii) About *z-z* axis

Effective slenderness ratio about $z-z$ axis $= \frac{kL}{\lambda_z} = \frac{0.65 \times 3200}{146.5} = 14.198$

$\because$ About $z-z$ axis, curve '*a*' is to be used

$\therefore$ From Table 5.8,

$$\chi = 0.997$$

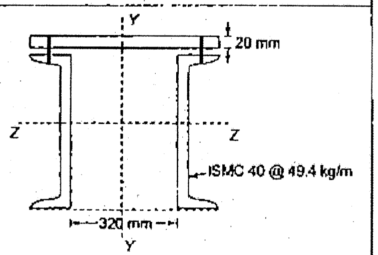
$\therefore$ Design compressive stress $(f_{cd}) = \left(\frac{f_y}{\gamma_{m0}}\right)\chi = \left(\frac{250}{1.1}\right)0.997 = 226.59$ N/mm^2

$\therefore$ Design compressive strength $(P_d) = f_{cd} \cdot A_g = 226.59 \times 9221$ N $= 2089.4$ kN

(Which is close to 2059.7 kN calculated using Eq.(5.17))

$\therefore$ Design compressive strength $(P_d) = 1900.8$ kN

Example 5.4 A column section is as shown in figure below. The length of the column is 4.8 m. Using steel of grade Fe410, compute the design compressive load.

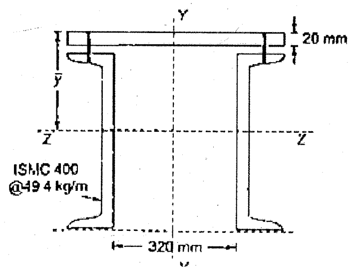


Solution:

For steel of grade Fe410, $f_y = 250$ N/mm^2

For ISMC 400 @ 49.4 kg/m

$A = 6293$ mm^2	$b_f = 100$ mm
$t_f = 15.3$ mm	$t_w = 8.6$ mm
$r_z = 154.8$ mm	$g = 60$ mm
$I_z = 15082.8 \times 10^4$ mm^4	$I_y = 504.8 \times 10^4$ mm^4
$C_{yy} = 24.2$ mm	



Gross area of column section

A_g = Area of two channels + Area of plate

$= 2 \times 6293 + (100 + 100 + 320)20$

$= 22986$ mm^2

Distance of neutral axis from top

$$\bar{y} = \frac{(320+100+100)20 \times 10 + 2 \times 6293\left(\frac{400}{2}+20\right)}{(320+100+100)20 + 2 \times 6293}$$

$= 124.99$ mm ≈ 125 mm

Moment of inertia of the section about $z-z$ axis I_{zz}

$$= 2\left[15082.8 \times 10^4 + 6293(220-125)^2\right] + \left[520 \times \frac{20^3}{12} + 520 \times 20(125-10)^2\right]$$

$= 41524.47 \times 10^4 + 13788.67 \times 10^4$

$= 55313.14 \times 10^4$ mm^4

Moment of inertia of the section about $y-y$ axis I_{yy}

$$= 2\left[504.8 \times 10^4 + 6293 \times (160+24.2)^2\right] + 20 \times \frac{520^3}{12}$$

$= 43713.44 \times 10^4 + 23434.67 \times 10^4$

$= 67148.11 \times 10^4$ mm^4

$\because \quad I_{zz} < I_{yy}$

$\therefore$ Minimum radius of gyration will be r_z

$\therefore$ Minimum radius of gyration $r_z = \sqrt{\frac{I_{zz}}{A}} = \sqrt{\frac{55313.14 \times 10^4}{22986}} = 155.13$ mm

Effective slenderness ratio $= \frac{l}{r_z} = \frac{4800}{155.13} = 30.94$

Overall flange thickness of section $t_f = 15.3 + 20 = 35.3$ mm < 40 mm

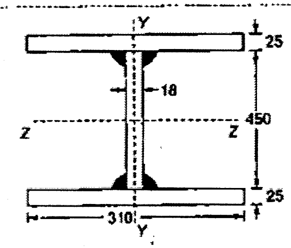
Thus design buckling curve is '*c*' both about *y-y* and *z-z* axis.

From Table 5.8,

$$f_{cd} = 209.78 \text{ N/mm}^2$$

$\therefore$ Design compressive load $(P_d) = f_{cd} \times A_g = 209.78 \times 22986$ N $= 4822$ kN

Example 5.5 Calculate the axial load carrying capacity of the welded column section as shown. Column is fixed at both the ends and it is of length 5 m. Use Fe410 grade of steel.



Solution:

For steel of grade Fe410, $f_u = 410$ N/mm^2, $f_y = 250$ N/mm^2

Gross area of the section, $A_g = 310 \times 25 \times 2 + 450 \times 18$

$= 23600$ mm^2

Moment of inertia of the section about $z-z$ axis

$$I_{zz} = 18 \times \frac{450^3}{12} + 2 \times \left[\frac{310 \times 25^3}{12} + 310 \times 25 \times \left(225 + \frac{25}{2}\right)^2\right]$$

$= 13668.75 \times 10^4 + 87510.42 \times 10^4$

$= 101179.17 \times 10^4$ mm^4

Moment of inertia of the section about *y-y* axis

$$I_{yy} = 2\times 25\times\frac{310^3}{12} + 450\times\frac{18^3}{12}$$

$$= 12412.92\times 10^4 + 21.87\times 10^4$$

$$= 12434.79\times 10^4\ \text{mm}^4$$

$\because$ $I_{yy} < I_{zz}$

$\therefore$ Minimum radius of gyration $r_y = \sqrt{\frac{I_{yy}}{A_b}} = \sqrt{\frac{12434.79\times 10^4}{23600}} = 72.59$ mm

$\therefore$ Effective slenderness ratio $= \frac{kL}{r_y} = \frac{0.65\times 5000}{72.59} = 44.772$

Now $t_f = 25$ mm < 40 mm

$\therefore$ Buckling curve '*c*' will be followed

For $\frac{kL}{r_y} = 44.772$ and $f_y = 250$ N/mm², design compressive stress as per Table 5.8.

$$f_{cd} = 190.84\ \text{N/mm}^2$$

$\therefore$ Design compressive strength (P_d)

$$f_{cd}\cdot A_e = 190.84\times 23600\ \text{N} = 4503.82\ \text{kN}$$

$\therefore$ Axial load carrying capacity $= \frac{4503.82}{1.5} = 3002.5\ \text{kN} \approx 3002\ \text{kN}$

Example 5.6 **Design a built up column with four angles, the column length being 10 m which supports an axial compressive load of 450 kN under working conditions. The column ends are held in position and restrained against rotation. Also design a suitable connecting system for the angles. Use steel of grade Fe410.**

Solution:

For Fe410, $f_u = 410$ N/mm², $f_y = 250$ N/mm²

Let bolts of grade 4.6 are used and thus $f_{ub} = 400$ N/mm²

Factor of safety for angle material $\gamma_{m0} = 1.10$

Factor of safety for bolt material $\gamma_{mb} = 1.25$

For preliminary design, let design compressive stress,

$$f_{cd} = 165\ \text{N/mm}^2$$

$\therefore$ Cross-sectional area required,

$$A = \frac{P}{f_{cd}} \quad \text{where } P = 1.5\times 450 = 675\ \text{kN}$$

$$= \frac{675\times 1000}{165} = 4090.91\ \text{N/mm}^2$$

$\because$ Four angles are to be used

$\therefore$ Area of one angle required $= \frac{4090.91}{4} = 1022.73$ mm²

Equal angle sections will be more desirable

Let ISA 90 × 90 × 6 is used as a trial section with section properties as follows

$$A = 1047\ \text{mm}^2$$
$$C_{xx} = C_{yy} = 24.2\ \text{mm}$$
$$\lambda_z = \lambda_y = 27.7\ \text{mm}$$
$$I_z = I_y = 80.1\times 10^4\ \text{mm}^4$$

Spacing of angles

For angle sections, buckling curve '*c*' will be applicable

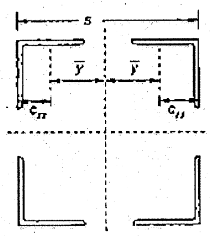
$\therefore$ From Table 5.8, for buckling curve '*c*' and design compressive stress of 165 N/mm², effective slenderness ratio is,

$$\lambda = 61.875$$

For laced columns, effective slenderness ratio (λ_e) = 1.05 × 61.875 = 64.97

$\because$ Column is restrained against position and rotation both and thus it is a fixed column

$\therefore$ Effective length of column $= kL = 0.65\times 10000 = 6500$ mm



Thus $\frac{kL}{r} = 64.97$

$\Rightarrow$ $r = \frac{6500}{64.97} = 100.046$ mm

Now $I = Ar^2 = (4\times 1047)(100.046)^2$
$= 41.919\times 10^6$ mm⁴

$\therefore$ $41.919\times 10^6 = 4\times 80.1\times 10^4 + (4\times 1047)\,\bar{y}^2$

$\Rightarrow$ $\bar{y} = 96.197$ mm

Thus spacing of angles, $S = (\bar{y} + C_{yy})$
$= 2(96.147 + 24.2) = 240.694\ \text{mm} \simeq 241$ mm

$\therefore$ Design compressive strength of column

$P_d = f_{cd}\cdot A = 165\times(4\times 1047)\text{N} = 691.02\ \text{kN} > 675\ \text{kN}$ (OK)

Connecting system for angles

Provide double lacing system with lacings inclined at 45° to the vertical. Bolting will be done at the centre of leg of angles.

$\therefore$ Spacing of lacing bars $a_1 = (241 - 45 - 45)\cot 45° = 151$ mm

$\therefore$ $\frac{a_1}{r_y} = \frac{151}{27.7} = 5.451 < 50$

Also $0.7\times 64.97 = 45.479$

Thus slenderness ratio of column in between the lacings

$= 5.4851$

< 50 (OK)

$< 0.7\lambda_e$ (OK)

Transverse shear to be carried by lacings

$V_t = 2.5\%$ of axial load

$= \frac{2.5}{100}\times 675\ \text{kN} = 16.875\ \text{kN}$

$\therefore$ Transverse shear per plane $= \dfrac{V_t}{N} = \dfrac{16.875}{2} = 8.4375 \text{ kN} \simeq 8.44 \text{ kN}$

Compressive force in lacing bar of double lacing system

$$= \frac{1}{2} \times \frac{V_t}{N} \operatorname{cosec} \theta = \frac{1}{2} \times 8.44 \times \operatorname{cosec} 45^\circ \text{ kN} = 5.97 \text{ kN}$$

Design of lacing section

Let 20 mm diameter bolts are used

$\therefore$ Minimum width of lacing flat = 3 × 20 = 60 mm

Thus thickness of lacing flat $\nless \dfrac{1}{60}(241 - 45 - 45) \operatorname{cosec} 45^\circ = 3.56 \text{ mm} \approx 8 \text{ mm}$ (say)

$\therefore$ Provide 60 ISF 8 mm flat section

Minimum radius of gyration, $r = \dfrac{t}{\sqrt{12}} = \dfrac{8}{\sqrt{12}} = 2.309$ mm

Effective slenderness ratio $= \dfrac{l}{r} = \dfrac{0.7(241 - 45 - 45)\operatorname{cosec} 45^\circ}{2.309} < 145$ (OK)

From Table 5.8, for buckling curve 'c' and effective slenderness ratio of 64.739, design compressive stress,

$$f_{cd} = 160.4176 \text{ N/mm}^2 \approx 160.42 \text{ N/mm}^2$$

$\therefore$ Design compressive strength of lacing flat,

$$P_d = f_{cd} \cdot A = 160.42 \times (60 \times 8) \text{N}$$
$$= 77 \text{ kN} > 5.97 \text{ kN} \quad \text{(OK)}$$

Tensile strength of lacing flat

Diameter of bolt hole for 20 mm dia. bolts, $d_0 = 22$ mm

Tensile strength of lacing flat is minimum of

(i) Fracture

$$0.9(B - d_0)t \cdot \frac{f_u}{\gamma_{m1}} = 0.9(60 - 22)8 \times \frac{410}{1.25} \text{N} = 89.74 \text{ kN}$$

(ii) Gross section yielding

$$\frac{A_g f_y}{\gamma_{m0}} = (60 \times 8)\frac{250}{1.1} \text{N} = 109.09 \text{ kN}$$

Thus tensile strength of lacing flat = 89.79 kN > 5.97 kN (OK)

Design of connection

Strength of 20 mm diameter bolt in single shear

$$= A_{nb} \frac{f_{ub}}{\sqrt{3}\gamma_{mb}} = \frac{245 \times 400}{\sqrt{3} \times 1.25} \text{N} = 45.2 \text{ kN}$$

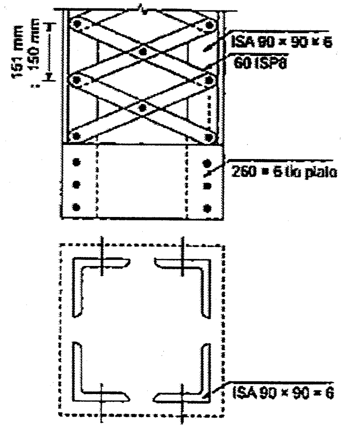
Strength of 20 mm diameter bolt in bearing (on 6 mm plate thickness) and assuming $k_b = 1$ ($\because$ thickness of angle = 6 mm)

$$= 2.5\, k_b \frac{dt f_u}{\gamma_{mb}} = 2.5 \times 1 \times 20 \times 6 \times \frac{410}{1.25} \text{N} = 98.4 \text{ kN}$$

$\therefore$ Strength of bolt = 45.2 kN

$\therefore$ Number of bolts required $= \dfrac{2 \times 5.97 \times \cot 45^\circ}{45.2} = 0.264 \simeq 1$

$\therefore$ Provide two 20 mm diameter bolts of grade 4.6 for connecting lacing flat to angle sections of column



Design of tie plate

Tie plates are provided at each end of the column

Effective depth of tie plate = 241 − 21 × 24.2

= 192.6 mm > 2 × 90 mm

Overall depth of tie plate = 192.6 + 2 × 1.5 × 22

= 258.6 mm ≃ 260 mm

Thus length of tie plate = 260 mm

Thickness of tie plate $= \dfrac{1}{50}(241 - 45 - 45)$

$= 3.02 \text{ mm} \simeq 6 \text{ mm}$ (say)

$\therefore$ Provide tie plate of 241 × 260 × 6 mm

Example 5.7 **Design a built up column to carry a factored axial load of 1000 kN which is 8.5 m long hinged at both the ends. Design the associated lacing system also. Use steel of grade Fe410 and bolts of grade 4.6. Only available section is the channel section. Explore all the possible arrangements of channel orientation.**

Solution:

For steel of grade Fe410, $f_u = 410$ N/mm² and $f_y = 250$ N/mm²

For bolts of grade 4.6, $f_{ub} = 400$ N/mm²

Partial factor of safety for material $\gamma_{m0} = 1.1$

Partial factor of safety for bolt material $\gamma_{mb} = 1.25$

Let design compressive stress $(f_{cd}) = 150$ N/mm²

$\therefore$ Area required $(A) = \dfrac{1000 \times 1000}{150} = 6666.67 \text{ mm}^2$

Try ISMC 250 @ 298.2 N/m

Thus area provided by 2 ISMC = 2 × 3867 = 7734 mm² > 6666.67 mm² (OK)

The advantage of built-up column is that the least radius of gyration can be increased so that its least radius of gyration can be made more atleast equal to r_{zz} i.e.

$$r_{min} = r_{zz}$$

This is done by adjusting the spacing between the sections.

Effective length = kL = 1 × 8.5 = 8.5 m

Now minimum radius of gyration $(\lambda_{min}) = r_z = 99.4$ mm

$\therefore$ Slenderness ratio $(\lambda) = \dfrac{kL}{r_z} = \dfrac{8500}{99.4} = 85.513$

Effective slenderness ratio $(\lambda_e) = 1.05 \times 85.513 = 89.79 < 180$ (OK)

From Table 5.8.

for $\left(\frac{kL}{r}\right)_e = 89.79$ and $f_y = 250$ N/mm² and for buckling curve '*c*'

design compressive stress $(f_{cd}) = 121.315$ N/mm²

∴ Design compressive strength $(P_{cd}) = f_{cd} \times A$

$= 121.315 \times (2 \times 3867)\text{N} = 938.25$ kN

< 1000 kN (Not OK)

Thus ISMC 250 is not safe

Try ISMC 300 @ 351.2 N/m

∴ Area provided by two channels $= 2 \times 4564 = 9128$ mm²

Least radius of gyration $(\lambda_{min}) = r_z = 118.1$ mm

$$\therefore \quad \frac{kL}{r} = \frac{8500}{118.1} = 71.97$$

∴ Effective slenderness ratio $\left(\frac{kL}{r}\right) = 1.05 \times 71.97 = 75.57$

From Table 5.8, for $\left(\frac{kL}{r}\right)_e = 75.57$, $f_y = 250$ N/mm² and for buckling curve '*c*'

design compressive stress $(f_{cd}) = 143.09$ N/mm²

∴ Design compressive strength $(P_{cd}) = f_{cd} \cdot A$

$= 143.09(2 \times 4564)\text{N} = 1306.13$ kN

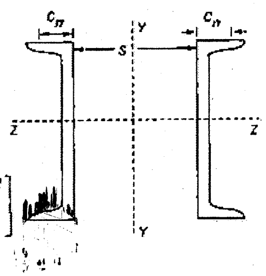
> 1000 kN (OK)

Thus ISMC 300 @ 351.2 N/m is safe

(a) Arrangement-1 : Channels are placed back-to-back.

The channels are so spaced that

$$MOI_z = MOI_y$$



$$\Rightarrow \quad 2I_z = 2\left[I_y + A\left(\frac{S}{2} + C_{yy}\right)^2\right]$$

$$\Rightarrow \quad 2 \times 6362.6 \times 10^4 = 2\left[310.8 \times 10^4 + 4564\left(\frac{S}{2} + 23.6\right)^2\right]$$

$$\Rightarrow \quad S = 183.103 \text{ mm} \simeq 183.5 \text{ mm}$$

Thus place channels back to back at a spacing of 183.5 mm

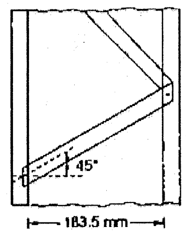
Design of lacings

Let lacing are inclined at 45° with the horizontal

Thus length of channel between lacings = (183.5 + 50 + 50)cot45° × 2 = 567 mm

∴ Slenderness ratio of portion of channel between lacings $= \frac{567}{r_y} = \frac{567}{26.1} = 21.724 < 50$ (OK)

Also $0.7\left(\frac{kL}{r}\right)_e = 0.7 \times 75.57 = 52.899 > 21.724$ (OK)



Design shear for lacing $(V) = 2.5\%$ of axial load $= \frac{2.5}{100} \times 1000 = 25$ kN

∴ Shear in each plane $= \frac{V}{N} = \frac{25}{2} = 12.5$ kN

∴ Compressive force in the lacing bars $= \frac{V}{N}\text{cosec}\,\theta = 12.5 \times \text{cosec}\,45° = 17.68$ kN

Section of lacing flat

Using 16 mm diameter bolts, minimum width of lacing flat required = 3 × 16 = 48 mm = 55 mm (say)

Thus minimum thickness of lacing flat

$= \frac{1}{40}$ (Distance between the inner end bolts)

$= \frac{1}{40}(183.5 + 50 + 50)\text{cosec}\,45°$

$= 10.02 \text{ mm} \simeq 12 \text{ mm (say)}$

Thus provide 55 ISF 12 mm as lacing flat

Minimum radius of gyration,

$$r = \frac{t}{\sqrt{12}} = \frac{12}{\sqrt{12}} = 3.464$$

Slenderness ratio, $\frac{l}{r} = \frac{283.5\,\text{cosec}\,45°}{3.464} = 115.74 < 145$ (OK)

For $\frac{l}{r} = 115.74$, $f_y = 250$ N/mm² and for buckling curve '*c*'

design compressive stress $(f_{cd}) = 88.34$ N/mm²

∴ Design compressive strength $(P_{cd}) = f_{cd} \cdot A$

$= 88.34 \times (55 \times 12)\text{N} = 58.3$ kN

> 17.68 kN (OK)

Check for tensile strength of lacing bar

$$0.9(B - d_0)\frac{t f_u}{\gamma_{m1}} = \frac{0.9(55 - 18)12 \times 410}{1.25} N = 131.07 \text{ kN}$$

$$\frac{A_g f_y}{\gamma_{m0}} = \frac{(55 \times 12)250 \text{ kN}}{1.1} = 150 \text{ kN}$$

Thus tensile strength of lacing bar is minimum of above two values i.e. 131.07 kN > 17.68 kN (OK)

Connection of lacing bar with channel section

Let two lacing bars are connected through one bolt

Thus bolt will be in double shear

Shear strength of bolt in double shear

$$= 2\times\frac{A_{nb}f_{ub}}{\sqrt{3}\,\gamma_{mb}} = \frac{2\times157\times400}{\sqrt{3}\times1.25}\text{N} = 58.01\text{ kN}$$

$$\text{Strength of bolt in bearing} = 2.5\,k_b\frac{dtf_u}{\gamma_{mb}} = \frac{2.5\times1\times16\times12\times410}{1.25}\text{N} = 157.44\text{ kN}$$

(Assume $k_b = 1$)

∴ Strength of 16 mm diameter bolt = 58.01 kN

Force coming on bolt from the two lacing flats

$$= 2\times\frac{V}{N}\cot\theta = 2\times12.5\times\cot45^\circ\text{ kN} = 25\text{ kN}$$

$$\therefore \text{Number of bolts required} = \frac{25}{58.01} = 0.43 \simeq 02 \text{ (say)}$$

∴ Provide 2-16 mm dia. bolts

Design of tie plates

Tie plates are provided at the ends of laced column

Effective depth of tie plate = $S + 2\,C_{yy}$

= 183.5 + 2 × 23.6 = 230.7 mm

Minimum edge distance for 16 mm bolt

= 1.5 d_0 = 1.5 × 18 = 27 mm ≃ 30 mm (say)

∴ Overall depth of tie plate = 230.7 + 30 + 30 = 290.7 mm = 300 mm (say)

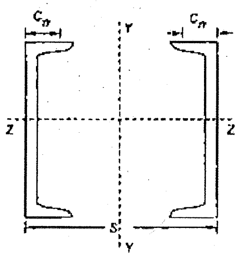
Length of tie plate = $S + 2\,b_f$

= 183.5 × 2 × 90 = 363.5 mm ≃ 364 mm

$$\text{Thickness of tie plate} = \frac{1}{50}(183.5+2\times50) = 5.67\text{ mm} \simeq 6\text{ mm (say)}$$

Provide a tie plate of 300 × 364 × 6 mm at the ends and connect it to channel with 16 mm diameter bolts.

(b) Arrangement-II : Channels are placed face-to-face



$MOI_z = MOI_y$

$$\Rightarrow \quad 2I_z = 2\left[I_y + A\left(\frac{S}{2}-C_{yy}\right)^2\right]$$

$$\Rightarrow \quad 2\times6362.6\times10^4 = 2\left[310.8\times10^4 + 4564\left(\frac{S}{2}-23.6\right)^2\right]$$

$$\Rightarrow \quad S = 277.5\text{ mm} \simeq 278\text{ mm}$$

Thus place channels face-to-face at a spacing of 278 mm

Design of lacings

Let lacings are inclined at 45°.

Thus length of column portion between two lacings = (278 − 50 − 50)cot45° × 2 = 356 mm

$$\therefore \text{Slenderness ratio of column portion between the lacings} = \frac{356}{r_y} = \frac{356}{26.1} = 13.64 < 50 \quad \text{(OK)}$$

Also $$0.7\left(\frac{KL}{r}\right)_e = 0.7\times75.57 = 52.899 > 13.64 \quad \text{(OK)}$$

As obtained in arrangement-I above, design compressive face in the lacing bars = 17.68 kN

Section of lacing flat

Here also, use 16 mm diameter bolts and thus minimum width of lacing flat required

= 3 × 16 = 48 mm ≃ 55 mm (say)

$$\text{Thickness of lacing flat} = \frac{1}{40}(278-50-50)\text{cosec}\,45^\circ = 6.29\text{ mm} \simeq 8\text{ mm (say)}$$

$$\text{Minimum radius of gyration } (r_{min}) = \frac{t}{\sqrt{12}} = \frac{8}{\sqrt{12}} = 2.309\text{ mm}$$

$$\therefore \quad \frac{l}{r} = \frac{(278-50-50)\text{cosec}\,45^\circ}{2.309} = 109.02 < 145 \quad \text{(OK)}$$

Provide 55 ISF 8 as lacing flat

for, $\frac{l}{r}$ = 109.02 and f_y = 250 N/mm² and for buckling curve 'c'

design compressive stress (f_{cd}) = 95.8 N/mm²

∴ Design compressive strength (P_{cd}) = $f_{cd} \cdot A$.

= 95.8 × (55 × 8)N

= 42.152 kN

> 17.68 kN (OK)

Check for tensile strength of lacing bar

$$0.9(B-d_0)\frac{tf_u}{\gamma_{m1}} = 0.9(55-18)\frac{8\times410}{1.25}N = 87.38\text{ kN}$$

$$\frac{A_g f_y}{\gamma_{m0}} = \frac{(55\times8)250}{1.1}N = 100\text{ kN}$$

Thus design tensile strength of flat = 87.38 kN > 17.68 kN (OK)

Connection of lacing bar with channel section

As in earlier case, shear strength of bolt in double shear = 58.01 kN

$$\text{Strength of bolt in bearing} = \frac{2.5k_b dtf_u}{\gamma_{mb}} = \frac{2.5\times1\times16\times8\times410}{1.25}\text{ N} = 104.96\text{ kN}$$

(Assuming k_b = 1)

Thus strength of bolt = 58.01 kN

$$\therefore \quad \text{No. of bolts required} = \frac{25}{58.01} = 0.43 \simeq 1 \text{ (say)}$$

Design of tie plate

Effective depth of tie plate = $S - 2\,C_{yy}$

= 278 − 2 × 23.6 = 230.8 mm

Minimum edge distance for 16 mm bolt

= 1.5 d_0 = 1.5 × 18 = 27 mm ≃ 30 mm (say)

∴ Overall depth of tie plate = 230.8 + 30 + 30 = 290.8 mm ≃ 300 mm (say)

Length of tie plate = 278 mm

$$\text{Thickness of tie plate} = \frac{1}{50}(278 - 50 - 50) = 3.56 \text{ mm} \simeq 6 \text{ mm}$$

Provide tie plate of size 278 × 300 × 6 at the ends.

As compared to tie plate size of Arrangement-I which is 300 × 364 × 6, it is seen that when channel are placed face to face, the size of the tie plate required gets reduced.

(c) If lacings are welded to channel

Let channels are placed back-to-back as per Arrangement-I.

Design compressive force in the lacing bar = 17.68 kN

Let same 55 mm wide lacing flats are used.

For welded lacing flat, effective length of flat = 0.7 × 183.5 cosec 45° = 181.66 mm

$$\text{Thickness of lacing flat} = \frac{1}{40} \times 183.5 \text{ cosec}45° = 6.49 \text{ mm} = 8 \text{ mm (say)}$$

$$\text{Min. radius of gyration} = \frac{t}{\sqrt{12}} = \frac{8}{\sqrt{2}} = 2.309$$

$$\therefore \quad \frac{l}{r} = \frac{181.66}{2.309} = 78.67 < 145 \qquad \text{(OK)}$$

for $\frac{l}{r} = 78.67$ and $f_y = 250$ N/mm² and for buckling curve 'c',

Design compressive stress (f_{cd}) = 138.128 N/mm²

∴ Design compressive strength (P_{cd})

$$= f_{cd} A$$
$$= 138.128(55 \times 8) \text{N}$$
$$= 60.78 \text{ kN} > 17.68 \text{ kN} \qquad \text{(OK)}$$

Overlap of lacing flat or channel > $4t$ = 4 × 8 = 32 mm ≃ 50 mm (say)

Welded connection of lacing bar with channel section

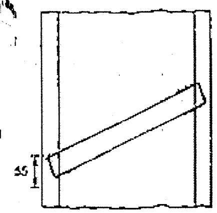
For ISMC 200, t_f = 13.6 mm

Let size of weld = 6 mm

$$\therefore \text{ Strength of weld per unit length} = \frac{0.7 \times 6 \times 410}{\sqrt{3} \times 1.5} \text{N/mm} = 662.8 \text{ N/mm}$$

$$\therefore \quad \text{Weld length required} = \frac{17.68 \times 1000}{662.8} = 26.67 \text{ mm}$$

Weld length provided = 50 + 50 + 55 = 155 mm



Design of tie plate

Overall depth of tie plate = $S + 2C_{yy}$

= 183.5 + 2 × 23.6 = 230.7 mm ≃ 231 mm

> $2b_f$(= 2 × 90 = 180 mm)

Length of tie plate = 183.5 + 2 × 50 = 283.5 mm ≃ 284 mm

$$\text{Thickness of tie plate} = \frac{1}{50} \times 283.5 = 5.67 \text{ mm} \simeq 8 \text{ mm (say)}$$

Provide tie plate of size 284 × 231 × 8 mm

Example 5.8 Design a column of height 3.7 m to carry a factored axial load of 5850 kN. The column is effectively held at both the ends and restrained in direction at one of the ends. Use plate sections if required. Use Fe410 steel.

Solution:

For Fe410 steel, f_u = 410 N/mm², f_y = 250 N/mm²

Let design compressive stress (f_{cd}) = 190 N/mm²

$$\therefore \text{ Cross-section area required } (A) = \frac{P}{f_{cd}} = \frac{5850 \times 1000}{190} = 30789.47 \text{ mm}^2$$

Try ISHB 450 @ 907 N/m

Area provided by ISHB450 = 111.89 × 10² mm² = 11189 mm²

b_f = 250 mm, t_f = 13.7 mm

∴ Area to be furnished by plates = 30789.47 − 11189 = 19600.47 mm²

$$\text{Using 25 mm thick plates, width of plate } (2b) = \frac{19600.47}{25} = 784.02 \text{ mm}$$

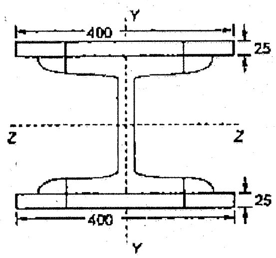
$$\therefore \quad \text{Width of one plate } (b) = \frac{784.02}{2} = 392.01 \text{ mm} \simeq 400 \text{ mm}$$

$$\therefore \quad \text{Overhang} = \frac{400 - 250}{2} = 75 \text{ mm}$$

$< 12t (= 12 \times 13.7 = 164.4 \text{ mm})$

Total area provided = 11189 + (400 × 25) 2

= 31189 mm² > 30789.47 mm²



For ISHB 450 @ 907 N/m,

I_z = 40349.9 × 10⁴ mm⁴

I_y = 3045 × 10⁴ mm⁴

$$I_z \text{ of built-up section} = 40349.9 \times 10^4 + 2\left[400 \times \frac{25^3}{12} + 400 \times 25 \times \left(\frac{450}{2} + \frac{25}{2}\right)^2\right]$$

$$= 40349.9 \times 10^4 \times 2\left[52.083 \times 10^4 + 56406.25 \times 10^4\right]$$

$$= 153266.6 \times 10^4 \text{ mm}^4$$

$$I_y \text{ of built-up section} = 3045 \times 10^4 + 2\left[25 \times \frac{400^3}{12}\right] = 29711.67 \times 10^4 \text{ mm}^4$$

$$\therefore \quad r_{min} = r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{29711.67 \times 10^4}{31189}} = 97.603 \text{ m}$$

Effective length = kL = 0.8 × 3.7 = 2.96 m

$$\therefore \quad \frac{kL}{r} = \frac{2960}{97.603} = 30.327$$

t_f = t_f of I-section + 25 = 13.7 + 25 = 38.7 mm < 40 mm

For kL/r and f_y = 250 N/mm² and using buckling curve 'c'

design compressive stress (f_{cd}) = 199.69 N/mm²

∴ Design compressive strength (P_{cd})

$$= f_{cd} \cdot A = 199.69 \times 31189 \text{ N} = 6228.13 \text{ kN}$$

$$> 5850 \text{ kN} \qquad \text{(OK)}$$

Objective Brain Teasers

Q.1 Which of the following beam sections is most preferred as a column section?
(a) ISBH (b) ISMB
(c) ISLB (d) ISWB

Q.2 The design compressive stress in an axially loaded compression member is given by

$$f_{cd} = \frac{\chi f_y}{\gamma_{mo}}$$

Here the factor χ is called as ________ and is equal to ________.
(a) Stress reduction factor, $\left[\phi + (\phi^2 - \lambda^2)\right]^{1/2}$
(b) Stress enhancement factor, $\frac{1}{\phi + (\phi^2 - \lambda^2)^{1/2}}$
(c) Stress reduction factor, $\frac{1}{\phi + (\phi^2 - \lambda^2)^{1/2}}$
(d) Stress enhancement factor, $\phi + \left(\phi^2 - \lambda^2\right)^{1/2}$

Q.3 A section is said to be plastic when outstand of compression flange is
(a) $> 9.4\,\epsilon$ (b) $> 8.4\,\epsilon$
(c) $< 9.4\,\epsilon$ (d) $< 8.4\,\epsilon$

Q.4 Under exactly identical conditions, battened column as compared to laced column is
(a) equal in strength (b) weaker in strength
(c) stronger in strength (d) data insufficient

Q.5 For a battened built-up column, minimum number of batten plates reqruied are
(a) Two (b) Six
(c) Five (d) Four

Q.6 The minimum thickness of lacing flat required for single lacing system is __________ than minimum thickness required for lacing flat in double lacing system under identical condition.
(a) 50% more (b) 50% less
(c) 66.67% more (d) 66.67% less

Q.7 The effective depth of end batterns should be
(i) More than twice the flange width of component columns
(ii) More than the distance between *CG* of component columns
(iii) Less than twice the flange width of component columns
(iv) Less than the distance between *CG* of component columns

Of the above statements, the correct ones are
(a) (i) and (iii) (b) (ii) and (iv)
(c) (i) and (ii) (d) (iii) and (iv)

Q.8 The expression for design compressive stress as given is IS 800 : 2007 is based on
(a) Rankine formula
(b) Perry Robertson formula
(c) Merchant Rankine formula
(d) Secant formula

Q.9 The maximum permissible slenderness ratio of single angle section corrected with one bolt only and is being used as a strut is
(a) 250 (b) 180
(c) 180 (d) 145

Q.10 Splices for a compression member are designed as
(a) Pedestal (b) Long column
(c) Compression block (d) Short column

Q.11 The four buckling curves as given by IS 800 : 2007 takes into account
(a) initial geometric imperfections and method of manufacture
(b) self weight of the member
(c) sectional geometric properties of the section
(d) All of the above

Q.12 Strain hardening in short columns
1. increases the collapse load
2. is taken into account for design purposes

of the above statements, the correct one(s) is(are):
(a) 1 only (b) 2 only
(c) Both (1) and (2) (d) None of (1) and (2)

Q.13 In which of the following types of column, the failure load is proportional to flexural rigidity (EI) of the column and independent of strength of steel?
(a) Short column (b) Pedestal
(c) Long column (d) All of these

Q.14 Which of the following types of section are preferred as compression members?
1. Compact 2. Semi-compact
3. Plastic 4. Slender

The correct ones are
(a) 1 and 2 (b) 2 and 4
(c) 3 and 4 (d) 1 and 3

Q.15 As per IS 800 : 2007, a separate provision is given for which of the following compression member?
(a) Double angle strut (b) Single angle strut
(c) Channel strut (d) I-section column

Q.16 The critical slenderness ratio for Euler's formula from yielding to buckling is
(a) 28.25 (b) 88.86
(c) 98.45 (d) 50.30

Answers

1. (a)	2. (c)	3. (c)	4. (b)	5. (d)
6. (a)	7. (c)	8. (b)	9. (c)	10. (d)
11. (a)	12. (a)	13. (c)	14. (d)	15. (b)
16. (b)				

Conventional Practice Questions

Q.1 Determine the design strength of ISA 100 × 100 × 8 which is 3.1 m long and is being used as a strut. The ends of the member are welded to 12 mm thick gusset plate.

Q.2 A column in building in 3.35 m long with both ends restrained in position and direction both along z-z and y-y axis. The column is required to carry a lead of 2080 kN. Design the column section.

Q.3 Design a signal angle discontinuous strut to carry a factored compression of 180 kN. The interconnection of the strut are 2.4 m apart.

Q.4 Design a continuous strut to carry a compressive load of 160 mm when effective length of the strut is 3.8 m.

Q.5 Design a double angle discontinuous strut to carry a load of 225 kN when interconnections are 2.75 m apart.

Q.6 Design a four angle built up column-section with lacing which is 6.8 m long and carries a factored load of 2500 kN.

Q.7 A built-up column section is required to carry a factored load of 2800 kN. The length of the member is 4.2 m. Design the column section with channel placed (i) back to back (ii) toe to toe.

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