CHAPTER : 2
MOTION IN A STRAIGHT LINE

We see a number of things moving around us. Humans, animals, vehicles can be seen moving on land. Fish, frogs and other aquatic animals move in water. Birds and aeroplanes move in air. Though we do not feel it, the earth on which we live also revolves around the sun as well as its own axis. It is, therefore, quite apparent that we live in a world that is very much in constant motion. Therefore to understand the physical world around us, the study of motion is essential. Motion can be in a straight line(1D), in a plane(2D) or in space(3D). If the motion of the object is only in one direction, it is said to be the motion in a straight line. For example, motion of a car on a straight road, motion of a train on straight rails, motion of a freely falling body, motion of a lift, and motion of an athlete running on a straight track, etc.

In this lesson you will learn about motion in a straight line. In the following lessons, you will study the laws of motion, motion in plane and other types of motion. You will also learn the concept of Differentiation and Integration.

OBJECTIVES

After studying this lesson, you should be able to,

- *distinguish between distance and displacement, and speed and velocity;*
- *explain the terms instantaneous velocity, relative velocity and average velocity;*
- *define acceleration and instantaneous acceleration;*
- *interpret position - time and velocity - time graphs for uniform as well as non-uniform motion;*
- *derive equations of motion with constant acceleration;*
- *describe motion under gravity;*
- *solve numericals based on equations of motion; and*
- *understand the concept of differentiation and integration.*

2.1 SPEED AND VELOCITY

We know that the total length of the path covered by a body is the **distance** travelled by it. But the difference between the initial and final position vectors of a body is called its **displacement**. Basically, **displacement is the shortest distance between the two positions and has a certain direction**. Thus, the displacement is a vector quantity but distance is a scalar. You might have also learnt that the rate of change of distance with time is called **speed** but the rate of change of displacement is known as **velocity**. Unlike speed, velocity is a vector quantity. For 1-D motion, the directional aspect of the vector is taken care of by putting + and – signs and we do not have to use vector notation for displacement, velocity and acceleration for motion in one dimension.

2.1.1 Average Velocity

When an object travels a certain distance with different velocities, its motion is specified by its average velocity. The **average velocity** of an object is defined as the displacement per unit time. Let x_1 and x_2 be its positions at instants t_1 and t_2, respectively. Then mathematically we can express average velocity as

$$\bar{v} = \frac{\text{displacement}}{\text{time taken}}$$

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad (2.1)$$

where $x_2 - x_1$ signifies change in position (denoted by Δx) and $t_2 - t_1$ is the corresponding change in time (denoted by Δt). Here the bar over the symbol for velocity ($\bar{v}$) is standard notation used to indicate an average quantity. Average velocity can be represented as v_{av} also. The average speed of an object is obtained by dividing the total distance travelled by the total time taken:

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} \qquad (2.2)$$

If the motion is in the same direction along a straight line, the average speed is the same as the magnitude of the average velocity. However, this is always not the case (see example 2.2).

Following examples will help you in understanding the difference between average speed and average velocity.

Example 2.1 : The position of an object moving along the x-axis is defined as $x = 20t^2$, where t is the time measured in seconds and position is expressed in metres. Calculate the average velocity of the object over the time interval from 3s to 4s.

Solution : Given,

$$x = 20t^2$$

Note that x and t are measured in metres and seconds. It means that the constant of proportionality (20) has dimensions ms^{-2}.

We know that the average velocity is given by the relation

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1}$$

At $t_1 = 3s$,

$$x_1 = 20 \times (3)^2$$
$$= 20 \times 9 = 180 \text{ m}$$

Similarly, for $t_2 = 4s$

$$x_2 = 20 \times (4)^2$$
$$= 20 \times 16 = 320 \text{ m}$$

$$\therefore \qquad \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(320 - 180)\text{ m}}{(4 - 3)\text{ s}} = \frac{140\text{ m}}{1\text{ s}} = 140 \text{ ms}^{-1}$$

Hence, average velocity = 140 ms^{-1}.

Example 2.2 : A person runs on a 300m circular track and comes back to the starting point in 200s. Calculate the average speed and average velocity.

Solution : Given,

Total length of the track = 300m.

Time taken to cover this length = 200s

Hence,

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{time taken}}$$

$$= \frac{300}{200} \text{ ms}^{-1} = 1.5 \text{ ms}^{-1}$$

As the person comes back to the same point, the displacement is zero. Therefore, the average velocity is also zero.

Note that in the above example, the average speed is not equal to the magnitude of the average velocity. Do you know the reason?

2.1.2 Relative Velocity

When we say that a bullock cart is moving at 10km h^{-1} due south, it means that the cart travels a distance of 10km in 1h in southward direction from its starting position. Thus it is implied that the referred velocity is with respect to some reference point. In fact, the velocity of a body is always specified with respect to some other body. Since all bodies are in motion, we can say that every velocity is relative in nature.

The relative velocity of an object with respect to another object is the rate at which it changes its position relative to the object / point taken as reference. For example, if υ_A and υ_B are the velocities of the two objects along a straight line, the relative velocity of B with respect to A will be $\upsilon_B - \upsilon_A$.

The rate of change of the relative position of an object with respect to the other object is known as the ***relative velocity*** *of that object with respect to the other.*

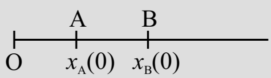
Importance of Relative Velocity

The position and hence velocity of a body is specified in relation with some other body. If the reference body is at rest, the motion of the body can be described easily .You will learn the equations of kinematics in this lesson. But what happens, if the reference body is also moving? Such a motion is seen to be of the two body system by a stationary observer. However, it can be simplified by invoking the concept of relative motion.

Let the initial positions of two bodies A and B be $x_A(0)$ and $x_B(0)$. If body A moves along positive x-direction with velocity υ_A and body B with velocity υ_B, then the positions of points A and B after t seconds will be given by

$$x_A(t) = x_A(0) + \upsilon_A t$$

$$x_B(t) = x_B(0) + \upsilon_B t$$



Therefore, the relative separation of B from A will be

$$x_{BA}(t) = x_B(t) - x_A(t) = x_B(0) - x_A(0) + (\upsilon_B - \upsilon_A)\, t$$

$$= x_{BA}(0) + \upsilon_{BA} t$$

where $\upsilon_{BA} = (\upsilon_B - \upsilon_A)$ is called the relative velocity of B with respect to A. Thus by applying the concept of relative velocity, a two body problem can be reduced to a single body problem.

Example 2.3 : A train A is moving on a straight track (or railway line) from North to South with a speed of 60km h^{-1}. Another train B is moving from South to North with a speed of 70km h^{-1}. What is the velocity of B relative to the train A?

Solution : Considering the direction from South to North as positive, we have

velocity (υ_B) of train B = + 70km h^{-1}

and, velocity (v_A) of train A = – 60km h^{-1}

Hence, the velocity of train B relative to train A

$$= v_B - v_A$$
$$= 70 - (-60) = 130\text{km h}^{-1}.$$

In the above example, you have seen that the relative velocity of one train with respect to the other is equal to the sum of their respective velocities. This is why a train moving in a direction opposite to that of the train in which you are travelling appears to be travelling very fast. But, if the other train were moving in the same direction as your train, it would appear to be very slow.

2.1.3 Acceleration

While travelling in a bus or a car, you might have noticed that sometimes it speeds up and sometimes it slows down. That is, its velocity changes with time. Just as the velocity is defined as the time rate of change of displacement, ***the acceleration is defined as time rate of change of velocity.*** Acceleration is a vector quantity and its SI unit is ms^{-2}. In one dimension, there is no need to use vector notation for acceleration as explained in the case of velocity. The average acceleration of an object is given by,

$$\text{Average acceleration } (\bar{a}) = \frac{\text{Final velocity - Initial velocity}}{\text{Time taken for change in velocity}}$$

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad (2.3)$$

In one dimensional motion, when the acceleration is in the same direction as the motion or velocity (normally taken to be in the positive direction), the acceleration is positive. But the acceleration may be in the opposite direction of the motion also. Then the acceleration is taken as negative and is often called deceleration or ***retardation***. So we can say that an increase in the rate of change of velocity is ***acceleration***, whereas the decrease in the rate of change of velocity is ***retardation***.

Example 2.4 : The velocity of a car moving towards the East increases from 0 to 12ms^{-1} in 3.0 s. Calculate its average acceleration.

Solution : Given,

$$v_1 = 0 \text{ m s}^{-1}$$
$$v_2 = 12 \text{ m s}^{-1}$$
$$t = 3.0 \text{ s}$$
$$a = \frac{(12.0\text{m s}^{-1})}{3.0\text{s}}$$
$$= 4.0 \text{ m s}^{-2}$$

INTEXT QUESTIONS 2.1

1. Is it possible for a moving body to have non-zero average speed but zero average velocity during any given interval of time? If so, explain.
2. A lady drove to the market at a speed of 8 km h^{-1}. Finding market closed, she came back home at a speed of 10 km h^{-1}. If the market is 2km away from her home, calculate the average velocity and average speed.
3. Can a moving body have zero relative velocity with respect to another body? Give an example.
4. A person strolls inside a train with a velocity of 1.0 m s^{-1} in the direction of motion of the train. If the train is moving with a velocity of 3.0 m s^{-1}, calculate his

   (a) velocity as seen by passengers in the compartment, and (b) velocity with respect to a person sitting on the platform.

2.2 POSITION - TIME GRAPH

If you roll a ball on the ground, you will notice that at different times, the ball is found at different positions. The different positions and corresponding times can be plotted on a graph giving us a certain curve. Such a curve is known as position-time curve. Generally, the time is represented along *x*-axis whereas the position of the body is represented along *y*-axis.

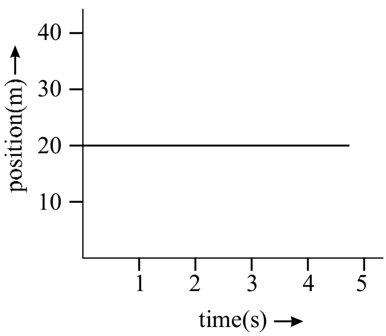


Fig. 2.1 : Position-time graph for a body at rest

Let us plot the position - time graph for a body at rest at a distance of 20m from the origin. What will be its position after 1s, 2s, 3s, 4s and 5s? You will find that the graph is a straight line parallel to the time axis, as shown in Fig. 2.1

2.2.1 Position-Time Graph for Uniform Motion

Now, let us consider a case where an object covers equal distances in equal intervals of time. For example, if the object covers a distance of 10m in each second for 5 seconds, the positions of the object at different times will be as shown in the following table.

Time (t) in s	1	2	3	4	5
Position (x) in m	10	20	30	40	50

In order to plot this data, take time along x-axis assuming 1cm as 1s, and position along y-axis with a scale of 1cm to be equal to 10m. The position-time graph will be as shown in Fig. 2.2

The graph is a straight line inclined with the x-axis. ***A motion in which the velocity of the moving object is constant is known as uniform motion.*** Its position-time graph is a straight line inclined to the time axis.

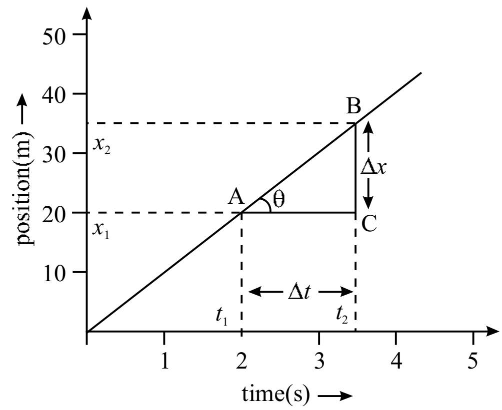


Fig. 2.2 : Position-time graph for uniform motion

In other words, we can say that when a moving object covers equal distances in equal intervals of time, it is in ***uniform motion.***

2.2.2 Position-Time Graph for Non-Uniform Motion

Let us now take an example of a train which starts from a station, speeds up and moves with uniform velocity for certain duration and then slows down before steaming in the next station. In this case you will find that the ***distances covered in equal intervals of time are not equal. Such a motion is said to be non-uniform motion.*** If the distances covered in successive intervals are increasing, the motion is said to be accelerated motion. The position-time graph for such an object is as shown in Fig.2.3.

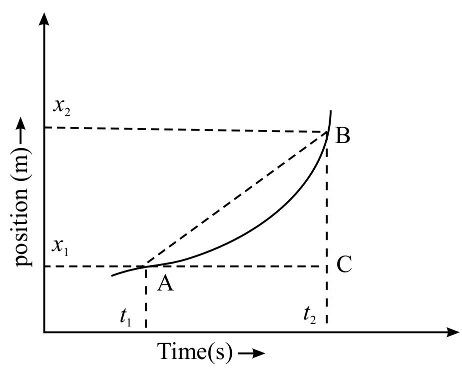


Fig. 2.3 : Position-time graph of accelerated motion as a continuous curve

Note that the position-time graph of accelerated motion is a continuous curve. Hence, the velocity of the body changes continuously. In such a situation, it is more appropriate to define average velocity of the body over an extremely small interval of time or instantaneous velocity. Let us learn to do so now.

2.2.3 Interpretation of Position - Time Graph

As you have seen, the position - time graphs of different moving objects can have different shapes. If it is a straight line parallel to the time axis, you can say that the body is at rest (Fig. 2.1). And the straight line inclined to the time axis shows that the motion is uniform (Fig.2.2). A continuous curve implies continuously changing velocity.

(a) Velocity from position - time graph : The slope of the straight line of position - time graph gives the average velocity of the object in motion. For determining the slope, we choose two widely separated points (say A and B) on the straight line (Fig.2.2) and form a triangle by drawing lines parallel to *y*-axis and *x*-axis. Thus, the average velocity of the object

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{BC}{AC} \qquad (2.4)$$

Hence, average velocity of object equals the slope of the straight line AB.

It shows that greater the value of the slope ($\Delta x/\Delta t$) of the straight line position - time graph, more will be the average velocity. Notice that the slope is also equal to the tangent of the angle that the straight line makes with a horizontal line, i.e., $\tan\theta = \Delta x/\Delta t$. Any two corresponding Δx and Δt intervals can be used to determine the slope and thus the average velocity during that time internal.

Example 2.5 : The position - time graphs of two bodies A and B are shown in Fig. 2.4. Which of these has greater velocity?

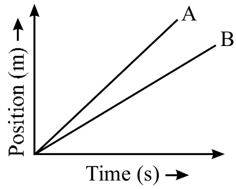


Fig. 2.4 : Position-time graph of bodies A and B

Solution : Body A has greater slope and hence greater velocity.

(b) Instantaneous velocity : As you have learnt, a body having uniform motion along a straight line has the same velocity at every instant. But in the case of non-uniform motion, the position - time graph is a curved line, as shown in Fig.2.5. As a result, the slope or the average velocity varies, depending on the size of the time intervals selected. The velocity of the particle at any instant of time or at some point of its path is called its instantaneous velocity.

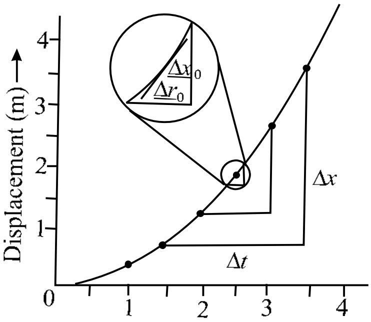


Fig. 2.5 : Displacement-time graph for non- uniform motion

Note that the average velocity over a time interval Δt is given by $\bar{v} = \frac{\Delta x}{\Delta t}$. As Δt is made smaller and smaller the average velocity approaches instantaneous velocity.

In the limit $\Delta t \to 0$, the slope $(\Delta x/\Delta t)$ of a line tangent to the curve at that point gives the instantaneous velocity. However, for uniform motion, the average and instantaneous velocities are the same.

Example 2.6 : The position - time graph for the motion of an object for 20 seconds is shown in Fig. 2.6. What distances and with what speeds does it travel in time intervals (i) 0 s to 5 s, (ii) 5 s to 10 s, (iii) 10 s to 15 s and (iv) 15 s to 17.5 s? Calculate the average speed for this total journey.

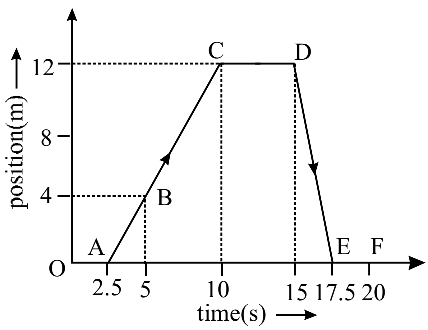


Fig. 2.6: Position-time graph

Solution :

(i) During 0 s to 5 s, distance travelled = 4 m

$$\therefore \text{ speed} = \frac{\text{Distance}}{\text{Time}} = \frac{4\text{ m}}{(5-0)\,s} = \frac{4\text{ m}}{5\,s} = 0.8\text{ m s}^{-1}$$

(ii) During 5 s to 10 s, distance travelled = 12 – 4 = 8 m

$$\therefore \text{ speed} = \frac{(12-4)\text{ m}}{(10-5)\text{ s}} = \frac{8\text{ m}}{5\text{ s}} = 1.6\text{ m s}^{-1}$$

(iii) During 10 s to 15 s, distance travelled = 12 – 12 = 0 m

$$\therefore \text{speed} = \frac{\text{Distance}}{\text{Time}} = \frac{0}{5} = 0$$

(iv) During 15 s to 17.5 s, distance travelled = 12 m

$$\therefore \text{ Speed} = \frac{12\text{ m}}{2.5\text{ s}} = 4.8\text{ m s}^{-1}$$

Now we would like you to pause for a while and solve the following questions to check your progress.

INTEXT QUESTIONS 2.2

1. Draw the position-time graph for a motion with zero acceleration.
2. The following figure shows the displacement - time graph for two students A and B who start from their school and reach their homes. Look at the graphs carefully and answer the following questions.

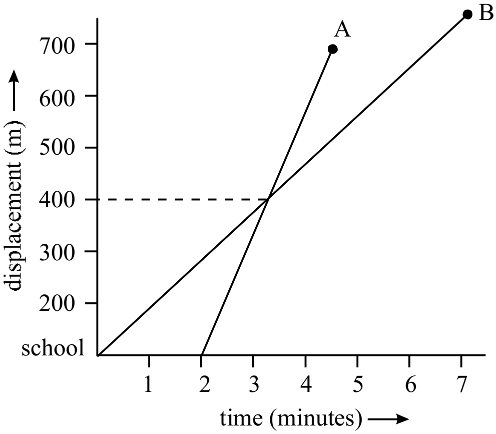
   (i) Do they both leave school at the same time?

   (ii) Who stays farther from the school?

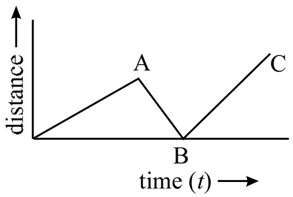
   (iii) Do they both reach their respective houses at the same time?

   (iv) Who moves faster?

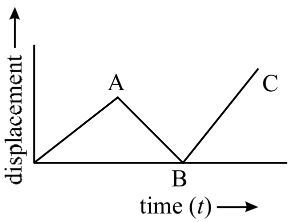
   (v) At what distance from the school do they cross each other?



3. Under what conditions is average velocity of a body equal to its instantaneous velocity?
4. Which of the following graphs is not possible? Give reason for your answer?



(a)



(b)

2.3 VELOCITY - TIME GRAPH

Just like the position-time graph, we can plot velocity-time graph. While plotting a velocity-time graph, generally the time is taken along the x-axis and the velocity along the y-axis.

2.3.1 Velocity-Time Graph for Uniform Motion

As you know, in uniform motion the velocity of the body remains constant, i.e., there is no change in the velocity with time. The velocity-time graph for such a uniform motion is a straight line parallel to the time axis, as shown in the Fig. 2.7.

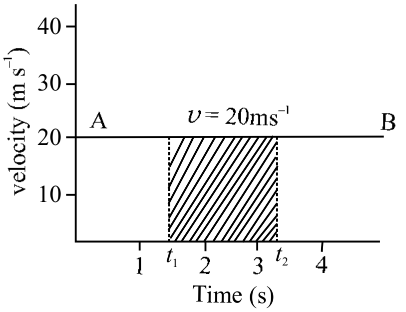


Fig. 2.7 : Velocity-time graph for uniform motion

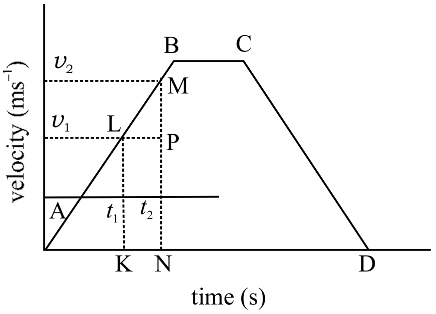


Fig. 2.8 : Velocity-time graph for motion with three different stages of constant acceleration

2.3.2 Velocity-Time Graph for Non-Uniform Motion

If the velocity of a body changes uniformly with time, its acceleration is constant. The velocity-time graph for such a motion is a straight line inclined to the time axis. This is shown in Fig. 2.8 by the straight line AB. It is clear from the graph that the velocity increases by equal amounts in equal intervals of time. The average acceleration of the body is given by

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} = \frac{MP}{LP}$$

$$= \text{slope of the straight line}$$

Since the slope of the straight line is constant, the average acceleration of the body is constant. However, it is also possible that the rate of variation in the velocity is not constant. Such a motion is called non-uniformly accelerated motion. In such a situation, the slope of the velocity-time graph will vary at every instant, as shown in Fig.2.9. It can be seen that θ_A, θ_B and θ_C are different at points A, B and C.

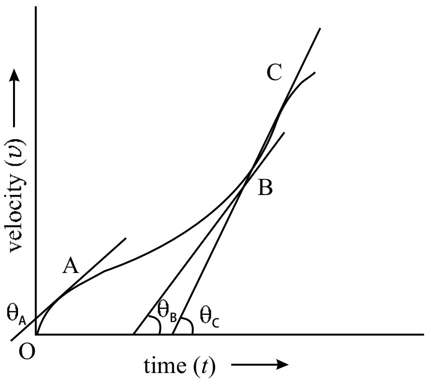


Fig. 2.9 : Velocity-time graph for a motion with varying acceleration

2.3.3 Interpretation of Velocity-Time Graph

Using $v-t$ graph of the motion of a body, we can determine the distance travelled by it and the acceleration of the body at different instants. Let us see how we can do so.

(a) Determination of the distance travelled by the body : Let us again consider the velocity-time graph shown in Fig. 2.10. The portion AB shows the motion with constant acceleration, whereas the portion CD shows the constantly retarded motion. The portion BC represents uniform motion (i.e., motion with zero acceleration).

For uniform motion, the distance travelled by the body from time t_1 to t_2 is given by $s = \upsilon(t_2 - t_1)$ = the area under the curve between t_1 and t_2 Generalising this result for Fig. 2.10, we find that the distance travelled by the body between time t_1 and t_2

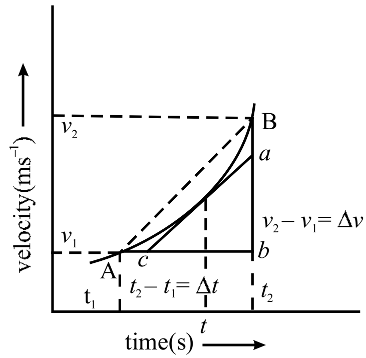


Fig. 2.10 : Velocity-time graph of non-uniformly accelerated motion

s = area of trapezium KLMN

= (½) × (KL + MN) × KN

= (½) × ($\upsilon_1 + \upsilon_2$) × ($t_2 - t_1$)

(b) Determination of the acceleration of the body : We know that acceleration of a body is the rate of change of its velocity with time. If you look at the velocity-time graph given in the Fig.2.10, you will note that the average acceleration is represented by the slope of the chord AB, which is given by

$$\text{average acceleration } (\bar{a}) = \frac{\Delta \upsilon}{\Delta t} = \frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}.$$

If the time interval Δt is made smaller and smaller, the average acceleration becomes instantaneous acceleration. Thus, instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta \upsilon}{\Delta t} = \frac{d\upsilon}{dt} = \text{slope of the tangent at } (t = t) = \frac{ab}{bc}$$

Thus, the slope of the tangent at a point on the velocity-time graph gives the acceleration at that instant.

Example 2.7 : The velocity-time graphs for three different bodies A,B and C are shown in Fig. 2.11.

(i) Which body has the maximum acceleration and how much?

(ii) Calculate the distances travelled by these bodies in first 3s.

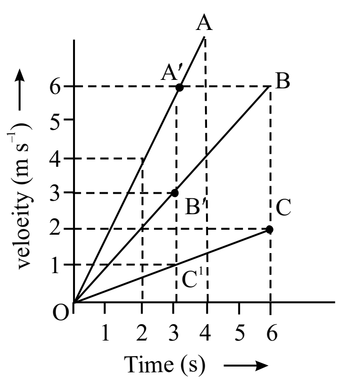


Fig. 2.11 : Velocity-time graph of uniformly accelerated motion of three different bodies

(iii) Which of these three bodies covers the maximum distance at the end of their journey?

(iv) What are the velocities at $t = 2$s?

Solution :

(i) As the slope of the v-t graph for body A is maximum, its acceleration is maximum:

$$a = \frac{\Delta v}{\Delta t} = \frac{6-0}{3-0} = \frac{6}{3} = 2 \text{ ms}^{-2}.$$

(ii) The distance travelled by a body is equal to the area of the v-t graph.

∴ In first 3s,

the distance travelled by A = Area OA′L

= (½) × 6 × 3 = 9m.

the distance travelled by B = Area OB′L

= (½) × 3 × 3 = 4.5 m.

the distance travelled by C = (½) × 1 × 3 = 1.5 m.

(iii) At the end of the journey, the maximum distance is travelled by B.

= (½) × 6 × 6 = 18 m.

(iv) Since v-t graph for each body is a straight line, instantaneous acceleration is equal to average acceleration.

At 2s, the velocity of A = 4 m s^{-1}

the velocity of B = 2 m s^{-1}

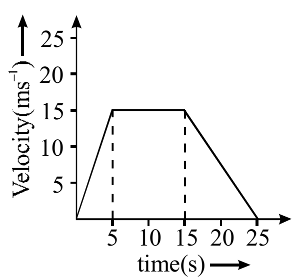
the velocity of C = 0.80 m s^{-1} (approx.)

INTEXT QUESTIONS 2.3

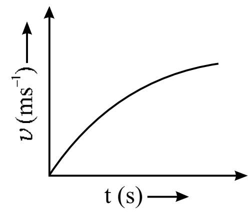
1. The motion of a particle moving in a straight line is depicted in the adjoining $v - t$ graph.

   (i) Describe the motion in terms of velocity, acceleration and distance travelled

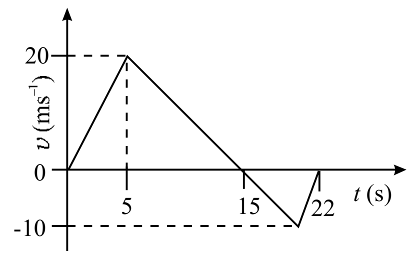
   (ii) Find the average speed.



2. What type of motion does the adjoining graph represent - uniform motion, accelerated motion or decelerated motion? Explain.



3. Using the adjoining v-t graph, calculate the (i) average velocity, and (ii) average speed of the particle for the time interval 0 – 22 seconds. The particle is moving in a straight line all the time.



2.4 EQUATIONS OF MOTION

As you now know, for describing the motion of an object, we use physical quantities like distance, velocity and acceleration. In the case of constant acceleration, the velocity acquired and the distance travelled in a given time can be calculated by using one or more of three equations. These equations, generally known as equations of motion for constant acceleration or kinematical equations, are easy to use and find many applications.

2.4.1 Equation of Uniform Motion

In order to derive these equations, let us take initial time to be zero i.e. $t_1 = 0$. We can then assume $t_2 = t$ to be the elapsed time. The initial position (x_1) and initial velocity (v_1) of an object will now be represented by x_0 and v_0 and at time t they will be called x and v (rather than x_2 and v_2). According to Eqn. (2.1), the average velocity during the time t will be

$$\bar{v} = \frac{x - x_0}{t}. \qquad (2.4)$$

2.4.2 First Equation of Uniformly Accelerated Motion

The first equation of uniformly accelerated motion helps in determining the velocity of an object after a certain time when the acceleration is given. As you know, by definition

$$\text{Acceleration } (a) = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1}$$

If at $t_1 = 0$, $v_1 = v_0$ and at $t_2 = t$, $v_2 = v$. Then

$$a = \frac{v - v_0}{t} \qquad (2.5)$$

$$\Rightarrow \qquad v = v_0 + at \qquad (2.6)$$

Example 2.8 : A car starting from rest has an acceleration of $10ms^{-2}$. How fast will it be going after 5s?

Solution : Given,

Initial velocity $\upsilon_0 = 0$

Acceleration $a = 10\ ms^{-2}$

Time $t = 5s$

Using first equation of motion

$$\upsilon = \upsilon_0 + at$$

we find that for $t = 5s$, the velocity is given by

$$\upsilon = 0 + (10\ ms^{-2}) \times (5s)$$

$$= 50\ ms^{-1}$$

2.4.3 Second Equation of Uniformly Accelerated Motion

Second equation of motion is used to calculate the position of an object after time t when it is undergoing constant acceleration a.

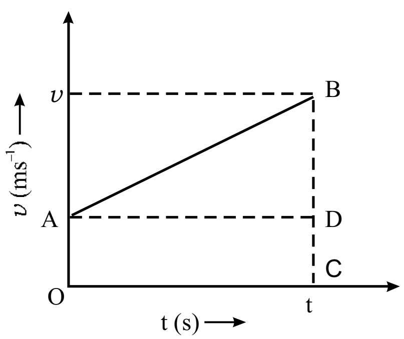


Fig. 2.12 : υ–t graph for uniformly accelerated motion

Suppose that at $t = 0$, $x_1 = x_0$; $\upsilon_1 = \upsilon_0$ and at $t = t$, $x_2 = x$; $\upsilon_2 = \upsilon$.

The distance travelled = area under $\upsilon - t$ graph

= Area of trapezium OABC

$$= \frac{1}{2}(CB + OA) \times OC$$

$$x - x_0 = \frac{1}{2}(\upsilon + \upsilon_0)\, t$$

Since $\upsilon = \upsilon_0 + at$, we can write

$$x - x_0 = \frac{1}{2}(\upsilon_0 + at + \upsilon_0)t$$

$$= \upsilon_0 t + \frac{1}{2} at^2$$

or $$x = x_0 + \upsilon_0 t + \frac{1}{2} at^2 \qquad (2.7)$$

Example 2.9 : A car A is travelling on a straight road with a uniform speed of 60 $km\ h^{-1}$. Car B is following it with uniform velocity of 70 $km\ h^{-1}$. When the distance between them is 2.5 km, the car B is given a decceleration of 20 $km\ h^{-1}$. At what distance and time will the car B catch up with car A?

Solution : Suppose that car B catches up with car A at a distance x after time t.

For car A, the distance travelled in t time, $x = 60 \times t$.

For car B, the distance travelled in t time is given by

$$x' = x_0 + v_0 t + ½\, at^2$$

$$= 0 + 70 \times t + ½\,(-20) \times t^2$$

$$x' = 70\, t - 10\, t^2$$

But the distance between two cars is

$$x' - x = 2.5$$

$$\therefore \qquad (70\, t - 10\, t^2) - (60\, t) = 2.5$$

or $$10\, t^2 - 10\, t + 2.5 = 0$$

It gives $t = ½$ hour

$$\therefore \qquad x = 70t - 10t^2$$

$$= 70 \times ½ - 10 \times (½)^2$$

$$= 35 - 2.5 = 32.5 \text{ km.}$$

2.4.4 Third Equation of Uniformly Accelerated Motion

The third equation is used in a situation when the acceleration, position and initial velocity are known, and the final velocity is desired but the time t is not known. From Eqn. (2.7.), we can write

$$x - x_0 = ½\,(v + v_0)\, t.$$

Also from Eqn. (2.6), we recall that

$$t = \frac{v - v_0}{a}$$

Substituting this in above expression we get

$$x - x_0 = ½\,(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$\Rightarrow \qquad 2a\,(x - x_0) = v^2 - v^0$$

$$\Rightarrow \qquad v^2 = v_0^2 + 2a\,(x - x_0) \qquad (2.8)$$

Thus, the three equations for constant acceleration are

$$v = v_0 + at$$

$$x = x_0 + v_0 t + ½\, at^2$$

and $$v^2 = v_0^2 + 2a\,(x - x_0)$$

Example 2.10 : A motorcyclist moves along a straight road with a constant acceleration of 4m s^{-2}. If initially she was at a position of 5m and had a velocity of 3m s^{-1}, calculate

(i) the position and velocity at time t = 2s, and

(ii) the position of the motorcyclist when its velocity is 5ms^{-1}.

Solution : We are given

$$x_0 = 5\text{m},\ \upsilon_0 = 3\text{m s}^{-1},\ a = 4\ \text{ms}^{-2}.$$

(i) Using Eqn. (2.7)

$$x = x_0 + \upsilon_0 t + \tfrac{1}{2} at^2$$
$$= 5 + 3 \times 2 + \tfrac{1}{2} \times 4 \times (2)^2 = 19\text{m}$$

From Eqn. (2.6)

$$\upsilon = \upsilon_0 + at$$
$$= 3 + 4 \times 2 = 11\text{ms}^{-1}$$

Velocity, $\upsilon = 11\text{ms}^{-1}$.

(ii) Using equation

$$\upsilon^2 = \upsilon_0^2 + 2a\,(x - x_0)$$
$$(5)^2 = (3)^2 + 2 \times 4 \times (x - 5)$$
$$\Rightarrow \quad x = 7\text{m}$$

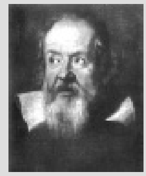
Hence position of the motor cyclist (x) = 7m.

2.5 MOTION UNDER GRAVITY

You must have noted that when we throw a body in the upward direction or drop a stone from a certain height, they come down to the earth. Do you know why they come to the earth and what type of path they follow? It happens because of the gravitational force of the earth on them. The gravitational force acts in the vertical direction. Therefore, motion under gravity is along a straight line. It is a one dimensional motion. ***The free fall of a body towards the earth is one of the most common examples of motion with constant acceleration.*** In the absence of air resistance, it is found that all bodies, irrespective of their size or weight, fall with the same acceleration. Though the acceleration due to gravity varies with altitude, for small distances compared to the earth's radius, it may be taken constant throughout the fall. For our practical use, the effect of air resistance is neglected.

The acceleration of a freely falling body due to gravity is denoted by g. At or near the earth's surface, its magnitude is approximately 9.8 ms^{-2}. More precise values, and its variation with height and latitude will be discussed in detail in lesson 5 of this book.

Galileo Galilei (1564 – 1642)



He was born at Pisa in Italy in 1564. He enunciated the laws of falling bodies. He devised a telescope and used it for astronomical observations. His major works are : Dialogues about the Two great Systems of the World and Conversations concerning Two New Sciences. He supported the idea that the earth revolves around the sun.

Example 2.11 : A stone is dropped from a height of 50m and it falls freely. Calculate the (i) distance travelled in 2 s, (ii) velocity of the stone when it reaches the ground, and (iii) velocity at 3 s i.e., 3 s after the start.

Solution : Given

Height $h = 50$ m and Initial velocity $v_0 = 0$

Consider, initial position (y_0) to be zero and the origin at the starting point. Thus, the y-axis (vertical axis) below it will be negative. Since acceleration is downward in the negative y-direction, the value of $a = -g = -9.8$ ms^{-2}.

(i) From Eqn. (2.7), we recall that

$$y = y_0 + v_0 t + ½ at^2$$

For the given data, we get

$$y = 0 + 0 - ½ gt^2 = -½ \times 9.8 \times (2)^2$$

$$= -19.6\text{m}.$$

The negative sign shows that the distance is below the starting point in downward direction.

(ii) At the ground $y = -50$m,

Using equation (2.8),

$$v^2 = v_0^2 + 2a (y - y_0)$$

$$= 0 + 2 (-9.8) (-50 - 0)$$

$$v = 9.9 \text{ ms}^{-1}$$

(iii) Using $v = v_0 + at$, at $t = 3$s, we get

$$\therefore \quad v = 0 + (-9.8) \times 3$$

$$v = -29.4 \text{ ms}^{-1}$$

This shows that the velocity of the stone at $t = 3$ s is 29.4 m s^{-1} and it is in downward direction.

Note : *It is important to mention here that in kinematic equations, we use certain sign convention according to which quantities directed upwards and rightwards are taken as positive and those downwards and leftward are taken as negative.*

2.6 CONCEPT OF DIFFERENTIATION AND INTEGRATION

All branches of Mathematics have been very useful tools in understanding and explaining the laws of Physics and finding the relations between different Physical quantities. You are already familiar with the use of Algebra and Trigonometry in this connection. In the further study of Physics, you will come across the use of Differentiation (or Differential Calculus) and Integration (or Integral Calculus). A brief and simple description of the concept of Differentiation and Integration is, therefore, being given below. You may consult books on Mathematics for more information on these topics.

We will often come across the following terms in this topic. Let us define these terms:

Constant: It is a quantity whose value does not change during mathematical operations, e.g. integers like 1, 2, 3, …., fractions, π, e, etc.

Variable: It is a quantity which can take different values during mathematical operations. A variable is generally denoted by x, y, z etc.

Function: 'y' is said to be a function of 'x', if for every value of 'x' there is definite value of 'y'.

Mathematically, it is represented by

$$y = f(x)$$

i.e. 'y' is a function of 'x'

Differential Coefficient: Of any variable 'y' with respect to any other variable 'x', is the instantaneous rate of change of 'y' with respect to 'x'.

Let 'y' be a function of 'x' i.e. $y = f(x)$. Suppose 'x' is increased by a very small amount δx or say there is a very small increament 'δx' in 'x'. Let there be a corresponding increament 'δy' in 'y'. Then, $y + \delta y$ is a function of $(x + \delta x)$

or $$y + \delta y = f(x + \delta x)$$

or $$\delta y = f(x + \delta x) - y$$

or $$\frac{\delta y}{\delta x} = \frac{f(x + \delta x) - f(x)}{\delta x}$$

The quantity $\frac{\delta y}{\delta x}$ is called increment ratio and represents the average rate of change of 'y' with respect to 'x' in the range between the time interval x and $(x + \delta x)$.

To find the instantaneous rate of change of 'y' with respect to 'x', we will have to calculate the limit of $\frac{\delta y}{\delta x}$ as δx tends to zero $(\delta x \to 0)$.

i.e. $$\underset{\delta x \to 0}{Lt} \frac{\delta y}{\delta x} = \underset{\delta x \to 0}{Lt} \frac{f(x+\delta x) - f(x)}{\delta x}$$

Thus, the instantaneous rate of change of 'y' with respect to 'x' is given by $\underset{\delta x \to 0}{Lt} \frac{\delta y}{\delta x}$. This is called the **differential coefficient** of 'y' with respect to 'x' and is denoted by $\frac{dy}{dx}$.

Integration

Integration is a mathematical process which is reverse of differentiation. In order to understand this concept, let a constant force **F** act on a body moving it through a distance S. Then, the work done by the force is calculated by the product W = **F·S**.

But, if the force is variable, ordinary algebra does not give any method to find the work done.

For example when a body is to be moved to a long distance up above the surface of the earth, the force of gravity on the body goes on changing as the body moves up. In such cases a method called integration is used to calculate the work done.

The work done by a variable force can be calculated as (see for details 6.2 work done by a variable force)

$$\mathrm{W} = \Sigma F(x)\Delta x$$

For infinitesimally small values of Δx,

$$W = \sum_{\lim \Delta x \to 0} F(x)dx$$

This may be written as

$$W = \int F(x)dx$$

This expression is called integral of function $F(x)$ with respect to x, where the symbol '$\int$' denotes integration.

Some often used formulae of Integration and Differentiation

(i) $\int x^n dx = \frac{x^{n+1}}{n+1}$ (for n $\neq 1$)	(i) $\frac{d}{dx} x^n = nx^{n-1}$
(ii) $\int x^{-1} dx = \int \frac{1}{x} dx = \log x$	(ii) $\frac{d}{dx}(\log x) = \frac{1}{x}$
(iii) $\int dx = \int x^0 dx = \frac{x^1}{1} = x$	(iii) $\frac{d}{dx}(x) = 1$
(iv) $\int cxdx = c\int xdx$ (c is a constant)	(iv) $\frac{d}{dx}(cu) = c\frac{d}{dx}(u)$
(v) $\int (u+v+w)dx = \int udx \pm \int vdx \pm \int wdx$	(v) $\frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$
(vi) $\int e^x dx = e^x$	(vi) $\frac{d}{dx}(e^x) = e^x$
(vii) $\int \sin xdx = -\cos x$	(vii) $\frac{d}{dx}\{\sin(x)\} = +\cos x$
(viii) $\int \cos xdx = \sin x$	(viii) $\frac{d}{dx}(\cos x) = -\sin x$
(ix) $\int \sec^2 xdx = \tan x$	(ix) $\frac{d}{dx}(\tan x) = \sec x$
(x) $\int \text{cosec}^2 xdx = -\cot x$	(x) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

A close look at the table shows that Integration and differentiation are converse mathematical operations.

Take a pause and solve the following questions.

INTEXT QUESTIONS 2.4

1. A body starting from rest covers a distance of 40 m in 4s with constant acceleration along a straight line. Compute its final velocity and the time required to cover half of the total distance.

2. A car moves along a straight road with constant aceleration of 5 ms^{-2}. Initially at 5m, its velocity was 3 ms^{-1} Compute its position and velocity at $t = 2$ s.

3. With what velocity should a body be thrown vertically upward so that it reaches a height of 25 m? For how long will it be in the air?

4. A ball is thrown upward in the air. Is its acceleration greater while it is being thrown or after it is thrown?

WHAT YOU HAVE LEARNT

- The ratio of the displacement of an object to the time interval is known as average velocity.
- The total distance travelled divided by the time taken is average speed.
- The rate of change of the relative position of an object with respect to another object is known as the relative velocity of that object with respect to the other.
- The change in the velocity in unit time is called acceleration.
- The position-time graph for a body at rest is a straight line parallel to the time axis.
- The position-time graph for a uniform motion is a straight line inclined to the time axis.
- A body covering equal distance in equal intervals of time, however small, is said to be in uniform motion.
- The velocity of a particle at any one instant of time or at any one point of its path is called its instantaneous velocity.
- The slope of the position-time graph gives the average velocity.
- The velocity-time graph for a body moving with constant acceleration is a straight line inclined to the time axis.
- The area under the velocity-time graph gives the displacement of the body.
- The average acceleration of the body can be computed by the slope of velocity-time graph.
- The motion of a body can be described by following three equations :

  (i) $v = v_0 + at$

  (ii) $x = x_0 + v_0 t + ½ at^2$

  (iii) $v^2 = v_0^2 + 2a.(x - x_0)$
- Elementary ideas about concepts of differentiation and integration.

ANSWERS TO INTEXT QUESTIONS

2.1

1. Yes. When body returns to its initial postion its velocity is zero but speed is non-zero.

2. Average speed = $\dfrac{2+2}{\dfrac{2}{8}+\dfrac{2}{10}} = \dfrac{4}{9} \times 20 = 8.89 \text{ km h}^{-1}$, average velocity = 0

3. Yes, two cars moving with same velocity in the same direction, will have zero relative velocity with respect to each other.
4. (a) 1 m s^{-1}
   (b) 2 m s^{-1}

2.2

1. See Fig. 2.2.
2. (i) A, (ii) B covers more distance, (iii) B, (iv) A, (v) When they are 3km from the starting point of B.
3. In the uniform motion.
4. (a) is wrong, because the distance covered cannot decrease with time or become zero.

2.3

1. (i) (a) The body starts with a zero velocity.
   (b) Motion of the body between start and 5th seconds is uniformly accelerated. It has been represented by the line OA.

$$a = \frac{15-0}{5-0} = 3 \text{ m s}^{-2}$$

   (c) Motion of the body between 5th and 10th second is a uniform motion (represented by AB). $a = \dfrac{15-15}{15-5} = \dfrac{0}{10} = 0 \text{ m s}^{-2}$.

   (d) Motion between 15th and 25th second is uniformly retarded. (represented by the line BC). $a = \dfrac{0-15}{25-15} = -1.5 \text{ m s}^{-2}$.

   (ii) (a) Average speed = $\dfrac{\text{Distance covered}}{\text{time taken}} = \dfrac{\text{Area of OA BC}}{(25-0)}$

$$= \frac{\left(\frac{1}{2}\times 15\times 5)\right) + (15\times 10) + \left(\frac{1}{2}\times 15\times 10\right)}{25} = \frac{525}{50} = 10.5 \text{ m s}^{-1}.$$

(b) Deccelerated Velocity decreases with time.

(c) Total distance covered $= \left(\frac{20\times15}{2}\right)\text{m} + \left(\frac{10\times7}{2}\right)\text{m} = 185$ m.

$\therefore$ average speed$= \left(\frac{185}{22}\right)\text{ms}^{-1} = 8.4\ \text{ms}^{-1}$.

Total displacement $= \left(\frac{20\times15}{2}\right)\text{m} - \left(\frac{10\times7}{2}\right)\text{m} = 115$ m.

$\therefore$ average velocity $= \frac{115}{22}\text{ms}^{-1} = 5.22\ \text{m s}^{-1}$.

2.4

1. Using $x = x_0 + \upsilon_0 t + ½\, at^2$

   $40 = \frac{1}{2} \times a \times 16$

   $\Rightarrow\ a = 5\ \text{ms}^{-2}$

   Next using $\upsilon^2 = \upsilon_0^2 + 2a\,(x - x_0)$

   $\upsilon = 20\ \text{m s}^{-1}$,

   $20 = 0 + \frac{1}{2} \times 5 \times t^2 \ \Rightarrow\ t = 2\sqrt{2}$ s

2. Using Eqn.(2.9), $x = 21$m, and using Eqn.(2.6), $\upsilon = 13\ \text{m s}^{-1}$.
3. At maximum height $\upsilon = 0$, using Eqn. (2.10), $\upsilon_0 = 7\sqrt{10}\ \text{ms}^{-1} = 22.6\ \text{m s}^{-1}$. The body will be in the air for the twice of the time it takes to reach the maximum height.
4. The acceleration of the ball is greater while it is thrown.