

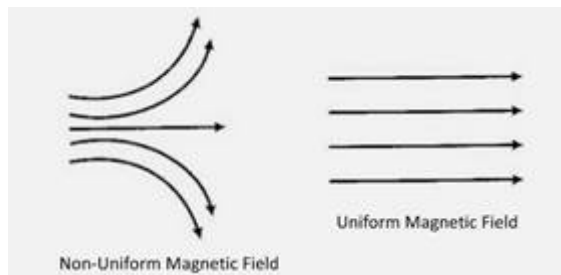
38. Electromagnetic Induction

Short Answer

1. Question

A metallic loop is placed in a non-uniform magnetic field. Will an emf be induced in the loop?

Answer



Given that the magnetic field is non-uniform. The magnetic field will not be uniform throughout the area of the loop. Due to non-uniformity, the magnetic flux lines will be random. In random motion, the field does not vary with time. If there is no change in the magnetic field, it can't induce emf in the loop. So if the metallic loop is placed in a non-uniform magnetic field, the field will not induce emf in it. Thus, Non-uniform magnetic field does not induce the emf.

2. Question

An inductor is connected to a battery through a switch. Explain why the emf induced in the inductor is much larger when the switch is opened as compared to the emf induced when the switch is closed.

Answer

In the above circuit, the inductor is connected to a battery through a switch. In that when the switch is closed, the current will induce in the circuit. The magnetic flux will be increased. Change in flux will induce emf. When the switch has opened the drop in the current occur is more than the increase in the current when the switch is closed.

Due to this large amount of emf is induced in the inductor when the switch is closed as compared to when the switch is opened.

3. Question

The coil of a moving-coil galvanometer keeps on oscillating for a long time if it is deflected and released. If the ends of the coil are connected together, the oscillation stops at once. Explain.

Answer

If the two ends of the coil of the moving-coil galvanometer is connected together, then it will act as a closed loop. thus, the coil no longer acts as an inductor. Therefore, all oscillations stop at once.

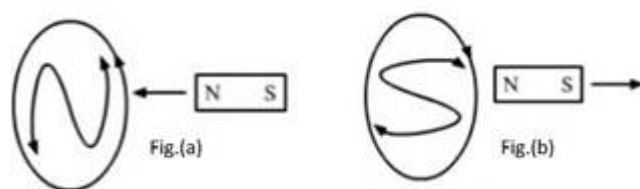
While If the ends of the coil are not connected, then the coil will act as an indicator, it will oscillate up to the current in it decays slowly.

4. Question

A short magnet is moved along the axis of conducting loops. Show that the loop repels the magnet if the magnet is approaching the loop and attracts the magnet if it is going away from the loop.

Answer

Lenz's law: The direction of the induced current is such that it opposes the magnetic field that has induced it.

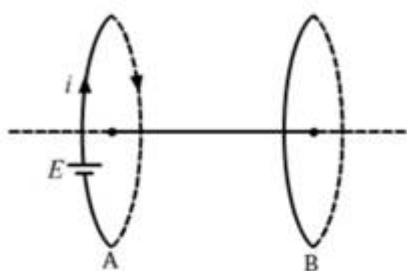


See fig.(a). In that, the north pole is facing towards the loop. According to Lenz's law, the direction of induced current will be anticlockwise. And the magnet is coming towards the loop. Then the flux through the loop will increase; it will create a magnetic field. This newly generated magnetic field will cancel the original magnetic field. Therefore the loops get repelled.

See fig.(b) in which the magnetic is moving away from the loop. It will lead to a decrease in the magnetic field intensity. Therefore, flux through the loop decreases. Therefore, flux through the loop decreases. According to Lenz's law in induced current produces a magnetic field in the opposite direction of the original field. Hence the loop attracts the magnet.

5. Question

Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. Will there be a current induced in the other loop? If yes, when does the current start and when does it end? Do the loops attract each other or do they repel?



Answer

Let us consider the loops as A and B. When the battery is connected to loop A; the current will flow in a clockwise direction. So the direction of the magnetic field due to the current will be towards left as seen from the loop B. Due to a sudden change in flux through loop A, a current will induce in loop B. But it will only be induced for a moment when the current suddenly jumps from zero to a constant value. After it attained a constant value, there will be no induced current in loop B.

According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So the induced current in loop B will be in the opposite direction to the magnetic field of loop A. So the direction of induced current in loop B will be in anticlockwise direction. And the current through loop B will end when the current through loop A becomes zero. Because the directions of the currents in the loops are opposite, they will repel each other.

6. Question

The battery discussed in the previous question is suddenly disconnected. It a current induced in the A pivoted aluminium bar falls much more slowly through a small region containing a magnetic field than a similar bar of insulating material. loop? If yes, when does it start and when does it end?

Answer

When the battery is suddenly disconnected, a current induced in B due to a sudden change in the flux through it. But it is only induced for a moment when the current suddenly falls to zero. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So that the induced current is such that it increases the decrease magnetic field. Therefore, if the current in loop A is in a clockwise direction, then the induced current in loop B will also be the clockwise direction. Hence two loops will attract each other.

Conclusion: If two circular loops are placed coaxially. A battery is connected to one of the loops. After some time, the battery is disconnected then the loops will attract each other.

7. Question

If the magnetic fields outside a copper box are suddenly changed, what happens to the magnetic field inside the box? Such low-resistivity metals are used to form enclosures which shield objects inside them against varying magnetic fields.

Answer

If the magnetic field is suddenly changing, it will induce eddy currents on the walls of the copper box. And because of these eddy currents there will be a magnetic field and that will be in the opposite direction. Copper is a good conductor of electricity, so a magnetic field due to eddy currents will have strong strength. This newly generated magnetic field will cancel the original magnetic field. So the magnetic field inside the box will become zero. In this way, the copper box will become a shield and protect the objects inside in it form varying magnetic fields.

8. Question

Metallic (non-ferromagnetic) and nonmetallic particles in solid waste may be separated as follows. The waste is allowed to slide down an incline over permanent magnets. The metallic particles slow down as compared to the nonmetallic ones and hence are separated. Discuss the role of eddy currents in the process.

Answer

When solid waste (metallic and non-metallic particles) allowed to slide over a permanent magnet, an emf will be induced in metallic particles. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So the induced emf in the metallic particles will oppose the downward motion along the inclined plane of the permanent magnet. And non-metallic particles are free from these effects. In this way, metallic particles slow down and get separated from the non-metallic particles.

9. Question

A pivoted aluminium bar falls much more slowly through a small region containing a magnetic field than a similar bar of insulating material. Explain.

Answer

Non-metallic or insulating materials are free of the effects of induced eddy currents or induced emf. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. Means the induced eddy current opposes its cause. That's why an aluminium bar will fall slowly through a small region containing a magnetic field.

10. Question

A metallic bob A oscillates through the space between the poles of an electromagnet (figure). The oscillations are more quickly damped when the circuit is on, as compared to the case when the circuit is off. Explain.



Answer

When the circuit is on, eddy currents will produce on the surface of the metallic Bob. Eddy currents will generate thermal energy. Thermal energy comes as the loss of the kinetic energy of the Bob. Therefore, oscillations are more quickly damped when the circuit is on compared to when the circuit is off the bob.

Hence, oscillation is more quickly damped when the circuit is on compared to when the circuit is off.

11. Question

Two circular loops are placed with their centres separated by a fixed distance. How would you orient the loops to have

- (a) the largest mutual inductance
- (b) the smallest mutual inductance?

Answer

- (a) The largest mutual induction

Mutual induction will be larger when the two loops are placed coaxially. Then the flux through a loop due to another loop is large. Hence, we will get the largest mutual inductance.

- (b) The mutual inductance will be small when the two loops are placed such that their axis are perpendicular to each other. The flux through the loop due to another loop will be small. Therefore, we will get the smallest mutual inductance.

Thus, we can conclude that to get the largest mutual inductance the loop has to place coaxially and to get smallest mutual inductance, the axis of the loops has to place perpendicularly.

12. Question

Consider the self-inductance per unit length of solenoids at its centre and that near its ends. Which of the two is greater?

Answer

Self-inductance is given by

$$L = \mu_0 n^2 A l$$

Self-inductance per unit length is given by

$$\frac{L}{l} = \mu_0 n^2 A$$

Where μ_0 is the permeability of the free space

n is the number of turns per unit length

A is Area of a cross section of the solenoid

L is the self-inductance

l is the length the wire

Therefore, self-inductance at the centre and that near its ends will be the same since the self -induction is independent of the distance of the point from the centre of the solenoid.

13. Question

Consider the energy density in a solenoid at its centre and that near its ends. Which of the two is greater?

Answer

Energy density is given by

$$\text{Energy density, } u = \frac{B^2}{2\mu_0}$$

Where

U is the energy density of the solenoid

B is the magnetic field intensity of the solenoid

μ_0 is the permeability of the free space.

In the solenoid, energy is stored in the form of magnetic field. For the constant flow of current, the magnetic field inside the solenoid is uniform. So, the energy density in a solenoid is constant. Nothing is greater. The energy density at all the points inside a solenoid is the same.

Objective I

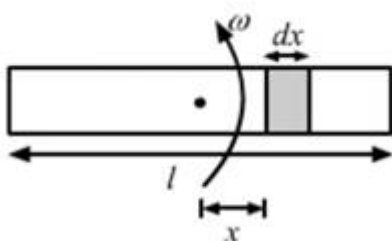
1. Question

A rod of length ℓ rotates with a small but uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the centre of the rod and an end is

- A. zero
- B. $1/8 \omega B \ell^2$
- C. $1/2 \omega B \ell^2$
- D. $B\omega\ell^2$

Answer

Let us consider a small element at a distance from the centre of the rod rotating with angular velocity about its axis perpendicular bisector.



Formula used: The emf induced is given by

$$\epsilon = Bvl$$

The emf is induced in the rod because of the small element is

$$d\epsilon = Bwxdx$$

Where $d\epsilon$ is the emf induced in that small element

B is the magnetic field

w is the angular velocity of the small element

dx is the length of the small element

The emf induced across the centre and the end of the rod is found by the integration of the above relation with respect to x from limit 0 to $\frac{l}{2}$

$$\int d\epsilon = \int_0^{l/2} Bwxdx$$

$$\epsilon = Bw \left[\frac{x^2}{2} \right]_0^{\frac{l}{2}}$$

$$\epsilon = \frac{Bwl^2}{8}$$

The potential difference between the centre of the rod and an end is $\frac{Bwl^2}{8}$

Thus, option B is correct.

2. Question

A rod of length ℓ rotates with a uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

A. zero

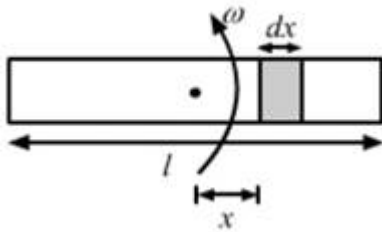
B. $\frac{1}{2} B\ell \omega^2$

C. $B\ell \omega^2$

D. $2B\ell \omega^2$

Answer

Let us consider a small element at a distance from the center of the rod rotating with angular velocity about its axis perpendicular bisector.



Formula used: The emf induced is given by

$$\epsilon = Bvl$$

The emf is induced in the rod because of the small element is

$$d\epsilon = Bwxdx$$

Where $d\epsilon$ is the emf induced in that small element

B is the magnetic field

w is the angular velocity of the small element

$dx \rightarrow$ length of the small element

The emf induced across the centre and the end of the rod is

$$\int d\epsilon = \int_0^{l/2} Bwxdx$$

$$\epsilon = Bw \left[\frac{x^2}{2} \right]_0^{l/2}$$

$$\epsilon = \frac{Bwl^2}{8}$$

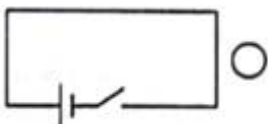
The potential difference between the two ends is

$$\frac{Bwl^2}{8} - \frac{Bwl^2}{8} = 0$$

The potential difference between the two ends of the rod is zero.

3. Question

Consider the situation shown in the figure. If the switch is closed and after some time it is opened again, the closed loop will show.



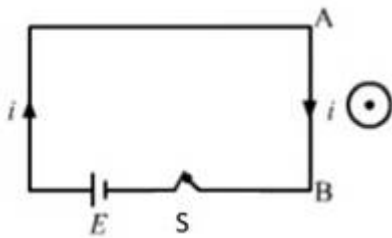
A. an anticlockwise current-pulse

- B. a clockwise current-pulse
- C. an anticlockwise current-pulse and then a clockwise current-pulse
- D. a clockwise current-pulse and then an anticlockwise current-pulse

Answer

Let us consider two points on the circuits A and B.

When the switch S is closed, the current will flow through A & B. Due to this magnetic field introduced in the loop.



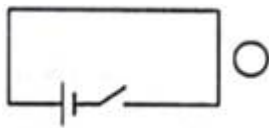
According to Lenz's law the induced current is such that it opposes the increase in the magnetic field that induces it.

So, the induced current will be in clockwise direction, and it opposes the increase in the magnetic field in the upward direction in the loop. When the switch S is opened, the current will fall due to this magnetic field in the loop will decrease. According to Lenz's law, the induced current will be in the anti-clockwise direction opposing the direction in the magnetic field in an upward direction in the loop.

If the switch is closed and after some time it is opened again, the closed loop will show a clockwise current-pulse and then an anticlockwise current-pulse

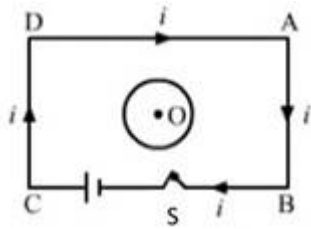
4. Question

Solve the previous question if the closed loop is completely enclosed in the circuit containing the switch.



- A. an anticlockwise current-pulse
- B. a clockwise current-pulse
- C. an anticlockwise current-pulse and then a clockwise current-pulse
- D. a clockwise current-pulse and then an anticlockwise current-pulse

Answer



According to Lenz's law, the induced current is such that it opposes the increase in the magnetic field that induces it. So the induced current will be anticlockwise when the switch S is closed. When the switch S is open, the current will suddenly fall. Then the magnetic field at the centre of the loop will decrease. Then the induced emf will be in the clockwise direction.

If the closed loop is completely enclosed in the circuit containing the switch.

5. Question

A bar magnet is released from rest along the axis of a very long, vertical copper tube. After some time, the magnet.

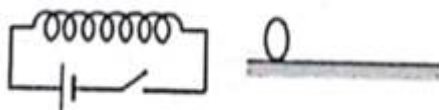
- A. will stop in tube
- B. will move with almost constant speed
- C. will move with an acceleration g
- D. will oscillate.

Answer

The flux linked with the copper tube will change because of the motion of the magnet. This will produce an eddy current in the body of the copper tube. According to the Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it. So the induced Current will be in the opposite direction to the magnet field of the bar magnet. And it will slow the fall of the magnet. Then negative acceleration nothing retarding force will act on the bar magnet. If the velocity of the magnet is increased then the retarding force will also increase. And it will happen up to this retarding force is equal to the force of gravity. Then the total force acting on the bar magnet will become zero. Then the magnet will move with almost a constant speed. So option B is correct.

6. Question

The figure shows a horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring is along the axis of the solenoid. As the switch is closed, the ring will

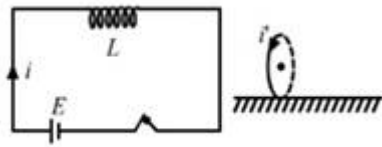


- A. remain stationary
- B. move towards the solenoid

C. move away from the solenoid

D. move towards the solenoid or away from it depending on which terminal (Positive or negative) of the battery is connected to the left end of the solenoid.

Answer



For the circuit

Emf introduced in the solenoid, $E = -L \frac{di}{dt}$

Where

L is the self-inductance of the solenoid

I is the current in the solenoid.

The direction of the current in the loop is clockwise. When the switch is closed the current will flow in the circuit. Therefore, the current on the solenoid will be increase. Then it will induce a current in the copper ring which is placed along a axis of solenoid. According to Lenz's law the induced current is such that it opposes the increase in the magnetic field that induces it. So the induce current in the copper ring will be anticlockwise. Because of the opposite direction of the currents the ring will repel. So it will move away from the solenoid.

Horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will move away fro the solenoid.

7. Question

Consider the following statements:

(A) An emf can be induced by moving a conductor in a magnetic field.

(B) An emf can be induced by changing the magnetic field.

A. Both A and B are true

B. A is true but B is false.

C. B is true but A is false

D. Both A and B are false

Answer

Formula used: emf is given by

$$\epsilon = vBl \text{ --- (1)}$$

Where v is the speed

B is the magnetic field

l is the length

That above formula states that an emf induced by moving a conductor of length l with some velocity v in a magnetic field B .

$$\epsilon = \frac{d\Phi}{dt} \text{ --- (2)}$$

Where Φ is the electric flux through the conductor

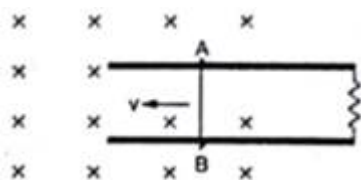
ϵ emf induced in the conductor

Formula (2) states that an emf can be induced by changing the magnetic field that causes the change in flux through a conductor in a loop.

So option A is the answer.

8. Question

Consider the situation shown in the figure. The wire AB is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a semicircular wire, the magnitude of the induced current will



A. increase

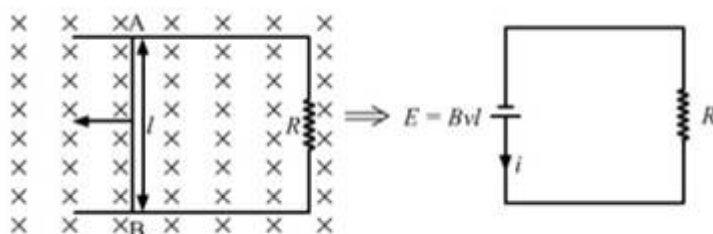
B. remain the same

C. decrease

D. increase or decrease depending on whether the semi-circle bulges towards the resistance or away from it.

Answer

Draw the equivalent circuit



The formula used: The emf induced across A and B is

$$\epsilon = vBl$$

Where v is the speed

B is the magnetic field

l is the length

The induced emf will serve as a voltage source. The direction of the current is anticlockwise, according to Lenz's law.

According to Lenz's law, the direction of the induced current is such that it opposes the magnetic field that has induced it.

$$P = \frac{\epsilon^2}{R}$$

Putting the value in the above equation, we get

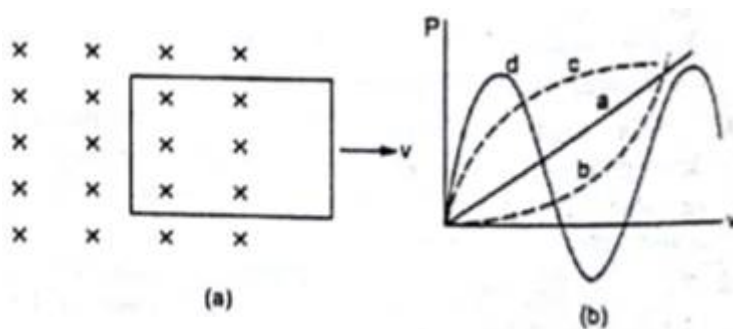
$$= \frac{(v^2 B^2 l^2)}{R}$$

The induced emf depends on the length of the wire but not on the shape of the wire.

If the wire is replaced by a semicircular wire, the induced emf will be same.

9. Question

The figure shows a conducting loop being pulled out of a magnetic field with speed v . Which of the four plots shown in the figure may represent the power delivered by the pulling agent as a function of the speed v ?



A. (a)

B. (b)

Answer

Formula used: The emf developed across the ends of the loop is given by

$$\epsilon = vBl$$

Where v is the speed

B is the magnetic field

L is the length

The power delivered to the loop is

$$P = \frac{\epsilon^2}{R}$$

Where P is the Power delivered to the loop

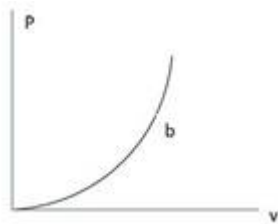
R is the resistance

ϵ is the emf induced

$$P = \frac{(vBl)^2}{R}$$

$$P = \frac{v^2 B^2 l^2}{R}$$

From the above equation, P is proportional to v^2 . Curve b showing this relation.

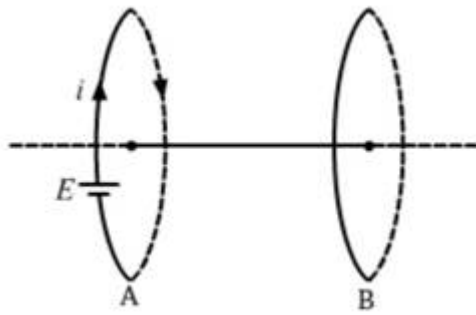


10. Question

Two circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops

- A. attract each other
- B. repel each other
- C. do not exert any force on each other
- D. attract or repel each other depending on the sense of the current.

Answer



By the right-hand screw rule, the magnetic field will be towards the left side. Due to the increase in resistance with temperature, the current through the loop will decrease with time. This change in current will induce the current in loop B. According to Lenz's law, the induced current is such that it opposes the increase in the magnetic field that induces it. So the direction of induced current in loop B will be in a clockwise direction. Because of the directions of currents, they will attract each other.

Thus, Option A is correct.

11. Question

A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is

- A. clockwise
- B. anticlockwise
- C. zero
- D. clockwise or anticlockwise depending on whether the resistance is increased or decreased.

Answer

The magnetic field inside the solenoid is parallel to its axis. If the plane of the loop contains the axis of the solenoid, the angle between the area vector of a circular loop and the magnetic field will be zero.

The formula used: The flux through the circular loop is given by

$$\Phi = BA \cos\theta$$

Where Φ is the electric flux

B is the magnetic field due to the solenoid

A is the Area of the circular loop

θ is the Angle between magnetic field and area vector

$$\Phi = BA \cos 0^\circ, \cos 0^\circ = 1$$

$$\Phi = BA = \text{constant}$$

Then the induced emf is given by

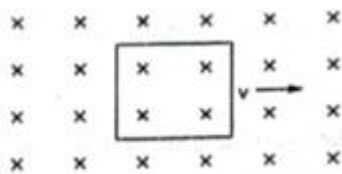
$$\begin{aligned}\epsilon &= \frac{d\Phi}{dt} \\ &= \frac{d}{dt}(BA)\end{aligned}$$

$$\epsilon = 0 \text{ Because } BA \text{ is constant}$$

The induced emf does not depend on the varying current through the solenoid. The induced emf will be zero for a constant flux through the loop. So no current will be induced in the loop.

12. Question

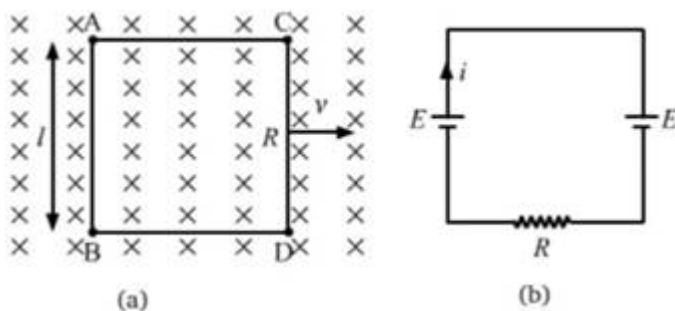
A conducting square loop of side ℓ and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop, as shown in the figure. The current induced in the loop is



- A. $B\ell v/R$ clockwise
- B. $B\ell v/R$ anticlockwise
- C. $2B\ell v/R$ anticlockwise
- D. zero

Answer

if we draw the equivalent circuit, it will be look like



The formula used: The emf induces across the ends AB and CD is given by

$$E = vBl$$

Where E is the emf

V is the velocity

B is the magnetic field

l is the length

Apply KVL in the loop in fig(b).

$$E - E + iR = 0$$

$$i = 0$$

Here the current I become zero means that there is no current induced in the loop. So the current induced in the loop will be zero.

Objective II

1. Question

A bar magnet is moved along the axis of a copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?

- A. The south pole faces the ring and the magnet moves towards it.
- B. The north pole faces the ring and the magnet moves towards it.
- C. The south pole faces the ring, and the magnet moves away from it.
- D. The north pole faces the ring, and the magnet moves away from it.

Answer

A bar magnet is moving along the axis of a copper ring. The movement of a magnet there will be a magnetic field. Then the current will be induced in the copper ring. It is observed that the current is in an anticlockwise direction looking from the magnet. Then the magnetic field induced in the copper ring will be towards the observer (We are observing the current direction from the magnet).

According to Lenz's law, the induced current is such that it opposes the increase in the magnetic field that induces it. If the south pole faces the ring and the magnet is moving towards the then the current induced in the ring will be the clockwise direction. In the same case if the magnet is moving away from the ring, then the current will be in the anticlockwise direction. If the magnet is moving towards the ring and north pole is facing the ring, then the current induced in the ring will be in the anticlockwise direction. If the magnet is moving away from the ring, in this case, the induced current will be in a clockwise direction. So option B and C are correct.

2. Question

A conducting rod is moved with a constant velocity v in a magnetic field. A potential difference appears across the two ends.

A. If $\vec{v} \parallel \vec{\ell}$

B. If $\vec{v} \parallel \vec{B}$

C. If $\vec{\ell} \parallel \vec{B}$

D. None of the above

Answer

The potential difference across the two ends is given by

$$\epsilon = vBl$$

Where ϵ is the emf or potential difference

$v \rightarrow$ velocity of conducting rod

B is the magnetic field

L is the length of the rod

Consider some conditions,

- i. The magnetic field is in the perpendicular direction to the velocity of the rod
- ii. The magnetic field is in the perpendicular direction to the length of the rod
- iii. The velocity of the rod is perpendicular to the direction to the length of the rod

In the above-mentioned conditions, only the potential difference across the two ends will be non zero.

In the given conditions in the A, B and C, the potential difference across the two ends is zero. So option D is correct.

3. Question

A conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the loop if

A. it is translated

B. it is rotated about its axis

C. it is rotated about a diameter

D. it is deformed

Answer

Conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf will be induced only when the magnetic flux changes. If there is no change in magnetic flux, then induced emf will be zero. An emf will be induced in the loop only when

i. The loop is rotated about a diameter

ii. Deforming the loop.

If the loop is deformed, the area of the loop inside the magnetic field changes. So magnetic flux will change leads to induce an emf. On rotating about its axis, the magnetic flux does not change. So an emf induced in the loop is zero.

Thus, Option C and D are correct.

4. Question

A metal sheet is placed in front of a strong magnetic pole. A force is needed to

A. hold the sheet there if the metal is magnetic

B. hold the sheet there if the metal is non-magnetic

C. move the sheet away from the pole with uniform velocity if the metal is magnetic.

D. move the sheet away from the pole with uniform velocity if the metal is non-magnetic.

Neglect any effect of paramagnetism, diamagnetism and gravity.

Answer

A metal sheet is placed in front of the strong magnetic pole. This pole has a strong magnetic field. A strong magnetic pole will attract the magnet if the metal is magnetic then a force is needed to hold the metal sheet. If the metal is magnetic or not if we want to move the metal sheet away from that strong magnetic pole we need some force to do it. Because of the movement, eddy currents will be induced in the sheet. These eddy currents produce thermal energy. Thermal energy comes at the cost of kinetic energy. Thus, the plates slow down. So we need a force to hold the sheet if the metal is magnetic. And also the sheet needs a force to move away from that strong magnetic field even it is nonmagnetic.

So option A, C and D are correct.

5. Question

A constant current i is maintained in a solenoid. Which of the following quantities will increase if an iron rod is inserted in the solenoid along its axis?

A. The magnetic field at the centre

B. magnetic flux linked with the solenoid

C. self-inductance of the solenoid

D. rate of Joule heating.

Answer

The iron rod has high permeability. So if we insert the iron rod in the solenoid along its axis, the magnetic flux inside the solenoid will increase. The self-inductance of the coil is proportional to the permeability of the material inside the solenoid. Because of an increase in the permeability inside the coil, the self-inductance will also increase. Thus, a constant current I will be maintained in the solenoid. Then

1. magnetic field at the centre
2. Magnetic flux linked with the solenoid
3. self-inductance of the solenoid is increased if an iron rod is inserted in the solenoid along its axis.

Thus, option A, B and C is the correct option.

6. Question

Two solenoids have identical geometrical construction, but one is made of thick wire and the other of thin wire.

Which of the following quantities are different for the two solenoids?

- A. self-inductance
- B. rate of Joule heating if the same current goes through them
- C. magnetic field energy if the same current goes through them.
- D. time constant if one solenoid is connected to one battery and the other is connected to another battery.

Answer

The two solenoids are identical. Therefore, the self-inductance of the two solenoids is the same.

The formula used: energy stored in the inductor is given by

$$U = \frac{1}{2} Li^2$$

Where U is the Magnetic energy

L is the self-Inductance

I is the Current

The current in both solenoids is the same. That implies magnetic field energy is also the same.

The power dissipated as heat is given by

$$P = \frac{i^2}{R}$$

Where P is the power

I is the current

R is the Resistance

The time constant is given by

$$t = \frac{L}{R}$$

Where t is the time constant

L is the self-Inductance

R is resistance is given by

$$R = \frac{\rho l}{A}$$

Where ρ is the resistivity

l is the length of the wire

A is the Area of cross section

The two solenoids are differed by the Area of the cross section A.

$$\Rightarrow R_{Thick} < R_{Thin}$$

$$\Rightarrow t_{Thick} > t_{Thin}$$

$$\Rightarrow P_{Thick} > P_{Thin}$$

Thus, the time constant and Joule heat is different for the two solenoids. And self-inductance and magnetic field are the same for the both.

7. Question

An LR circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection?

- A. Current in the circuit
- B. Magnetic field energy in the inductor.
- C. Power delivered by the battery
- D. Emf induced in the inductor

Answer

The formula used: At time t , the current in the LR circuit is given by

$$I = \frac{\epsilon}{R} \left(1 - e^{-\frac{tR}{L}} \right)$$

Where I is the Current

ϵ is the emf

R is the resistance

L is the self-inductance

At time $t = 0$ the current is given by

$$I = \frac{\epsilon}{R} \left(1 - e^{-\frac{0 \times R}{L}} \right)$$

At $t=0$, the current in the circuit is zero.

The magnetic field energy in the inductor is given by

$$U = \frac{1}{2} Li^2$$

Current is zero at $t=0$, So magnetic field energy is also zero.

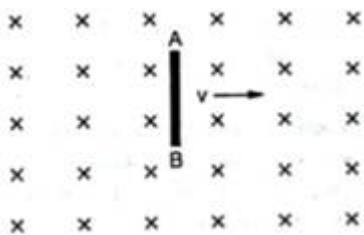
Therefore, power delivered by the battery is also zero at $t=0$.

At $t=0$, the current is on the verge to start growing in the circuit. So there will be an induced emf at that time to oppose the growing current.

In the LR circuit, at time $t=0$ there will be an induced emf in the inductor. Exactly at $t=0$, the current, magnetic field and power delivered by the battery are zero.

8. Question

A rod AB moves with a uniform velocity v in a uniform magnetic field, as shown in the figure.



A. The rod becomes electrically charged.

B. The end A becomes positively charged.

C. The end B becomes positively charged.

D. The rod becomes hot because of Joule heating.

Answer

Due to electromagnetic induction, the emf will be induced in the rod. Induced emf is given by the formula end A becomes more positive because the direction of the induced emf is from A to B. This is because of the magnetic field exerts a same force equal to qvB on each of the electrons, where $q = -1.6 \times 10^{-16} \text{ C}$. According to Fleming's left hand rule (If a conductor is placed in a magnetic field then the force acting on the conductor is in the perpendicular direction to the magnetic field and current direction) the, end A becomes positive and, end B becomes negatively charged.

Conclusion: A rod AB moves with a uniform velocity v in a uniform magnetic field then the end A becomes positively charged.

9. Question

L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. Which of the following combinations have dimensions of frequency?

A. $\frac{1}{RC}$

B. $\frac{R}{L}$

C. $\frac{1}{\sqrt{LC}}$

C. C/L

Answer

The formula used: For RC circuit the time constant is

$$\tau = RC$$

Where τ is the time constant

R is the resistance

C is the capacitance

Frequency $f = \frac{1}{\tau}$

Then the frequency of the RC circuit is given by

$$f_1 = \frac{1}{RC}$$

For LR circuit the time constant is given by

$$\tau = \frac{L}{R}$$

Where τ is the time constant

L is the inductance

R is the resistance

Then the frequency is given by

$$f_2 = \frac{R}{L}$$

Let's take a multiplication of f_1 and f_2

$$f_1 \times f_2 = \frac{1}{RC} \times \frac{R}{L}$$

$$f_1 f_2 = \frac{1}{LC}$$

$$\Rightarrow \sqrt{f_1 f_2} = \frac{1}{\sqrt{LC}}$$

This combination $\sqrt{f_1 f_2} = \frac{1}{\sqrt{LC}}$ has a dimension of frequency.

The above combination has a dimension of frequency

i. RC has a frequency component $f_1 = \frac{1}{RC}$

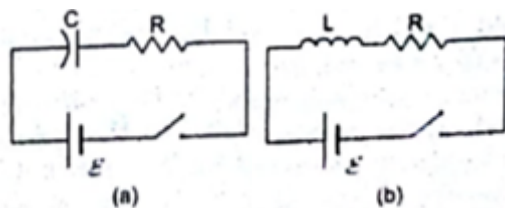
ii. LR has a frequency component $f_2 = \frac{R}{L}$

iii. $f_1 f_2$ has a frequency component $\sqrt{f_1 f_2} = \frac{1}{\sqrt{LC}}$

Thus, option A, B and C are correct.

10. Question

The switches in figure (a) and (b) are closed at $t = 0$ and reopened after a long time at $t = t_0$.



A. The charge on C just after $t = 0$ is ϵC .

B. The charge on C long after $t = 0$ is ϵC .

C. The current in L just before $t = t_0$ is ϵ/R

D. The current in L long after $t = t_0$ is ϵ/R .

Answer

Formula used:

The charge on capacitor at time t , when the switch is closed is

$$Q = \epsilon C \left(1 - e^{-\frac{t}{RC}} \right)$$

Where Q is the Charge

ϵ is the emf

T is the Time period

R is the Resistance

C is the Capacitance

At $t=0$, the charge on the capacitor is

$$Q = \epsilon C \left(1 - e^{-\frac{0}{RC}} \right) = 0$$

After a long time $t_0 = \infty$, the charge on the capacitor is

$$Q = \epsilon C \left(1 - e^{-\frac{\infty}{RC}} \right) = \epsilon C$$

The current in inductor L is given by

$$I = \frac{\epsilon}{R} \left(1 - e^{-\frac{tR}{L}} \right)$$

Where I is the current

ϵ is the voltage or potential difference

R is the Resistance

L is the inductance

T is the time period

At initial we take, $t=0$, the current in the inductor is given by

$$I = \frac{\epsilon}{R} \left(1 - e^{-\frac{0 \times R}{L}} \right)$$

$$I = 0$$

After a long time $t_0 = \infty$ is

$$I = \frac{\epsilon}{R} \left(1 - e^{-\frac{t_0 \times R}{L}} \right)$$

$$I = \frac{\epsilon}{R} \left(1 - e^{-\frac{\infty \times R}{L}} \right)$$

$$I = \frac{\epsilon}{R}$$

The charge on C long after $t=0$ is ϵC

The current in L long after $t = t_0$ is $\frac{\epsilon}{R}$

Exercises

1. Question

Calculate the dimension of

$$(a) \int \vec{E} \cdot d\vec{l}, (b) vB\ell (c) \frac{d\phi_3}{dt}.$$

The symbols have their usual meanings.

Answer

(a) using faraday's law of induction

$$\int \vec{E} \cdot d\vec{l} = \epsilon$$

Where ϵ is the emf or voltage

$\int \vec{E} \cdot d\vec{l}$ has thus the dimensions of voltage

Voltage is given by formula

$$V = \frac{W}{Q}$$

Where

W=work done

Q=charge

Dimensions of W= $[ML^2T^{-2}]$

Dimensions of Q= $[AT]$

The dimensions of voltage can be given as

$$V = \frac{[ML^2T^{-2}]}{[AT]}$$

$$V = [ML^2T^{-3}A^{-1}] \dots (i)$$

(b) vBl is the motional emf developed due to motion of conductor of length l with velocity v in a magnetic field (B)

Therefore, vBl has same dimensions as of voltage

Therefore, by eqn.(i)

$$vBl = [ML^2T^{-3}A^{-1}]$$

(c) by faraday's law of electromagnetic induction

$$\epsilon = - \frac{d\phi}{dt}$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

thus $\frac{d\phi}{dt}$ has same dimensions as of emf or voltage

therefore, by eqn.(i)

$$\frac{d\phi}{dt} = [ML^2A^{-1}T^{-3}]$$

Therefore $\int \vec{E} \cdot d\vec{l}$, vBl and $\frac{d\phi}{dt}$ all have same dimensional formula equal to $[ML^2A^{-1}T^{-3}]$

2. Question

The flux of magnetic field through a closed conducting $\phi = at^2 + bt + c$.

(a) Write the S.I. units of a , b and c .

(b) If the magnitudes of a , b , and c are 0.20, 0.40 and 0.60 respectively, find the induced emf at $t = 2s$.

Answer

Given:

flux of magnetic field $\phi = at^2 + bt + c$

magnitudes of a , b and $c = 0.20, 0.40$ and 0.60

(a) We know that,

Dimensions on both sides of an equation are equal and only terms with same dimensions can be added

Therefore

Dimensions of ϕ , at^2 , bt and c are same

Thus units of a are given by

$$a = \frac{\phi}{t^2} = \frac{\frac{\phi}{t}}{t}$$

Now,

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt} \dots (i)$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

therefore, units of $\frac{\phi}{t}$ is same as voltage i.e. Volt

$$\therefore \text{unit of } a = \text{Volt/sec}$$

Unit of b is given by

$$b = \frac{\phi}{t}$$

Using eqn.(i)

$$\text{unit of } b = \text{Volt}$$

SI unit of c is given by

$$c = \phi$$

$$\therefore \text{SI unit of } c \Rightarrow \text{SI unit of flux} \Rightarrow \text{Weber}$$

(b)

We know that

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

here ϵ is given by

$$\epsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(at^2 + bt + c) = -(2at + b)$$

Putting the values of a, b and t=2sec we have

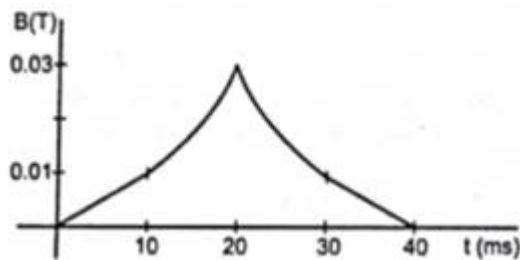
$$\epsilon = 2 \times 0.2 \times 2 + 0.4 = 1.2 \text{ Volt}$$

Therefore, the induced emf at t=2sec is given by 1.2Volts

3. Question

(a) The magnetic field in a region varies as shown in figure. Calculate the average induce emf in a conducting loop of area $2.0 \times 10^{-3} \text{ m}^2$ placed perpendicular to the field in each of the 10 ms intervals shown.

(b) In which intervals is the emf not constant? Neglect the behavior near the ends of 10 ms intervals.



Answer

Given:

$$\text{Area of loop} = 2.0 \times 10^{-3} \text{ m}^2$$

We know that

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ =emf produced

ϕ =flux of magnetic field

so, the average induced emf in the conducting loop between time intervals t_1 and t_2 is given by

$$\epsilon = \frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} \dots (ii)$$

Also

Magnetic flux through the circular ring of area A is given by

$$\phi = \vec{B} \cdot \vec{A}$$

Since loop is placed perpendicular to the field,

$$\phi = BA$$

$$(i) \ t_1 = 0 \text{ and } t_2 = 10ms$$

Using eqn.(ii)

$$\epsilon = - \frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = - \frac{A(B_2 - B_1)}{t_2 - t_1} \dots (iii)$$

Putting the values of t_1, t_2, B_1 and B_2 we get

$$\epsilon = -2 \times \frac{10^{-3}(0.01)}{10 \times 10^{-3}} = -2 \times 10^{-3} = -2mV$$

$$(ii) \ t_1 = 10ms \text{ and } t_2 = 20ms$$

Using eqn.(ii)

$$\epsilon = - \frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = - \frac{A(B_2 - B_1)}{t_2 - t_1} \dots (iii)$$

Putting the values of t_1, t_2, B_1 and B_2 we get

$$\epsilon = -2 \times \frac{10^{-3}(0.02)}{10 \times 10^{-3}} = -4 \times 10^{-3} = -4mV$$

$$(iii) \ t_1 = 20ms \text{ and } t_2 = 30ms$$

Using eqn.(ii)

$$\epsilon = - \frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = - \frac{A(B_2 - B_1)}{t_2 - t_1} \dots (iii)$$

Putting the values of t_1, t_2, B_1 and B_2 we get

$$\epsilon = -2 \times \frac{10^{-3}(-0.02)}{10 \times 10^{-3}} = 4 \times 10^{-3} = 4mV$$

$$(iv) \ t_1 = 30ms \text{ and } t_2 = 40ms$$

Using eqn.(ii)

$$\epsilon = - \frac{\Delta\phi_2 - \Delta\phi_1}{t_2 - t_1} = - \frac{A(B_2 - B_1)}{t_2 - t_1} \dots (iii)$$

Putting the values of t_1, t_2, B_1 and B_2 we get

$$\epsilon = -2 \times \frac{10^{-3}(-0.01)}{10 \times 10^{-3}} = 2 \times 10^{-3} = 2\text{mV}$$

\therefore emf across the intervals

0-10ms = -2mV

10-20ms = -4mV

20-30ms = 4mV

30-40ms = 2mV

From the graph we can see that flux varies in a nonlinear fashion between time intervals 10-20 ms and 20-30 ms and hence the derivative of flux wrt time is not constant in the given interval

\therefore emf is not constant in time intervals 10-20 ms and 20-30ms

4. Question

A conducting circular loop having a radius of 5.0 cm, is placed perpendicular to a magnetic field of 0.50 T. It is removed from the field in 0.50 s. Find the average emf produced in the loop during this time.

Answer

Given:

Radius of circular loop = 5.0 cm

Magnetic field intensity = 0.50 T

Area of circular loop $A = \pi r^2$

Initial magnetic flux through the loop is given by

$$\phi_1 = BA \cos 0^\circ = BA$$

(As loop is placed perpendicular to magnetic field)

After loop is removed from the field after time $\Delta t = 0.50$ s the magnetic flux

$$\phi_2 = 0$$

(As no magnetic field passes through the loop)

We know that,

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1) / \Delta t$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Putting the values of ϕ_1 , ϕ_2 and Δt we get

$$\epsilon = -\frac{0 - BA}{\Delta t} = \frac{B\pi r^2}{\Delta t} = 0.50 \times \pi \times \frac{(5 \times 10^{-2})^2}{0.50} = 25\pi \times 10^{-4}$$
$$= 7.8 \times 10^{-3} \text{ Volt}$$

$$\epsilon = 7.8 \times 10^{-3} \text{ Volt}$$

Therefore, average induced emf produced in the loop during this time interval is 7.8×10^{-3} Volts.

5. Question

A conducting circular loop of area 1 mm^2 is placed co-planarly with a long, straight wire at a distance of 20 cm from it. The straight wire carries an electric current which changes from 10 A to zero in 0.1 s. Find the average emf induced in the loop in 0.1 s.

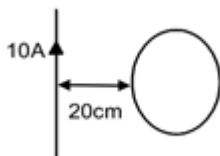
Answer

Given:

Area of circular loop $A = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$

Separation between wire and loop $d = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$

Current through the wire $i = 10 \text{ A}$



We know that magnetic field (B) due to an infinitely long wire carrying a current i at a perpendicular distance d from the wire is given by

$$B = \frac{\mu_0 i}{2\pi d}$$

Also,

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Magnetic flux due to magnetic field B through cross section area A is given by

$$\phi = \vec{B} \cdot \vec{A}$$

Since magnetic field due to long wire is perpendicular to the circular loop, initial magnetic flux through the loop is given by

$$\phi = BA = \frac{\mu_0 i}{2\pi d} A$$

After time interval of 0.1s current in the wire becomes zero and hence magnetic field

$$\therefore \phi_2 = 0$$

Average induced emf is given by

$$\begin{aligned} \epsilon &= -\frac{(\phi_2 - \phi_1)}{\Delta t} = -\frac{0 - BA}{\Delta t} = \frac{\frac{\mu_0 i}{2\pi d} A}{\Delta t} \\ &= 4\pi \times 10^{-7} \times 10 \times \frac{10^{-6}}{2\pi \times 2 \times 10^{-1} \times 10^{-1}} = 10^{-10} \text{ Volt} \end{aligned}$$

$$\epsilon = 10^{-10} \text{ V}$$

Therefore, average emf induced in the loop in time interval of 0.1s is given by 10^{-10} V

6. Question

A square-shaped copper coil has edges of length 50 cm and contains 50 turns. It is placed perpendicular to a 1.0T magnetic field. It is removed from the magnetic field in 0.25 s and restored in its original place in the next 0.25 s. Find the magnitude of the average emf induced in the loop during

(a) its removal, (b) its restoration and (c) its motion.

Answer

Given:

Length of side of square = $50 \text{ cm} = 0.5 \text{ m}$

No. of turns = 50

Intensity of magnetic field = 1.0T

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1) / \Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Magnetic flux due to magnetic field B through cross section area A is given by

$$\phi = \vec{B} \cdot \vec{A}$$

(a) During removal

Initial magnetic flux through the loop

$$\phi_1 = B.A = 50 \times 0.5 \times 0.5 = 12.5 \text{ Tm}^2$$

Final magnetic flux through the loop

$$\phi_2 = 0$$

Time interval $\Delta t = 0.25\text{s}$

Using eqn.(i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - 12.5}{0.25} = 50\text{V}$$

Magnitude of average emf during removal is 50V

(b) During restoration

Initial magnetic flux through the loop

$$\phi_1 = 0$$

Final magnetic flux through the loop

$$\phi_2 = BA = 1 \times 50 \times 0.5 \times 0.5 = 12.5 \text{ Tesla} - \text{m}^2$$

Time interval $\Delta t = 0.25\text{s}$

Using eqn.(i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{12.5 - 0}{0.25} = -50\text{V}$$

Magnitude of average emf during restoration is 50V

(c) During the motion

Initial magnetic flux through the loop

$$\phi_1 = 0$$

Final magnetic flux through the loop

$$\phi_2 = 0$$

Time interval $\Delta t = 0.25 + 0.25 = 0.50\text{s}$

Using eqm. (i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = 0$$

Therefore, magnitude of average emf during its motion is zero

7. Question

Suppose the resistance of the coil in the previous problem is 25Ω . Assume that the coil moves with uniform velocity during its removal and restoration. Find the thermal energy developed in the coil during

- (a) its removal
- (b) its restoration and
- (c) its motion.

Answer

Given

Resistance of the coil $r = 25\Omega$

- (a) during its removal

the emf induced in the *loop* $\epsilon = 50V$

so the current flowing through the loop is given by

$$i = \frac{\epsilon}{r} = \frac{50}{25} = 2A$$

Thermal energy developed in the coil (H) during its removal in time interval $\Delta t = 0.25s$ is given by

$$H = i^2 r \Delta t = 4 \times 25 \times 0.25 = 25J$$

Therefore, energy developed in the coil during removal is 25J

- (b) during its restoration

The magnitude of emf induced in the loop $\epsilon = 50V$

so the current flowing through the loop is given by

$$i = \frac{\epsilon}{r} = \frac{50}{25} = 2A$$

Thermal energy developed in the coil (H) during its removal in time interval $\Delta t = 0.25s$ is given by

$$H = i^2 r \Delta t = 4 \times 25 \times 0.25 = 25J$$

Therefore, energy developed in the coil during restoration is 25J

(c) during motion

total thermal energy developed = energy developed during removal + energy developed during restoration

$$\Rightarrow H = 25 + 25 = 50J$$

As heat energy is scalar quantity, it is algebraically added

Therefore, total energy developed in coil during its motion is 50J

8. Question

A conducting loop of area 5.0 cm^2 is placed in a magnetic field which varies sinusoidally with time as $B = B_0 \sin \omega t$ where $B_0 = 0.20 \text{ T}$ and $\omega = 300 \text{ s}^{-1}$. The normal to the coil makes an angle of 60° with the field. Find

(a) the maximum emf induced in the coil,

(b) the emf induced at $\tau = (\pi/900) \text{ s}$ and

(c) the emf induced at $t = (\pi/600) \text{ s}$.

Answer

Given:

$$\text{Area of conducting loop} = 5.0 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

$$\text{Variation of magnetic field with time } B = B_0 \sin \omega t$$

$$\text{Angle of field with normal to coil } \theta = 60^\circ$$

Magnetic flux due to magnetic field B through cross section area A is given by

$$\phi = \vec{B} \cdot \vec{A} = BA \cos \theta$$

Here flux through the loop is given by

$$\phi = BA \cos 60^\circ = B_0 \sin \omega t \times \cos 60^\circ. \text{ (i)}$$

Also,

by faraday's law of electromagnetic induction

$$\epsilon = - \frac{d\phi}{dt}$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

using eqn.(i)

we get

$$\epsilon = -\frac{d}{dt}\left(\frac{B_0 A \sin \omega t}{2}\right) = -\frac{B_0 A \omega \cos \omega t}{2} \dots (ii)$$

Since maximum value of $\cos \omega t = 1$

Therefore, maximum value of magnitude of emf induced in the loop is given by

$$\epsilon_{max} = \frac{B_0 A \omega}{2}$$

Putting the values of B_0 and ω we get,

$$\epsilon_{max} = 0.2 \times 5 \times 10^{-4} \times \frac{300}{2} = 0.015V$$

Therefore, maximum value of emf induced in the coil is 0.015V

(b) from eqn.(ii), we have

$$\epsilon = -\frac{B_0 A \omega \cos \omega t}{2}$$

At $t = \pi/900$ s magnitude of induced emf is given by

$$\epsilon = 0.015 \times \cos\left(300 \times \frac{\pi}{900}\right) = \frac{0.015}{2} = 7.5 \times 10^{-3}V$$

Therefore magnitude of induced emf at $t = \pi/900$ is $7.5 \times 10^{-3}V$

(c) from eqn.(ii), we have

$$\epsilon = -\frac{B_0 A \omega \cos \omega t}{2}$$

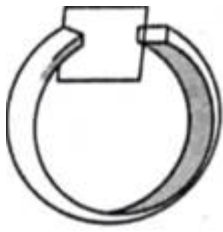
At $t = \pi/600$ s magnitude of induced emf is given by

$$\epsilon = 0.015 \times \cos\left(300 \times \frac{\pi}{600}\right) = 0.015 \times \cos\left(\frac{\pi}{2}\right) = 0V$$

Therefore magnitude of induced emf at $t = \pi/600$ is 0V

9. Question

Figure shows a conducting square loop placed parallel to the pole-faces of a ring magnet. The pole-faces have an area of 1 cm^2 each and the field between the poles is 0.10 T. The wires making the loop are all outside the magnetic field. If the magnet is removed in 1.0 s, what is the average emf induced in the loop?



Answer

Given:

Area of pole faces $A = 1\text{cm}^2 = 10^{-4}\text{m}^2$

Magnetic field intensity $= 0.1\text{T}$

Time taken to remove the magnet completely $\Delta t = 1\text{s}$

Initial magnetic flux through the loop is given by formula

$$\phi = \vec{B} \cdot \vec{A}$$

Since magnetic field through the square loop is perpendicular to the loop above eqn. reduces to

$$\phi_1 = BA \dots (i)$$

When the magnet is removed after 1s the magnetic flux passing through the square loop becomes zero

$$\therefore \phi_2 = 0 \dots (ii)$$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots (iii)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Using eqns.(i), (ii) and (iii) we get,

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - BA}{\Delta t} = \frac{BA}{\Delta t}$$

Putting the values of B, A and Δt in the above eqn.

$$\epsilon = 0.1 \times \frac{10^{-4}}{1} = 10^{-5}\text{V}$$

Therefore average emf induced in the square loop is 10^{-5}V

10. Question

A conducting square loop having edges of length 2.0 cm is rotated through 180° about a diagonal in 0.20 s. A magnetic field B exists in the region which is perpendicular to the loop in its initial position. If the average induced emf during the rotation is 20 mV, find the magnitude of the magnetic field

Answer

Given:

Average induced emf in the loop $\epsilon = 20\text{mV} = 2 \times 10^{-2}\text{V}$

Time taken to rotate the loop $\Delta t = 0.2\text{s}$

Edge length of square loop $= 2\text{cm} = 0.02\text{m}$

Area of square loop $A = 0.02^2 = 4 \times 10^{-4}$

We know that,

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Magnetic flux(ϕ) through the loop is given by the formula

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

Where B=magnetic field intensity

A=area of cross section

θ =angle between area vector and magnetic field

Initially, angle between area vector and magnetic field is 0°

Therefore, initial flux through the coil is

$$\phi_1 = BA \cos 0^\circ = BA$$

When it is rotated by 180° flux passing through the coil is given by

$$\phi_2 = BA \cos 180^\circ = -BA$$

Putting these values in eqn.(i) we get,

$$\epsilon = -\frac{(\phi_2 - \phi_1)}{\Delta t} = -\frac{-BA - BA}{\Delta t} = \frac{2BA}{\Delta t}$$

Putting the values of ϵ , B and Δt in the above eqn.

$$2 \times 10^{-2} = 2 \times B \times 4 \times \frac{10^{-4}}{0.2}$$

$$B = 20 \times \frac{10^{-3}}{4 \times 10^{-3}} = 5T$$

Therefore, magnitude of magnetic field intensity is 5T

11. Question

A conducting loop of face-area A and resistance R is placed perpendicular to a magnetic field B. The loop is withdrawn completely from the field. Find the charge which flows through any cross-section of the wire in the process. Note that it is independent of the shape of the loop as well as the way it is withdrawn.

Answer

Given:

Face area of loop =A

Resistance of loop=R

Magnetic field intensity =B

Magnetic flux(ϕ) through the loop is given by the formula

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

Where B=magnetic field intensity

A=area of cross section

θ =angle between area vector and magnetic field

initially loop is perpendicular to the applied magnetic field hence initial flux is

$$\phi_1 = BA \cos 0^\circ = BA$$

Finally, when the loop is withdrawn from the field flux is given by

$$\phi_2 = 0$$

Now,

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots(i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively

Using eqn.(i) we get

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - BA}{\Delta t} = \frac{BA}{\Delta t}$$

Current flowing in the loop is calculated by using formula

$$i = \frac{\epsilon}{R} = \frac{BA}{R\Delta t}$$

Hence the charge (Q) flowing through the loop is

$$Q = i\Delta t = \frac{BA}{R\Delta t} \times \Delta t = \frac{BA}{R}$$

Therefore, charge flowing through any cross-section of the wire is BA/R

12. Question

A long solenoid of radius 2 cm has 100 turns/cm and carries a current of 5 A. A coil of radius 1 cm having 100 turns and a total resistance of 20 Ω is placed inside the solenoid coaxially. The coil is connected to a galvanometer. If the current in the solenoid is reversed in direction, find the charge flown through the galvanometer.

Answer

Given:

Radius of solenoid $r = 2\text{cm} = 0.02\text{m}$

No. of turns in the solenoid $n = \frac{100}{\text{cm}} = 10000\text{m}^{-1}$

Current in the solenoid $i = 5\text{A}$

Radius of second coil $r' = 1\text{cm} = 0.01\text{m}$

No. of turns in the coil $N = 100$

Resistance of the coil $R = 20\Omega$

We know that,

Magnetic field inside solenoid (B) is given by formula

$$B = \mu_0 ni$$

Where,

n=no. of turns per unit length

i=current through solenoid

Magnetic flux(ϕ) through the coil is given by the formula

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

Where B=magnetic field intensity

A=area of cross section of the coil

θ =angle between area vector and magnetic field

magnetic field inside solenoid is perpendicular to the coil

initially flux through the coil is given by

$$\phi_1 = BA \cos 0^\circ = \mu_0 n i \times \pi r'^2 \times N = \mu_0 N n i \pi r'^2$$

When the current in the solenoid is reversed in direction of magnetic field gets reversed and flux through the coil now ϕ_2 becomes

$$\phi_2 = BA \cos 180^\circ = -BA = -\mu_0 N n i \pi r'^2$$

$$\phi_2 = 0$$

Now,

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1) / \Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively

Putting these values in eqn.(i) we get

$$\epsilon = \frac{2\mu_0 N n i \pi r'^2}{\Delta t}$$

Current (i) through the coil of resistance R can be calculated as

$$i = \frac{\epsilon}{R} = \frac{2\mu_0 N n i \pi r'^2}{R \Delta t}$$

Hence the charge (Q) passing through the coil in time Δt is

$$Q = i \Delta t = \frac{2\mu_0 N n i \pi r'^2}{R}$$

Putting the values of μ_0 , I, N, $n \pi r'$ and R in above eqn.

$$Q = 2 \times 4\pi \times 10^{-7} \times 100 \times 10^4 \times 5 \times 3.14 \times \frac{10^{-4}}{20} = 2 \times 10^{-4} C$$

Therefore flowing through the galvanometer is $2 \times 10^{-4} \text{ C}$

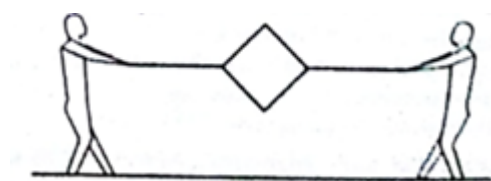
13. Question

Figure shown a metallic square frame of edge a in a vertical plane. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the figure. Two boys pull the opposite corners of the square to deform it into a rhombus. They start pulling the corners at $t = 0$ and displace the corners at a uniform speed u .

(a) Find the induced emf in the frame at the instant when the angles at these corners reduced to 60° .

(b) Find the induced current in the frame at this instant if the total resistance of the frame is R .

(c) Find the total charge which flows through a side of the frame by the time the square is deformed into a straight line.



Answer

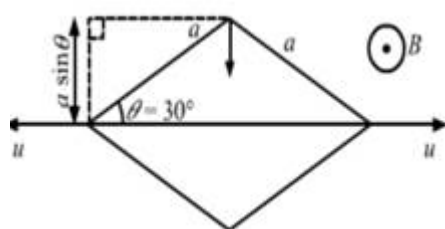
Given:

Edge length of square frame $= a$

Magnetic field intensity B

Speed of corners of rhombus $= u$

(a) when the angles at the corner reduce to 60°



We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

Motional emf is produced in each side of rhombus and the effective length of each side is the length perpendicular to velocity of corners.

From the fig. the effective length of each side is

$$l_{eff} = a \sin 30^\circ = \frac{a}{2}$$

Since velocity is perpendicular to magnetic field the equation of emf induced in each side given by

$$\epsilon = vBl = \frac{uBa}{2}$$

Total emf induced in all four side is

$$\epsilon = 4 \times \frac{uBa}{2} = 2uBa$$

Therefore, induced emf in the frame when the angles at the corner reduces to 60° is $2uBa$

(b) total resistance of the frame =R

hence current flowing in the frame is

$$i = \frac{\epsilon}{R} = \frac{2uBa}{R}$$

Therefore, current flowing in the frame at this instant is $2uBa/R$

(c) initially the frame is in form of square of side a and area a^2

at this time flux through this frame is given by

$$\phi_1 = B.A = Ba^2$$

Finally, when the frame reduces to straight line flux passing through the frame reduces to zero

$$\phi_2 = 0$$

Average emf induced in the frame in time t is given by

$$\epsilon = -\frac{\phi_2 - \phi_1}{t} = -\frac{0 - Ba^2}{t} = \frac{Ba^2}{t}$$

The current flowing through the frame is then given by

$$i = \frac{\epsilon}{R} = \frac{Ba^2}{Rt}$$

Where R is the resistance of frame

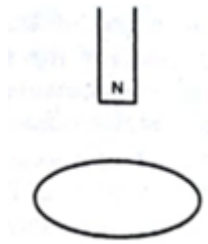
Hence the charge (Q) flowing through the side of frame in time t is

$$Q = it = \frac{Ba^2}{R}$$

Therefore total charge flowing through side of frame by the time the frame reduces to straight line is Ba^2/R

14. Question

The north pole of a magnet is brought down along the axis of a horizontal circular coil (figure). As a result, the flux through the coil changes from 0.35 weber to 0.85 weber in an interval of half a second. Find the average emf induced during this period. Is the induced current clockwise or anticlockwise as you look into the coil from the side of the magnet?



Answer

Given:

Initial flux $\phi_1 = 0.35Wb$

Final flux $\phi_2 = 0.85Wb$

Time interval $\Delta t = 0.5s$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Putting the values in eqn.(i) we get,

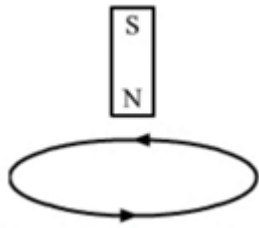
$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0.85 - 0.35}{0.5} = -1V$$

Magnitude of induced emf = 1V

From Lenz's law,

The direction of induced current is such that it opposes the change that has induced it.

According to fig.



The flux in the downward direction increases, so the current is induced such that flux in upward direction increases. Therefore, current is induced in anticlockwise direction

Therefore, average emf induced in the coil is 1V in anti-clockwise direction

15. Question

A wire-loop confined in a plane is rotated in its own plane with some angular velocity. A uniform magnetic field exists in the region. Find the emf induced in the loop.

Answer

Let the area of the wire-loop be A and magnetic field intensity be B .

When the wire rotates in its own plane the area through which flux passes remains same and B is also constant.

Hence, the flux passing through the loop remains constant and is given by

$$\phi = BA$$

Now,

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

as flux is constant (independent of time), its derivative with respect to time is zero. Therefore, emf induced in the loop is zero

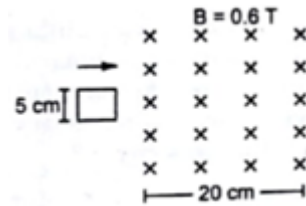
therefore, zero emf is induced in the wire-loop

16. Question

Figure shows a square loop of side 5 cm being moved towards right at a constant speed of 1 cm/s. The front edge enters the 20 cm wide magnetic field at $t = 0$. Find the emf induced in the loop at

(a) $t = 2s$, (b) $t = 10s$

(c) $t = 22s$, (d) $t = 30s$



Answer

Given:

Side length of square loop = $5\text{ cm} = 0.05\text{ m}$

Speed of square loop = $1\text{ cm s}^{-1} = 0.01\text{ m s}^{-1}$

Width of magnetic field = $20\text{ cm} = 0.2\text{ m}$

Magnetic field intensity = 0.6 T

(a) $t = 2s$

distance moved by the loop = $0.01 \times 2 = 0.02\text{ m}$

area of the loop under magnetic field =

area of rectangle of length 0.05 m and width 0.02 m

$$\Rightarrow 0.02 \times 0.05 = 10^{-3}\text{ m}^2$$

Now,

Initial magnetic flux through the loop $\phi_1 = 0$ (at $t=0$)

Final magnetic flux through the loop is given by

$$\phi = \vec{B} \cdot \vec{A}$$

$$\therefore \phi_2 = 0.6 \times 10^{-3} = 6 \times 10^{-4}\text{ Wb}$$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Putting the values of ϕ_1 , ϕ_2 and $\Delta t = 2s$ in eqn.(i),

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{6 \times 10^{-4} - 0}{2} = -3 \times 10^{-4}\text{ V}$$

Therefore magnitude of induced emf at t=2s is $3 \times 10^{-4}V$

(b) t=10s

distance moved by the square loop $0.01 \times 10 = 0.1m$

at this moment, square loop is completely inside the magnetic field and area of loop through which flux pass = $0.05 \times 0.05 = 25 \times 10^{-4}m^2$

so the flux linkage does not changes with time $\therefore \Delta\phi = 0$

and thus from eqn.(i)

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = 0$$

Therefore, magnitude of induced emf in the coil at t=10s is zero

(c) t=22s

distance moved by the loop $0.01 \times 22 = 0.22m$

the loop is moving out of the field, the area of loop under the field is
= $(2 \times 5 \times 10^{-4}m^2)$

the magnetic flux acting on the loop is

$$= -0.6 \times 2 \times 5 \times 10^{-4} Tm^2$$

(- sign as the flux has decreased)

The induced emf is

$$\epsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{6 \times 10^{-4} - 0}{2} = -3 \times 10^{-4}V$$

Therefore magnitude of induced emf at t=22s is $3 \times 10^{-4}V$

(d) t=30s

distance moved by the square loop = $0.01 \times 30 = 0.3m$

at this time, square loop is completely outside the magnetic field and the area of loop through which flux passes = 0

hence the flux linkage through the loop remains zero $\therefore \Delta\phi = 0$

and thus from eqn.(i)

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = 0$$

17. Question

Find the total heat produced in the loop of the previous problem during the interval 0 to 30 s if the resistance of the loop is $4.5 \text{ m}\Omega$.

Answer

Given:

Resistance of the loop $R = 4.5 \text{ m}\Omega = 4.5 \times 10^{-3} \Omega$

Time interval = 30s

As heat produced is a scalar quantity total heat is found by algebraically adding heat produced in different time intervals

$$\therefore H_T = H_a + H_b + H_c + H_d$$

Where

H_a = heat produced during time interval 0-5s

H_b = heat produced during time interval 5-20s

H_c = heat produced during time interval 20-25s

H_d = heat produced during time interval 25-30s

Now,

(i) during time interval 0-5s emf produced in the loop is given by

$$\epsilon = 3 \times 10^{-4} \text{ V}$$

Current in the coil is

$$i = \frac{\epsilon}{R} = 3 \times \frac{10^{-4}}{(4.5 \times 10^{-3})} = 6.7 \times 10^{-2} \text{ A}$$

Heat produced in the coil is given by the formula

$$H = i^2 R t \dots (i)$$

where

i=current

R=resistance

t=time interval

putting the values of i, R and t in above eqn. we get,

$$H_a = (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5$$

Emf induced in the time interval 5-20s and 25-30s = 0

So current in the coil during this time

$$i = 0$$

Heat produced in the coil is given by eqn.(i)

$$H_b = H_d = 0$$

Emf induced in the time interval 20-25s is same as that induced at 5s

$$\epsilon = 3 \times 10^{-4} V$$

Hence the current and heat produced during this interval is same and given by

$$H_c = (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5$$

Total heat produced is given by

$$H_T = H_a + H_b + H_c + H_d$$

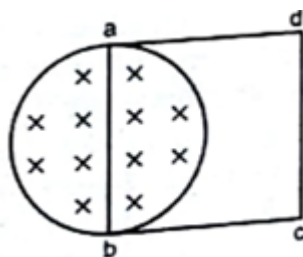
$$H_T = H_a + H_c = 2 \times (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5 J$$

$$H_T = 2 \times 10^{-4} J$$

Therefore total heat produced in the loop during interval 0-30s is $2 \times 10^{-4} J$

18. Question

A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure. A uniform wire of length 80 cm and resistance 4.0Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0.010 T/s , find the current induced in the frame.



Answer

Given:

Radius of cylindrical region $r = 10 \text{ cm} = 0.1 \text{ m}$

Length of wire = 80 cm

Resistance of the wire $R = 4\Omega$

Rate of increase of magnetic field $= 0.01 \text{ T/s} = \frac{dB}{dt}$

Area of loop inside magnetic field = area of semicircle of radius 0.1m

$$\Rightarrow A = \frac{\pi r^2}{2}$$

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

Since magnetic field is perpendicular to the loop the flux becomes

$$\phi = BA \cos 0^\circ = BA$$

Rate of change of magnetic field wrt. time is given by

$$\frac{d\phi}{dt} = \frac{d(BA)}{dt} = A \cdot \frac{dB}{dt}$$

(since area of cross section in magnetic field does not change with time, A remains constant)

Now,

by faraday's law of electromagnetic induction

$$\epsilon = - \frac{d\phi}{dt} \dots (i)$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

using eqn.(i) the emf induced in the loop is given by

$$\epsilon = - \frac{d\phi}{dt} = -A \frac{dB}{dt}$$

Hence the current through the loop (i) of resistance R is

$$i = \frac{\epsilon}{R} = - \frac{A}{R} \frac{dB}{dt}$$

Putting the values of A, R and $\frac{dB}{dt}$ in the above eqn. the magnitude of current is

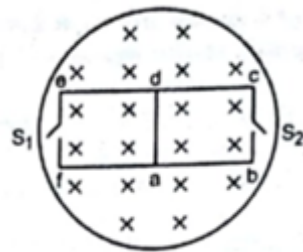
$$i = \frac{\pi r^2}{2R} \frac{dB}{dt} = 3.14 \times 0.01 \times \frac{0.01}{2 \times 4} = 3.9 \times 10^{-5} A$$

Therefore current induced in the frame is $3.9 \times 10^{-5} A$

19. Question

The magnetic field in the cylindrical region shown in figure increases at a constant rate of 20.0 mT/s . Each side of the square loop abcd and defa has a length of 1.00 cm and a resistance of 4.00Ω . Find the current (magnitude and since) in the wire ad if

- (a) the switch S_1 is closed but S_2 is open,
- (b) S_1 is open but S_2 is closed,
- (c) both S_1 and S_2 are open and
- (d) both S_1 and S_2 are closed.



Answer

Given:

Rate of increase of magnetic field $= 20 \text{ mT s}^{-1} = 0.02 \text{ T s}^{-1}$

Side length of square loop $= 1 \text{ cm} = 0.01 \text{ m}$

Resistance of each side $= 4 \Omega$

Area of the coil adef = area of coil abcd $= 10^{-4} \text{ m}^2$

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

Since magnetic field is perpendicular to the loop the flux becomes

$$\phi = BA \cos 0^\circ = BA$$

Rate of change of magnetic field wrt. time is given by

$$\frac{d\phi}{dt} = \frac{d(BA)}{dt} = A \cdot \frac{dB}{dt}$$

(since area of cross section in magnetic field does not change with time, A remains constant)

Now,

by faraday's law of electromagnetic induction

$$\epsilon = - \frac{d\phi}{dt} \dots (i)$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

using eqn.(i) the emf induced in the loop is given by

$$\epsilon = - \frac{d\phi}{dt} = -A \frac{dB}{dt}$$

Hence the current through the loop (i) of resistance R is

$$i = \frac{\epsilon}{R} = - \frac{A}{R} \frac{dB}{dt} \dots (ii)$$

(a) when the switch S_1 is closed but S_2 is open

no current flows through loop abcd

net resistance of the loop adef $R = 4 \times 4 = 16\Omega$

area of loop adef $= 10^{-4} m^2$

using eqn.(ii) current can be given by

$$i = - \frac{A}{R} \frac{dB}{dt} = \frac{10^{-4}}{16} \times 0.02 = 1.25 \times 10^{-7}$$

As the magnetic field increases, the flux of magnetic field increases in downward direction so by Lenz's law

The direction of induced current is such that it opposes the change that has induced it

Therefore, current flows in anticlockwise direction (**along ad**) to increase the magnetic flux in upward direction

(b) S_1 is open but S_2 is closed

No current flows in loop adef

Net resistance of loop abcd $= 4 \times 4 = 16\Omega$

Area of loop abcd $= 10^{-4} m^2$

using eqn.(ii) current can be given by

$$i = -\frac{A}{R} \frac{dB}{dt} = \frac{10^{-4}}{16} \times 0.02 = 1.25 \times 10^{-7}$$

As the magnetic field increases, the flux of magnetic field increases in downward direction so by Lenz's law

Therefore, current flows in anticlockwise direction (**along da**) to increase the magnetic flux in upward direction.

(c) When both S_1 and S_2 is open

No current flows in both the loop adef and abcd

And hence current in wire ad is zero

$$\therefore i = 0$$

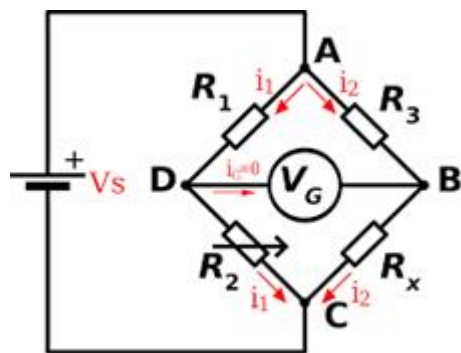
(d) When both S_1 and S_2 is closed

The circuit forms a **balanced Wheatstone Bridge** and the current flowing through the wire ad is zero

$$\therefore i = 0$$

Concept of wheat stone bridge:

When the circuit forms a Wheatstone bridge in balanced condition then the current through galvanometer (i_G) becomes zero

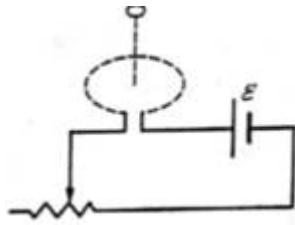


20. Question

Figure shows a circular coil of N turns and radius a , connected to a battery of emf ϵ through a rheostat. The rheostat has a total length L and resistance R . The resistance of the coil is r . A small circular loop of radius a' and resistance r' is placed coaxially with the coil. The center of the loop is at a distance x from the center of the coil. In the beginning, the sliding contact of the rheostat is at the left end and then onwards it is moved towards right at a constant speed v . Find the emf induced in the small circular loop at the instant

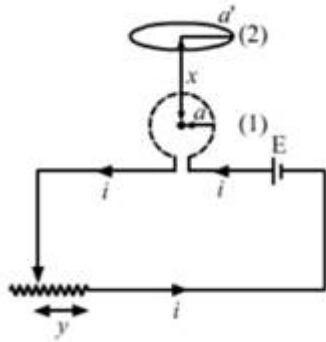
(a) the contact begins to slide and

(b) it has slid through half the length of the rheostat.



Answer

Given:



Area of coil (2) of radius $a' = \pi a'^2$

We know that magnetic field due to coil (1) at the center of coil (2) is

$$B = \frac{\mu_0 N i a^2}{2(a^2 + x^2)^{3/2}}$$

Where

N =no. of turns in coil (1)

i = current in coil (1)

a =radius of coil (1)

x =distance of center of coil (2) from center of coil (1)

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = B A \cos \theta$$

Since the magnetic field due to coil (1) is parallel to axis of coil (2) $\theta = 0^\circ$ and flux through the coil (2) is given by

$$\phi = B A \cos 0^\circ = B A = \frac{\mu_0 N i a^2}{2(a^2 + x^2)^{3/2}} \pi a'^2$$

Now,

by faraday's law of electromagnetic induction

$$e = -\frac{d\phi}{dt} \dots (i)$$

Where

e = emf produced

ϕ = flux of magnetic field

using eqn.(i) emf induced in the coil (2) is given by

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{3/2}} \pi a'^2 \frac{di}{dt} \dots (ii)$$

Let y be the distance of sliding contact from its right end

Given,

Total length of rheostat = L

Total resistance of rheostat = R

When the sliding contact is at a distance y from its right end then the resistance (R') of the rheostat is given by

$$R' = \frac{R}{L} y$$

So the current i flowing through the circuit is given by

$$i = \frac{\epsilon}{R' + r} = \frac{\epsilon}{\frac{R}{L} y + r}$$

Where r is the resistance of the coil and ϵ is the emf of battery

Putting value of i in eqn.(ii) we get,

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{3/2}} \pi a'^2 \times \frac{d\left(\frac{\epsilon}{\frac{R}{L} y + r}\right)}{dt}$$
$$\Rightarrow e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{3/2}} \pi a'^2 \times \left[\epsilon \times \frac{-\frac{R}{L} \frac{dy}{dt}}{\left(\frac{R}{L} y + r\right)^2} \right]$$

Since $\frac{dy}{dt} = v$

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{-\frac{R}{L} v}{\left(\frac{R}{L} y + r\right)^2} \right]$$

(a) When the contact begins to slide $y = L$

Therefore, magnitude of emf induced is

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{\left(\frac{R}{L} L + r\right)^2} \right]$$

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{(R + r)^2} \right]$$

(b) When the contact has slid through half the length of rheostat $y = \frac{L}{2}$

Therefore, magnitude of emf induced is

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{\left(\frac{R}{L} \frac{L}{2} + r\right)^2} \right]$$

$$e = \frac{\mu_0 N a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \pi a'^2 \times \left[\epsilon \times \frac{\frac{R}{L} v}{\left(\frac{R}{2} + r\right)^2} \right]$$

21. Question

A circular coil of radius 2.00 cm has 50 turns. A uniform magnetic field $B = 0.200 \text{ T}$ exists in the space in a direction parallel to the axis of the loop. The coil is now rotated about a diameter through an angle of 60.0° . The operation takes 0.100s.

(a) Find the average emf induced in the coil.

(b) If the coil is a closed one (with the two ends joined together) and has a resistance of 4.00Ω , calculate the net charge crossing a cross-section of the wire of the coil.

Answer

Given:

Radius of coil $r = 2\text{cm} = 0.02\text{m}$

No. of turns in the coil $N = 50$

Magnetic field intensity $B = 0.2T$

We know that,

Flux (ϕ) of magnetic field (B) through the loop of cross section area A in the magnetic field is given by

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = NBA \cos \theta$$

Where N=no. of turns in the coil

Since magnetic field is perpendicular to the loop the flux becomes

$$\phi = NBA \cos 0^\circ = NBA$$

Initial flux through the coil is given by

$$\phi_1 = NBA$$

After 0.1 s the coil is rotated through an angle of $60^\circ = \theta$

Finally, the flux through the coil becomes

$$\phi_2 = NBA \cos 60^\circ = \frac{NBA}{2}$$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Using eqn.(i) emf induced in the coil is given by

$$\epsilon = -\frac{\frac{NBA}{2} - NBA}{\Delta t} = \frac{NBA}{2\Delta t}$$

Putting the values of N, B, A and Δt in above eqn. we get

$$\epsilon = 50 \times 0.2 \times \frac{\pi(0.02)^2}{2 \times 0.1} = 6.28 \times 10^{-3} V$$

Therefore average emf induced in the coil is $6.28 \times 10^{-3} V$

(b) the current through the coil (i) is calculated using formula

$$i = \epsilon/R$$

Hence the charge(Q) crossing the cross-section of the wire in time interval Δt is

$$Q = i \times \Delta t = \frac{\epsilon \Delta t}{R}$$

Putting the values of ϵ , R and Δt we get,

$$Q = 6.28 \times 10^{-3} \times \frac{0.1}{4} = 1.57 \times 10^{-4} C$$

Therefore charge crossing cross-section of the wire in the coil is
 $1.57 \times 10^{-4} C$

22. Question

A closed coil having 100 turns is rotated in a uniform magnetic field $B = 4.0 \times 10^{-4} T$ about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 25 cm^2 and its resistance is 4.0Ω . Find

- (a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field,
- (b) the average emf in a full turn and
- (c) the net charge displaced in part (a).

Answer

Given:

No. of turns in the coil $N = 100$

Magnetic field intensity $B = 4 \times 10^{-4} T$

Angular velocity of rotation $\omega = 300 \text{ rev min}^{-1} = 2\pi \times 5 = 10\pi \text{ rad s}^{-1}$

Area of the coil $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$

Resistance of the coil $R = 4 \Omega$

Magnetic flux through the circular coil ϕ can be given by formula

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = NBA \cos \theta \dots (i)$$

Where B=magnetic field intensity

A=area of cross section

N=no. of turns in the coil

θ =angle between area vector and magnetic field

(a) Initially, angle between area vector and magnetic field is 0°

Therefore, initial flux through the coil is

$$\phi_1 = NBA$$

When it is rotated by 180° flux passing through the coil is given by

$$\phi_2 = NBA \cos 180^\circ = -NBA$$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1)/\Delta t \dots(i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Average induced emf is then given by

$$\epsilon = -\frac{-NBA - NBA}{\Delta t} = \frac{2NBA}{\Delta t} \dots\dots(ii)$$

Now,

Angular velocity of coil $\omega = 10\pi \text{ rads}^{-1}$

Time taken to complete half revolution i.e. rotate by π radian

$$\Delta t = \frac{1}{10} \text{ s}$$

Putting the values of N, B, A and Δt in eqn.(ii)

$$\epsilon = 2 \times 100 \times 4 \times 10^{-4} \times 25 \times \frac{10^{-4}}{0.1} = 2 \times 10^{-3} \text{ V}$$

Therefore average emf induced in the coil in half a turn is $2 \times 10^{-3} \text{ V}$

(b) In a full turn coil returns to its original position

$$\therefore \phi_1 = \phi_2 = NBA$$

And hence emf induce in the coil using eqn.(ii) is

$$\epsilon = -\frac{NBA - NBA}{\Delta t} = 0$$

Therefore, average emf induced in full turn in the coil is zero

(c) Emf induced in the coil in part (a) is

$$\epsilon = 2 \times 10^{-3} \text{ V}$$

Hence the current i flowing through the coil of resistance R is

$$i = \frac{\epsilon}{R} = 2 \times \frac{10^{-3}}{4} = 5 \times 10^{-4} A$$

So the charge displaced in time interval $\Delta t = 0.1s$ is

$$Q = i\Delta t = 5 \times 10^{-4} \times 0.1 = 5 \times 10^{-5} C$$

Therefore net charge displaced in part (a) is $5 \times 10^{-5} C$

23. Question

A coil of radius 10 cm and resistance 40Ω has 1000 turns. It is placed with its plane vertical and its axis parallel to the magnetic meridian. The coil is connected to a galvanometer and is rotated about the vertical diameter through an angle of 180° . Find the charge which flows through the galvanometer if the horizontal component of the earth's magnetic field is $B_H = 3.0 \times 10^{-5} T$.

Answer

Given:

Radius of coil $r = 10cm$

Resistance of the coil $R = 40\Omega$

No. of turns in the coil $N = 1000$

Horizontal component of earth's magnetic field $= 3.0 \times 10^{-5} T$

Angle of rotation $\theta = 180^\circ$

Magnetic flux through the circular coil ϕ can be given by formula

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = NBA \cos \theta \dots (i)$$

Where B = magnetic field intensity

A = area of cross section

N = no. of turns in the coil

θ = angle between area vector and magnetic field

Initially, angle between area vector and magnetic field is 0°

Therefore, initial flux through the coil is

$$\phi = NBA$$

When it is rotated by 180° flux passing through the coil is given by

$$\phi = NBA \cos 180^\circ = -NBA$$

Now,

by faraday's law of electromagnetic induction

$$\epsilon = -\frac{d\phi}{dt}$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

therefore, emf produced in the coil is given by

$$\epsilon = -\frac{-NBA - NBA}{dt} = \frac{2NBA}{dt} \dots\dots\dots (ii)$$

Current passing through the loop (i) of resistance R is

$$i = \frac{\epsilon}{R} = \frac{2NBA}{Rdt} \text{ using eq.(ii)}$$

Charge flowing through the galvanometer (Q) in time dot is given by formula

$$Q = i \cdot dt = \frac{2NBA}{Rdt} \times dt = \frac{2NBA}{R}$$

Putting the values of N, B, A and R we get,

$$Q = 2 \times 1000 \times 3 \times 10^{-5} \times \pi \times \frac{10^{-2}}{40} = 4.71 \times 10^{-5} C$$

Therefore charge which flows through the galvanometer is $4.71 \times 10^{-5} C$

24. Question

A circular coil of one turn of radius 5.0 cm is rotated about a diameter with a constant angular speed $B = 0.010 \text{ T}$ exists in a direction perpendicular to the axis of rotation. Find

- (a) the maximum emf induced,
- (b) the average emf induced in the coil over a long period and
- (c) the average of the squares of emf induced over a long period.

Answer

Given:

Radius of circular coil $r = 5.0 \text{ cm} = 5 \times 10^{-2} \text{ m}$

Magnetic field intensity $B = 0.01 \text{ T}$

Angular speed of coil $\omega = 80 \text{ rev min}^{-1} = 2\pi \times \frac{80}{60} \text{ rads}^{-1}$

Magnetic flux through the circular coil ϕ can be given by formula

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

$$\phi = BA \cos \omega t \dots (i)$$

As $\theta = \omega t$

Where ω = angular velocity of loop

Where θ is the angle between magnetic field and area vector of loop.

by faraday's law of electromagnetic induction

$$\epsilon = - \frac{d\phi}{dt}$$

Where

ϵ = emf produced

ϕ = flux of magnetic field

putting the value of eqn.(i) in above eqn. we get,

$$\epsilon = - \frac{d}{dt} (BA \cos \omega t) = \omega BA \sin \omega t \dots (ii)$$

Since maximum value of $\sin \omega t$ is equal to 1

$$\therefore \epsilon_{\max} = BA\omega$$

Putting the values of B, A and ω we get,

$$\epsilon = 0.01 \times \pi \times 25 \times 10^{-4} \times 2\pi \times \frac{80}{60} = 6.66 \times 10^{-4} \text{ V} \dots \dots \dots (iii)$$

Therefore maximum emf induced in the circular coil is $6.66 \times 10^{-4} \text{ V}$

(b) from eqn.(ii) emf induced in the coil is given by

$$\epsilon = BA\omega \sin \omega t$$

Average value of induced emf is given by formula

$$\epsilon_{av} = \left(\int_0^T BA\omega \sin \omega t \, dt \right) / \int_0^T dt = BA\omega \left[-\frac{\cos \omega t}{\omega} \right]_0^T$$

Where $T = 2\pi/\omega$ is the time taken by the coil to complete one revolution

$$\Rightarrow \epsilon_{av} = -\frac{BA\omega}{\omega} [\cos\omega t]_0^{\frac{2\pi}{\omega}} = 0$$

Therefore, average induced emf is zero

(c). the average of squares of induced emf is given by the formula

$$\begin{aligned}\epsilon_{av}^2 &= \left(\int_0^T B^2 A^2 \omega^2 \sin^2 \omega t \cdot dt \right) / \int_0^T dt \\ \Rightarrow \epsilon_{av}^2 &= B^2 A^2 \omega^2 \int_0^T \sin^2 \omega t \, dt / \int_0^T dt \\ \Rightarrow \epsilon_{av}^2 &= \frac{B^2 A^2 \omega^2}{2} \int_0^T (1 - \cos 2\omega t) \cdot dt / \int_0^T dt \\ \Rightarrow \epsilon_{av}^2 &= \frac{B^2 A^2 \omega^2 \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T}{2T} \\ \Rightarrow \epsilon_{av}^2 &= \frac{B^2 A^2 \omega^2}{2T} \left[T - \frac{\sin 4\pi - \sin 0}{2\omega} \right] = \frac{B^2 A^2 \omega^2}{2}\end{aligned}$$

Putting the values of B, A and ω we get

$$\epsilon_{av}^2 = \frac{(6.66 \times 10^{-4})^2}{2} = 2.2 \times 10^{-7} V^2 \text{ using eqn.(iii)}$$

Therefore average of squares of emf produced is $2.2 \times 10^{-7} V^2$

25. Question

Suppose the ends of the coil in the previous problem are connected to a resistance of 100 Ω . Neglecting the resistance of the coil, find the heat produced in the circuit in one minute.

Answer

Given:

Resistance of the coil $R=100\Omega$

Time period $T=1\text{min}=60\text{s}$

From previous question induced emf is given by

$$\epsilon = BA\omega \sin\omega t$$

Current in the coil i is given by

$$i = \frac{\epsilon}{R} = \frac{BA\omega \sin\omega t}{R} \dots (i)$$

Heat produced in the circuit is calculated by the following formula

$$H = \int_0^T i^2 R dt$$

Using eqn.(i) we get,

$$H = \int_0^T B^2 A^2 \omega^2 \sin^2 \frac{\omega t}{R^2} R dt$$

$$\Rightarrow H = \frac{B^2 A^2 \omega^2}{2R} \int_0^T (1 - \cos 2\omega t) dt$$

$$H = \frac{B^2 A^2 \omega^2}{2R} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$H = \frac{B^2 A^2 \omega^2 T}{2R}$$

Putting the values of B, A, ω , T and R we have

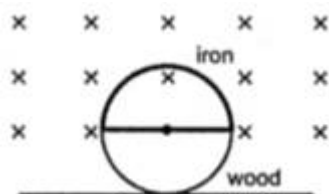
$$H = \frac{60}{2R} \times \pi^2 r^4 \times B^2 \times \left(80 \times \frac{2\pi}{60} \right)^2$$

$$H = 1.33 \times 10^{-7} J$$

Therefore heat produced in the circuit in one minute is $1.33 \times 10^{-7} J$

26. Question

Figure shows a circular wheel of radius 10.0 cm whose upper half, shown dark in the figure, is made of iron and the lower half of wood. The two junctions are joined by an iron rod. A uniform magnetic field B of magnitude $2.00 \times 10^{-4} T$ exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes 2.00 seconds for the iron part to come down and the wooden part to go up, find the average emf induced during this period.



Answer

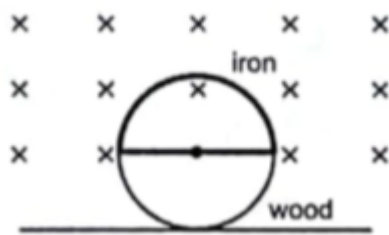
Given:

Radius of circular wheel $r = 10 \text{ cm} = 0.1 \text{ m}$

Magnetic field intensity $B = 2 \times 10^{-4} T$

Area of semicircular part $A = \frac{\pi r^2}{2}$

Initially



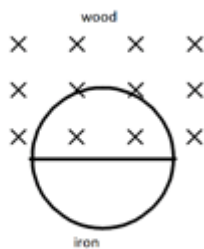
Magnetic flux through the wheel ϕ can be given by formula

$$\phi = \vec{B} \cdot \vec{A}$$

Since area vector and magnetic field is parallel we get flux

$$\phi_1 = BA = \frac{B\pi r^2}{2} = 2 \times 10^{-4} \times 3.14 \times \frac{(0.1)^2}{2} = 3.14 \times 10^{-6} \text{ Wb}$$

After time interval $\Delta t = 2\text{s}$



Flux through the wheel

$$\phi_2 = 0$$

Average induced emf in time interval Δt is given by

$$\epsilon = -(\phi_2 - \phi_1) / \Delta t \dots (i)$$

Where

ϕ_1 and ϕ_2 are flux across the cross section at time intervals t_1 and t_2 respectively.

Therefore, average induced emf ϵ is

$$\epsilon = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{0 - 3.14 \times 10^{-6}}{2} = 1.57 \times 10^{-6} \text{ V}$$

Therefore average induced emf in the wheel is $1.57 \times 10^{-6} \text{ V}$

27. Question

A 20 cm long conducting rod is set into pure translation with a uniform velocity of 10 cm s^{-1} perpendicular to its length. A uniform magnetic field of magnitude 0.10 T exists in a direction perpendicular to the plane of motion.

- (a) Find the average magnetic force on the free electrons of the rod.
- (b) For what electric field inside the rod, the electric force on a free electron will balance the magnetic force? How is this electric field created?
- (c) Find the motional emf between the ends of the rod.

Answer

Given:

Length of rod $l = 20\text{cm} = 0.2\text{m}$

Velocity of rod $v = 10\text{cm s}^{-1} = 0.1\text{m s}^{-1}$

Magnetic field intensity $B = 0.1\text{T}$

(a) we know that,

A charge q moving with velocity v inside a magnetic field B experiences a force F given by

$$\vec{F} = q.(\vec{v} \times \vec{B}) \dots (i)$$

Since velocity of rod is perpendicular to magnetic field the above eqn. reduces to

$$F = qvB$$

To find the force on a free electron $q = e = 1.6 \times 10^{-19}\text{C}$

Putting the values of q , v and B we get,

$$F = 1.6 \times 10^{-19} \times 0.1 \times 0.1 = 1.6 \times 10^{-21}\text{N}$$

Therefore average force experienced by a free electron is $1.6 \times 10^{-21}\text{N}$

(b) we know that,

force experienced by a charge particle having charge q in presence of an electric field E is given by

$$\vec{F} = q. \vec{E}$$

To balance this force with the magnetic force, equating above eqn. with eqn.(i),

$$q\vec{E} = q.(\vec{v} \times \vec{B}) = qvB$$

$$\Rightarrow E = vB$$

Putting the values of v and B we get,

$$E = 0.1 \times 0.1 = 0.01\text{Vm}^{-1}$$

This electric field is created due to emf produced due to motion of conducting rod, as a result of which the free electrons in the rod experiences a force

Therefore, electric field needed to balance magnetic force is 0.01Vm^{-1}

(c) We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

Since \vec{v} and \vec{B} are perpendicular and their cross product is parallel to \vec{l} , eqn.(i) reduces to

$$\epsilon = vBl$$

Putting the values of v , B and l we get,

$$\epsilon = 0.1 \times 0.1 \times 0.2 = 2 \times 10^{-3} \text{V}$$

Therefore motional emf between the ends of the rod is $2 \times 10^{-3} \text{V}$

28. Question

A metallic meter stick moves with a velocity of 2ms^{-1} in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0.2 T . Find the emf induced between the ends of the stick.

Answer

Given:

Velocity of meter stick $v = 2 \text{ms}^{-1}$

Intensity of magnetic field $B = 0.2 \text{T}$

Length of stick $= 1 \text{m}$

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

Since \vec{v} and \vec{B} are perpendicular and their cross product is parallel to \vec{l} , eqn.(i) reduces to

$$\epsilon = vBl$$

Putting the values of v , B and l we get,

$$\epsilon = 2 \times 0.2 \times 1 = 0.4V$$

Therefore, emf induced between the ends of a stick is 0.4V

29. Question

A 10 m wide spacecraft moves through the interstellar space at a speed $3 \times 10^7 \text{ m s}^{-1}$. A magnetic field $B = 3 \times 10^{-10} \text{ T}$ exists in the space in a direction perpendicular to the plane of motion. Treating the spacecraft as a conductor, calculate the emf induced across its width.

Answer

Given:

Speed of spacecraft $v = 3 \times 10^7 \text{ m/s}$

Magnetic field intensity $B = 3 \times 10^{-10} \text{ T}$

Width of spacecraft $l = 10 \text{ m}$

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

Since \vec{v} and \vec{B} are perpendicular and their cross product is parallel to \vec{l} , eqn.(i) reduces to

$$\epsilon = vBl$$

Putting the values of v , B and l we get,

$$\epsilon = 3 \times 10^7 \times 3 \times 10^{-10} \times 10 = 9 \times 10^{-2} V$$

$$\epsilon = 0.09V$$

Therefore, emf induced across the width of spacecraft is 0.09V

30. Question

The two rails of a railway track, insulated from each other and from the ground, are connected to a millivoltmeter. What will be the reading of the millivoltmeter when a train travels on the track at a speed of 180 km h^{-1} ? The vertical component of earth's magnetic field is $0.2 \times 10^{-4} \text{ T}$ and the rails are separated by 1 m.

Answer

Given:

Vertical component of earth's magnetic field $B = 0.2 \times 10^{-4} \text{ T}$

Speed of train $v = 180 \text{ km h}^{-1} = 50 \text{ m s}^{-1}$

Separation between rails $l = 1 \text{ m}$

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l} \dots (i)$$

Since \vec{v} and \vec{B} are perpendicular and their cross product is parallel to \vec{l} , eqn.(i) reduces to

$$\epsilon = vBl$$

Putting the values of v , B and l we get,

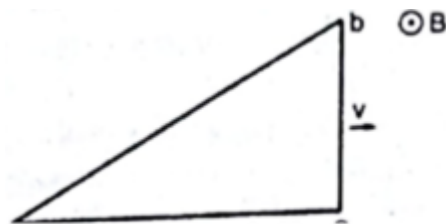
$$\epsilon = 50 \times 0.2 \times 10^{-4} \times 1 = 10^{-3} \text{ V} = 1 \text{ mV}$$

Therefore, the reading of millivoltmeter is 1mV

31. Question

A right-angled triangle abc , made from a metallic wire, moves at a uniform speed v in its plane as shown in figure. A uniform magnetic field B exists in the perpendicular direction. Find the emf induced

- (a) in the loop abc ,
- (b) in the segment bc ,
- (c) in the segment ac and
- (d) in the segment ab .



Answer

Given:

Speed of right-angled triangle $= v$

We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l} \dots (i)$$

(a) for loop abc since the flux of magnetic field through the triangle does not change hence the emf induced in loop abc is zero

$$\therefore \epsilon_{abc} = 0$$

Therefore, emf induced in loop abc is zero

(b) in the segment bc

bc is perpendicular to velocity. emf induced in the segment bc is given by

$$\epsilon_{bc} = (\vec{v} \times \vec{B}) \cdot \vec{l} = (v\hat{i} \times B\hat{k}) \cdot (bc(-\hat{j})) = vB\hat{j} \cdot \hat{j}(bc) = vB(bc)$$

Therefore, emf induced in segment bc is $vB(bc)$

(c) in the segment ac

ac is parallel to velocity. Emf induced in the segment ac is given by

$$\epsilon_{ac} = (\vec{v} \times \vec{B}) \cdot \vec{l} = (v\hat{i} \times B\hat{k}) \cdot (ac\hat{i}) = vB(-\hat{j}) \cdot ac(\hat{i}) = 0$$

Therefore, emf induced in the segment ac is zero

(d) in the segment ab

the effective length of ab perpendicular to velocity is given by bc $(-\hat{j})$

emf induced in the segment ab is given by

$$\epsilon_{ab} = (\vec{v} \times \vec{B}) \cdot \vec{l} = (v\hat{i} \times B\hat{k}) \cdot (bc(-\hat{j}) + ac\hat{i}) = vB\hat{j} \cdot \hat{j}(bc) + 0 = vB(bc)$$

Therefore, emf induced in the segment ab is $vB(bc)$

32. Question

A copper wire bent in the shape of a semicircle of radius r translates in its plane with a constant velocity v . A uniform magnetic field B exists in the direction perpendicular to the plane of the wire. Find the emf induced between the ends of the wire if

(a) the velocity is perpendicular to the diameter joining free ends,

(b) the velocity is parallel to this diameter.

Answer

Given:

Radius of semicircular wire= r

Velocity = v

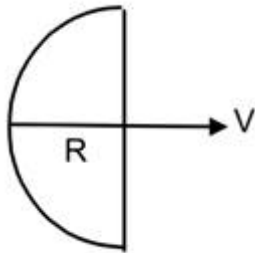
We know that,

motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

In the case of semicircular wire, l denotes the effective length of wire perpendicular to velocity.

(a) when the velocity is perpendicular to diameter joining free ends



the effective length of wire perpendicular to velocity is given by length of diameter

$$\therefore l = 2R$$

Therefore, induced emf in the wire is given by

$$\epsilon = vB \times 2R = 2BvR$$

Therefore, induced emf in this case is $2BvR$

(b) when the velocity is parallel to the diameter



The effective length of wire parallel to velocity is zero

$$\therefore l = 0$$

Therefore, induced emf in the wire is given by

$$\epsilon = vB \times 0 = 0$$

Therefore, induced emf in this case is zero

33. Question

A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1.0 T that exists in the space. Find the emf induced between the ends of the rod if the speed of translation is 20 cm s^{-1} .

Answer

Given:

Length of the wire = 10 cm

Angle of length of wire with velocity = 60°

Magnetic field intensity = 1.0 T

Speed of wire $v = 20 \text{ cm/s} = 0.2 \text{ m/s}$

(a) we know that,

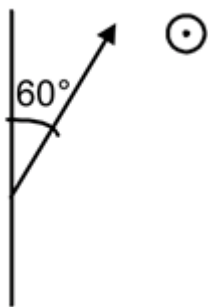
motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

It is given that plane of motion is perpendicular to electric field i.e. angle between \vec{v} and $\vec{B} = 90^\circ$

And the angle between velocity and length of wire = 60°

$$\therefore \epsilon = Bv l \sin 60^\circ$$



We take only that component of length vector which is perpendicular to velocity vector

Putting the values of B , v , l we get

$$\epsilon = 1 \times 0.2 \times 0.1 \times \frac{\sqrt{3}}{2} = 17.32 \times 10^{-3} \text{ V}$$

Therefore the emf induced in the rod is $1.732 \times 10^{-2} \text{ V}$

34. Question

A circular copper-ring of radius r translates in its plane with a constant velocity v . A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring.

(a) Between which pair of points is the emf maximum?

(b) Between which pair of points is the emf minimum?

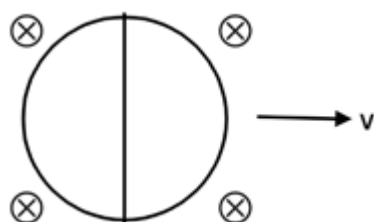
What is the value of this minimum emf?

Answer

Given:

Radius of ring = R

Velocity of ring = v



(b) we know that motional emf produced due to a conductor of length l moving with velocity v in a magnetic field B is given by

$$\epsilon = (\vec{v} \times \vec{B}) \cdot \vec{l} = B \cdot (\vec{v} \times \vec{l}) \dots (i)$$

This value is maximum when length between the points is perpendicular to the velocity of the rod

Thus the emf is the highest between the end points of diameter perpendicular to the velocity and the value of this emf is given by

$$\epsilon = B v \times 2R = 2BvR$$

Therefore, maximum value of emf is $2BvR$

(c) the value of eqn.(i) is minimum when length between the points is parallel to the velocity of the rod

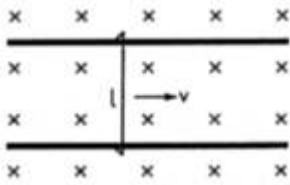
Thus the emf is lowest between the end points of diameter parallel to the velocity and this value of emf is given by

$$\epsilon = Bv \times 2R \times \sin 0^\circ = 0$$

Therefore, minimum value of emf is 0

35. Question

Figure shows a wire sliding on two parallel, conducting rails placed at a separation ℓ . A magnetic field B exists in a direction perpendicular to the plane of the rails. What force is necessary to keep the wire moving at a constant velocity v ?

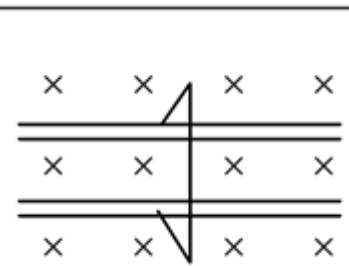


Answer

Given:

Velocity of wire = v

Separation between rail = l



We know that,

Force experienced by a wire of length l carrying current I in a magnetic field B and placed perpendicular to magnetic field is given by

$$F = ilB$$

Now since here there is no formation of closed circuit the circuit is open and the current flowing in the wire = 0

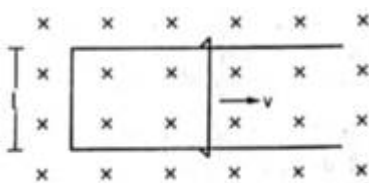
Therefore, force experienced by wire due to magnetic field = 0

Hence net force on wire becomes zero and it moves with constant velocity v

Therefore, no external force is needed to move the wire with velocity v

36. Question

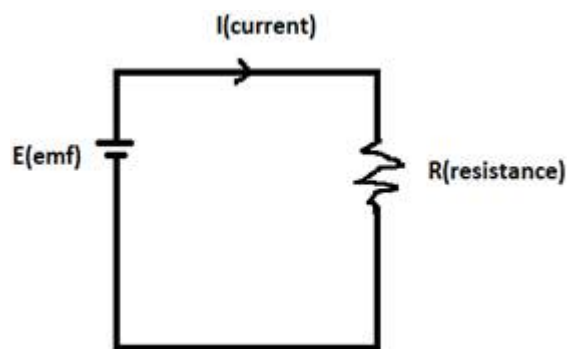
Figure shows a long U-shaped wire of width ℓ placed in a perpendicular magnetic field B . A wire of length ℓ is slid on the U-shaped wire with a constant velocity v towards right. The resistance of all the wires is r per unit length. At $t = 0$, the sliding wire is close to the left edge of the U-shaped wire. Draw an equivalent circuit diagram, showing the induced emf as a battery. Calculate the current in the circuit.



Answer

Current will flow from the left edge to the right, that is, in the clockwise direction. Therefore, the induced emf will also flow along the clockwise direction.

Diagram showing induced emf as a battery:



Given:

Resistance per unit length = r

Length of wire = l

Velocity with which the wire moves = v

Formula used:

We know, that $E = Blv$... (i), where E = emf, B = magnetic field, v = velocity with which the wire moves, l = length of wire.

Now, total resistance (R) = $r \times l'$... (ii), where r = resistance per unit length, l' = total length of loop

Horizontal length of loop = vt , where v = velocity, t = time.

Hence, total length of loop (l') = $2(l + vt)$

Therefore, $R(\text{total resistance}) = 2r(l + vt)$... (iii), where r = resistance per unit length, l = length of wire

By Ohm's law, we know that $E = IR$, where E = emf, I = current, R = total resistance. Hence, from (iii), we get, $E = 2Ir(l + vt)$ (iv)

Equation (i) and (iv), we get:

$$Blv = 2Ir(l + vt)$$

$$\Rightarrow I = \frac{Blv}{2r(l+vt)}$$

$$\text{Current in the circuit } I = \frac{Blv}{2r(l+vt)} \text{ (Answer)}$$

37. Question

Consider the situation of the previous problem.

(a) Calculate the force needed to keep the sliding wire moving with a constant velocity v .

(b) If the force needed just after $t = 0$ is F_0 , find the time at which the force needed will be $F_0/2$.

Answer

Formula used:

(a) Magnetic force on a current carrying wire $\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$, where I = current, l = length of wire, B = magnetic field.

Since l and B are perpendicular to each other, magnetic force F becomes $F = IlB$... (i)

Now, from the previous problem, $I = \frac{Blv}{2r(l+vt)}$... (ii)

Now, the force needed to keep the sliding wire from moving will be equal to the magnetic force, but in the opposite direction.

Let this force be F' .

Hence, $F' = F = IlB$, where F = magnetic force, I = current, L = length of wire, B = magnetic field.

Substituting the value of I from (ii):

$$F' = \frac{Blv}{(2r(l+vt))} \times lB = \frac{B^2 l^2 v}{2r(L+vt)}$$

Hence, force required to keep the wire from sliding = $\frac{B^2 l^2 v}{2r(L+vt)}$ (Ans)

(b) Now, just after time $t = 0$, the force required to stop the wire from sliding will be $F_0 = \frac{B^2 l^2 v}{2rl}$... (i) (substituting $t = 0$), from the previous part of this question.

Now, let the time taken for the required force to be $\frac{F_0}{2}$ be $t = T$.

Hence, from the previous question, substituting $t = T$,

$$\frac{FF_0}{2} = \frac{BlB^2lv^2v}{4r} \dots (ii)$$

Substituting the value of F_0 from (i), we get

$$\frac{BlB^2lv^2v}{4rl} = \frac{B^2 l^2 v}{2rl}$$

$$\Rightarrow 2r(l + vT) = 4rl$$

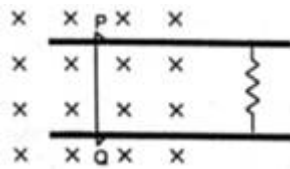
$$\Rightarrow l + vT = 2l \Rightarrow T = \frac{l}{v}$$

Time taken for the force to reduce to $\frac{F_0}{2} = \frac{l}{v}$ (Ans)

38. Question

Consider the situation shown in figure. The wire PQ has mass m , resistance r and can slide on the smooth, horizontal parallel rails separated by a distance ℓ . The resistance of the rails is negligible. A uniform magnetic field B exists in the rectangular region and a resistance R connects the rails outside the field region. At $t = 0$, the wire PQ is pushed towards right with a speed v_0 . Find

- (a) the current in the loop at an instant when the speed of the wire PQ is v ,
- (b) the acceleration of the wire at this instant,
- (c) the velocity v as a function of x and
- (d) the maximum distance the wire will move.



Answer

Given:

Mass of PQ = m

Resistance of PQ = r

Length of PQ between the two rails = l

Magnetic field = B

Resistance connected to the rails = R

Velocity with which PQ is pushed towards right at $t=0 = v_0$

Formula used:

(a) By Ohm's law, $E = IR'$, where E = emf, I = current, R' = total resistance.

Hence, current $I = \frac{E}{R'} \dots$ (i)

Now, emf induced due to the moving rod in the magnetic field $E = Blv \dots$ (ii), where B = magnetic field, l = length of rod, v = velocity of rod

Also, total resistance $R' = r + R \dots$ (iii), where r = resistance of PQ, R = resistance attached to the rails.

Hence, substituting the values of E and R' from (ii) and (iii) in (i), we get

$$I = \frac{Blv}{r+R}$$

Therefore, current in the loop when the speed of the wire PQ is $v = \frac{Blv}{r+R}$. (Ans)

(b) Now, magnetic force on a current carrying wire $F = IlB$... (i), where I = current, l = length of wire, B = magnetic field.

From the previous part, the value of current at an instant when velocity = v is $I = \frac{Blv}{r+R}$... (ii)

Therefore, from (i) and (ii), magnetic force $F = \frac{B^2 l^2 v}{r+R}$... (iii)

According to Newton's second law of motion, $F = ma$... (iv), where F = force, m = mass, a = acceleration.

Hence, equating (iii) and (iv):

$$ma = \frac{B^2 l^2 v}{r+R} \Rightarrow a = \frac{B^2 l^2 v}{m(r+R)}$$

Therefore, acceleration of the wire at this instant = $a = \frac{B^2 l^2 v}{m(r+R)}$ (Ans)

(c) Velocity v' can be expressed as $v = v_0 - at$... (i), where v_0 = initial velocity, a = acceleration, t = time. We put a negative sign before at since the force is opposite to velocity, and it

Now, from the previous part, we can write acceleration $a = \frac{B^2 l^2 v}{m(r+R)}$... (ii)

Hence, from (i) and (ii), we can write $v = v_0 - \frac{B^2 l^2 v}{m(r+R)} t$

But, distance travelled $x = vt$, where v = velocity, t = time.

Therefore, velocity v as a function of x is $v = v_0 - \frac{B^2 l^2 v}{m(r+R)} t$ (Ans)

(d) We know that, $a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{B^2 l^2 v}{m(r+R)} t \frac{dv}{dx}$, where a = acceleration,

v = velocity, x = distance, t = time.

Now, from the part (b), acceleration as a function of time

$$dx = \frac{dv m(R+r)}{B^2 l^2}$$

$$\Rightarrow x = \frac{m(R+r)v_0}{B^2 l^2}$$

Now, the wire can travel maximum distance when its velocity is v_0 .

Hence, integrating on both sides, we get

$$\Rightarrow \chi = \frac{m(r+R)v_0}{B^2 l^2}$$

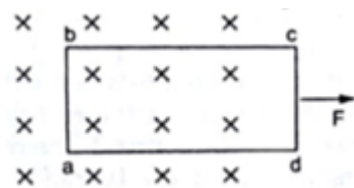
Therefore, maximum distance travelled by the wire

$$\chi = \frac{m(r+R)v_0}{B^2 l^2} \text{ (Ans)}$$

39. Question

A rectangular frame of wire abcd has dimensions 30 cm × 80 cm and a total resistance of 2.0 Ω. It is pulled out of a magnetic field $B = 0.020$ T by applying a force of 3.2×10^{-6} N (figure). It is found that the frame moves with constant speed. Find

- (a) this constant speed,
- (b) the emf induced in the loop,
- (c) the potential difference between the points a and b and (d) the potential difference between the points c and d.



Answer

Given:

Length $ab = cd = 30 \text{ cm} = 0.3 \text{ m}$

Length $bc = ad = 80 \text{ cm} = 0.8 \text{ m}$

Total resistance $R = 2 \text{ } \Omega$

Magnetic field $B = 0.02 \text{ T}$

Force $F = 3.2 \times 10^{-6} \text{ N}$

Formula used:

(a) Magnetic force on a current carrying wire $F = IlB$... (i), where I = current, l = length of wire, B = magnetic field.

Hence, current $I = \frac{F}{lB}$... (ii)

Now, emf $E = Blv$... (iii), where B = magnetic field, l = length of wire, v = constant velocity with which it is moving

Also, by Ohm's law, $E = IR$... (iv), where I = current,

R = resistance.

Hence, equating (iii) and (iv) and substituting I from (ii), we get

$Blv = \frac{FR}{lB}$, where B = magnetic field, v = velocity, F = force, R = resistance, l = length of wire.

Here, since the force is applied on the side cd, we consider

$l = 30 \text{ cm} = 0.3 \text{ m}$ (the shorter length).

Hence,

Substituting the given values, we get

$$v = \frac{(3.2 \times 10^{-6})^2}{(0.3 \times 0.02)^2} \text{ ms}^{-1} = 0.18 \text{ ms}^{-1}$$

Constant speed with which the frame moves = **0.18 ms^{-1}** (Ans)

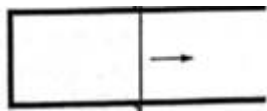
(b) Emf induced in the loop $E = Blv$, where B = magnetic field, l = length of the wire which is moving, v = velocity

Hence, $E = (0.02 \times 0.3 \times 0.18) \text{ V} = 0.001 \text{ V}$

Emf induced in loop = **0.001 V** (Ans)

40. Question

Figure shows a metallic wire of resistance 0.20Ω sliding on a horizontal, U-shaped metallic rail. The separation between the parallel arms is 20 cm. An electric current of $2.0 \mu\text{A}$ passes through the wire when it is slid at a rate of 20 cm s^{-1} . If the horizontal component of the earth's magnetic field is $3.0 \times 10^{-5} \text{ T}$, calculate the dip at the place.



Answer

Given:

Resistance of wire (R) = 0.2Ω

Length of wire (l) = $20 \text{ cm} = 0.2 \text{ m}$

Current (I) = $2 \mu\text{A} = 2 \times 10^{-6} \text{ A}$

Velocity with which the wire moves (v) = $20 \text{ cm s}^{-1} = 0.2 \text{ ms}^{-1}$

Horizontal component of earth's magnetic field(B_H) = 3.0×10^{-5} T

Formula used:

Angle of dip $\delta = \tan^{-1}\left(\frac{B_V}{B_H}\right)$... (i), where B_V = vertical component of earth's magnetic field, B_H = horizontal component of earth's magnetic field.

Now, emf induced in the wire $E = B_V l v$... (ii), where B_V = vertical component of earth's magnetic field, l = length of wire, v = velocity with which the wire moves

Also, by Ohm's law, $E = IR$... (iii), where E = emf, I = current, v = velocity.

Equating (ii) and (iii) we get

$$B_V l v = IR \Rightarrow B_V = \frac{IR}{l v}$$

Substituting the given values, we get

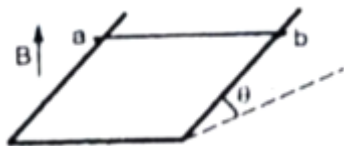
$$B_V = \frac{2 \times 10^{-6} \times 0.2}{0.2 \times 0.2} T = 10^{-5} T$$

Hence, angle of dip $\Rightarrow \delta = \tan^{-1}\left(\frac{1}{3}\right)$ (Ans)

41. Question

A wire ab of length ℓ , mass m and resistance R slides on a smooth, thick pair of metallic rails joined at the bottom as shown in figure. The plane of the rails makes an angle θ with the horizontal. A vertical magnetic field B exists in the region. If the wire slides on the rails at a constant speed v , show that

$$B = \sqrt{\frac{mg R \sin \theta}{v l^2 \cos^2 \theta}}$$



Answer

Given:

Length of $ab = l$

Mass = m

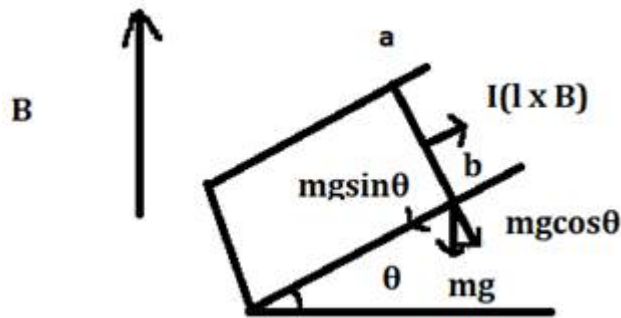
Resistance of $ab = R$

Angle between the plane of the rails and the horizontal = θ

Magnetic field = B

Velocity with which the wire slides along the rails = v

Diagram:



Formula used:

Emf induced in the wire ab $\mathcal{E} = Bl'v$... (i), where

\mathcal{E} = emf, B = magnetic field, l' = component of l perpendicular to the magnetic field = $l \cos \theta$, v = velocity

Magnetic force on ab $\mathbf{F} = I (\mathbf{l} \times \mathbf{B})$, where I = current, l = length of ab , B = magnetic field

Now, the angle between l and B is $90^\circ - \theta$.

Therefore, $F = IlB \sin (90^\circ - \theta) = IlB \cos \theta$... (ii)

This force will be equal to the $\sin \theta$ component of the weight of the wire.

Hence, $F = mg \sin \theta$... (iii)

Now, $I = \frac{\mathcal{E}}{R} = Bl' \frac{v}{R} = \frac{Blv \cos \theta}{R}$... (iv), where I = current, \mathcal{E} = emf, R = resistance, B = magnetic field, v = velocity, l = length of wire, θ = angle between plane of rod and horizontal

Therefore, substituting this in (ii), we get

$$F = \frac{B^2 l^2 v \cos^2 \theta}{R} \dots (v)$$

Hence, equating (iii) and (v), we get

$$mg \sin \theta = \frac{B^2 l^2 v \cos^2 \theta}{R}$$

$$\Rightarrow \text{Magnetic field } B = \sqrt{\frac{mgR \sin \theta}{vl^2 \cos^2 \theta}} \text{ (proved)}$$

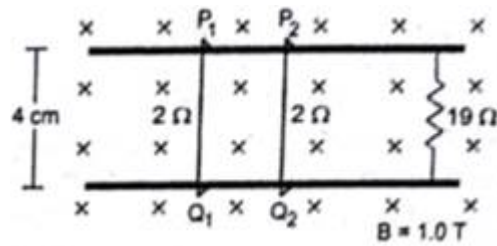
42. Question

Consider the situation shown in figure. The wires P_1Q_1 and P_2Q_2 are made to slide on the rails with the same speed 5 cm s^{-1} . Find the electric current in the 19Ω

resistor if

(a) both the wires move towards right and

(b) If P_1Q_2 moves towards left but P_2Q_2 moves towards right.



Answer

Given:

Velocity with which the wires move (v) = $5 \text{ cm s}^{-1} = 0.05 \text{ m s}^{-1}$

Resistance (R) = 19Ω

Resistance of each of the wires (r) = 2Ω

Length of wire (l) = $4 \text{ cm} = 0.04 \text{ m}$

Magnetic field (B) = 1 T

Formula used:

Emf induced $\mathbf{E = Blv}$... (i) where B = magnetic field, l = length of wire, v = velocity

Also, by Ohm's law, emf $\mathbf{E = IR}$... (ii), where I = current, R = total resistance.

(a) When the wires slide in the same direction, we have two parallel sources of emf with current flowing in the same direction.

Hence, from (i), net emf $\mathbf{E = (1 \times 0.04 \times 0.05) V = 2 \times 10^{-3} V}$

Net parallel resistance of the 2Ω wires = $\frac{2 \times 2}{2 + 2} \Omega = 1 \Omega$

Hence, total resistance (R) = $(1 + 19) \Omega = 20 \Omega$

Therefore, substituting these values in (ii), we get

Current $\mathbf{I = \frac{2 \times 10^{-3}}{20} A = 10^{-4} A}$ (Ans)

(b) When the wires slide in opposite directions, the two parallel sources of emf have opposing directions. Hence, the net emf is 0.

Therefore, the net current is also 0. (Ans)

43. Question

Suppose the 19Ω resistor of the previous problem is disconnected. Find the current through P_2Q_2 in the two situations.

(a) and (b) of that problem.

Answer

(a) When the 19Ω resistor is removed and the wires move in the same direction, their polarity remains the same. Hence, the circuit remains incomplete and the current through P_2Q_2 is 0. (Ans)

(b) When the wires slide in opposite directions, the polarity of one of the wires reverses and current flows.

In this case, emf $E = Blv$, where B = magnetic field, l = length, v = velocity.

Hence, $E = (1 \times 0.04 \times 0.05) \text{ V} = 2 \times 10^{-3} \text{ V}$

But here, resistance $R = 2\Omega$ only

Therefore, Current flowing through the wire $I = \frac{E}{R} = 10^{-3} \text{ A}$ (Ans)

44. Question

Consider the situation shown in figure. The wire PQ has a negligible resistance and is made to slide on the three rails with a constant speed of 5 cm s^{-1} . Find the current in the 10Ω resistor when the switch S is thrown to (a) the middle rail (b) the bottom rail.



Answer

Given:

Speed(v) = $5 \text{ cm s}^{-1} = 0.05 \text{ m s}^{-1}$

External resistance(R) = 10Ω

Magnetic field(B) = 1 T

Formula used:

Induced emf $E = Blv$... (i), where B = magnetic field, l = length of sliding wire, v = velocity

(a) When the switch S is thrown to the middle rail, length of sliding wire $l = 2 \text{ cm} = 0.02 \text{ m}$

Hence, induced emf in this case from (i) is

$$E = (1 \times 0.02 \times 0.05) \text{ V} = 10^{-3} \text{ V}$$

Given resistance $R = 10\Omega$

Therefore, current flowing through the resistor $I = \frac{E}{R}$

where $E = \text{emf}$, $R = \text{resistance}$.

$$I = \frac{10^{-3}}{10} \text{ A} = 10^{-4} \text{ A} = 0.1 \text{ mA (Ans)}$$

(b) When the switch S is thrown to the bottom rail, length of sliding wire (l') = 4 cm = 0.04 m

Hence, induced emf $E' = Bl'v = (1 \times 0.04 \times 0.05) \text{ V} = 2 \times 10^{-3} \text{ V}$, where $B =$ magnetic field, $l' =$ length of sliding wire, $v =$ velocity

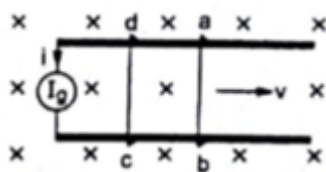
Resistance $R = 10\Omega$

Therefore, current flowing through the resistor $I' = E'/R$, where $E' = \text{emf}$, $R =$ resistance

$$I = \frac{2 \times 10^{-3}}{10} \text{ A} = 2 \times 10^{-4} \text{ A} = 0.2 \text{ mA (Ans)}$$

45. Question

The current generation I_g , shown in figure, sends a constant current i through the circuit. The wire cd is fixed and ab is made to slide on the smooth, thick rails with a constant velocity v towards right. Each of these wires has resistance r . Find the current through the wire cd .



Answer

Given:

Initial current passing through the circuit = i

Velocity with which the wire ab moves = v

Resistance of each wire = r

Formula used:

Induced emf due to moving of wire ab $E' = Blv$... (i), where $B =$ magnetic field, l = length of sliding wire, $v =$ velocity

Initial emf in the wire $E_0 = ir$, where i = current, r = resistance of wire ab.

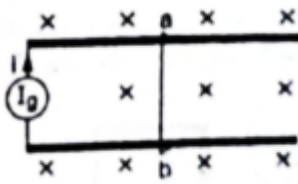
Hence, net emf(E) = $E_0 - E' = ir - Blv$... (ii)

Now, net resistance = $2r$

Hence, current passing through the wire cd = $\frac{E}{2r} = \frac{ir - Blv}{2r}$ (Ans)

46. Question

The current generator I_g , shown in figure, sends a constant current i through the circuit. The wire ab has a length ℓ and mass m and can slide on the smooth, horizontal rails connected to I_g . The entire system lies in a vertical magnetic field B . Find the velocity of the wire as a function of time.



Answer

Given:

Initial current = i

Length of sliding wire ab = l

Mass = m

Magnetic field = B

Formula used:

Magnetic force on the wire ab $F = ilB$... (i), where i = current, l = length of sliding wire, B = magnetic field

Now, velocity v can be written as $v = u + at$... (ii), where u = initial velocity = 0 (in this case), a = acceleration, t = time.

Hence, acceleration $a = v/t$... (iii)

Now, according to Newton's 2nd law of motion, $F = ma$... (iv), where F = force, m = mass, a = acceleration.

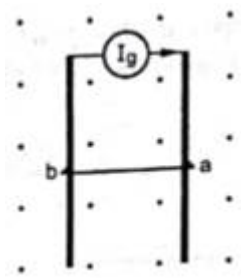
Substituting (iii) in (iv) and equating (i) and (iv), we get

$$ilB = \frac{mv}{t} \Rightarrow v = \frac{ilBt}{m}$$

Hence, velocity of the wire as a function of time is $v = \frac{ilBt}{m}$ (Ans)

47. Question

The system containing the rails and the wire of the previous problem is kept vertically in a uniform horizontal magnetic field B that is perpendicular to the plane of the rails figure. It is found that the wire stays in equilibrium. If the wire ab is replaced by



another wire of double its mass, how long will it take in falling through a distance equal to its length?

Answer

Given:

Initial mass = m

Magnetic field = B

Length of sliding wire = l

Formula used:

Magnetic force $F = ilB$... (i), where i = current, l = length of sliding wire, B = magnetic field.

At equilibrium, this magnetic force balances the weight of the wire mg acting downward, where m = mass, g = acceleration due to gravity.

Therefore, $mg = ilB$... (ii)

Now, when the wire ab is replaced by another wire of mass $2m$, the weight acting downward will be $2mg$, where g = acceleration due to gravity.

Hence, net force = $2mg - ilB$... (iii) where $2m$ = mass, g = acceleration due to gravity, i = current, l = length of sliding wire, B = magnetic field.

According to Newton's law of motion, $F = m'a$... (iv), where F = net force, m' = mass, acceleration

In this case, $m' = 2m$.

Therefore, equating (iii) and (iv), we get

$$2mg - ilB = 2ma$$

$$\Rightarrow a = \frac{2mg - ilB}{2m} \text{ ... (v)}$$

Now, distance travelled can be expressed as $s = ut + \frac{1}{2}at^2$... (vi),
 where s = distance travelled, u = initial velocity = 0 (in this case), t = time, a = acceleration.

Now, distance travelled = l (given)

Therefore, from (v), (vi) becomes:

$$l = \frac{1}{2} \times \frac{2mg - Blv}{2m} t^2$$

$$\Rightarrow t = \sqrt{\frac{4mll}{2mg - Blv}}$$

But, from (i), $ilB = mg$.

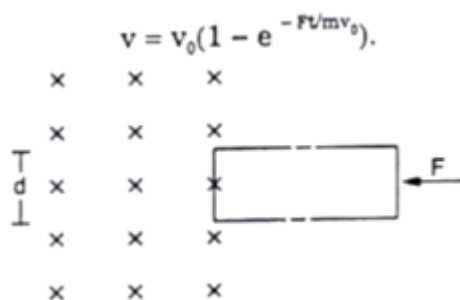
$$\text{Therefore, } t = \sqrt{\frac{4mll}{2mg - mg}} \Rightarrow t = \sqrt{\frac{2l}{g}}$$

$$\text{Required time taken} = t = \sqrt{\frac{2l}{g}} \text{ (Ans)}$$

48. Question

The rectangular wire-frame, shown in figure has a width d , mass m , resistance R and a large length. A uniform magnetic field B exists to the left of the frame. A constant force F starts pushing the frame into the magnetic field at $t = 0$.

- Find the acceleration of the frame when its speed has increased to v .
- Show that after some time the frame will move with a constant velocity till the whole frame enters into the magnetic field. Find this velocity v_0 .
- Show that the velocity at time t is given by



Answer

Given:

Length of sliding wire = width of frame = d

Mass = m

Resistance = R

Magnetic field = B

Initial force = F

Formula used:

(a) Induced emf(when it attains a speed v) $E = Bdv$... (i), where B = magnetic field, d = width of frame, v = velocity

Therefore, induced current $I = \frac{E}{R}$, where E = induced emf, R = resistance \Rightarrow

$$I = \frac{Bdv}{R} \dots (ii)$$

Now, magnetic force acting on the wire $F' = IdB$... (iii), where I = current, d = length of sliding wire = width of frame, B = magnetic field

$$\text{Substituting (ii) in (iii), } F' = \frac{B^2 d^2 v}{R} \dots (iv)$$

Now, as the magnetic force is in opposite direction to applied force, net force =

$$F' - F = F - \frac{B^2 d^2 v}{R} \dots (v)$$

But, from Newton's 2nd law of motion, **net force = ma** ... (vi), where m = mass, a = acceleration

Equating (v) and (vi):

$$ma = F - \frac{B^2 d^2 v}{R} \Rightarrow a = F - \frac{B^2 d^2 v}{Rm}$$

$$\text{Acceleration of the frame at speed } v = F - \frac{B^2 d^2 v}{Rm} \text{ (Ans)}$$

(b) For the velocity to be constant, acceleration needs to be 0.

Hence, from previous part,

$\frac{F}{m} - \frac{B^2 d^2 v_0}{mR} = 0$ where F = external force, m = mass, B = magnetic field, d = width of frame, v_0 = constant velocity, R = resistance

$$\Rightarrow v_0 = \frac{FR}{B^2 d^2}$$

$$\text{Constant velocity } v_0 = \frac{FR}{B^2 d^2} \text{ (Ans)}$$

$$\text{(c) From part (a), acceleration } a = \frac{F}{m} - \frac{B^2 d^2 v}{mR}$$

Now, acceleration $a = dv/dt$, where v = velocity, t = time

$$\text{Hence, } \frac{dv}{dt} = \frac{F}{m} - \frac{B^2 d^2 v}{mR} = \frac{FR - B^2 d^2 v}{mR}$$

$$\Rightarrow \frac{dv}{FR - B^2 d^2 v} = \frac{dt}{mR}$$

Integrating with proper limits, we get

$$\int_0^v \frac{dv}{FR - B^2 d^2 v} = \int_0^t \frac{dt}{mR}$$

$$\Rightarrow \left[\frac{-1}{B^2 d^2} \ln|FR - B^2 d^2 v| \right]_0^v = \frac{t}{mR} \Rightarrow \ln|FR - B^2 d^2 v| - \ln(RF) = \frac{(-B^2 d^2 t)}{mR}$$

$$\Rightarrow \frac{FR - B^2 d^2 v}{RF} = e^{-\frac{B^2 d^2 t}{mR}}$$

$$\Rightarrow B^2 d^2 v = RF \left(1 - e^{-\frac{B^2 d^2 t}{mR}} \right)$$

$$\Rightarrow v = \frac{RF}{B^2 d^2} \left(1 - e^{-\frac{B^2 d^2 t}{mR}} \right)$$

But, from previous part (b), we found out that $v_0 = \frac{FR}{B^2 d^2} \Rightarrow$

$$F = \frac{v_0 B^2 d^2}{R}$$

$$\text{Hence, } v = \frac{R x v_0 B^2 d^2}{(B^2 d^2 R) \left(1 - e^{-\frac{B^2 d^2 t}{mR}} \right)}$$

$$\Rightarrow v = v_0 \left(1 - e^{-\frac{B^2 d^2 t}{mR}} \right) = v_0 \left(1 - e^{-\frac{Ft}{Mv_0}} \right) \text{ (proved)}$$

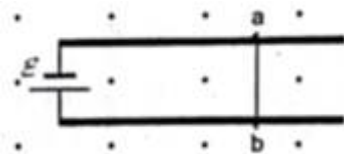
49. Question

Figure shows a smooth pair of thick metallic rails connected across a battery of emf ϵ having a negligible internal resistance. A wire ab of length ℓ and resistance r can slide smoothly on the rails. The entire system lies in a horizontal plane and is immersed in a uniform vertical magnetic field B . At an instant t , the wire is given a small velocity v towards right.

(a) Find the current in it at this instant. What is the direction of the current?

(b) What is the force acting on the wire at this instant?

(c) Show that after some time the wire ab will slide with a constant velocity. Find this velocity.



Answer

Given:

Emf of the battery = ϵ

Length of sliding wire = l

Resistance = r

Magnetic field = B

Velocity = v

Formula used:

(a) Induced emf $\mathcal{E}' = Blv$... (i), where B = magnetic field, l = length of sliding wire, v = velocity

Therefore, net emf = $\epsilon - Blv$

Hence, current in the wire $(i) = \frac{\epsilon - Blv}{r}$ (Ans)

(b) Magnetic force acting on the wire $F = ilB$... (ii), where i = current, l = length of sliding wire, B = magnetic field.

Hence, from (a), $i = \frac{\epsilon - Blv}{r}$

Therefore, force $F = \frac{(\epsilon - Blv)lB}{r}$ (Ans)

(c) At constant velocity, net force will be 0.

Hence, $\frac{(\epsilon - Blv)lB}{r} = 0$

$\Rightarrow \epsilon - Blv = 0 \Rightarrow v = \frac{\epsilon}{Bl}$

Hence, value of velocity = $\frac{\epsilon}{Bl}$ (Ans)

50. Question

A conducting wire ab of length ℓ , resistance r and mass m starts sliding at $t = 0$ down a smooth, vertical, thick pair of connected rails as shown in figure. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the rails.

(a) Write the induced emf in the loop at an instant t when the speed of the wire is v .

(b) What would be the magnitude and direction of the induced current in the wire?

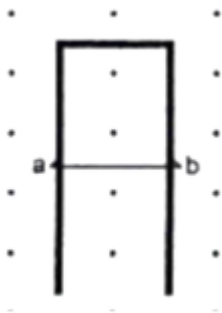
(c) Find the downward acceleration of the wire at this instant.

(d) After sufficient time, the wire starts moving with a constant velocity. Find this velocity v_m .

(e) Find the velocity of the wire as a function of time.

(f) Find the displacement of the wire as a function of time.

(g) Show that the rate of heat developed in the wire is equal to the rate at which the gravitational potential energy is decreased after steady state is reached.



Answer

Given:

Length of sliding wire $ab = l$

Resistance = r

Mass = m

Magnetic field = B

Speed = v

Formula used:

(a) Induced emf in the loop $E = Blv$ (Ans), where B = magnetic field, l = length, v = velocity

(b) Induced current in the wire $I = \frac{E}{r} = \frac{Blv}{r}$ (Ans), where E = emf, r = resistance

As the wire is moving, the magnetic flux is increasing. Hence, the direction of the current will be such as to oppose the increase in flux. Hence, the current will move from b to a .

(c) Magnetic force on the wire (upward) $F = IlB$... (i), where I = current, l = length of sliding wire, B = magnetic field.

Weight acting downward = mg

Hence, net downward force = $IlB - mg$... (ii)

According to Newton's 2nd law of motion, net force = ma ... (iii), where m = mass, a = acceleration.

Hence, equating (ii) and (iii), $ma = mg - IlB$

But, $I = E/r$, where I = current, E = emf, r = resistance $\Rightarrow I = Blv/r$, where B = magnetic field, l = length of sliding wire, v = velocity

$$\Rightarrow \text{acceleration } a = \left(g - \frac{B^2 l^2 v}{mr} \right) \text{ (Ans)}$$

(d) When the wire will move with constant velocity (let it be v_0), acceleration will be 0.

Hence, from part (d) of this question,

$$g - \frac{B^2 l^2 v_0}{mr} = 0$$

$$\Rightarrow \text{constant velocity } v_0 = \frac{mrg}{B^2 l^2} \text{ (Ans)}$$

(e) From part (c),

$$a = g - \frac{B^2 l^2 v_0}{mr}$$

But, $a = dv/dt$, where v = velocity, t = time

$$\text{Therefore, } \frac{dv}{dt} = g - \frac{B^2 l^2 v_0}{mr}$$

$$\Rightarrow \frac{dv}{mrg - B^2 l^2 v} = \frac{dt}{mr}$$

Integrating with proper limits, we get

$$\int_0^v \frac{dv}{\frac{mrg}{B^2 l^2} - v} = \int_0^t \frac{dt}{mr}$$

$$\Rightarrow \left[-\ln \left| \frac{mrg}{B^2 l^2} - v \right| \right]_0^v = \frac{t}{mr} \Rightarrow \frac{B^2 l^2 v}{mrg} = 1 - e^{-\frac{B^2 l^2 t}{mr}}$$

$$\Rightarrow v = \frac{mrg}{(B^2 l^2) \left(1 - e^{-\frac{B^2 l^2 t}{mr}} \right)}$$

$$\text{But, from the previous part, } v_0 = \frac{mrg}{B^2 l^2}$$

$$\text{Therefore, velocity as a function of time : } v = v_0 \left(1 - e^{-\frac{B^2 l^2 t}{mr}} \right) \text{ (Ans)}$$

(f) Now, v can be written as dx/dt , where x = position, t = time

$$\text{Therefore, from previous part, } v = \frac{dx}{dt} = v_0 \left(1 - e^{-\frac{B^2 l^2 t}{mr}} \right)$$

$$\Rightarrow dx = v_0 \left(1 - e^{-\frac{B^2 l^2 t}{mr}} \right) dt$$

Integrating with suitable limits, we get:

$$\int_0^x dx = \int_0^t v_0 \left(1 - e^{-\frac{B^2 l^2 t}{mr}} \right) dt$$

$$\Rightarrow x = v_0 \left[t + \frac{\frac{v_0}{g}}{\left(e^{\frac{gt}{v_0}} - 1 \right)} \right]$$

$$\Rightarrow \text{Displacement as a function of time } x = v_0 t - \frac{v_0^2}{g \left(1 - e^{-\frac{gt}{v_0}} \right)} \text{ (Ans)}$$

$$(g) \text{ Then, } \frac{d}{dt} mgs = mg \frac{ds}{dt} = mgv_m \left(1 - e^{-\frac{gt}{v_m}} \right)$$

$$\frac{dH}{dt} = i^2 R = R \left(\frac{lBv}{R} \right)^2$$

$$\frac{dH}{dt} = \frac{l^2 B^2}{R} \left(v_m \left(1 - e^{-\frac{gt}{v_m}} \right) \right)^2$$

After steady state, $t \rightarrow \infty$

$$\frac{d}{dt} mgs = mgv_m \quad \frac{dH}{dt} = \frac{l^2 B^2}{R} v_m^2$$

$$\frac{dH}{dt} = \frac{l^2 B^2}{R} v_m \times \frac{mgR}{l^2 B^2}$$

$$\frac{dH}{dt} = mgv_m$$

Hence after steady state,

$$\frac{dH}{dt} = \frac{d}{dt} mgs$$

So, the rate of heat developed in the wire is equal to the rate at which the gravitational potential energy is decreased after steady state is reached.

51. Question

A bicycle is resting on its stand in the east-west direction and the rear wheel is rotated at an angular speed of 100 revolutions per minute. If the length of each spoke is 30.0 cm and the horizontal component of the earth's magnetic field is $2.0 \times 10^{-5} \text{ T}$, find the emf induced between the axis and the outer end of a spoke. Neglect centripetal force acting on the free electrons of the spoke.

Answer

Given:

Angular speed(ω) = 100 revolutions/minute $\times 2\pi$ = 100 revolutions/60 sec $\times 2\pi$ = $10\pi/3$ revolutions/sec

Length of each spoke(l) = 30 cm = 0.3 m

Magnetic field(B) = 2.0×10^{-5} T

Formula used:

Induced emf $E = Blv$... (i), where B = magnetic field, l = length of spoke, v = velocity

Now, linear speed of the spoke $v = \omega r$, where ω = angular speed, r = distance from the axis to the outer end.

Here, $r = \frac{l}{2}$, where l = length of spoke

Hence, $v = \frac{\omega l}{2}$

Therefore, emf induced $E = \frac{B\omega l^2}{2}$ (from (i))

$$\Rightarrow E = \left(\frac{2 \times 10^{-5} \times 10 \frac{\pi}{3} \times (0.3)^2}{2} \right) \text{V} = 9.42 \times 10^{-6} \text{ V (Ans)}$$

52. Question

A conducting disc of radius r rotates with a small but constant angular velocity ω about its axis. A uniform magnetic field B exists parallel to the axis of rotation. Find the motional emf between the center and the periphery of the disc.

Answer

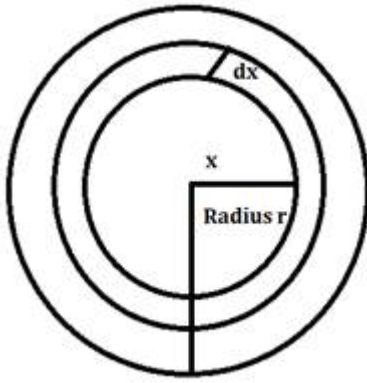
Given:

Radius = r

Angular velocity = ω

Magnetic field = B

Diagram:



Formula used:

In this case, the velocity will increase radially.

Let us consider a strip of width dx at a distance x from the centre.

Hence, induced emf of this portion will be $dE = Blv = Bdx \times xw$, where B = magnetic field, dx = width of the element, x = distance of the element from the centre, w = angular velocity

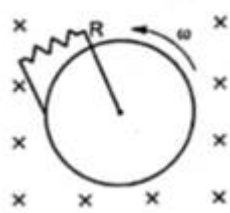
Hence, integrating on both sides using proper limits, we get

$$\int_0^E dE = \int_0^r Bxw dx$$

$$\Rightarrow \text{Total motional emf } E = \frac{Bwr^2}{2} \text{ (Ans)}$$

53. Question

Figure shows a conducting disc rotating about its resistance R is connected between the centre and the rim. Calculate the current in the resistor. Does it enter the disc or leave it at the centre? The radius of the disc is 5.0 cm, angular speed $\omega = 10 \text{ rad/s}$, $B = 0.40 \text{ T}$ and $R = 10 \Omega$.



Answer

Given:

Resistance(R) = 10Ω

Radius(r) = 5 cm = 0.05 m

Angular speed(w) = 10 rad/s

Magnetic field(B) = 0.4 T

Formula used:

We consider a rod of length 5 cm from the centre and rotating with same ω .

Hence, length of sliding rod (l) = 5 cm = 0.05 m.

Now, induced emf $E = Blv$... (i), where B = magnetic field, l = length of sliding rod, v = velocity

Now, velocity $v = \frac{l}{2} \omega$... (ii), where l = length of rod, ω = angular velocity

From Ohm's law, current through resistor $R(I) = E/R = Bl^2\omega/2R$ (from (i) and (ii)), where E = emf, R = resistance

$$\text{Hence, } I = \frac{0.4 \times 0.05^2 \times 10}{2 \times 10} \text{ A} = 0.5 \text{ mA (Ans)}$$

Since the disc is rotating anticlockwise, the emf induced is such that the centre is at a higher potential than the periphery. Hence, the current leaves from the centre.

54. Question

The magnetic field in a region is given by $\vec{B} = k \frac{\vec{B}_0}{L} y$ where L is a fixed length. A

conducting rod of length L lies along the Y -axis between the origin and the point $(0, L, 0)$. If the rod moves with a velocity $\vec{v} = v_0 \hat{i}$, find the emf induced between the ends of the rod.

Answer

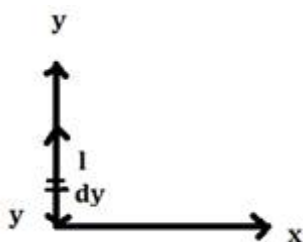
Given:

$$\vec{B} = k \frac{\vec{B}_0}{L} y$$

Where B = magnetic field, y = distance from origin on y axis, L = fixed length.

Velocity of rod = $v_0 \hat{i}$

Diagram:



Formula used:

Now, we consider a small element dy at a distance y from the origin.

Emf induced in the element $dE = Bvdy$, where B = magnetic field, v = velocity, dy = length of element

$$\Rightarrow dE = \frac{B_0}{L} y v_0 dy$$

Integrating on both sides with proper limits, we get

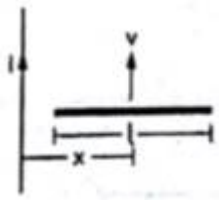
$$\int_0^E dE = \int_0^L \frac{B_0}{L} y v_0 dy$$

$$\Rightarrow \text{Total emf } E = \frac{B_0 v_0 L^2}{2L}$$

$$E = \frac{B_0 v_0 L}{2} \text{ (Ans)}$$

55. Question

Figure shows a straight, long wire carrying a current i and a rod of length ℓ coplanar with the wire and perpendicular to it. The rod moves with a constant velocity v in a direction parallel to the wire. The distance of the wire from the centre of the rod is x . Find the motional emf induced in the rod.



Answer

Given:

Current = i

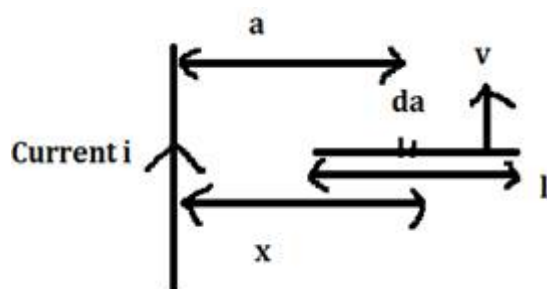
Length of rod = l

Velocity = v

Distance of centre of rod from wire = x

Hence, the two ends of the rod are at distances $x - \frac{l}{2}$ and $x + \frac{l}{2}$ from the wire

Diagram:



Formula used:

We consider an element of length da at a distance of 'a' from the wire.

Now, magnetic field due to an infinite current carrying wire at a distance a
 $(B) = \frac{\mu_0 i}{2\pi a}$... (i), where μ_0 = magnetic permeability of vacuum, i = current, a = distance from wire.

Therefore, emf induced in the element $da = dE = B d a v$... (ii), where B = magnetic field, da = element, v = velocity

Hence, putting (i) in (ii), we get

$$dE = \frac{\mu_0 i v da}{2\pi a}$$

Integrating on both sides and putting suitable limits, we get

$$\int_0^E dE = \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} \frac{\mu_0 i da v}{2\pi a}$$

$$\Rightarrow E = \frac{\mu_0 i v}{2\pi} \left[\ln|a| \right]_{x-\frac{l}{2}}^{x+\frac{l}{2}} = \frac{\mu_0 i v}{2\pi} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| \text{ (ans)}$$

56. Question

Consider a situation similar to that of the previous problem except that the ends of the rod slide on a pair of thick metallic rails laid parallel to the wire. At one end the rails are connected by resistor of resistance R .

- (a) What force is needed to keep the rod sliding at a constant speed v ?
- (b) In this situation what is the current in the resistance R ?
- (c) Find the rate of heat developed in the resistor.
- (d) Find the power delivered by the external agent exerting the force on the rod.

Answer

Given:

Resistance = R

Constant velocity = v

Formula used:

From previous question, induced emf

$$E = \frac{\mu_0 i v}{2\pi} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| = \frac{\mu_0 i v}{2\pi} \ln \left| \frac{2x+l}{2x-l} \right| \text{ w, induced current } I = \frac{E}{R} = \frac{\mu_0 i v}{2\pi R} \ln \left| \frac{x+l/2}{x-l/2} \right| \dots$$

(i), where E = emf, R = resistance, μ_0 = magnetic permeability of vacuum, i = current in the wire, v = velocity of sliding rod, x = distance of centre of rod from wire, l = length of rod.

Now, magnetic force on element $da = dF = I da B \dots$ (ii), where I = induced current, da = element, B = magnetic field due to infinitely straight wire

$B = \frac{\mu_0 i}{2\pi a} \dots$ (iii), where μ_0 = magnetic permeability of vacuum, i = current in wire, a = distance from wire.

Hence, (ii) becomes

$$dF = \frac{\frac{\mu_0 i v}{2\pi R} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| da \times \frac{\mu_0 i}{2\pi a \left(\frac{\mu_0 i}{2\pi} \right)^2 x \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right|} v}{R} \times \frac{da}{a} \quad \text{integrating with suitable limits, we get}$$

$$\int_0^F dF = \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} \frac{\left(x+\frac{l}{2} \right) \left(\frac{\mu_0 i}{2\pi} \right)^2 x \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| v}{R} \times \frac{da}{a}$$

=> Force needed to keep the wire sliding at constant velocity v

$$F = \left(\frac{\mu_0 i}{2\pi} \right)^2 \left(\ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| \right)^2 \times \frac{v}{R} \text{ ns}$$

(b) Current $I = E/R = \frac{\mu_0 i v}{2\pi R} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right|$ here E = emf, R = resistance, μ_0 = magnetic permeability of vacuum, i = current in the wire, v = velocity of sliding rod, x = distance of centre of rod from wire, l = length of rod. (Ans)

(c) Rate of heat developed in the resistor = Power(P) = $I^2 R$, where I = current, R = resistance

From previous part, $I = \frac{\mu_0 i v}{2\pi R} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right|$ therefore, rate of heat developed =

$$\left(\frac{\mu_0 i v}{2\pi R} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| \right)^2 R \frac{1}{R \left(\frac{\mu_0 i v}{2\pi} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| \right)^2} \text{ (ans)}$$

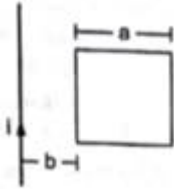
(d) Power delivered by external agent = rate of heat developed in resistor =

$$\frac{1}{R \left(\frac{\mu_0 i v}{2\pi} \ln \left| \frac{x+\frac{l}{2}}{x-\frac{l}{2}} \right| \right)^2} \text{ (ans)}$$

57. Question

Figure shows a square frame of wire having a total resistance r placed co-planarly with long, straight wire. The wire carries a current I given by $i = i_0 \sin \omega t$. Find

- (a) the flux of the magnetic field through the square frame,
- (b) the emf induced in the frame and
- (c) the heat developed in the frame in the time interval 0 to $\frac{20\pi}{\omega}$.



Answer

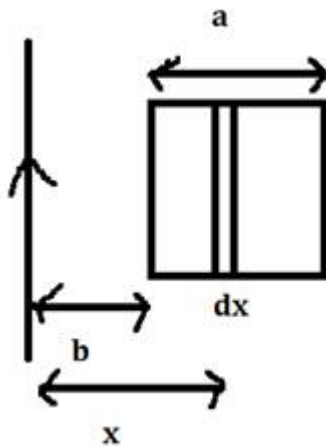
Given:

Current in wire $i = i_0 \sin \omega t$

Length of each side of square loop = a

Distance of one edge from wire = b

Diagram:



Formula used:

Magnetic flux $\Phi = \int B \cdot da \dots$ (i), where Φ = magnetic flux, B = magnetic field, da = area element

Magnetic field due to a long current carrying wire at distance x $B = \frac{\mu_0 i}{2\pi x} \dots$ (ii), where μ_0 = magnetic permeability of vacuum, i = current, x = distance from wire

We consider a strip of width dx at a distance x from the wire.

Now, area element $da = a \, dx$, where a = length of loop, dx = width element

Hence, from (i) and (ii),

$$\text{Flux } \Phi = \int_b^{a+b} B \cdot da = \int_b^{a+b} \frac{\mu_0 i a dx}{2\pi x} = \frac{\mu_0 i a}{2\pi} \ln\left(\frac{a+b}{b}\right) \text{ (ans)}$$

(b) Emf induced in frame $E = \frac{d\Phi}{dt}$, where Φ = flux, t = time

From previous part, $\Phi = \frac{\mu_0 i a}{2\pi} \ln\left(\frac{a+b}{b}\right)$ re, $i = i_0 \sin \omega t$

$$\Rightarrow E = \frac{d\Phi}{dt} = \frac{\mu_0 a}{2\pi} \ln\left(\frac{a+b}{b}\right) i_0 \omega \cos \omega t \text{ (Ans)}$$

(c) Heat developed in wire (H) = $i^2 r t$, where i = current through frame, r = resistance, t = time

$$\text{From previous } i = E/r = \frac{\mu_0 a}{2\pi r} \ln\left(\frac{a+b}{b}\right) i_0 \omega \cos \omega t$$

where E = emf, r = resistance

$$\text{Hence } H = \frac{\mu_0 a}{2\pi r} \ln\left(\frac{a+b}{b}\right) i_0 \omega \cos \omega t$$

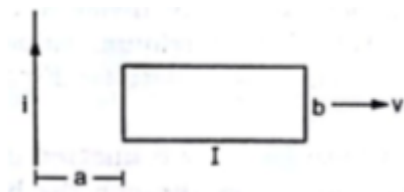
$$\text{Now, Given: } t = \frac{20\pi}{\omega}$$

$$\text{Hence, } H = \frac{\mu_0 a r}{2\pi} \ln\left(\frac{a+b}{b}\right) i_0 \omega X \frac{20\pi \cos(20\pi)}{\omega} = \frac{5\mu_0^2 a^2 i_0^2 \omega}{2\pi r} \ln^2\left(\frac{b+a}{b}\right)$$

58. Question

A rectangular metallic loop of length ℓ and width b is placed

coplanarly with a long wire carrying a current i figure. The loop is moved perpendicular to the wire with a speed v in the plane containing the wire and the loop. Calculate the emf induced in the loop when the rear end of the loop is at a distance a from the wire. Solve by using Faraday's law for the flux through the loop and also by replacing different segments with equivalent batteries.



Answer

Given: Length of the rectangular loop = ℓ

Breadth of the rectangular loop = b

Current in the wire = i

Speed of loop = v

We have to find the emf induced in the loop by Faraday's Law.

Faraday's law states that whenever the magnetic flux through a closed surface changes, there will be an induced emf produced in the loop that encloses the surface. The mathematical relation is

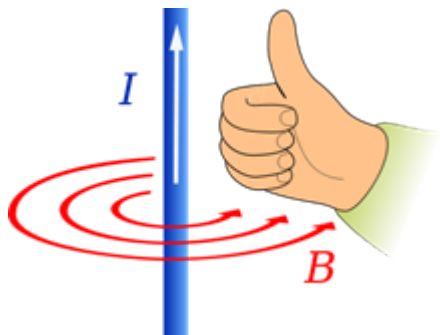
$$\mathcal{E} = - \frac{d(\phi_B)}{dt}$$

where \mathcal{E} is the magnitude of the induced emf, ϕ_B is the magnetic flux through the surface. The negative sign arises because the induced emf will be produced such that it will oppose the change of magnetic flux. The magnetic flux is given by

$$\phi_B = \oint \vec{B} \cdot d\vec{S}$$

where B is the magnetic field and dS is a small area element on the surface.

The wire which is near the loop is responsible for the magnetic field. The wire is carrying current in the upward direction so by Fleming's right hand thumb rule, the magnetic field will be perpendicular to the plane of the paper in the inward direction.



We have to now calculate the magnetic field acting across the loop.

Let us consider a small rectangular element of length b and width dx at a distance of x from wire. The magnetic field on this element due to the current carrying wire is given by Ampere circuital law.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl \cos\theta = \oint B dl$$

($\cos \theta = 1$ because the \vec{dl} vector and \vec{B} vector are both acting in ward in the plane of paper so $\theta = 0^\circ$, here dl is a small current carrying element of the circular amperian loop)

$$\oint B dl = B \oint dl = B \times 2\pi r$$

$$B \times 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$

The magnetic flux through this element will be

$$d\phi_B = \oint \vec{B} \cdot \vec{dS}$$

$$d\phi_B = \oint B dS \cos \theta$$

$$d\phi_B = \oint B dS = B b dx$$

($\cos \theta = 1$ because the magnetic field across the area and the normal vector of this area element both point in the same direction so $\theta = 0^\circ$ so $\cos \theta = 1$)

The total magnetic flux through the loop will be the flux through

Infinite such elementss from $x=a$ to $x=a+l$

$$\phi_B = \int_{x=a}^{x=a+l} d\phi_B$$

$$\phi_B = \int_{x=a}^{x=a+l} B b dx$$

$$\phi_B = \int_{x=a}^{x=a+l} \frac{\mu_0 i}{2\pi r} b dx$$

$$\phi_B = \frac{\mu_0 i b}{2\pi} \int_{x=a}^{x=a+l} \frac{1}{x} dx \text{ (taking constants out of the integral)}$$

$$\phi_B = \frac{\mu_0 i b}{2\pi} [\ln x]_a^{a+l}$$

$$\phi_B = \frac{\mu_0 i b}{2\pi} [\ln(a+l) - \ln(a)]$$

$$\phi_B = \frac{\mu_0 ib}{2\pi} \left[\ln \frac{a+l}{a} \right]$$

The flux through the loop is $\frac{\mu_0 ib}{2\pi} \left[\ln \frac{a+l}{a} \right]$

The emf induced will be $\mathcal{E} = - \frac{d\left(\frac{\mu_0 ib}{2\pi} \left[\ln \frac{a+l}{a} \right]\right)}{dt}$

$$\mathcal{E} = - \frac{d\left(\frac{\mu_0 ib}{2\pi} \left[\ln \frac{a+l}{a} \right]\right)}{dt}$$

$$\mathcal{E} = - \left(\frac{\mu_0 ib}{2\pi}\right) \frac{d\left(\left[\ln \frac{a+l}{a} \right]\right)}{dt} \text{ (taking constants out of the differential)}$$

$$\mathcal{E} = - \left(\frac{\mu_0 ib}{2\pi}\right) \left(\frac{a}{a+l}\right) \times \frac{(a+l) \frac{da}{dt} - a \frac{d(l+a)}{dt}}{a^2}$$

$$\mathcal{E} = - \left(\frac{\mu_0 ib}{2\pi}\right) \left(\frac{a}{a+l}\right) \times \frac{(a+l)v - av}{a^2}$$

(because $\frac{da}{dt} = v$ as the rate of change of Distance from wire is the loop speed)

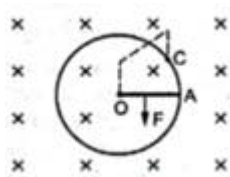
$$\mathcal{E} = - \left(\frac{\mu_0 ib}{2\pi}\right) \left(\frac{1}{a+l}\right) \times \frac{lv}{a}$$

$$\mathcal{E} = \frac{-\mu_0 iblv}{2\pi a(a+l)}$$

The magnitude of the induced emf is $\frac{\mu_0 iblv}{2\pi a(a+l)}$.

59. Question

Figure shows a conducting circular loop of radius a placed in a uniform, perpendicular magnetic field B . A thick metal rod OA is pivoted at the centre O . The other end of the rod touches the loop at A . The centre O and a fixed point C on the loop are connected by a wire OC of resistance R . A force is applied at the middle point of the rod OA perpendicularly, so that the rod rotates clockwise at a uniform angular velocity ω . Find the force.



Answer

Given:

Radius = a

Magnetic field = B

Resistance = R

Angular velocity = ω

Formula used:

Let us consider an element of length dr at a distance r from the centre.

Hence, induced emf on this portion $dE = Blv = Bdr\omega r \dots$ (i), where B = magnetic field, dr = length of element, ω = angular velocity, r = distance from centre (since $v = \omega r$)

Hence, integrating on both sides with suitable limits, we get

$$\int_0^a dE = \int_0^a B\omega r dr \Rightarrow E = \frac{B\omega a^2}{2}$$

Now, current $I = \frac{E}{R}$, where E = emf, resistance = R

$$\Rightarrow I = \frac{B\omega a^2}{2R}$$

Hence, force on the rod $F = IlB = IaB$ (where I = current, a = length of rod, B = magnetic field) $\Rightarrow F = \frac{B^2\omega a^3}{2R}$ (Ans)

60. Question

Consider the situation shown in the figure of the previous problem. Suppose the wire connecting O and C has zero resistance but the circular loop has a resistance R uniformly distributed along its length. The rod OA is made of rotate with a uniform angular speed ω as shown in the figure. Find the current in the rod when $\angle AOC = 90^\circ$.

Answer

Given:

Resistance of circular loop = R

$\angle AOC = 90^\circ$

Angular velocity = ω

Formula used:

From the previous problem, emf $E = \frac{B\omega a^2}{2R} \dots$ (i), where B = magnetic field, ω = angular velocity, a = radius

Now, since $\angle AOC = 90^\circ$, the major and minor segments of the arc AC consist of parallel combination of resistances of R/4 and 3R/4 respectively (since the resistance is divided in the ratio of the angle at the centre).

Hence, equivalent resistance $R' = \frac{\frac{R}{4} \times \frac{3R}{4}}{R} = \frac{3R}{16}$

Therefore, current through the rod $I = \frac{E}{R'}$, where E = emf, R' = equivalent resistance

$$\Rightarrow I = \frac{\frac{Bwa^2}{\frac{3R}{16}}}{\frac{3R}{16}} = \frac{8Bwa^2}{3R} \text{ (Ans)}$$

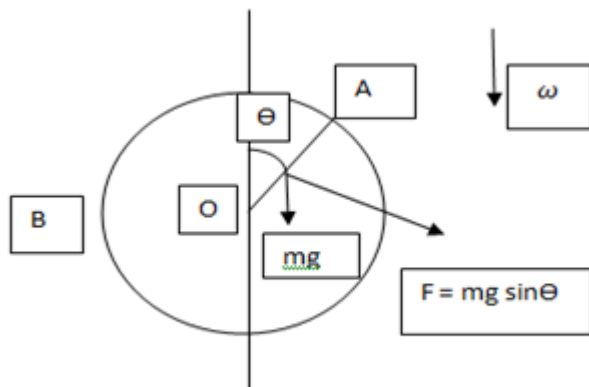
61. Question

Consider a variation of the previous problem figure. Suppose the circular loop lies in a vertical plane. The rod has a mass m. The rod and the loop have negligible resistances but the wire connecting O and C has a resistance R. The rod is made to rotate with a uniform angular velocity ω in the clockwise direction by applying a force at the midpoint of OA in a direction perpendicular to it. Find the magnitude of this force when the rod makes an angle θ with the vertical.

Answer

When the circular loop is in the vertical plane, it tends to rotate in the clockwise direction because of its weight.

Let the force applied be F and its direction be perpendicular to the rod. The component of mg along F is $mg \sin \theta$. The magnetic force is in perpendicular and opposite direction to $mg \sin \theta$.



Now, Current in the rod will be

$$I = \frac{Ba^2\omega}{2R}$$

The force on the rod will be

$$F_B = IBl = \frac{B^2a^2\omega}{2R}$$

So, the net force will be

$$F - \frac{B^2a^2\omega}{2R} + mg \sin \theta$$

The net force passes through the centre of mass of the rod. Net torque on the rod about the centre O will be

$$\tau = \left(F - \frac{B^2 a^2 \omega}{2R} + mg \sin \theta \right) \frac{OA}{2}$$

Because the rod rotates with a constant angular velocity, the net torque on it is zero.

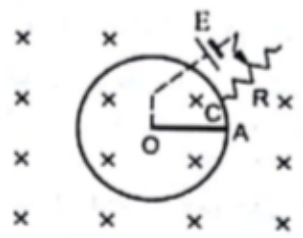
Thus, $\tau = 0$

$$\left(F - \frac{B^2 a^2 \omega}{2R} + mg \sin \theta \right) \frac{OA}{2} = 0$$

$$F = \frac{B^2 a^2 \omega}{2R} + mg \sin \theta$$

62. Question

Figure shows a situation similar to the previous problem. All parameters are the same except that a battery of emf ϵ and a variable resistance R are connected between O and C. Neglect the resistance of the connecting wires. Let θ be the angle made by the rod from the horizontal position (shown in the figure), measured in the clockwise direction. During the part of the motion $0 < \theta < \pi/4$ the only forces acting on the rod are gravity and the forces exerted by the magnetic field and the pivot. However, during the part of the motion, the resistance R is varied in such a way that the rod continues to rotate with a constant angular velocity ω . Find the value of R in terms of the given quantities.



Answer

Given:

Emf = ϵ

Resistance = r

Angle made by rod = θ

Angular velocity = w

Formula used:

From the previous questions, induced emf $e = \frac{Bwa^2}{2} \dots$ (i), where B = magnetic field, w = angular velocity, a = radius

Hence, Total emf = $E + e = E + \frac{Bwa^2}{2}$

Total current $i = \text{total emf}/\text{resistance} = \frac{E + \frac{Bwa^2}{2}}{R} = \frac{Bwa^2 + 2E}{2R} \dots \text{(ii)}$, where $R =$ resistance

'Now, net force on the rod $F = (mg\cos\theta - ilB) \dots \text{(iii)}$,

where $mg\cos\theta$ = component of weight along rod, where m = mass, g = acceleration due to gravity, θ = angle made by rod with horizontal, and ilB = Magnetic force, where i = current, l = length of rod, B = magnetic field.

Since the rod rotates with uniform angular velocity, net torque about $O = 0$.

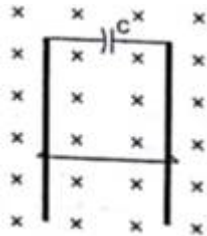
Hence, torque = net force x distance from line of action = $(mg\cos\theta - ilB) \left(\frac{a}{2}\right) = 0$, where a = radius of rod

Therefore, $mg\cos\theta = ilB \dots \text{(iii)}$

Hence, $R = \frac{Bwa^2 + 2E}{2R} (a \times B) \Rightarrow R = \frac{Bwa^2 + 2EaB}{2mg\cos\theta} \text{(Ans)}$

63. Question

A wire of mass m and length ℓ can slide freely on a pair of smooth, vertical rails figure. A magnetic field B exists in the region in the direction perpendicular to the plane of the rails. The rails are connected at the top end by a capacitor of capacitance C . Find the acceleration of the wire neglecting any electric resistance.



Answer

Given:

Mass = m

Length = l

Magnetic field = B

Capacitance = C

Formula used:

Induced emf $E = Blv \dots \text{(i)}$, where B = magnetic field, l = length, v = velocity

Also, we know that $E = \frac{q}{C}$... (ii), where q = charge, C = capacitance.

Hence, $Blv = \frac{q}{C} \Rightarrow q = CBlv$.. (ii)

Therefore, current $i = \frac{dq}{dt}$, where q = charge, t = time

$\Rightarrow i = \frac{CBldv}{dt} = CBla$... (iii), where a = acceleration

Therefore, net force on rod = weight - magnetic force = $mg - ilB$.. (iv), where m = mass, g = acceleration due to gravity, i = current, l = length, B = magnetic force

From newton's second law of motion, $F = ma$... (v), where f= force, m = mass, a = acceleration

Therefore, $ma = mg - ilB = mg - CB^2l^2a$ (from (iii))

$\Rightarrow a(m + CB^2l^2) = mg$

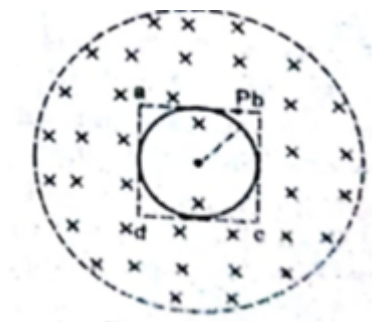
$\Rightarrow \text{acceleration } a = \frac{mg}{m + CB^2l^2}$ (Ans)

64. Question

A uniform magnetic field B exists in a cylindrical region, shown dotted in figure. The magnetic field increases at a constant rate dB/dt . Consider a circle of radius r coaxial with the cylindrical region.

(a) Find the magnitude of the electric field E at a point on the circumference of the circle.

(b) Consider a point P on the side of the square circumscribing the circle. Show that the component of the induced electric field at P along ba is the same as the magnitude forum in part (a).



Answer

Given:

Magnetic field = B

Rate of increase of magnetic field = dB/dt

Radius = r

Formula used:

(a) Induced emf $E' = \frac{d\Phi}{dt}$... (i), where Φ = magnetic flux, t = time

Now, $\Phi = B.A$ where B = magnetic field, A = area

Hence, $E' = \frac{d(B.A)}{dt} = \frac{A dB}{dt}$... (ii)

For the circular loop, $A = \pi r^2$... (iii), where A = area, r = radius

Let the electric field be E

Hence, $\int E \cdot dr = E'$... (iv), where dr = element of length, E' = emf

Hence, for this loop, $\int dr = 2\pi r$, where r = radius

$$\Rightarrow E \times 2\pi r = \pi r^2 \frac{dB}{dt}$$

$$\Rightarrow E = \frac{r}{2} \frac{dB}{dt} \text{ (Ans)}$$

(b) When the square is considered, $A = (2r)^2 = 4r^2$, where A = area, r = radius

In this case, $\int dr = 2r \times 4$ (perimeter of square)

Hence, from $\int E \cdot dr = E'$, where E = electric field, dr = length element, E' = emf, we get

$$E \times 2r \times 4 = \frac{dB}{dt} \times 4r^2$$

$$\Rightarrow \text{electric field } E = \frac{r}{2} \frac{dB}{dt} \text{ (Ans)}$$

65. Question

The current in an ideal, long solenoid is varied at a uniform rate of 0.01 A s^{-1} . The solenoid has 2000 turns/m and its radius is 6.0 cm.

(a) Consider a circle of radius 1.0 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 2.0 seconds.

(b) Find the electric field induced at a point on the circumference of the circle.

(c) Find the electric field induced at a point outside the solenoid at a distance 8.0 cm from its axis.

Answer

Given:

Rate of variation of current $\left(\frac{dI}{dt}\right) = 0.01 \text{ A s}^{-1}$.

No of turns/m (n) = 2000

Radius(r) = 6 cm = 0.06 m

Formula used:

(a) Radius of circle(r') = 1 cm = 0.01 m

Time(t) = 2 s

For two seconds, change of current $\Delta i = (2 \times 0.01 \text{ A.}) = 0.02 \text{ A}$

Magnetic flux $\Phi = B.A$, where B = magnetic field, A = area

Area of circle(A) = $\pi(0.01)^2 = \pi \times 10^{-4} \text{ m}^2$

Magnetic field of a solenoid $B = \mu_0 n \Delta i$, where μ_0 = magnetic permeability of vacuum, n = number of turns per unit length, Δi = change in current

Hence, flux $\Phi = B.A = \mu_0 n \Delta i \times A$

$$\Rightarrow \Phi = (4\pi \times 10^{-7} \times 2000 \times 0.02) \times \pi \times 10^{-4} \text{ Tm}^2 = 1.6 \times 10^{-8} \text{ Wb}$$

Hence, $\frac{d\Phi}{dt}$ in 1 second = **$0.785 \times 10^{-8} \text{ Wb}$** (Ans)

(b) $\int E.dr = E' = \frac{d\Phi}{dt}$, where E = electric field, dr = line element, E' = emf, Φ = flux, t = time

Hence, in this case, this becomes

$E \times 2\pi r = \frac{d\Phi}{dt}$, where r = radius of circle

$$\Rightarrow E = \frac{0.785 \times 10^{-8}}{2\pi \times 10^{-2}} = \mathbf{1.2 \times 10^{-7} \text{ Vm}^{-1}} \text{ (Ans)}$$

(c) For the point located outside,

$$\frac{d\Phi}{dt} = \frac{\mu_0 n di}{dt} \times A = (4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times 0.06^2) \text{ Wbs}^{-1}$$

Φ = flux, t = time, μ_0 = magnetic permeability of vacuum, n = number of turns per unit length, di/dt = rate of change in current

$$\int E.dl = \frac{d\Phi}{dt} \Rightarrow \frac{E = \frac{d\Phi}{dt}}{2\pi r}, \text{ where } E = \text{electric field, } r = \text{radius of circle (since } \int dl = 2\pi r)$$

$$\text{Hence, } E = \frac{(4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times 0.06^2)}{\pi \times 0.082} \times \frac{dB}{dt}$$

$$\Rightarrow E = 5.64 \times 10^{-7} \text{ Vm}^{-1} \text{ (Ans)}$$

66. Question

An average emf of 20 V is induced in an inductor when the current in it is changed from 2.5 A in one direction to the same value in the opposite direction in 0.1 s. Find the self-inductance of the inductor.

Answer

Given:

$$\text{Emf}(E) = 20 \text{ V}$$

$$\text{Rate of change of current} \left(\frac{di}{dt} \right) = \frac{2.5 - (-2.5)}{0.1} \text{ A/s} = 50 \text{ As}^{-1}$$

Formula used:

$$\text{Emf } E = \frac{L di}{dt}, \text{ where } L = \text{self inductance, } di/dt = \text{rate of change of current}$$

Substituting the values, we get

$$20 = L \times 50$$

$$\Rightarrow \text{self inductance } L = 0.4 \text{ H (Ans)}$$

67. Question

A magnetic flux of 8×10^{-4} weber is linked with each turn of a 200-turn coil when there is an electric current of 4A in it. Calculate the self-inductance of the coil.

Answer

Given:

$$\text{Magnetic flux}(\Phi) = 8 \times 10^{-4} \text{ Wb}$$

$$\text{Number of turns}(n) = 200$$

$$\text{Current}(i) = 4 \text{ A}$$

Formula used:

$$L = \left(\frac{n\Phi}{i} \right), \text{ where } L = \text{self inductance, } n = \text{number of turns, } \Phi = \text{flux, } i = \text{current}$$

Putting the values, we get

$$L = \frac{200 \times 8 \times 10^{-4}}{4} \text{ H} = 0.04 \text{ H (Ans)}$$

68. Question

The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at a rate of 0.8 A s^{-1} . Find the emf induced in it.

Answer

Given:

Number of turns(n) = 240

Length of solenoid(l) = 12 cm = 0.12 m

Radius(r) = 2 cm = 0.02 m

Rate of change of current(di/dt) = 0.8 A s^{-1}

Formula used:

$L = \mu_0 n^2 A / l$, where L = self inductance, μ_0 = magnetic permeability of vacuum n = number of turns, A = area, l = length

Now, emf $E = \frac{L di}{dt}$, where L = self inductance, $\frac{di}{dt}$ = rate of change of current

Putting the values:

$$E = \frac{4\pi \times 10^{-7} \times 240^2 \times \pi \times 0.02^2 \times 0.8}{0.12} = 6 \times 10^{-4} \text{ V (Ans)}$$

69. Question

Find the value of t/τ for which the current in an LR circuit builds up to

- (a) 90%
- (b) 99%
- (c) 99.9%

of the steady-state value.

Answer

For a series LR circuit, the current across the inductor varies as a function of time. The current across the inductor at time t will be

$$i = i_0 \left(1 - e^{-\frac{tR}{L}} \right) \dots (i)$$

where i_0 is the current at time $t=0$ (also called the steady state value), R is the resistance of the resistor and L is the inductance of the inductor.

We can define a quantity called the time constant for a series LR circuit. It is given as

$$\tau = L/R$$

So equation(i) becomes

$$i = i_0 \left(1 - e^{-\frac{t}{\tau}}\right) \dots (ii)$$

We have to find the values of $\frac{t}{\tau}$ for three different values of current

(a)- when i is 90% of i_0

$$90\% \text{ of } i_0 \text{ is } \frac{90}{100} i_0, \text{ so } i = \frac{9}{10} i_0$$

Putting these values in eq(ii)

$$\frac{9}{10} i_0 = i_0 \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\frac{9}{10} = \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$e^{-\frac{t}{\tau}} = 1 - \frac{9}{10}$$

$$e^{-\frac{t}{\tau}} = \frac{1}{10}$$

Taking natural logarithm on both sides

$$-\frac{t}{\tau} = \ln \frac{1}{10}$$

$$-\frac{t}{\tau} = \ln(1) - \ln(10)$$

$$-\frac{t}{\tau} = -\ln(10) \text{ (as } \ln(1) \text{ is equal to 0)}$$

$$\frac{t}{\tau} = \ln(10) = 2.30$$

The value of $\frac{t}{\tau}$ for which the current is 90% of steady state

value is 2.3.

(b)- when i is 99% of i_0

$$99\% \text{ of } i_0 \text{ is } \frac{99}{100} i_0, \text{ so } i = \frac{99}{100} i_0$$

Putting these values in eq(ii)

$$\frac{99}{100}i_0 = i_0 \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\frac{99}{100} = \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$e^{-\frac{t}{\tau}} = 1 - \frac{99}{100}$$

$$e^{-\frac{t}{\tau}} = \frac{1}{100}$$

Taking natural logarithm on both sides

$$-\frac{t}{\tau} = \ln \frac{1}{100}$$

$$-\frac{t}{\tau} = \ln(1) - \ln(100)$$

$$-\frac{t}{\tau} = -\ln(100) \text{ (as } \ln(1) \text{ is equal to } 0)$$

$$\frac{t}{\tau} = \ln(100) = 4.60$$

The value of $\frac{t}{\tau}$ for which the current is 99% of steady state value is 4.6.

(c)- when i is 90% of i_0

$$99.9\% \text{ of } i_0 \text{ is } \frac{999}{1000}i_0, \text{ so } i = \frac{999}{1000}i_0$$

Putting these values in eq(ii)

$$\frac{999}{1000}i_0 = i_0 \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\frac{999}{1000} = \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$e^{-\frac{t}{\tau}} = 1 - \frac{999}{1000}$$

$$e^{-\frac{t}{\tau}} = \frac{1}{1000}$$

Taking natural logarithm on both sides

$$-\frac{t}{\tau} = \ln \frac{1}{1000}$$

$$-\frac{t}{\tau} = \ln(1) - \ln(1000)$$

$$-\frac{t}{\tau} = -\ln(1000) \text{ (as } \ln(1) \text{ is equal to } 0)$$

$$\frac{t}{\tau} = \ln(1000) = 6.90$$

The value of $\frac{t}{\tau}$ for which the current is 99.9% of steady state

value is 6.9.