HOTS (Higher Order Thinking Skills)

Que 1. Two circles touch internally. The sum of their area is 116 πcm^2 and distance between their centres is 6 cm. Find the radii of the circles.

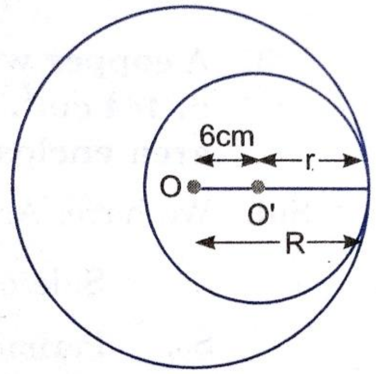

Fig. 12.49

Sol. Let R and r be the radii of the circles [Fig. 12.49].

Then, according to question,

$\Rightarrow \quad \pi R^2 + \pi r^2 = 116\pi$

$\Rightarrow \quad R^2 + r^2 = 116$ ….(i)

Distance between the centres = 6 cm

$\Rightarrow \quad OO' = 6$ cm

$\Rightarrow \quad R - r = 6$ …..(ii)

Now, $(R + r)^2 + (R - r)^2 = (R^2 + r^2)$

Using the equation (i) and (ii), we get

$(R + r)^2 + 36 = 2 \times 116$

$\Rightarrow \quad (R + r)^2 = (2 \times 116 - 36) = 196$

$\Rightarrow \quad R + r = 14$ ….(iii)

Solving (ii) and (iii), we get R = 10 and r = 4.

Hence, radii of the gives circles are 10 cm and 4 cm respectively.

Que 2. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Sol. Let the radius of the wheel be r cm.

Distance covered by the wheel in one revolution = $\frac{Distance\ moved}{Number\ of\ revolutions} = \frac{11}{5000} km$

$= \frac{11}{5000} \times 1000 \times 100\ cm\ = 220\ cm$

$\therefore$ Circumference of the wheel = 220 cm

$\Rightarrow \quad 2\pi r = 220$ cm $\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = \frac{220 \times 7}{2 \times 22} \quad \Rightarrow$ r = 35 cm

∴ Diameter = 2r cm = $(2 \times 35)\ cm = 70\ cm$
Hence, the diameter of the wheel is 70 cm.

Que 3. Find the area of the shaded design of Fig. 12.50, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter (use $\pi = 3.14$).

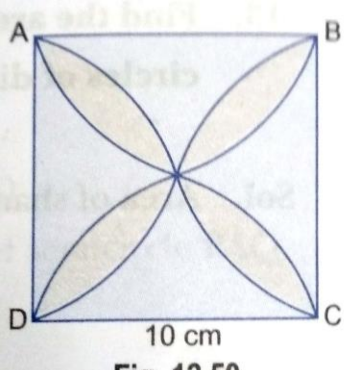

Fig. 12.50

Sol. Let us mark the four unshaded regions as I, II, III and IV (Fig. 12.50).
Area of 1 + Area of III
= Area of ABCD – Area of two semicircles of radius 5 cm each

$= \left(10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2\right)\ cm^2$

$= (100 - 3.14 \times 25)\ cm^2 = (100 - 78.5)\ cm^2 = 21.5\ cm^2$

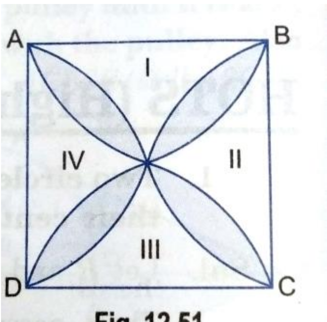

Fig. 12.51

Similarly, Area of II + Area of IV = 21.5 cm^2
So, Area of the shaded design
= Area of ABCD – Area of (I + II + III + IV)
= (100 – 2 × 21.5) cm^2
= (100 – 43) cm^2 = 57 cm^2

Que 4. A copper wire, when bent in the form of a square, enclosed an area of 484 cm^2. If the same wire is bent in the form of a circle, find the area enclosed by it.

Sol. We have, Area of the square = a^2 = 484 cm^2
∴ Side of the square = $\sqrt{484}\ cm = 22\ cm$
So, Perimeter of the square = 4 (side) = (4 × 22) cm = 88 cm
Let r be the radius of the circle. Then, according to question,
Circumference of the circle = Perimeter of the square

$\Rightarrow \quad 2\pi r = 88 \quad \Rightarrow \quad 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow \quad r = \frac{88 \times 7}{2 \times 22} \quad \Rightarrow \quad r = 14\ cm$

$\therefore$ Area of the circle $= \pi r^2 = \left\{\frac{22}{7} \times (14)^2\right\} \text{cm}^2 = 616\ \text{cm}^2$

Que 5. Two circles touch externally. The sum of their areas is 130 π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

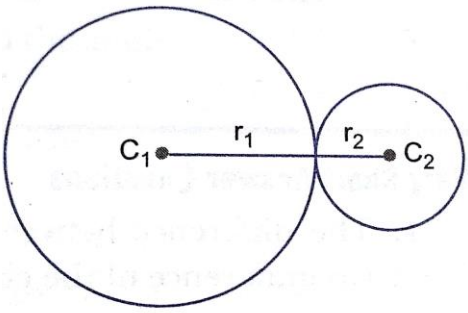

Fig. 12.52

Sol. If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

Let the radii of the two circles be r_1 cm and r_2 cm respectively {Fig. 12.52].

Let C_1 and C_2 be the centres of the given circles. Then,

$$C_1C_2 = r_1 + r_2$$

$\Rightarrow \quad 14 = r^1 + r^2 \quad [\because\ C_1C_2 = 14\ cm\ given]$

$\Rightarrow \quad r_1 + r_2 = 14 \quad \ldots(i)$

It is given that the sum of the areas of two circles is equal to 130 π cm^2.

$\therefore \quad \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow \quad r_1^2 + r_2^2 = 130 \quad \ldots(ii)$

Now, $\quad (r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \quad 14^2 = 130 + 2r_1r_2$ [using (*i*) and (*ii*)]

$\Rightarrow \quad 196 - 130 = 2r_1r_2$

$\Rightarrow \quad r_1r_2 = 33 \quad \ldots(iii)$

Now, $\quad (r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$\Rightarrow \quad (r_1 - r_2)^2 = 130 - 2 \times 33$ [using (*ii*) and (*iii*)]

$\Rightarrow \quad (r_1 - r_2)^2 = 64$

$\Rightarrow \quad r_1 - r_2 = 8 \quad \ldots(iv)$

Solving (*i*) and (*iv*), we get $r_1 = 11$ cm and $r_2 = 3$ cm.

Hence, the radii of the two circles are 11 cm and 3 cm.