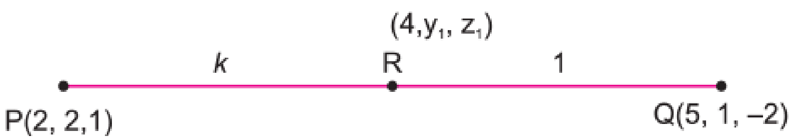
Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. The *x*-coordinate of a point on the line joining the point *P*(2, 2, 1) and *Q*(5, 1, –2) is 4. Find its z-coordinate.

Ans.



Let required point be $R(4, y_1, z_1)$

Which divides PQ in ratio $k:1$

By section formula

$4=\frac{5k+2}{k+1}$

$\Rightarrow\ 4k+4=5k+2 \qquad \Rightarrow\ k=2$

$\therefore\ z_1=\frac{2\times(-2)+1\times1}{2+1}=\frac{-4+1}{3}=\frac{-3}{3}=-1$

Q.2. Find 'λ' when the projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.

Ans.

We know that projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\Rightarrow\ 4=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \qquad ...(i)$

Now, $\vec{a}\cdot\vec{b}=2\lambda+6+12=2\lambda+18$

Also $|\vec{b}|=\sqrt{2^2+6^2+3^2}=\sqrt{4+36+9}=7$

Putting in (i), we get

$4=\frac{2\lambda+18}{7} \quad \Rightarrow \quad 2\lambda=28-18 \quad \Rightarrow \quad \lambda=\frac{10}{2}=5$

Q.3. What are the direction cosines of a line, which makes equal angles with the co-ordinate axes?

Ans.

Let α be the angle made by line with coordinate axes.

$\Rightarrow$ Direction cosines of line are $\cos\alpha, \cos\alpha, \cos\alpha$

$$\Rightarrow \quad \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow \quad 3\cos^2\alpha = 1 \quad \Rightarrow \quad \cos^2\alpha = \frac{1}{3}$$

$$\Rightarrow \quad \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Hence, the direction cosines, of the line equally inclined to the coordinate axes are

$$\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$$

[**Note:** If l, m, n are direction cosines of line, then $l^2+m^2+n^2 = 1$]

Q.4. For what value of *p*, is $(\hat{i} + \hat{j} + \widehat{k})$ ***p*** **a unit vector?**

Ans.

Let, $\vec{a} = p(\hat{i} + \hat{j} + \hat{k})$

Magnitude of $\vec{a}$ is $|\vec{a}|$

$$|\vec{a}| = \sqrt{(p)^2 + (p)^2 + (p)^2} = \pm\sqrt{3}p$$

As $\vec{a}$ is a unit vector $|\vec{a}| = 1$,

$$\therefore \Rightarrow \quad \pm\sqrt{3}p = 1 \quad \Rightarrow \quad p = \pm\frac{1}{\sqrt{3}}$$

Q.5. Find the value of $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$ **.**

Ans.

$$(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$$

$$\begin{vmatrix}\hat{i} & j & \hat{k}\\ 2 & 6 & 27\\ 1 & 3 & p\end{vmatrix}=\vec{0} \qquad \Rightarrow \qquad (6p-81)\hat{i}-(2p-27)\hat{j}+0\hat{k}=\vec{0}$$

$$\Rightarrow \quad 6p=81 \qquad \Rightarrow \qquad p=\frac{81}{6}=\frac{27}{2}$$

Q.6. Write the position vector of the mid-point of the vector joining the points *P*(2, 3, 4) and *Q*(4, 1, –2).

Ans.

Let $\vec{a}$, $\vec{b}$ be position vector of points $P(2, 3, 4)$ and $Q(4, 1, -2)$ respectively.

$$\therefore \quad \vec{a}=2\hat{i}+3\hat{j}+4\hat{k} \quad \text{and} \quad \vec{b}=4\hat{i}+\hat{j}+2\hat{k}$$

$\therefore$ Position vector of mid point of P and $Q=\frac{\vec{a}+\vec{b}}{2}=\frac{6\hat{i}+4\hat{j}+2\hat{k}}{2}=3\hat{i}+2\hat{j}+\hat{k}$

Q.7. If $|\overline{a}|$ = *a*, then find the value of the following:

$$\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2$$

Ans.

Let $\overrightarrow{a}$ makes angle α, β, γ with x, y and z axis.

$\therefore \left|\overrightarrow{a} \times \hat{i}\right| = \left|\overrightarrow{a}\right|.1.\ \sin\ \alpha\ = a\ \sin\alpha$

Similarly, $|\overrightarrow{a} \times \hat{j}| = a\ \sin\ \beta$

and $|\overrightarrow{a} \times \hat{k}| = a \sin\gamma$

$\therefore |\overrightarrow{a} \times \hat{i}|^2 + |\overrightarrow{a} \times \hat{j}|^2 + |\overrightarrow{a} \times \hat{k}|^2 = a^2 \sin^2 \alpha + a^2 \sin \beta + a^2 \sin^2 \gamma$

$= a^2[\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma]$

$= a^2 [1 - \cos^2 \alpha + 1 - \cos^2 \beta + 1 - \cos^2 \gamma]$

$= a^2 [3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 g)]$

$= a^2(3 - 1) \qquad \left[\begin{array}{l} \because\ l^2 + m^2 + n^2 = 1 \\ \Rightarrow\ \cos^2\alpha\ +\ \cos^2\beta + \cos^2\gamma\ = 1 \end{array}\right]$

$= 2a^2$

Q.8. The vectors $\overrightarrow{a} = 3\hat{i} + x\hat{j}$ and $\overrightarrow{b} = 2\hat{i} + \hat{j} + y\hat{k}$ **are mutually perpendicular. If** $|\overrightarrow{a}| = |\overrightarrow{b}|$ **, then find the value of *y* .**

Ans.

$\because \vec{a}$ and $\vec{b}$ are mutually perpendicular.

$\therefore \vec{a} \cdot \vec{b} = 0$

$\Rightarrow (3\hat{i} + x\hat{j}).(2\hat{i} + \hat{j} + y\hat{k}) = 0$

$\Rightarrow 6 + x + 0.\ y = 0$

$\Rightarrow 6 + x = 0 \qquad \Rightarrow x = -6$

Again, $|\vec{a}| = |\vec{b}|$

$\Rightarrow \sqrt{3^2 + x^2} = \sqrt{2^2 + 1 + y^2}$

$\Rightarrow \sqrt{9 + 36} = \sqrt{5 + y^2} \qquad [\because x = -6]$

$\Rightarrow \sqrt{45} = \sqrt{5 + y^2} \qquad \Rightarrow y^2 = 45 - 5$

$\Rightarrow y = \pm\sqrt{40} = \pm 2\sqrt{10}$

Q.9. Find the value of $\vec{a} \cdot \vec{b}$ if $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$.

Ans.

$\because \quad |\vec{a} \times \vec{b}| = 16 \quad \Rightarrow \quad |\vec{a}||\vec{b}| \sin\theta = 16$

$\Rightarrow \quad 10 \times 2 \sin\theta = 16 \qquad \Rightarrow \sin\theta = \frac{16}{20} = \frac{4}{5}$

$\Rightarrow \quad \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{16}{25}} = \pm\frac{3}{5}$

$\therefore \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta = \pm 10 \times 2 \times \frac{3}{5} = \pm 2$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. Find the unit vector in the direction of the sum of the vectors

$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = -\hat{i} + \hat{j} + 3\hat{k}$

Ans.

Let $\overrightarrow{c}$ be the sum of $\overrightarrow{a}$ and $\overrightarrow{b}$

$$\overrightarrow{c} = \overrightarrow{a} + \overrightarrow{b}$$

$$= (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k})$$

$$|\overrightarrow{c}| = \hat{i} + 5\hat{k}$$

$$= \sqrt{(1)^2 + 0^2 + (5)^2} = \sqrt{1 + 25} = \sqrt{26}$$

Therefore required unit vector is

$$\frac{\hat{i} + 5\hat{k}}{\sqrt{26}} = \frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{k}$$

Q.2. Write the direction ratio's of the vector $\overrightarrow{a} = \hat{i} + \hat{j} - 2\hat{k}$ and hence calculate its direction cosines.

Ans.

We know that, the direction ratio's *a, b, c* of a vector $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$ are just the respective components *x, y* and *z* of the vector. So, for the given vector, we have $a = 1$, $b = 1$ and $c = -2$. Further, if *l, m* and *n* are the direction cosines of the given vector, then

$$l = \frac{a}{|\overrightarrow{r}|} = \frac{1}{\sqrt{6}}, \quad m = \frac{b}{|\overrightarrow{r}|} = \frac{1}{\sqrt{6}},$$

$$n = \frac{c}{|\overrightarrow{r}|} = \frac{-2}{\sqrt{6}} \text{as } |\overrightarrow{r}| = \sqrt{6}$$

Thus, the direction cosines are $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$

Q.3. Find the value of λ for which the two vectors

$2\hat{i} - \hat{j} + 2k$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$

and are perpendicular to each other.

Ans.

Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + \lambda\hat{j} + \hat{k}$

$\because$ $\vec{a}$ is perpendicular to $\vec{b}$

$\Rightarrow \vec{a} . \vec{b} = 0$

$\Rightarrow (2\hat{i} - \hat{j} + 2\hat{k}). (3\hat{i} + \lambda\hat{j} + \hat{k}) = 0$

$\Rightarrow 2 \times 3 + (-1) \times (\lambda) + 2 \times 1 = 0$

$\Rightarrow 6 - \lambda + 2 = 0 \quad \Rightarrow \lambda = 8$

Q.4. Find the area of a parallelogram whose adjacent sides are
$\hat{i} + \hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$.

Ans.

Let $\vec{a}$ and $\vec{b}$ be adjacent sides of parallelogram such that

$\vec{a} = \hat{i} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

$\therefore$Area of parallelogram = $|\vec{a} \times \vec{b}|$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = (0-1)\hat{i} - (1-2)\hat{j} + (1-0)\hat{k}$

$= -\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Area of parallelogram $= \sqrt{(-1)^2 + 1^2 + 1^2}$

$= \sqrt{1+1+1} = \sqrt{3}$ sq. units.