HOTS (Higher Order Thinking Skills)

Que 1. The dimension of a rectangle ABCD are 51 cm x 25 cm. A trapezium PBCQ with its parallel sides QC and PB in the ratio 9: 8, is cut off from the rectangle as shown in the Fig. 12.12. If the area of the trapezium PBCQ is $\frac{5}{6}$ th part of the area of the area of the rectangle, Find the lengths of QC and PB.

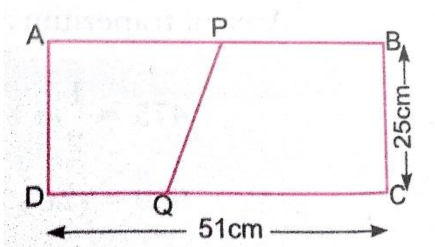

Fig. 12.12

Sol. Area of rectangle ABCD = AB x BC = 51 x 25 = 1275 cm^2

Area of trapezium PBCQ = $\frac{5}{6}$ 1275 = $\frac{6375}{6}$ cm^2

Let QC = 9x cm and PB = 8x cm

∴ Area of trapezium PBCQ = rrr (QC + PB) x BC

⇒ $\frac{6375}{6} = \frac{1}{2}(9x + 8x) \times 25$

⇒ $\frac{17x \times 25}{2} = \frac{6375}{6}$

⇒ $x = \frac{6375}{6} \times \frac{2}{17 \times 25}$

⇒ $x = 5$

∴ QC = 9 X 5 cm = 45 cm and PB = 8 X 5 cm = 40 cm

Que 2. Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

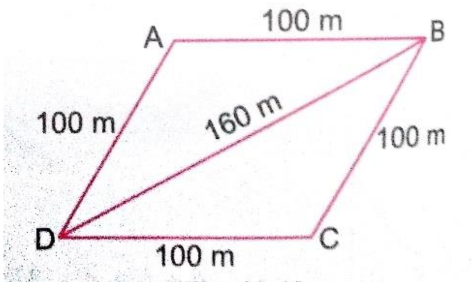

Fig. 12.13

Sol. Let ABCD be the filed. Give perimeter = 400 m

So, each side = $\frac{400}{4}$ = 100 m

Diagonal BD = 160 m

Let a = 100 m, b = 100 m c = 160 m

∴ $s = \frac{a+b+c}{2} = \frac{100+100+160}{2}$ = 180 m

Therefore, Area of ΔABD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{180(180-100)(180-100)(180-160)}$

$= \sqrt{180 \times 80 \times 80 \times 20}$ = 4800 m^2

Alternative method:

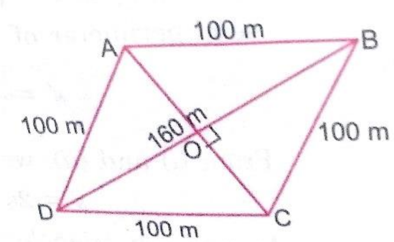

Fig. 12.14

As the diagonals of rhombus bisect each other: Therefore

$OD = \frac{1}{2} BD = \frac{1}{2} \times 160 = 80$ m

$OC = \frac{1}{2} AC$

In ΔOCD, we have,

$CD^2 = OC^2 + OD^2$

$100^2 = OC^2 + 80^2$ ⇒ $OC^2 = 10000 – 6400$

⇒ $OC^2 = 3600$ ⇒ OC = 60 m

Therefore, area of ΔBCD = $\frac{1}{2}$ (BD x OC) = $\frac{1}{2}$ x 160 x 60 = 4800 m^2

∴ Each of them will get 4800 m^2 of area for their crops.

Que 3. A field is in the shape of a trapezium, its parallel sides are 25 m and 10 m and non-parallel sides are 14 m and 13 m. Find the area of the field.

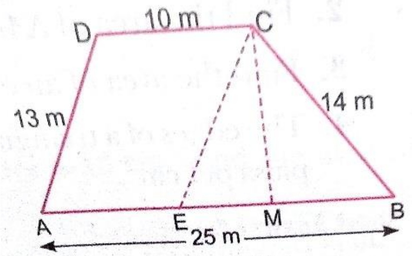

Fig. 12.15

Sol. Let ABCD be a trapezium, with parallel sides AB = 25 m, CD = 10 cm and non-parallel sides

BC = 14 m and AD = 13 m.

Draw CM ⊥ AB and CE || AD.

For ΔBCE

BC = 14 m

CE = AD = 13 m

BE = AB – AE

= 25 – 10 = 15 m (∵ AE = CD = 10 m)

Now, Let a = 14 m, b = 13 m and c = 15 m

∴ Semi-perimeter (s) = $\frac{a+b+c}{2} = \frac{14+13+15}{2}$ = 21 m

∴ Area of Δ BCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-14)(21-13)(21-15)}$

$= \sqrt{21 \times 7 \times 8 \times 6} = \sqrt{3 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 3}$

= 2 x 2 x 3 x 7 = 84 m^2

Also, area (Δ BCE) = $\frac{1}{2}$ x BE x CM ⇒ 84 = $\frac{1}{2}$ x 15 x CM

⇒ CM = $\frac{2 \times 84}{15}$ ⇒ CM = $\frac{56}{5}$ m

Now, area of parallelogram AECD = base x altitude

= AE x CM = 10 x $\frac{56}{5}$ m^2 = 112 m^2

∴ Area of trapezium = area of parallelogram AECD + area of ΔBCE

= 112 + 84 = 196 m^2.

Que 4. If each side of a triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle.

Sol. Let a, b, c be the side of the given triangle and s be its semi-perimeter.

Then, s = $\frac{a+b+c}{2}$ …(i)

∴ Area of the given triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ = Δ . Say

According to the question, the sides of the new triangle will be 2a, 2b and 2c. Let s' be the semi-perimeter of the new triangle.

S' = $\frac{2a+2b+2c}{2}$ = a + b + c …(ii)

From (i) and (ii), we get

S' = 2s

Area of new triangle = $\sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

= $\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

= $\sqrt{16s(s-a)(s-b)(s-c)}$

= $\sqrt{s(s-a)(s-b)(s-c)}$ = 4Δ

Therefore, the required ratio is 4: 1