High Order Thinking Skills (HOTS) Questions

Q.1. Hot air becomes cooler as it rises appreciably. Why?

Ans. As hot air rises up, it reaches region of reduced pressure. The air suffers adiabatic expansion.

From first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

For adiabatic process

$$\Delta Q = 0$$

$$\Delta U + \Delta W = 0$$

Since, ΔU becomes negative. Therefore, air becomes cool.

Q.2. What is the coefficient of performance (β) or a Carnot refrigerator working 30°C and 0°C?

Ans. Given: $T_2 = 0^oC = 273$ K

$T_1 = 30^oC = 273 + 30 = 303$ K

$\beta = ?$

From the relation, $\beta = \frac{T_2}{T_1 - T_2}$,

or $\beta = \frac{273}{303-273} = \frac{273}{30}$

$= 9.1$

Q.3. A certain volume of dry air at NTP is allowed to expand 4 times of its original volume under (a) isothermal conditions, (b) adiabatic conditions. Calculate the final pressure and temperature in each case if γ = 1.4.

Ans. Given: Suppose $V_1 = V$

$V_2 = 4V$

$P_1 = 76$ cm of Hg

$P_2 = ?$

$\gamma = 1.4$

$T_1 = 273\text{K}$

$T_2 = ?$

(a) For isothermal expansion:

$$P_1V_1 = P_2V_2$$

or $$P_2 = P_1 \frac{V_1}{V_2} = 76.\frac{V}{4V}$$

$$= 19 \text{ cm of Hg.}$$

As the process is isothermal, therefore the final temperature will be the same as the initial temperature,

i.e., $$\mathrm{T}_2 = 273 \text{ K}$$

(b) Adiabatic expansion: From the relation

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

∴ $$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$= 76 \times \left(\frac{1}{4}\right)^{0.04}$$

$$= 76 \times (0.25)^{1.4}$$

$$= 10.91 \text{ cm of Hg}$$

From the relation,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

⇨ $$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

⇨ $$T_2 = 273 \left(\frac{1}{4}\right)^{0.04}$$

$$= \frac{273}{(4)^{0.04}} = 156.8 \text{ K}$$

Q.4. One litre of hydrogen at 27^{o}C and 10^{6} dyne cm^{-2} pressure expands isothermally until its volume in doubled and then adiabatically until redoubled. Find the final temperature, pressure and work done in each case, γ = 1.4.

Ans. Given: $P_1 = 10^6$ dyne cm^{-2}

$V_1 = 10^3$ cm^3 = 1 litre

$T_1 = 27^{o}$C = 300 K

$V_2 = 2V_1 = 2000$ cm^3 for isothermal expansion

$T_2 = ?$

For adiabatic expansion $P_2 = ?$

$V_1' = 2000$ cm^3

$V_2' = 2V_1' = 4000$ cm^3

(a) For isothermal expansion:

$$P_2V_2 = P_1V_1$$

or $$P_2 = \frac{P_1V_1}{V_2}$$

$$= \frac{10^6 \times 10^3}{2 \times 10^3}$$

$= 5 \times 10^5$ dyne cm^{-2}

$T_2 = T_1$ for isothermal expansion = 300 K = 27°C Work done during isothermal expansion is given by

$$W_{iso} = 2.303 \text{ RT} \log_{10}\left(\frac{V_2}{V_1}\right)$$

or $$W_{iso} = 2.303 P_1V_1 \log_{10}\left(\frac{V_2}{V_1}\right)$$

or $$W_{iso} = 2.303 \times 10^6 \times 10^3 \log_{10}$$

or $$W_{iso} = 2.303 \times 10^9 \times 0.3010$$

or $W_{iso} = 6.93 \times 10^8$ erg.

(b) For adiabatic expansion:

$$P_2 = P_1\left(\frac{V_1'}{V_2'}\right)^\gamma$$

$$= 10^6\left(\frac{2000}{4000}\right)^{1.4}$$

or $$P_2 = 10^6 \times \left(\frac{1}{2}\right)^{1.4}$$

or $P_2 = 1.895 \times 10^5$ dyne cm^{-2}

From relation $T^1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$, we get

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{1}{2}\right)^{1.4}$$

$= 227.4$K $= -45.6$°C

Work done during adiabatic expansion is given by

$$W_{adi} = (T_1 - T_2)\frac{R}{1-\gamma}$$

$$= \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{10^6 \times 2 \times 10^3 - 1.895 \times 10^5 \times 4 \times 10^3}{1.4 - 1}$$

or $$W_{adi} = \frac{20 \times 10^8 - 1.895 \times 4 \times 10^8}{0.4}$$

or $W_{adi} = 31.05 \times 10^8$ erg.

Q.5. Two friends want to increase the temperature of a gas without adding heat to it. Is it possible to increase the temperature of a gas without adding heat to it?

Ans. Yes, during adiabatic compression the temperature of a gas increase while no heat is given to it.

In adiabatic compression,

$$dQ = 0$$

∴ From first law of thermodynamics

$$dU = dQ - dW$$

$$dU = -dW$$

In compression work is done on the gas i.e., work done is negative. Therefore,

$$dU = +ve.$$

Hence, internal energy of the gas increases due to which its temperature increases.

Q.6. A person of mass 60 kg wants to lose 5 kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is bunt on expanding 7000 kcal, how many times must he go up and down to reduce his weight by 5 kg?

Ans. Height (h) = 10 m

Energy produced by burning 1 kg of fat

= 7000 kcal

∴ Energy produced by burning 5 kg fat

= 5 × 7000 = 35000 kcal

= 35×10^6 cal

Energy utilized in going up and down one time

$$= mgh + \frac{1}{2}mgh = \frac{3}{2}mgh$$

$$= \frac{3}{2} \times 60 \times 10 \times 10 \text{ J}$$

= 9000 J

$$= \frac{90000}{4.2} \text{ cal}$$

$$= \frac{3000}{1.4} \text{ cal}$$

∴ Number of times, the person has to go up and down the stairs:

$$= \frac{35 \times 10^6}{\left(\frac{3000}{1.4}\right)}$$

$= \frac{3.5 \times 1.4 \times 10^6}{3000}$

$= 16.3 \times 10^3$ times.

Q.7. Consider a Carnot's cycle operating between $T_1 = 500$ K and T_2 = 300 K producing 1 KJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.

Ans. Given: Temperature of the source

$T_1 = 500$ K

Temperature of the sink

$T_2 = 300$ K

Work done per cycle

W = 1 kJ = 1000 J

Heat transferred to the engine per cycle

$Q_1 = ?$

Efficiency of Carnot engine

$$(\eta) = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{2}{5}$$

and $\eta = \frac{W}{Q_1}$

or $Q_1 = \frac{W}{\eta} = \frac{1000}{(2/5)} = 2500$ J.