CBSE Class 09

Mathematics

Sample Paper 8 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

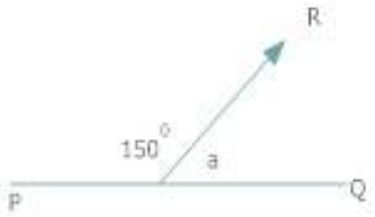
Section A

1. $2^{\frac{4}{3}}$ is same as
   a. $\sqrt[3]{2^4}$
   b. $\sqrt[3]{4}$
   c. $\sqrt[4]{2^3}$
   d. none of these
2. A polynomial of degree 3 in x has at most
   a. 3 terms
   b. 1 term

c. 5 terms

d. 4 terms

3. In the given figure, the measure of $\angle a$ is:



a. 150°

b. 30°

c. 15°

d. 50°

4. The construction of $\triangle ABC$, given that BC = 5 cm, $\angle B = 60^0$ is not possible when the difference of AB and AC is equal to_____.

a. 4 cm.

b. 4.2 cm

c. 5.9 cm.

d. 3 cm.

5. A polynomial containing three non-zero terms is called a _____.

a. monomial

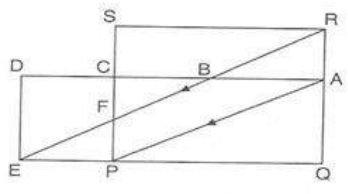
b. trinomial

c. none of these

d. binomial

6. PQRS and ADEQ are rectangles. $RE \parallel AP$. If $ar\ (ACPQ) = 25\ cm^2$ and

$ar(ABEP) = 10\ cm^2$, then ar (PQRS) is



a. $25\,cm^2$.

b. $35\,cm^2$.

c. $10\,cm^2$.

d. $30cm^2$.

7. A linear polynomial

   a. May have two zero

   b. May have one zero

   c. May have more than one zero

   d. has one and only one zero

8. If side of equilateral triangle is 25 m. Its area is

   a. $5\sqrt{3}$ sq cm

   b. $\sqrt{3}$ sq cm

   c. $\frac{625}{4}\sqrt{3}$ sq cm

   d. $54\sqrt{3}$ sq cm

9. The edge of a cube whose volume is equal to the volume of a cuboid of dimensions $36\ cm \times 75\ cm\ \times 80\,cm\ is$

   a. 60 cm.

   b. 42 cm.

c. 48 cm.

d. 36 cm.

10. Which of the following cannot be the probability of an event?

    a. $\frac{17}{16}$

    b. 0.1

    c. $\frac{1}{3}$

    d. 3%

11. Fill in the blanks:

    Rational number $\frac{42}{100}$ in decimal form is _____.

12. Fill in the blanks:

    If (2, 0) is a solution of the linear equation 2x + 3y = k, then the value of k is _____.

OR

Fill in the blanks:

If x = 1, y = 1 and 9k + 12ky = 63, then the value of k is _____.

13. Fill in the blanks:

    There are _____ quadrants in 2D Geometry.

14. Fill in the blanks:

    The length of the complete circle is called its _____.

15. Fill in the blanks:

    A cuboid is 12 cm long, 9 cm broad and 8 cm high. Then its total surface area is _____ cm^2.

16. Simplify: $(0.001)^{1/3}$

17. Evaluate: $25^3 - 75^3 + 50^3$

18. If the radius of a sphere is doubled what will happen to its surface area?

OR

The surface area of cuboid is 1792 sq cm. If its length, breadth and height are in the ratio 4 : 2 : 1, then find the length of the cuboid.

19. ABCD is a rectangle with $\angle ABD = 40^o$. Determine $\angle DBC$.

20. Find the value of λ, if $x = -\lambda$ and $y = \frac{5}{2}$ is a solution of the equation $x + 4y - 7 = 0$.

21. Prove that: $9^{3/2} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-1/2} = 15$

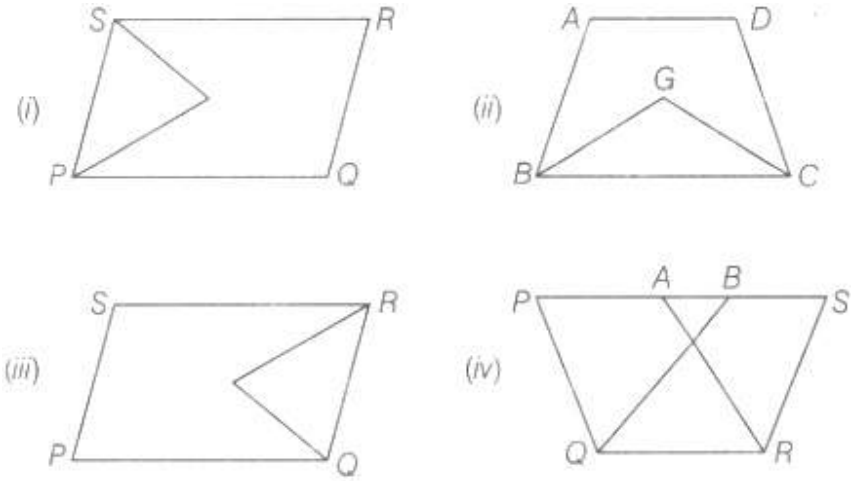
22. Draw the graph of each of the line a equations in two variables: $y = 3x$.

23. If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3.$

OR

If $x + \frac{1}{x} = 4$, then find the value of $x^2 + \frac{1}{x^2}$.

24. Which of the following figures is an example of figure having same base and same parallels?



25. The marks scored by 55 students in a test are given below:

Marks:							

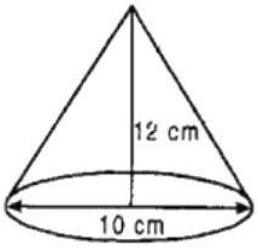
	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of students	2	6	13	17	11	4	2

Find the cumulative frequency distribution.

OR

The marks obtained by 35 students in an examination are given below:
370, 290, 318, 175, 170, 410, 378, 405, 380, 375, 315, 305, 325, 275, 241, 288, 261, 355, 402, 380,178, 253, 428, 240, 210,175,154, 405, 380, 370, 306, 460, 328, 440, 425.
Form a cumulative frequency table with class intervals of length 50.

26. Sameera wants to celebrate the fifth birthday of her daughter with a party. She bought thick paper to make the conical party caps. Each cap is to have a base diameter of 10 cm and height 12 cm. A sheet of the paper is 25 cm by 40 cm and approximately 82% of the sheet can be effectively used for making the caps after cutting. What is the minimum number of sheets of paper that Sameera would need to buy, if there are to be 15 children at the party? (Use π = 3.14)



27. Prove that $\sqrt{3}$ is an irrational number.

OR

If $4^{2x-1} - 16^{x-1} = 384$, find the value of x.

28. The following table gives measures (in degrees) of two acute angles of a right triangle

X	10	20	30	40	50	60	70	80
Y	80	70	60	50	40	30	20	10

Plot the point and join them.

29. Plot the graph of each of the following equations using the same pair of axes :

    i. y = 2x + 3
    ii. $y = 2x - \frac{3}{2}$
    iii. 2x– y = 0are these lines parallel?

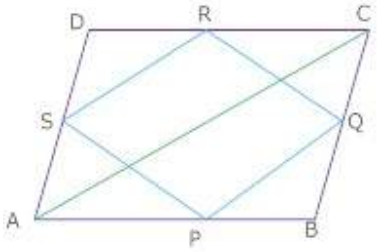
OR

Draw the graph of the following linear equation: 2y + 5 = 0

30. Construct a triangle ABC in which BC = 7 cm, $\angle$B = 75^{o} and AB + AC = 13 cm.

31. In fig ABCD is a quadrilateral P, Q, R and S are the mid points of the sides AB, BC, CD and DA, AC is diagonal. Show that

    i. SR||AC
    ii. PQ=SR
    iii. PQRS is a parallelogram
    iv. PR and SQ bisect each other



32. P is a point on the bisector of $\angle$ ABC. If the line through P, parallel to BA meets BC at Q, prove that BPQ is an isosceles triangle.

OR

Prove that if one angle of a triangle is equal to the sum of the other two angles, the triangle is right-angled:

33. A traffic signal board indicating 'school ahead' is an equilateral triangle with side 'a' find the area of the signal board using heron's formula. Its perimeter is 180 cm, what will be Its area?

34. The percentage of marks obtained by a student in monthly unit tests are given below.

Test	I	II	III	IV	V	VI
Percentage of marks	52	60	65	75	80	72

Find the probability that in the next test the student gets

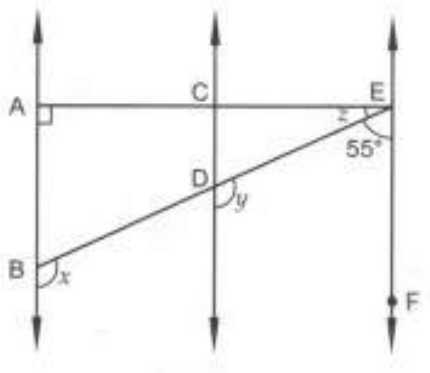
i. more than 70% marks,
ii. less than 70% marks,
iii. at least 60% marks.

35. Two circles intersect at P and Q. Through P, two straight lines APB and CPD are drawn to meet the circles at A, B, C and D. AC and DB when produced meet at O. Show that OAQB is a cyclic quadrilateral.

OR

AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

36. Fig., AB || CD and CD || EF. Also, EA ⊥ AB. If ∠BEF = 55°, find the values of x, y, and z.



37. The remainder of the polynomial $5 + bx - 2x^2 + ax^3$, when divided by (x - 2) is twice the remainder when it is divided by (x + 1). Show that 10a + 4b = 9.

OR

If $y^3 + ay^2 + by + 6$ is divisible by y - 2 and leaves remainder 3 when divided by y - 3, find the values of a and b.

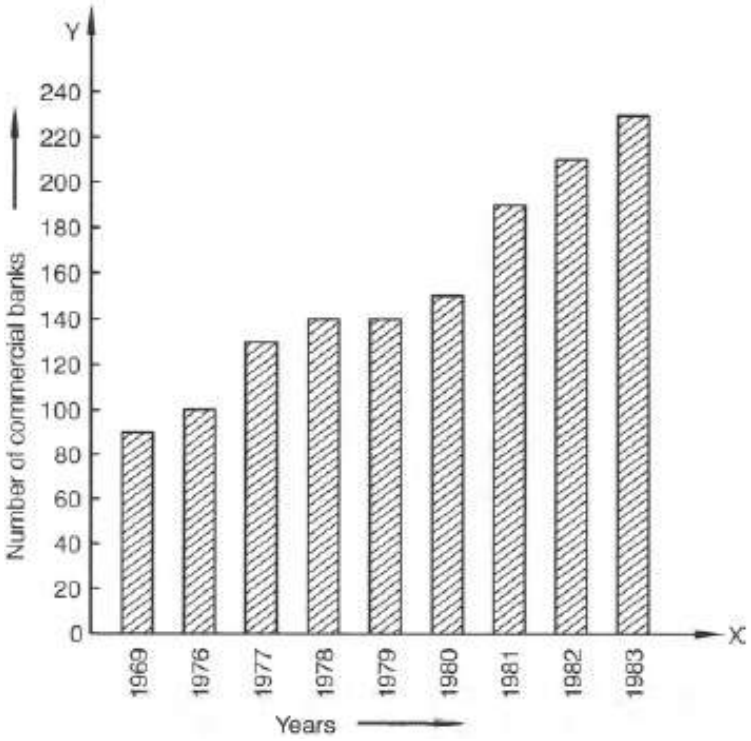
38. Shanti sweets stall was placing an order for making cardboard boxes for packing their sweets two sizes of boxes were required. The bigger of dimensions $25cm \times 20cm \times 5cm$ and the smaller of dimensions $15cm \times 12cm \times 5cm$ for all the overlaps, 5% of the total surface area is required extra. If the cost of cardboard is ₹ 4 for 1000 cm^2. Find the cost of cardboard required for supplying 250 boxes of each kind.

OR

A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. Find the volume of steel used in making the bowl.

39. In Δ ABC, locate a point in its interior which is equidistant from all the sides of the ΔABC.

40. Read the bar graph shown in the figure and answer the following questions:



i. What is the information given by the bar graph
ii. What was the number of commercial banks in 1977?
iii. What is the ratio of the number of commercial banks in 1969 to that in 1980?
iv. State whether true or false: The number of commercial banks in 1983 is less than double the number of commercial banks in 1969.

CBSE Class 09

Mathematics

Sample Paper 8 (2019-20)

Solution

Section A

1. (a) $\sqrt[3]{2^4}$

   Explanation:

   $\sqrt[3]{2^4}$

   $=(2^4)^{\frac{1}{3}}$

   $=2^{\frac{4}{3}}$

2. (a) 3 terms

   Explanation: 3 terms of not more than the power of 3

3. (b) 30°

   Explanation: In the given figure

   150° + ∠a = 180° (linear - pair)

   ∠a = 180° - 150°

   Therefore

   ∠a = 30°

4. (c) 5.9 cm.

   Explanation: The construction of $\triangle ABC$ is not possible when difference of the other two sides of a triangle is greater than its base i.e BC<AB-AC.

5. (b) trinomial

   Explanation: A polynomial containing three non-zero terms is called a trinomial.

   Example: $5x^2 + 2x + 3, ax^2 + bx + c, 3x + 2y - 3$

6. (b) $35\, cm^2$.

   Explanation:

   Since ABEP and ARFP are on the same base AP and between the same parallels, therefore,

$\text{area}\,(\text{ABEP}) = \text{area}\,(\text{ARFP}) = 10$ sq. cm

Also, since ARSC and ARFP are on the same base AR and between the same parallels, therefore,

$\text{area}\,(\text{ARSC}) = \text{area}\,(\text{ARFP}) = 10$ sq. cm

Therefore, $\text{area}\,(\text{ABFP}) = \text{area}\,(\text{ARSC}) = 10$ sq. cm

$\text{area}\,(\text{PQRS}) = \text{area}\,(\text{ACPQ}) + \text{area}\,(\text{ARSC}) = 25 + 10 = 35\text{cm}^2$

7. (d) has one and only one zero

   Explanation: A polynomial which has one and only one zero, is called a linear polynomial.

8. (c) $\frac{625}{4}\sqrt{3}$ sq cm

   Explanation:

   Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{Side})^2$

   = $\frac{\sqrt{3}}{4}(25)^2$ = $\frac{625\sqrt{3}}{4}$ sq.cm

9. (a) 60 cm.

   Explanation: Volume of cube = volume of cuboid

   Volume of cube = 36×75×80

   Edge³ = 216000 cm³

   Edge = $(216000)^{1/3}$

   = 60 cm

10. (a) $\frac{17}{16}$

    Explanation: The probability of an event lies between 0 and 1 (or 0-100% expressed in percentage). It cannot be greater than 1 and the value of is greater than 1,so,it cannot be the probability of an event.

11. 0.$\underline{42}$

12. 4

OR

3

13. Four

14. circumference

15. 552

16. It is given that,

$(0.001)^{\frac{1}{3}} = \left(\frac{1}{1000}\right)^{\frac{1}{3}}$

$= \left(\frac{1}{10^3}\right)^{\frac{1}{3}}$

$= \frac{1}{10^{3\times\frac{1}{3}}}$

$= \frac{1}{10}$

$= 0.1$

17. Let a = 25, b = -75 and c = 50

Then,

$a + b + c = 25 - 75 + 50 = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (25)^3 + (-75)^3 + (50)^3 = 3 \times 25 \times (-75) \times 50$

$= -75 \times 75 \times 50$

$= -5625 \times 50$

$= -281250$

18. If radius of sphere = r

So, surface area of sphere = $4\pi r^2$

When radius is doubled so, new radius = 2r

So, new surface area = $4\pi(2r)^2 = 4\pi \times 4r^2$

$= 4(4\pi r^2)$

= 4 × original surface area.

Hence new surface area becomes 4 times the original surface area.

OR

If length, breadth and height are in the ratio 4 : 2 : 1

Let the common multiple is x

So, height = x cm, then breadth = 2x cm, length = 4x cm

The surface area of cuboid is 1792 sq cm.

The surface area of cuboid= 2(lb + bh + lh) = 1792

$2[4x\ (2x) + 2x\ (x) + 4x(x)] = 1792$

$2(8x^2 + 2x^2 + 4x^2) = 1792$

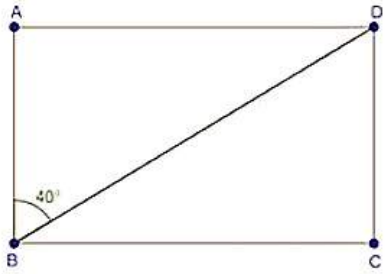
$28x^2 = 1792$

$\Rightarrow x^2 = \frac{1792}{28} = 64$

$\Rightarrow x = 8$

Length = $8 \times 4 = 32$ cm

19.



We have,

$\angle ABC = 90^o$

$\Rightarrow \angle ABD + \angle DBC = 90^o$

$\Rightarrow 40^o + \angle DBC = 90^o$

$\Rightarrow \angle DBC = 50^o$

20. We have,

$x + 4y - 7 = 0$

It is given that $x = -\lambda$ and $y = \frac{5}{2}$ is a solution of the equation $x + 4y - 7 = 0$.

$\therefore -\lambda + 4 \times \frac{5}{2} - 7 = 0$

$\Rightarrow -\lambda + 10 - 7 = 0$

$\Rightarrow -\lambda = -3$

$\Rightarrow \lambda = 3$

21. We have,

$9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$

= $\left(3^2\right)^{\frac{3}{2}}$ - 3 $\times$ 1 - $\left(\frac{1}{9^2}\right)^{-\frac{1}{2}}$

= $3^{2\times\frac{3}{2}}$ - 3 - $\left(9^{-2}\right)^{-\frac{1}{2}}$

= 3^3 - 3 - $9^{-2\times\left(-\frac{1}{2}\right)}$

= 3^3 - 3 - 9

= 27 - 3 - 9

= 27 - 12

= 15

22. y=3x

if $x = 0 \Rightarrow y = 0$

$x = 1 \Rightarrow y = 3$

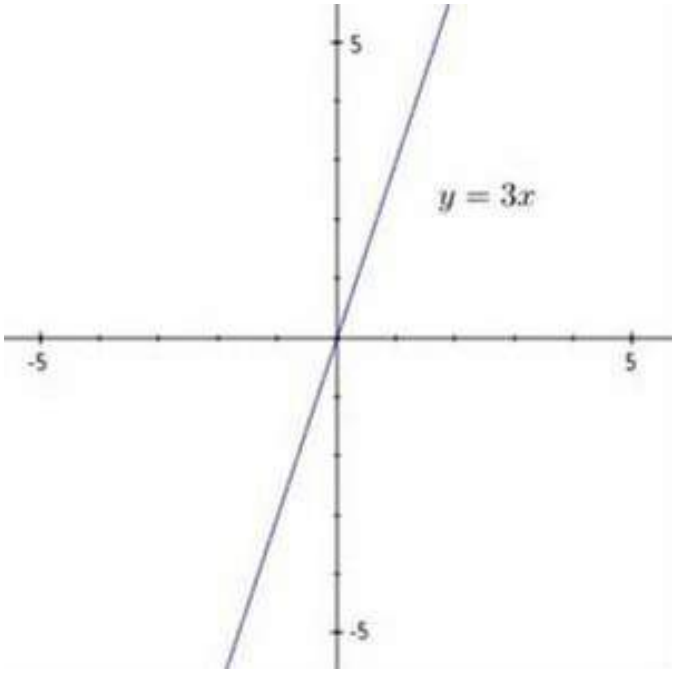
$x = 2 \Rightarrow y = 6$

x = 0, y = 0; x = 1, y = 3 and x = 2, y = 6

are the solutions of the linear equation y = 3x.

We can optionally consider the given below table for plotting the linear equation y = 3x on the graph.

x	0	1	2
y	0	3	6



23. Given, $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=3$

L.H.S. $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{a^3+b^3+c^3}{abc}$

We know that, if $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

$=\frac{3abc}{abc}$

$=3$

= R.H.S.

OR

According to the question,

$x+\frac{1}{x}=4$

$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2=4^2 \Rightarrow \quad x^2+\frac{1}{x^2}+2x\times\frac{1}{x}=16$

$\Rightarrow \quad x^2+\frac{1}{x^2}=16-2=14$

$\therefore \quad x^2+\frac{1}{x^2}=14$

24. Only figure (iv) is an example of figures having same base QR and between same parallels QR and PS.

25.

Marks	No. of students	Marks	Cumulative Frequency
0-5	2	less than 5	2

5-10	6	less than 10	8
10-15	13	less than 15	21
15-20	17	less than 20	38
20-25	11	less than 25	49
25-30	4	less than 30	53
30-35	2	less than 35	55
	N = 55		

OR

Cumulative frequency distribution of marks

Class interval	**Frequency**	**Cumulative frequency**
150 - 200	5	5
200- 250	3	8
250 - 300	5	13
300 - 350	6	19
350 - 400	8	27
400 - 450	7	34
450 - 500	1	35

26. Diameter of base of conical cap = 10 cm

∴ Radius of conical cap (r) = 5 cm

Slant height of cone (l) $= \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$ = 13 cm

Curved surface area of a cap = $\pi rl = 3.14 \times 5 \times 13 = 204.1\ cm^2$

Curved surface area of 15 caps = 15 × 204.1 = 3061.5cm^2

Area of a sheet of paper used for making caps $= 25 \times 40 = 1000\ cm^2$

82% of sheet is used after cutting = 82% of 1000 cm^2

$= \frac{82}{100} \times 1000 = 820 \text{ cm}^2$

Number of sheet $= \frac{3061.5}{820} = 3.73$

Hence 4 sheets area needed.

27. To prove whether $\sqrt{3}$ is rational or irrational, we find the square root of $\sqrt{3}$ by long division method.

	1.732050807
1	3.00 00 00 00 00 00 00 00
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
346405	1760000
	1732025
34641008	279750000
	277128064
3464101607	26219360000
	24248711249
	1970648751

$\therefore \sqrt{3} = 1.732050807$

We observe that the decimal representation of $\sqrt{3}$ is neither terminating nor repeating. Hence, $\sqrt{3}$ is an irrational number.

OR

Given, $4^{2x-1} - 16^{x-1} = 384$

$\Rightarrow 4^{2x-1} - 4^{2(x-1)} = 384$

$4^{2x-1} - \frac{4^{2x-2+1}}{4} = 384$

$\Rightarrow 4^{2x-1} - \frac{4^{2x-1}}{4} = 2^7 \times 3$

$\Rightarrow 4^{2x-1}\left(1 - \frac{1}{4}\right) = 2^7 \times 3$

$\Rightarrow 2^{2(2x-1)} \times \frac{3}{4} = 2^7 \times 3$

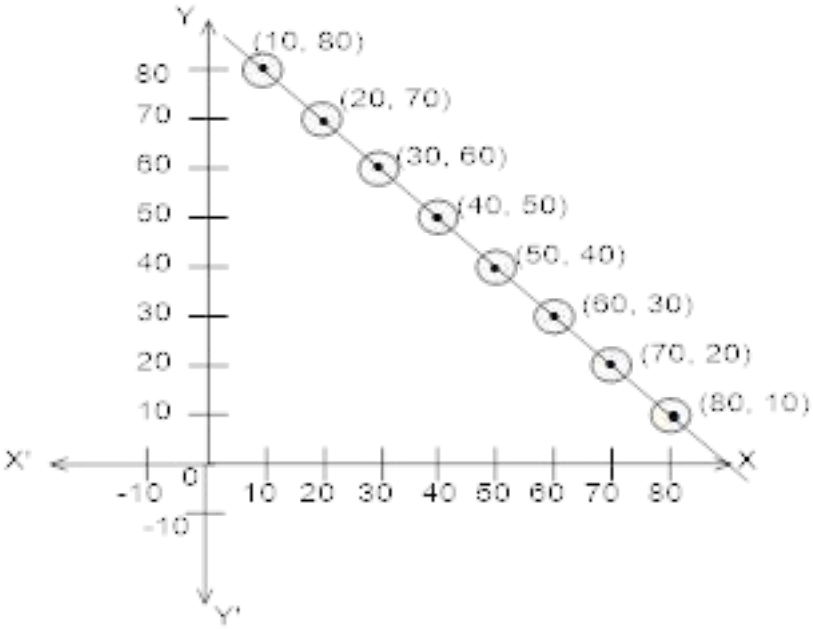
$\Rightarrow 2^{4x-2} = 2^7 \times 3 \times \frac{2^2}{3} = 2^9$

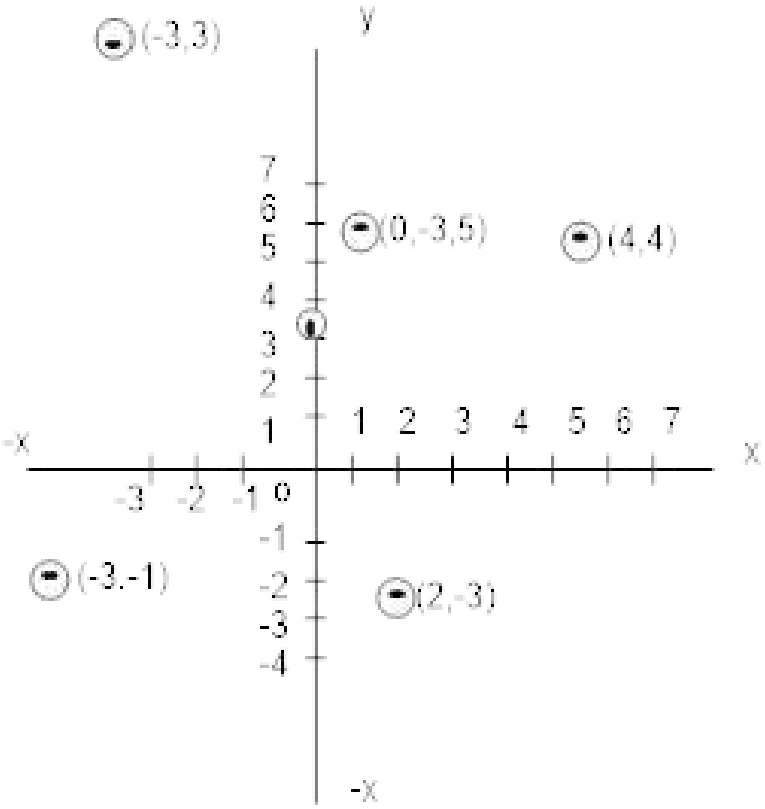
Equating the exponents, we get

$4x - 2 = 9$

$\Rightarrow x = \frac{11}{4}.$

28.





29. i. $y = 2x + 3$

$y = 2x + 3$

x	0	–1

y	3	1

We plot the points (0, 3) and (–1, 1) on the graph paper and join the same by a ruler to get the line which is the graph of the equation y = 2x + 3.

ii. $y = 2x - \frac{3}{2}$

x	0	1
y	$\frac{-3}{2}$	$\frac{1}{2}$

We plot the points $(0, \frac{-3}{2})$ and $(1, \frac{1}{2})$ on the graph paper and join the same by a ruler to get the line which is the graph of the equation $y = 2x - \frac{3}{2}$

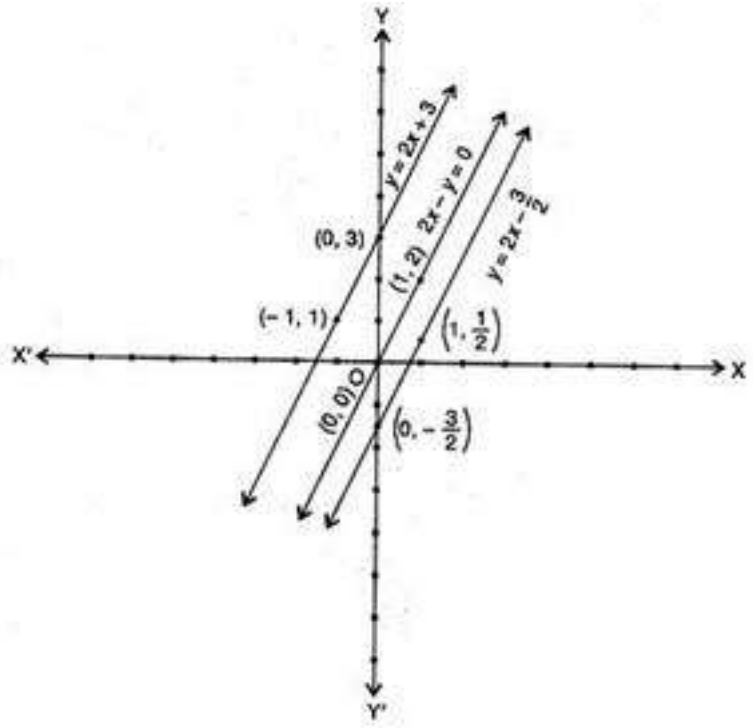
iii. 2x– y = 0

$\Rightarrow$ y = 2x

x	0	1
y	0	2

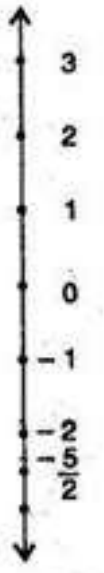
We plot the points (0, 0) and (1, 2) on the graph paper and join the same by a ruler to get the line which is the graph of the equation 2x– y = 0

From the graph, we see that these three lines are parallel.



OR

I. In one variable



II. In two variables

The given equation is

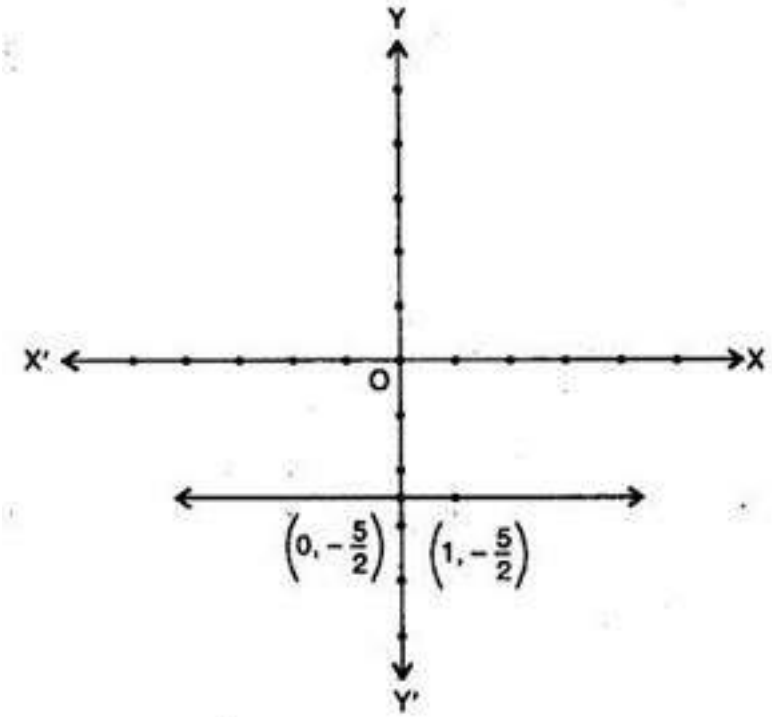
2y + 5 = 0

$\Rightarrow y = -\frac{5}{2}$

x	0	1

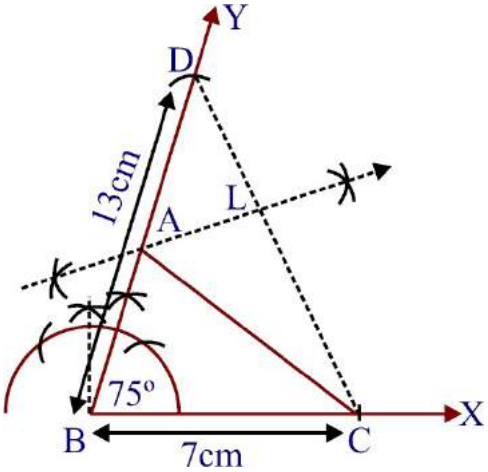
y	$-\frac{5}{2}$	$-\frac{5}{2}$

We plot the points $(0, -\frac{5}{2})$ and $(1, -\frac{5}{2})$ on the graph paper and join the same by a ruler to get the line which is the graph of the equation. $y = -\frac{5}{2}$



30. Given: In $\triangle$ABC, BC = 7 cm, $\angle$B = 75^o and AB + AC = 13 cm.
Required: To construct the triangle ABC.
Steps of construction :

i. Draw the base BC = 7 cm.



ii. At the point B construct an angle YBC = 75^o.

iii. Cut an arc from B as centre and radius equal to AB + AC = 13 cm. on the ray BY.

Name it D.

iv. Join DC.

v. Draw the perpendicular bisector of line segment DC which intersects BD at some point name it A.

vi. Join AC.

ABC is the required triangle.

31. In $\triangle$ABC, P and Q are the mid-points of the sides AB and BC respectively

i. $\therefore$ PQ$\|$AC and PQ = $\frac{1}{2}$ AC

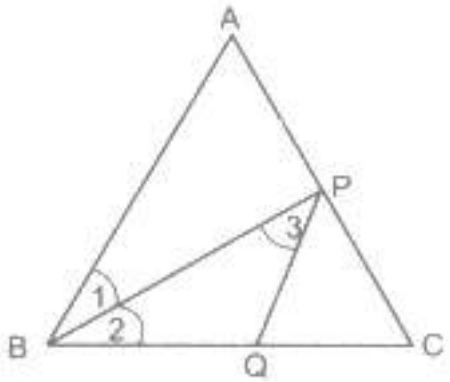
ii. Similarly SR$\|$AC and SR = $\frac{1}{2}$ AC

$\therefore$ PQ$\|$SR and PQ = SR

iii. Hence PQRS is a Parallelogram.

iv. PR and SQ bisect each other.

32. We draw diagram according to given conditions in the given problem. We have to prove that BPQ is an isosceles triangle.



$\angle 1 = \angle 2$ .....(1) [$\because$ BP is the bisector of $\angle ABC$]

Now, PQ is parallel to BA and BP cuts them

$\therefore \angle 1 = \angle 3$ [Alternate angles]...(2)

From (1) and (2), we get

$\angle 2 = \angle 3$

In ΔBPQ we have

$\angle 2 = \angle 3$ [Proved above]

$\therefore PQ = BQ$ [$\because$ Side opposite to equal angles are equal]

Hence, BPQ is an isosceles triangle.

OR

$\angle$ A + $\angle$B + $\angle$C = 180^0 Sum of three angles of triangle is 180°] .....(1)

Given that: $\angle \mathrm{A} + \angle \mathrm{C} = \angle \mathrm{B} \rightarrow (2)$

From (1) and (2)

$\angle \mathrm{B} + \angle \mathrm{B} = 180^\circ$

$\Rightarrow \angle B = \frac{180^\circ}{2} = 90^\circ$

Hence ΔABC is right angled.

33. $S = \frac{a+a+a}{2}$ units $= \frac{3a}{2} units$

$\therefore$Area of triangle = $\sqrt{\frac{3a}{2} \times (\frac{3a}{2} - a)(\frac{3a}{2} - a)(\frac{3a}{2} - a)}$

$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$

$= \frac{a^2}{4}\sqrt{3}$ sq units

Now, perimeter = 180 cm

$\therefore$ each side $= \frac{180}{3} = 60cm$

Using above derived formula

$\therefore$ Area of signal board = $\frac{\sqrt{3}}{4}$ $(60)^2$ sq cm

$= 900\ \sqrt{3}$ sq cm

34. i. Number of tests in which the student scored more than 70% marks = 3

$\therefore$ P (more than 70% marks) = $\frac{3}{6} = \frac{1}{2}$

ii. Number of tests in which the student scored less than 70% marks = 3

$\therefore$ P (less than 70% marks) = $\frac{3}{6} = \frac{1}{2}$

iii. Number of tests in which the student scored at least 60% marks = 5

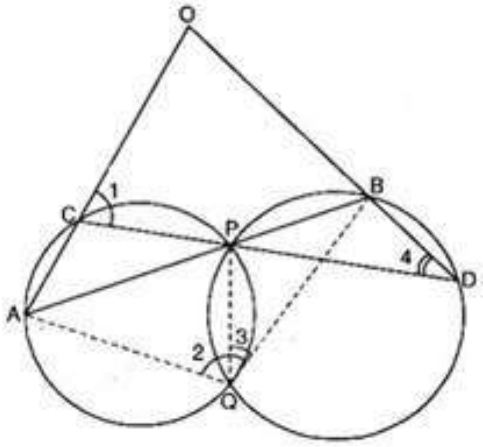
$\therefore$ P (at least 60% marks) = $\frac{5}{6}$

35. Given: Two circles intersect at P and Q. Through P, two straight lines APB and CPD are drawn to meet the circle at A, B, C and D. AC and DB when produced meet at O.

To Prove : OAQB is a cyclic quadrilateral

Construction : Join PQ, AQ and BQ.

Proof: PCAQ is a cyclic quadrilateral.



$\therefore$ exterior $\angle 1$ = opposite interior $\angle 2$

Also, $\angle 4 = \angle 3$ ..... (2) | $\angle$S in the same segment

Adding (1) and (2), we get

$\angle 1 + \angle 4 = \angle 2 + \angle 3 = \angle AQB$

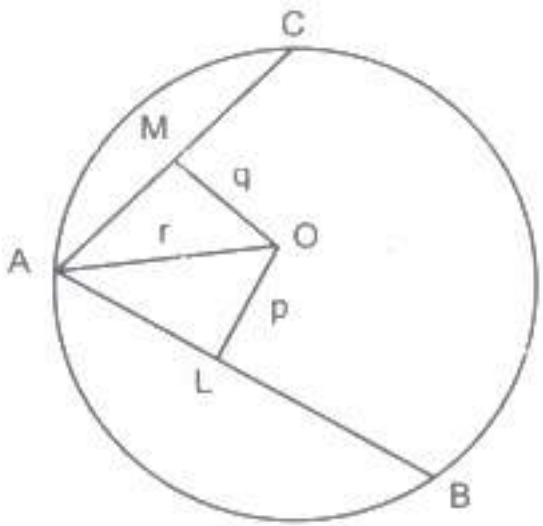
But $\angle 1 + \angle 4 + \angle O = 108^o$ [$\angle$S of $\triangle$OCD]

$\therefore \angle AQB + \angle O = 180^o$

$\therefore$ OAQB is a cyclic quadrilateral. Proved

OR

Here it is given that the circle with centre O and radius r in which there are two chords such that AB = 2AC. $OL \perp AB$ and $OM \perp AC$. Also OL = p and OM = q.



We need to prove that $4q^2 = p^2 + 3r^2$

Since the perpendicular from the centre bisects the chord In right angled ΔAOL , we have

$r^2 = AL^2 + p^2$ (by pythagoreous theorem)

$\Rightarrow AL^2 = r^2 - p^2$

$\therefore \left(\frac{1}{2}AB\right)^2 = r^2 - p^2 \Rightarrow \frac{1}{4}AB^2 = r^2 - p^2$

$\Rightarrow AB^2 = 4\left(r^2 - p^2\right)$

$\Rightarrow (2AC)^2 = 4\left(r^2 - p^2\right)$ [$\because$ AB = 2AC]

$\Rightarrow 4AC^2 = 4\left(r^2 - p^2\right)$ .....(1)

Again, in right ΔAOM we have

$r^2 = AM^2 + q^2 \Rightarrow AM^2 = r^2 - q^2$

Since $\perp$from the centre to a chord bisects the chord

$\therefore \left(\frac{1}{2}AC\right)^2 = r^2 - q^2 \Rightarrow \frac{1}{4}AC^2 = r^2 - q^2$

$\Rightarrow AC^2 = 4\left(r^2 - q^2\right)$ .....(2)

From equations (1) and (2), we obtain

$4\left\{4\left(r^2 - q^2\right)\right\} = 4\left(r^2 - p^2\right)$

$\Rightarrow 4r^2 - 4q^2 = r^2 - p^2 \Rightarrow 4q^2 = 3r^2 + p^2$

Hence, $4q^2 = p^2 + 3r^2$

36. Since corresponding angles are equal.

$\therefore$ x = y ... (i)

We know that the interior angles on the same side of the transversal are supplementary.

$\therefore$ y + 55^o = 180^o

$\Rightarrow$ y = 180^o - 55^o = 125^o

So, x = y = 125^o

Since AB || CD and CD || EF.

$\therefore$ AB || EF

$\Rightarrow \angle EAB + \angle FEA = 180^o$ [$\because$ Interior angles on the same side of the transversal EA are supplementary]

$\Rightarrow 90^o + z + 55^o = 180^o$

$\Rightarrow z = 35^o$

37. Let f (x) = 5 + bx - $2x^2$ + ax^3

When f (x) is divided by (x - 2), then the remainder is f (2).

When f (x) is divided by (x + 1), then the remainder is f (-1).

Now, f (2) = 5 + b (2) - 2 $(2)^2$ + a$(2)^3$ = 8a + 2b - 3

and f (-1) = 5 + b (-1) - 2$(-1)^2$+ a $(-1)^3$

= - a - b + 3

According to the question, f (2) = 2f(-1)

$\therefore$ 8a + 2b - 3 = 2(- a - b + 3)

$\Rightarrow$ 8a + 2b - 3= -2a - 2b + 6

$\Rightarrow$ 10a + 4b = 9

Hence proved.

OR

Let $p(y) = y^3 + ay^2 + by + 6$

p(y) is divisible by y – 2

Then P (2) = 0

$2^3 + a \times 2^2 + b \times 2 + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4\text{a} + 2\text{b} = -14$

$2\text{a} + \text{b} = -7 \ (\text{i})$

If p (y) is divided by y-3 remainder is 3

$\therefore \ \text{p}\ (3) = 3$

$3^3 + a \times 3^2 + b \times 3 + 6 = 3$

9a+3b=-30

3a+b=-10 ---(ii)

Subtracting (i) from (ii)

-a = 3 and a = -3

Put a = -3 in eq (i)

$2 \times -3 + b = -7$

-6+b=-7

b=-7+6

b=-1

38. Given dimensions of bigger box

$= 25cm \times 20cm \times 5cm$

Total surface area of bigger box

$= 2\left[25 \times 20 + 20 \times 5 + 25 \times 5\right] cm^2$

$= 2\left[500 + 100 + 125\right] cm^2$

$= 2 \times 725 = 1450cm^2$

Extra cardboard for packing = 5 of $1450cm^2$

= $\frac{5}{100} \times 1450 = 72.5cm^2$

Cardboard used for making box $= 1450 + 72.5 = 1522.5cm^2$

Dimensions of smaller box $= 15cm \times 12cm \times 5cm$

Total surface area of smaller box $= 2\left[15 \times 12 + 12 \times 15 + 15 \times 5\right] cm^2$

$= 2\left[180 + 60 + 75\right] cm^2$

$= 2 \times 315cm^2 = 630cm^2$

Extra cardboard for packing $= 5$ % of 630

=0.05x630=31.5

Total area of cardboard = $630 + 31.5 = 661.5cm^2$

Total cardboard used for making both boxes

$= \left(1522.5 + 661.5\right) cm^2 = 2184cm^2$

Cardboard used for making 250 boxes $= 250 \times 2184 = 546000cm^2$

Cost of cardboard= $\frac{4}{1000} \times 546000$

= ₹ 2184

OR

We have,

r = Inner radius of the bowl = 4 cm

R = Outer radius of the bowl = (4 + 0.5) cm = 4.5 cm

Volume of the inner hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 4 \times 4 \times 4$ cm^3

Volume of the outer hemisphere = $\frac{2}{3}\pi R^3 = \frac{2}{3} \times \frac{22}{7} \times 4.5 \times 4.5 \times 4.5$ cm^3

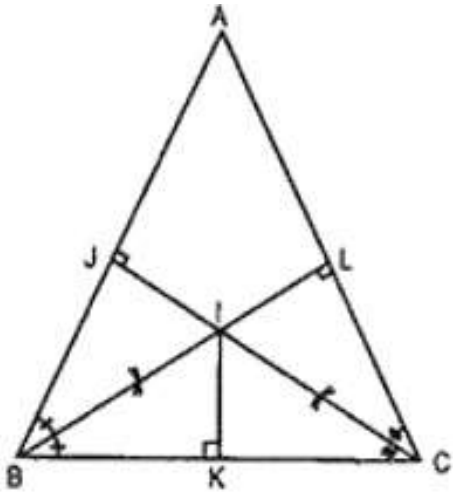
$\therefore$ Volume of steel used = $\left(\frac{22}{7} \times \frac{22}{7} \times 4.5 \times 4.5 \times 4.5 - \frac{2}{3} \times \frac{22}{7} \times 4 \times 4 \times 4\right)$ cm^3

= $\frac{2}{3} \times \frac{22}{7} \times$ {$(4.5)^3$ - $(4)^3$} cm^3

= $\frac{44}{21} \times$ (91.125 - 64) cm^3 = $\frac{44}{21} \times$ 27.125 cm^3 = 56.83 cm^3

39. The point which is equidistant from all the sides of a triangle is known as its incentre and it is also the point of intersection of the angular bisectors of angles of the triangle. Hence we will proceed with finding the incentre of the given triangle.

Let ABC be a triangle.



Draw bisectors of $\angle$ B and $\angle$ C.

Let these angular bisectors intersect each other at point I.

Draw IK $\perp$ BC

Also, draw IJ $\perp$ AB and IL $\perp$ AC.

Join BI & CI

In Δ BIK and Δ BIJ,

$\angle$ IKB = $\angle$ IJB = 90° [By construction]

$\angle$ IBK = $\angle$ IBJ

[$\because$ BI is the bisector of $\angle$ B (By construction)]

BI = BI [Common side]

$\therefore \Delta BIK \cong \Delta BIJ$ [ASA congruency criterion of triangle]

$\therefore$ IK = IJ [C.P.C.T.] .....(i)

Now, in ΔCIK and ΔCIL,

$\angle IKC = \angle ILC$ (each 90°)

$\angle ICK = \angle ICL$ [since, CI is angular bisector of $\angle C$]

CI = CI [common side]

Hence, $\Delta ClK \cong \Delta ClL$ [ASA congruency criterion of triangle]

$\therefore$ IK = IL [C.P.C.T.] .....(ii)

From eq (i) and (ii),

IK = IJ = IL

Hence, I is the point of intersection of angular bisectors of any two angles of Δ ABC and is also equidistant from the sides of the triangle.

40.
i. The given bar graph represents the number of commercial banks in India during some years.
ii. The number of commercial banks in 1977 was 130.
iii. The ratio of the number of commercial banks in 1969 to that in 1980 = $\frac{90}{150}$
= 3 : 5
iv. The number of commercial banks in 1983 = 230.
The number of commercial banks in 1980 = 150.
Clearly, the number of commercial banks in 1983 is not less than double the number of commercial banks in 1969.
So, the given statement is false.