Complex Numbers and Quadratic Equations

Question 1.
Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further assume that the origin, z_1 and z_2 form an equilateral triangle. Then
(a) $a^2 = b$
(b) $a^2 = 2b$
(c) $a^2 = 3b$
(d) $a^2 = 4b$

Answer: (c) $a^2 = 3b$
Given, z_1 and z_1 be two roots of the equation $z^2 + az + b = 0$
Now, $z_1 + z_2 = -a$ and $z_1 \times z_2 = b$
Since z_1 and z_2 and z_3 from an equilateral triangle.
$\Rightarrow z_1^2 + z_2^2 + z_3^2 = z_1 \times z_2 + z_2 \times z_3 + z_1 \times z_3$
$\Rightarrow z_1^2 + z_2^2 = z_1 \times z_2$ {since $z_3 = 0$}
$\Rightarrow (z_1 + z_2)^2 - 2z_1 \times z_2 = z_1 \times z_2$
$\Rightarrow (z_1 + z_2)^2 = 2z_1 \times z_2 + z_1 \times z_2$
$\Rightarrow (z_1 + z_2)^2 = 3z_1 \times z_2$
$\Rightarrow (-a)^2 = 3b$
$\Rightarrow a^2 = 3b$

Question 2.
The value of i^i is
(a) 0
(b) $e^{-\pi}$
(c) $2e^{-\pi/2}$
(d) $e^{-\pi/2}$

Answer: (d) $e^{-\pi/2}$
Let $A = i^i$

$\Rightarrow \log A = i \log i$
$\Rightarrow \log A = i \log(0 + i)$
$\Rightarrow \log A = i [\log 1 + i \tan^{-1} \infty]$
$\Rightarrow \log A = i [0 + i \pi/2]$
$\Rightarrow \log A = -\pi/2$
$\Rightarrow A = e^{-\pi/2}$

Question 3.
The value of $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)}$ is
(a) 13 i
(b) -13 i
(c) 17 i
(d) -17 i

Answer: (c) 17 i
Given, $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)}$
$= \sqrt{\{(-1) \times (25)\}} + 3\sqrt{\{(-1) \times 4\}} + 2\sqrt{\{(-1) \times 9\}}$
$= \sqrt{(-1)} \times \sqrt{(25)} + 3\{\sqrt{(-1)} \times \sqrt{4}\} + 2\{\sqrt{(-1)} \times \sqrt{9}\}$
$= 5i + 3 \times 2i + 2 \times 3i$ {since $\sqrt{(-1)} = i$}
$= 5i + 6i + 6i$
$= 17 i$
So, $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)} = 17 i$

Question 4.
If the cube roots of unity are 1, ω and ω^2, then the value of $(1 + \omega / \omega^2)^3$ is
(a) 1
(b) -1
(c) ω
(d) ω^2

Answer: (b) -1
Given, the cube roots of unity are 1, ω and ω^2
So, $1 + \omega + \omega^2 = 0$
and $\omega^3 = 1$
Now, $\{(1 + \omega)/ \omega^2\}^3 = \{-\omega^2/ \omega^2\}^3 = \{-1\}^3 = -1$

Question 5.
If $\{(1 + i)/(1 - i)\}^n = 1$ then the least value of n is
(a) 1
(b) 2

(c) 3
(d) 4

Answer: (d) 4
Given, $\{(1 + i)/(1 – i)\}^n = 1$
$\Rightarrow [\{(1 + i) \times (1 + i)\}/\{(1 – i) \times (1 + i)\}]^n = 1$
$\Rightarrow [\{(1 + i)^2\}/\{(1 – i^2)\}]^n = 1$
$\Rightarrow [(1 + i^2 + 2i)/\{1 – (-1)\}]^n = 1$
$\Rightarrow [(1 – 1 + 2i)/\{1 + 1\}]^n = 1$
$\Rightarrow [2i/2]^n = 1$
$\Rightarrow i^n = 1$
Now, i^n is 1 when n = 4
So, the least value of n is 4

Question 6.
The value of $[i^{19} + (1/i)^{25}]^2$ is
(a) -1
(b) -2
(c) -3
(d) -4

Answer: (d) -4
Given, $[i^{19} + (1/i)^{25}]^2$
$= [i^{19} + 1/i^{25}]^2$
$= [i^{16} \times i^3 + 1/(i^{24} \times i)]^2$
$= [1 \times i^3 + 1/(1 \times i)]^2$ {since $i^4 = 1$}
$= [i^3 + 1/i]^2$
$= [i^2 \times i + 1/i]^2$
$= [(-1) \times i + 1/i]^2$ {since $i^2 = -1$}
$= [-i + 1/i]^2$
$= [-i + i^4 /i]^2$
$= [-i + i^3]^2$
$= [-i + i^2 \times i]^2$
$= [-i + (-1) \times i]^2$
$= [-i – i]^2$
$= [-2i]^2$
$= 4i^2$
$= 4 \times (-1)$
$= -4$
So, $[i^{19} + (1/i)^{25}]^2 = -4$

Question 7.
If z and w be two complex numbers such that $|z| \leq 1$, $|w| \leq 1$ and $|z + iw| = |z – iw| = 2$, then z equals {w is congugate of w}
(a) 1 or i
(b) i or – i
(c) 1 or – 1
(d) i or – 1

Answer: (c) 1 or – 1
Given $|z + iw| = |z – iw| = 2$ {w is congugate of w}
$\Rightarrow |z – (-iw)| = |z – (iw)| = 2$
$\Rightarrow |z – (-iw)| = |z – (-iw)|$
So, z lies on the perpendicular bisector of the line joining -iw and -iw.
Since, -iw is the mirror in the x-axis, the locus of z is the x-axis.
Let $z = x + iy$ and $y = 0$
$\Rightarrow |z| < 1$ and $x^2 + 0^2 < 0$
$\Rightarrow -1 \leq x \leq 1$
So, z may take value 1 or -1

Question 8.
The value of $\{-\sqrt{(-1)}\}^{4n+3}$, $n \in N$ is
(a) i
(b) -i
(c) 1
(d) -1

Answer: (a) i
Given, $\{-\sqrt{(-1)}\}^{4n+3}$
$= \{-i\}^{4n+3}$ {since $\sqrt{(-1)} = i$}
$= \{-i\}^{4n} \times \{-i\}^3$
$= \{(-i)^4\}^n \times (-i^3)$ {since $i^4 = 1$}
$= 1^n \times (-i \times i^2)$
$= -i \times (-1)$ {since $i^2 = -1$}
$= i$

Question 9.
Find real θ such that $(3 + 2i \times \sin \theta)/(1 – 2i \times \sin \theta)$ is real
(a) π
(b) nπ

(c) $n\pi/2$
(d) $2n\pi$

Answer: (b) $n\pi$
Given,
$(3 + 2i \times \sin\theta)/(1 - 2i \times \sin\theta) = \{(3 + 2i \times \sin\theta)\times(1 - 2i \times \sin\theta)\}/(1 - 4i^2 \times \sin^2\theta)$
$(3 + 2i \times \sin\theta)/(1 - 2i \times \sin\theta) = \{(3 - 4\sin^2\theta) + 8i \times \sin\theta\}/(1 + 4\sin^2\theta)$ ………… 1
Now, equation 1 is real if $\sin\theta = 0$
$\Rightarrow \sin\theta = \sin n\pi$
$\Rightarrow \theta = n\pi$

Question 10.
If $i = \sqrt{(-1)}$ then $4 + 5(-1/2 + i\sqrt{3}/2)^{334} + 3(-1/2 + i\sqrt{3}/2)^{365}$ is equals to
(a) $1 - i\sqrt{3}$
(b) $-1 + i\sqrt{3}$
(c) $i\sqrt{3}$
(d) $-i\sqrt{3}$

Answer: (c) $i\sqrt{3}$
Given, $4 + 5(-1/2 + i\sqrt{3}/2)^{334} + 3(-1/2 + i\sqrt{3}/2)^{365}$
$= 4 + 5w^{334} + 3w^{365}$ {since $w = -1/2 + i\sqrt{3}/2$}
$= 4 + 5w + 3w^2$ {since $w^3 = 1$}
$= 4 + 5(-1/2 + i\sqrt{3}/2) + 3(-1/2 - i\sqrt{3}/2)$ {since $w^2 = (-1/2 - i\sqrt{3}/2)$}
$= i\sqrt{3}$

Question 11.
The real part of the complex number $\sqrt{9} + \sqrt{(-16)}$ is
(a) 3
(b) -3
(c) 4
(d) -4

Answer: (a) 3
Given, $\sqrt{9} + \sqrt{(-16)} = \sqrt{9} + \sqrt{(16)} \times \sqrt{(-1)}$
$= 3 + 4i$ {since $i = \sqrt{(-1)}$}
So, the real part of the complex number is 3

Question 12.
The modulus of $5 + 4i$ is
(a) 41

(b) -41
(c) √41
(d) -√41

Answer: (c) √41
Let Z = 5 + 4i
Now modulus of Z is calculated as
$|Z| = \sqrt{(5^2 + 4^2)}$
$\Rightarrow |Z| = \sqrt{(25 + 16)}$
$\Rightarrow |Z| = \sqrt{41}$
So, the modulus of 5 + 4i is √41

Question 13.
The modulus of 1 + i√3 is
(a) 1
(b) 2
(c) 3
(d) None of these

Answer: (b) 2
Let Z = 1 + i√3
Now modulus of Z is calculated as
$|Z| = \sqrt{\{1^2 + (\sqrt{3})^2\}}$
$\Rightarrow |Z| = \sqrt{(1 + 3)}$
$\Rightarrow |Z| = \sqrt{4}$
$\Rightarrow |Z| = 2$
So, the modulus of 1 + i√3 is 2

Question 14.
The value of $\{-\sqrt{(-1)}\}^{4n+3}$, n ∈ N is
(a) i
(b) -i
(c) 1
(d) -1

Answer: (a) i
Given, $\{-\sqrt{(-1)}\}^{4n+3}$
$= \{-i\}^{4n+3}$ {since √(-1) = i}
$= \{-i\}^{4n} \times \{-i\}^3$
$= \{(-i)^4\}^n \times (-i^3)$ {since $i^4 = 1$}

$= 1^n \times (-i \times i^2)$
$= -i \times (-1)$ {since $i^2 = -1$}
$= i$

Question 15.
If ω is cube root of unity (ω ≠ 1) , then the least value of n where n is a positive integer such that $(1 + \omega^2)^n = (1 + \omega^4)^n$ is
(a) 2
(b) 3
(c) 5
(d) 6

Answer: (b) 3
Given ω is an imaginary cube root of unity.
So $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$
Now, $(1 + \omega^2)^n = (1 + \omega^4)^n$
$\Rightarrow (-1)^n \times (\omega)^n = (1 + \omega \times \omega^3)^n$
$\Rightarrow (-1)^n \times (\omega)^n = (1 + \omega)^n$
$\Rightarrow (-1)^n \times (\omega)^n = (-\omega^2)^n$
$\Rightarrow (-1)^n \times (\omega)^n = (-1)^n \times \omega^{2n}$
$\Rightarrow \omega^n = \omega^{2n}$
Since $\omega^3 = 1$, So the least value of n is 3

Question 16.
The value of $i^9 + i^{10} + i^{11} + i^{12}$ is
(a) i
(b) 2i
(c) 0
(d) 1

Answer: (c) 0
Given, $i^9 + i^{10} + i^{11} + i^{12}$
= i9 (1 + i + i2 + i3)
= i9 (1 + i – 1 – i) {since i2 = (-1) and i4 = 1}
= i9 × 0
= 0

Question 17.
If a = cos α + i sin α and b = cos β + i sin β , then the value of 1/2(ab + 1/ ab) is

(a) $\sin(\alpha + \beta)$
(b) $\cos(\alpha + \beta)$
(c) $\sin(\alpha - \beta)$
(d) $\cos(\alpha - \beta)$

Answer: (b) $\cos(\alpha + \beta)$
Given $a = \cos\alpha + i\sin\alpha$ and $b = \cos\beta + i\sin\beta$
Now, $1/a = 1/(\cos\alpha + i\sin\alpha)$
$\Rightarrow 1/a = \{1 \times (\cos\alpha - i\sin\alpha)/\{(\cos\alpha + i\sin\alpha) \times (\cos\alpha + i\sin\alpha)\}$
$\Rightarrow 1/a = (\cos\alpha - i\sin\alpha)/(\cos^2\alpha + i\sin^2\alpha)$
$\Rightarrow 1/a = (\cos\alpha - i\sin\alpha)$
Again, $1/b = 1/(\cos\beta + i\sin\beta)$
$\Rightarrow 1/b = \{1 \times (\cos\beta - i\sin\beta)/\{(\cos\beta + i\sin\beta) \times (\cos\beta + i\sin\beta)\}$
$\Rightarrow 1/b = (\cos\beta - i\sin\beta)/(\cos^2\beta + i\sin^2\beta)$
$\Rightarrow 1/b = (\cos\beta - i\sin\beta)$
Now, $ab = (\cos\alpha + i\sin\alpha) \times (\cos\beta + i\sin\beta)$
$\Rightarrow ab = \cos\alpha \times \cos\beta + i\cos\alpha \times \sin\beta + i\sin\alpha \times \cos\beta - \sin\alpha \times \sin\beta$
Again, $1/ab = (\cos\alpha - i\sin\alpha) \times (\cos\beta - i\sin\beta)$
$\Rightarrow 1/ab = \cos\alpha \times \cos\beta - i\cos\alpha \times \sin\beta - i\sin\alpha \times \cos\beta - \sin\alpha \times \sin\beta$
Now, $ab + 1/ab = \cos\alpha \times \cos\beta + i\cos\alpha \times \sin\beta + i\sin\alpha \times \cos\beta - \sin\alpha \times \sin\beta + \cos\alpha \times \cos\beta - i\cos\alpha \times \sin\beta - i\sin\alpha \times \cos\beta - \sin\alpha \times \sin\beta$
$\Rightarrow ab + 1/ab = 2(\cos\alpha \times \cos\beta - \sin\alpha \times \sin\beta)$
$\Rightarrow 1/2(ab + 1/ ab) = 2(\cos\alpha \times \cos\beta - \sin\alpha \times \sin\beta)/2$
$\Rightarrow 1/2(ab + 1/ ab) = \cos\alpha \times \cos\beta - \sin\alpha \times \sin\beta$
$\Rightarrow 1/2(ab + 1/ ab) = \cos(\alpha + \beta)$

Question 18.
The polar form of $-1 + i$ is
(a) $\sqrt{2}(\cos\pi/2 + i \times \sin\pi/2)$
(b) $\sqrt{2}(\cos\pi/4 + i \times \sin\pi/4)$
(c) $\sqrt{2}(\cos 3\pi/2 + i \times \sin 3\pi/2)$
(d) $\sqrt{2}(\cos 3\pi/4 + i \times \sin 3\pi/4)$

Answer: (d) $\sqrt{2}(\cos 3\pi/4 + i \times \sin 3\pi/4)$
The polar form of a com plex number = $r(\cos\theta + i \times \sin\theta)$
Given, complex number = $-1 + i$
Let $x + iy = -1 + i$
Now, $x = -1$, $y = 1$
Now, $r = \sqrt{\{(-1)^2 + 1^2\}} = \sqrt{(1 + 1)} = \sqrt{2}$
and $\tan\theta = y/x$
$\Rightarrow \tan\theta = 1/(-1)$

$\Rightarrow \tan \theta = -1$
$\Rightarrow \theta = 3\pi/4$
Now, polar form is $\sqrt{2}(\cos 3\pi/4 + i \times \sin 3\pi/4)$

Question 19.
For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is
(a) 0
(b) 2
(c) 7
(d) 17

Answer: (b) 2
Given For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$
Now, $mod(z_1) = 12$ represents a circle centred at 0 and radius 12
$mod(z_2 - 3 - 4i) = 5$ represents a circle centred at (3, 4) and radius 5
This circle passes through the origin. Distance of diametrically opposite end is 10
So, the minimum value $(z_1 - z_2) = 2$

Question 20.
The value of $(1 - i)^2$ is
(a) i
(b) -i
(c) 2i
(d) -2i

Answer: (d) -2i
Given, $(1 - i)^2 = 1 + i^2 - 2i$
$= 1 + (-1) - 2i$
$= 1 - 1 - 2i$
$= -2i$
