CBSE Test Paper 04
Chapter 5 Arithmetic Progression

1. Which term of the A.P. 121, 117, 113, ... is its first negative term? **(1)**
   a. 32
   b. 33
   c. 30
   d. 31
2. The A.P. whose third term is 16 and the difference of 5th term from the 7th term is 12, then the A.P. is **(1)**
   a. 4, 11, 18, 25, ...
   b. 4, 14, 24, 34, ...
   c. 4, 10, 16, 22, ...
   d. 4, 6, 8, 10, ...
3. If $a_1 = 4\ and\ a_n = 4a_{n-1} + 3,\ n > 1,$ then the value of a_4 is **(1)**
   a. 320
   b. 329
   c. 319
   d. 300
4. In an A.P. it is given that a = 5, d = 3 and a_n = 50, then the value of 'n' is **(1)**
   a. 16
   b. 18
   c. 20
   d. 15
5. If the second term of an AP is 13 and its fifth term is 25, then its 7th term is **(1)**
   a. 37
   b. 33
   c. 39
   d. 35
6. In the A.P. 2, x, 26 find the value of x. **(1)**
7. Find 9^{th} term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$ **(1)**
8. If they form an AP, find the common difference d and write three more terms. 0.2,

0.22, 0.222, 0.2222, ... **(1)**

9. If sum of first n terms of an AP is $2n^2 + 5n$. Then find S_{20}. **(1)**
10. Find the sum of each of the following APs: 0.6, 1.7, 2.8,... to 100 terms. **(1)**
11. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. **(2)**
12. In a certain A.P. 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term. **(2)**
13. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P. **(2)**
14. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 7 - 3n$. **(3)**
15. The 12th term of an AP is -13 and the sum of its first four terms is 24. Find the sum of its first 10 terms. **(3)**
16. Prove that the 11^{th} term of an A.P. cannot be $n^2 + 1$. Justify your answer. **(3)**
17. There are 25 trees at equal distances of 5m in a line with a water tank, the distance of the water tank from the nearest tree being 10m. A gardener waters all the trees separately, starting from the water tank and returning back to the water tank after watering each tree to get water for the next. Find the total distance covered by the gardener in order to water all the trees. **(3)**

18. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find n and a_n, if a = 2, d = 8 and $S_n = 90$. **(4)**
19. Find the sum of all natural numbers between 200 and 300 which are divisible by 4. **(4)**
20. In a garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row. How many rows are there of rose plants? Also find the total number of rose plants in the garden. **(4)**

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Solution

1. a. 32

   Explanation: Here $a = 121, d = 117 - 121 = -4$

   For the first negative term, $a_n < 0$

   $\Rightarrow 121 + (n-1)(-4) < 0$

   $\Rightarrow 121 - 4n + 4 < 0$

   $\Rightarrow 125 - 4n < 0$

   $\Rightarrow n > \frac{125}{4}$

   $\Rightarrow n > 31\frac{1}{4}$

   Therefore 32nd term is the first negative term.

2. c. 4, 10, 16, 22, ...

   Explanation: Given: $a_3 = 16 \Rightarrow a + 2d = 16$ ....(i)

   And $a_7 - a_5 = 12 \Rightarrow a + 6d - a - 4d = 12$

   $\Rightarrow 2d = 12 \Rightarrow d = 6$

   Putting value of d in eq. (i), we get

   $a + 2 \times 6 = 16$

   $\Rightarrow a = 4$

   Therefore, A.P. is 4, 10, 16, 22, ...

3. c. 319

   Explanation: Given: $a_1 = 4$ and $a_n = 4a_{n-1} + 3,\ n > 1,$

   $\therefore a_2 = 4a_{2-1} + 3 = 4a_1 + 3 = 4 \times 4 + 3 = 16 + 3 = 19$

   and$a_3 = 4a_{3-1} + 3 = 4a_2 + 3 = 4 \times 19 + 3 = 76 + 3 = 79$

   and $a_4 = 4a_{4-1} + 3 = 4a_3 + 3 = 4 \times 79 + 3 = 316 + 3 = 319$

4. a. 16

   Explanation: Given: a = 5, d = 3 and a_n = 50

   $\therefore a_n = a + (n-1)d$

   $\Rightarrow 50 = 5 + (n-1) \times 3$

   $\Rightarrow 45 = (n-1) \times 3$

   $\Rightarrow \frac{45}{3}$ = n-1

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 16$

5. b. 33

   Explanation: Given: $a_2 = 13$

   $\Rightarrow a + (2 - 1)\,d = 13$

   $\Rightarrow a + d = 13$ ....(i)

   And $a_5 = 25$

   $\Rightarrow a + (5 - 1)\,d = 25$

   $\Rightarrow a + 4d = 25$ ....(ii)

   Solving eq. (i) and (ii),

   we get $a = 9$ and $d = 4$

   $\therefore a_7 = a + (7 - 1)\,d$

   = $9 + (7 - 1) \times 4$

   = 9 + 6 $\times$ 4

   = 9 + 24 = 33

6. $2, x$ and 26 are in A.P.

   $\therefore x - 2 = 26 - x$

   or, $x + x = 26 + 2$

   or, $2x = 28$

   or, $x = \frac{28}{2}$ = 14

7. Given, A.P $= \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$

   First term(a) = $\frac{3}{4}$

   Common difference (d) = $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$

   As we know,

   $a_n = a + (n - 1)d$

   $\Rightarrow a_9 = \frac{3}{4} + (9 - 1) \times \frac{2}{4}$

   $= \frac{3}{4} + 8 \times \frac{2}{4}$

   $= \frac{3}{4} + \frac{16}{4} = \frac{19}{4}$

8. 0.2, 0.22, 0.222, 0.2222, 000

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

As $a_2 - a_1 \neq a_3 - a_2$, the given list of numbers does not form an AP.

9. $S_n = 2n^2 + 5n$
$S_{20} = 2(20)^2 + 5(20)$
$= 2(400) + 100 = 900.$

10. Here, $a = 0.6,\ d = 1.7 - 0.6 = 1.1\ and\ n = 100$
Now we know that, $S_n = \frac{n}{2}[2a + (n - 1)d]$
Therefore, $S_n = \frac{100}{2}[2 \times 0.6 + (100 - 1)(1.1)]$
$= 50[1.2 + 108.9]$
$= 50 \times 110.1$
$= 5505$

11. We have,
$a_4 = 0$
$a + 3d = 0$
$3d = -a$
or $-3d = a$..........(i)
Now,
$a_{25} = a + 24d$
$= -3d + 24d$ [Putting value of a from eq(i)]
$= 21d$...........(ii)
$a_{11} = a + 10d$
$= -3d + 10d$
$= 7d$.........(iii)
From eq(ii) and (iii), we get
$a_{25} = 21d$
$a_{25} = 3(7d)$
$a_{25} = 3a_{11}$
Hence Proved

12. Let the 1st term be a and common difference be d.

    an = a + (n-1)d

    According to the question, $a_{32} = 2a_{12}$

    a +31d = 2 (a + 11d)

    a + 31d = 2a + 22d

    a - 2a = 22d - 31d

    a = 9d

    a_{70} = a + 69d = 9d + 69d = 78d

    a_{31} = a + 30d = 9d + 30d = 39d

    a_{70} = 78d

    a_{70} = 2(39d)

    $a_{70} = 2a_{31}$

    $a_{70} = 2a_{31}$ **Hence Proved**.

13. Let a be the first term and d be the common difference of the given A.P. Then,

    $\therefore . S_6 = 42$

    $\frac{6}{2}(2a + 5d) = 42$

    $2a + 5d = 14.........(i)$

    It is given that

    $a_{10} : a_{30}$ = 1: 3

    $\Rightarrow \quad \frac{a+9d}{a+29d} = \frac{1}{3}$

    $3a + 27d = a + 29d$

    $2a = 2d$

    $a = d.........(ii)$

    From (i) and(ii) we get

    a = d = 2

    Hence $a_{13} = 2 + 12 \times 2 = 26$ and $a_1 = 2$

14. Given, $a_n = 7 - 3n$

    Put n = 1, $a_1 = 7 - 3 \times 1 = 7 - 3 = 4$

    Put n = 2, $a_2 = 7 - 3 \times 2 = 7 - 6 = 1$

    Common difference(d) = 1 - 4 = -3

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{25} = \frac{25}{2}[2 \times 4 + (25-1)(-3)]$

$= \frac{25}{2}[8 - 72]$

$= \frac{25}{2} \times -64$

= -800

15. 12^{th} term = T_{12} = -13

$\Rightarrow$a + 11d = -13...(i)

S_4 = 24

$\Rightarrow \frac{4}{2}$ [2a + 3d] = 24

$\Rightarrow$2[2a + 3d] = 24

$\Rightarrow$2a + 3d = 12...(ii)

Multiplying equation (i) by 2, we get

2a + 22d = -26...(iii)

Subtracting (ii) from (i), we get

19d = -38

$\Rightarrow$d = -2

$\Rightarrow$a + 11(-2) = -13...[from (i)]

$\Rightarrow$a = 9

$\therefore$ Sum fo first 10 terms, $S_{10} = \frac{10}{2}$ [2(9) + 9(-2)] = 5[18 - 18] = 5 $\times$ 0 = 0

16. Let n^{th} term of A.P.

$a_n = n^2 + 1$

Putting the value of n = 1, 2 ,3 ,.....we get

$a_1 = 1^2 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

$a_3 = 3^2 + 1 = 10$

The obtained sequence = $2, 5, 10, 17$.......

Its common difference

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

or, 5 - 2 $\neq$ 10 - 5 $\neq$ 17 - 10

$\therefore 3 \neq 5 \neq 7$

Since the sequence has no. common difference

Hence, $n^2 + 1$ is not a form of n^{th} term of an A.P.

17. Assume that gardener is standing near the well initially, and he did not return to the well after watering the last tree.

    distance covered by gardener to water 1^{st} tree and return to the initial position=10m + 10m = 20m.

    Distance covered by gardener to water 2^{nd} tree and return to the initial position =15m + 15m =3 0m.

    Distance covered by gardener to water 3^{rd} tree and return to the initial position =20m + 20m = 40m.

    ∴ Distance covered by the gardener to water the plants are in AP.

    Here a = 20, d = 10

    Total distance covered by the gardener is given by Sn, where n = 25.

    $\Rightarrow$Sn $=\frac{25}{2}$ [2(20) + (25 - 1)10] = 3500.

    Thus, the total distance covered by the gardener is 3500m.

18. Given that, a = 2, d = 8 and S_n = 90.

    As, $S_n = \frac{n}{2}[2a + (n-1)d]$

    $90 = \frac{n}{2}[4 + (n-1)8]$

    90 = n[2 + (n - 1)4]

    90 = n[2 + 4n - 4]

    $90 = n(4n - 2) = 4n^2 - 2n$

    $4n^2 - 2n - 90 = 0$

    $4n^2 - 20n + 18n - 90 = 0$

    4n(n - 5) + 18(n - 5) = 0

    (n - 5)(4n + 18) = 0

    Either n = 5 or $n = -\frac{18}{4} = \frac{-9}{2}$

    However, n can neither be negative nor fractional.

    Therefore, n = 5

    $a_n = a + (n - 1)d$

    $a_5 = 2 + (5 - 1)8$

= 2 + 4(8)

= 2 + 32 = 34

19. All the numbers between 200 and 300 which are divisible by 4 are 204, 208, 212, 216, ..., 296

Here, a_1 = 204

a_2 = 208

a_3 = 212

a_4 = 216

$\therefore$ a_2 - a_1 = 208 - 204 = 4

a_3 - a_2 = 212 - 208 = 4

a_4 - a_3 = 216 - 212 = 4

$\because$ a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... (= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 204

d = 4

l = 296

Let the number of terms be n. Then,

l = a + (n - 1)d

$\Rightarrow$ 296 = 204 + (n - 1)4

$\Rightarrow$ 92 = (n - 1)4

$\Rightarrow$ n - 1 = 23

$\Rightarrow$ n = 23 + 1

$\Rightarrow$ n = 24

$\therefore S_n = \frac{n}{2}(a + l)$

$= \left(\frac{24}{2}\right)(204 + 296)$

=(12) (500)

= 6000.

Hence, the sum of all the natural numbers between 200 and 300 which are divisible by 4 is 6000.

20. The number of rose plants in the 1^{st}, 2^{nd}, ... are 23,21,19,..........5

a = 23, d = 21 - 23 = - 2 , a_n = 5

∴ a_n = a + (n - 1)d

or, 5 = 23 + (n - 1)(-2)

or, 5 = 23 -2n + 2

or, 5 = 25 -2n

or, 2n = 20

or, n =10

Total number of rose plants in the flower bed,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$$

$$= 5[46 - 20 + 2]$$

S_{10} = 5(46 - 18)

= 5 (28)

S_{10} = 140