13. Logarithms

(i) Definition: $a^x = b$ can be represented in logarithmic form as $\log_a b = x$

(ii) $\log a = x$ means that $10^x = a$.

(iii) $10^{\log\ a} = a$ (The basic logarithmic identity).

(iv) $\log (ab) = \log a + \log b$, $a > 0, b > 0$

(v) $\log \frac{a}{b}$ = log a -log b, a > 0, b > 0.

(vi) log a^n = n (log a) (Logarithm of a power).

(vii) $\log_x y = \frac{\log_m y}{\log_m x}$ (Change of base rule).

(viii) $\log_x y = \frac{1}{\log_y x}$.

(ix) $\log_x 1 = 0$ ($x \neq 0, 1$).

(x) The natural numbers 1, 2, 3,… are respectively the logarithms of 10, 100, 1000, … to the base 10.

(xi) The logarithm of '0' and negative numbers are not defined.

The logarithm of a number to the base '10' is known as common logarithm and the logarithm of a number to the base 'e' is

known as natural logarithm.

Characteristic And Mantissa Of Common Logarithms:

The integral part of the common logarithm of a number x > 0 is called the **Characteristic** and the fractional part is called the **Mantissa**.

e.g. the logarithm of 2 to the base 10 is 0.3010, where 0 is the characteristic and 3010 is the mantissa.

Any positive number 'x' can be written in the form $x = a10^n$, where $1 < a < 10$ and n is an integer. The number n is called the order of the number x. e.g. 30 can be written as $3*10^1$

and similarly 300 can be written $3*10^2$. The same rule applies to fractions as well where the value of n will be negative.

The characteristic of the logarithm of the given number ‘x’ will be ‘n’ and the mantissa will be the logarithm of ‘a’. Therefore, while log 2 = 0.3010, log 20 will be 1.3010 as ‘n’ in this case is ‘1’.

- Thus, the value of the characteristic of the logarithm of a number will help determine the number of integral digits the number has = characteristic + 1.