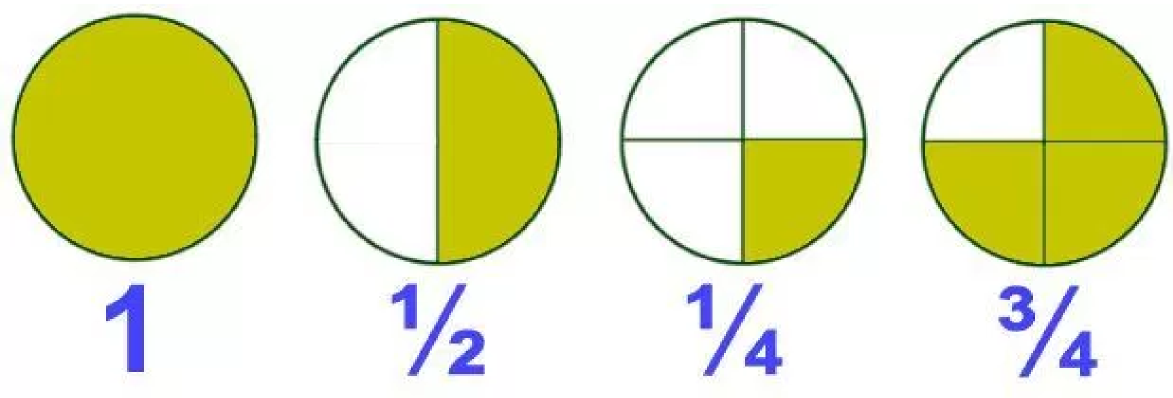
9. Fractions and Percents

What is Fraction and How many Types of Fractions are there

Fraction

A number that compares part of an object or a set with the whole, especially the quotient of two whole numbers is written in the form of xly is called a **fraction**. The fraction 1/3, which means 1 divided by 3, can be represented as 1 pencil out of a box of 3 pencils.
A fraction is a (i) part of a whole. (ii) part of a collection.

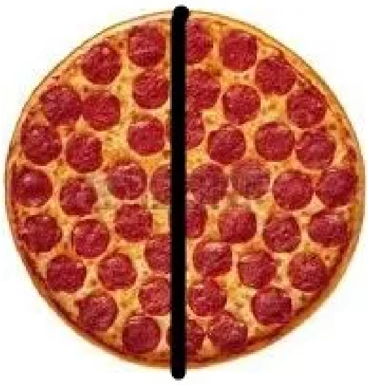


A fraction comprises two numbers separated by a horizontal line. The number above-the horizontal line is called the numerator and the number below the horizontal line is called the denominator of the fraction.

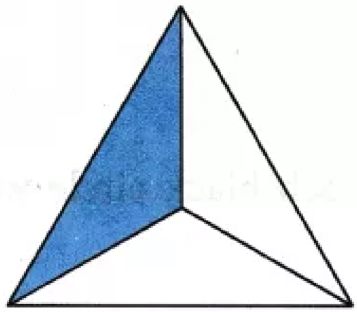
$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\leftarrow 4}{\leftarrow 7}$$

Fraction as a part of a whole
A fraction is a part of a whole. Imagine a pizza cut into slices. All of the slices make 1 whole pizza. Each slice is a fraction of a pizza.
Tanya and Sanya want to share a pizza equally

They decide to cut the pizza from the middle and divide it into two equal parts. Each part is called the half of the whole and written as $\frac{1}{2}$. Both the sisters get equal share. The $\frac{1}{2}$ part of the whole is a fraction.



Similarly, we can take many examples from our daily life to show fraction as a part of a whole.



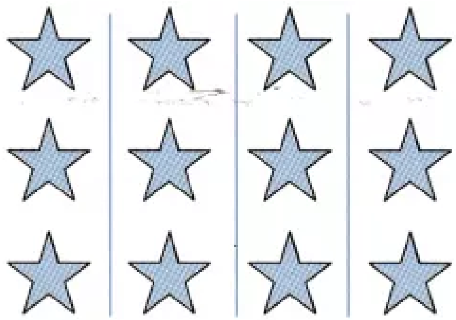
In this figure we have divided a triangle into 3 equal parts. The shaded part shows one part out of three, i.e., $\frac{1}{3}$. Here, $\frac{1}{3}$ is a fraction, which is a part of the whole triangle.

Fraction is a part of a collection

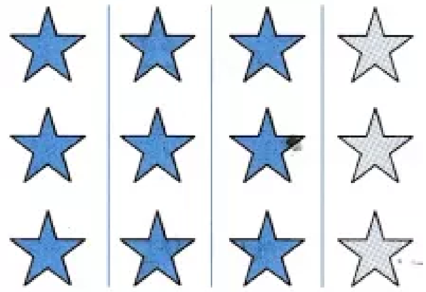
A fraction represents parts of a collection, the numerator being the number of parts we have and the denominator being the total number of parts in the collection.

Let us take a collection of 12 stars and we want to shade $\frac{3}{4}$ of the collection.

In order to find $\frac{3}{4}$ out of the 12 stars, we divide the 12 stars into four equal parts.



Each part contains 3 stars. Now, we can shade 3 parts out of 4 parts.

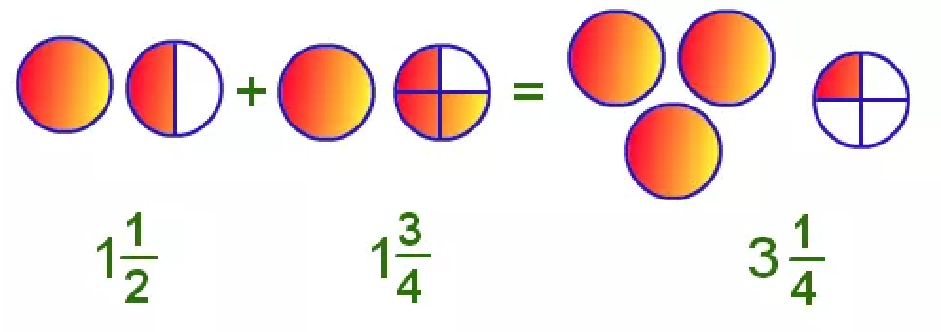


On counting, we find that the total number of shaded stars is 9.

In other words, $\frac{3}{4}$ of 12 stars = 9 stars.

Types of fractions

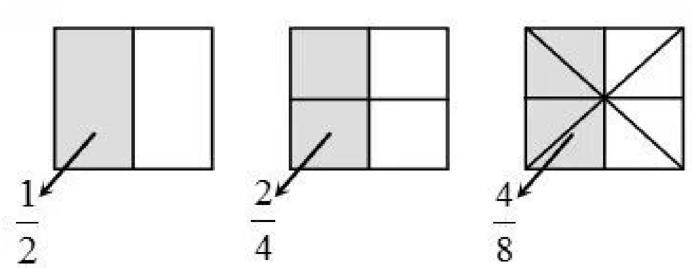
1. **Like fractions:** Fractions having the same denominators are called like fractions.
   Examples: $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}, \frac{6}{7}$ etc. are like fractions.
2. **Unlike fractions:** Fractions having different denominators are called unlike fractions.
   Examples: $\frac{2}{3}, \frac{5}{7}, \frac{6}{8}, \frac{1}{3}$ etc. are unlike fractions.
3. **Unit fraction:** A fraction having a numerator as 1 is called a unit fraction.
   Examples: $\frac{1}{3}, \frac{1}{9}, \frac{1}{8}, \frac{1}{5}$ etc. are all unit fractions
4. **Proper fraction:** A fraction, whose numerator is smaller than its denominator is called a proper fraction.
   Examples: $\frac{2}{3}, \frac{5}{7}, \frac{1}{6}, \frac{3}{9}$ etc. are all proper fractions.
5. **Improper fraction:** A fraction, whose numerator is greater than or equal to its denominator is called an improper fraction.
   Examples: $\frac{4}{3}, \frac{7}{5}, \frac{9}{9}$ etc. are all improper fractions.
6. **Mixed fraction:** A fraction, which is a combination of a whole number and a proper fraction is called a mixed fraction. All improper fractions can be written in the form of mixed fractions.
   Example: $2\frac{1}{4}$ is a mixed fraction, since 2 is a 4 whole number and $\frac{1}{4}$ is a proper fraction.

7. **Equivalent Fraction:** If $\frac{c}{d} = \frac{m \times a}{m \times b}$, then the fractions $\frac{a}{b}$ and $\frac{c}{d}$ are called equivalent fractions because they represent the same portion of the whole.

   For example, $\frac{4}{6} = \frac{2 \times 2}{3 \times 2}$; $\frac{15}{48} = \frac{5 \times 3}{16 \times 3}$

   For example, the shaded parts of each of the following figures are same but they are represented by different fractional numbers.



They are called equivalent fractions.

So we write $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, etc.

8. **Decimal fractions:** A fraction whose denominator is any of the number 10,100,1000 etc. is called a decimal fraction.

   For example : $\frac{8}{10}, \frac{11}{100}, \frac{17}{1000}$ etc. are decimal fractions.

9. **Vulgar fractions:** A fraction whose denominator is a whole number, other than 10,100,1000 etc. is called a vulgar fractions.

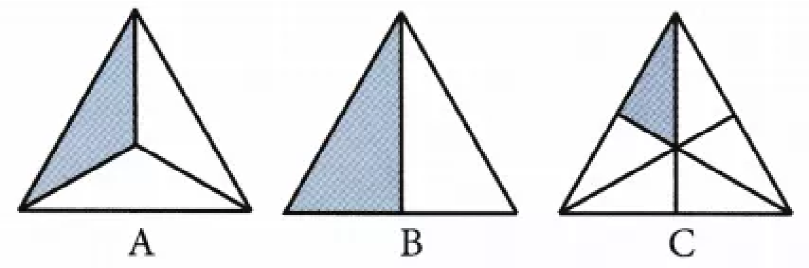
   For example $\frac{2}{7}, \frac{3}{8}, \frac{11}{17}$ etc. are vulgar fractions.

What is Comparing and Ordering of Fractions

Comparison of fractions are divided into three categories.

1. Fraction with the same numerator

Let us consider the following fractions with the same numerator: $\frac{1}{3}, \frac{1}{2}, \frac{1}{6}$
The pictorial representation of these fractions are given below:

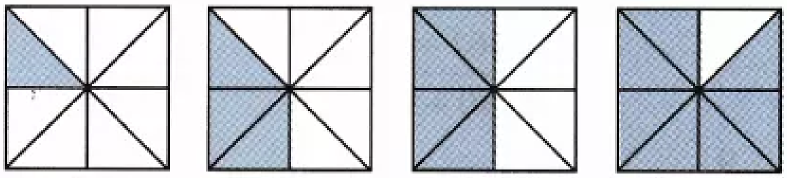


Shaded portion is $\frac{1}{3}$ Shaded portion is $\frac{1}{2}$ Shaded portion is $\frac{1}{6}$

By looking at the shaded portion in these pictures, we can conclude that
shaded area of B > shaded area of A > shaded area of C
or $\frac{1}{2} > \frac{1}{3} > \frac{1}{6}$
Thus, we conclude that if two or more fractions have the same numerator, then the fraction with a smaller denominator is greater.

2. Fraction with the same denominator

Let us consider the following fractions with the same denominator $\frac{1}{8}, \frac{3}{8}, \frac{4}{8}, \frac{7}{8}$
The pictorial representation of these fractions are given below:



A. Shaded portion $\frac{1}{8}$ B. Shaded portion $\frac{3}{8}$ C. Shaded portion $\frac{4}{8}$ D. Shaded portion $\frac{7}{8}$

By looking at the shaded portion in these pictures, we can easily say that
shaded area of D > shaded area of C > shaded area of B > shaded area of A
$\frac{7}{8} > \frac{4}{8} > \frac{3}{8} > \frac{1}{8}$
Thus, we conclude that if two or more fractions have the same denominator, then the fraction with the greater numerator is the greater fraction.

3. Fractions with different numerators and denominators

To compare the fractions with different numerators and denominators, first we find the LCM of their denominators. Then, we make the denominator of each fraction equal to the LCM by multiplying with a suitable number.

Example 1: Compare the fractions $\frac{3}{4}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$

Solution: LCM of denominators 4,6,7, and 8 = 168
Hence, the given fractions can be written as

$$\frac{3}{4} = \frac{3 \times 42}{4 \times 42} = \frac{126}{168}$$

$$\frac{5}{6} = \frac{5 \times 28}{6 \times 28} = \frac{140}{168}$$

$$\frac{6}{7} = \frac{6 \times 24}{7 \times 24} = \frac{144}{168}$$

$$\frac{7}{8} = \frac{7 \times 21}{8 \times 21} = \frac{147}{168}$$

given fractions can be written as

$$\frac{147}{168} > \frac{144}{168} > \frac{140}{168} > \frac{126}{168}$$

$$\frac{7}{8} > \frac{6}{7} > \frac{5}{6} > \frac{3}{4}$$

Example 2: Arrange the following fractions in ascending order:

Solution: LCM of denominators 4,2,8, and 16
Hence, the given fractions can be written as

$$\frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$$

$$\frac{3}{2} = \frac{3 \times 8}{2 \times 8} = \frac{24}{16}$$

$$\frac{7}{8} = \frac{7 \times 2}{8 \times 2} = \frac{14}{16}$$

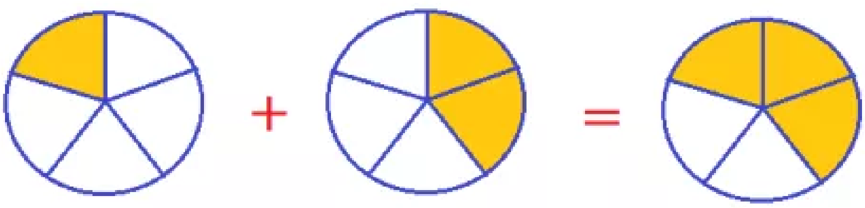
$$\frac{1}{16} = \frac{1 \times 1}{16 \times 1} = \frac{1}{16}$$

$\therefore$ Ascending order is $\frac{1}{16} < \frac{4}{16} < \frac{14}{16} < \frac{24}{16}$

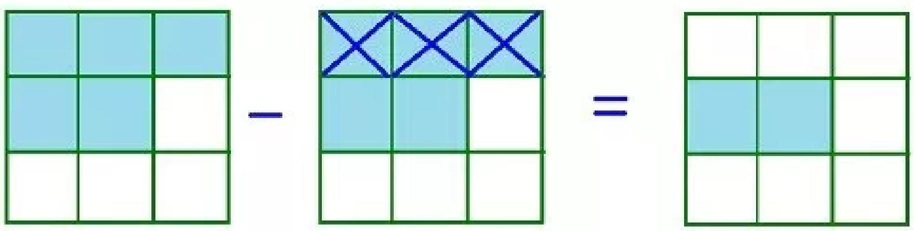
or, $\frac{1}{16} < \frac{1}{4} < \frac{7}{8} < \frac{3}{2}$

For adding and subtracting like fractions, we follow these steps:
Step 1. Add/subtract the numerators with common denominator.
Step 2. Reduce the fraction to its lowest term.
Step 3. If the result is an improper fraction, convert it into a mixed fraction.

$$\text{Addition/ subtraction of fractions} = \frac{\text{Sum/Difference of numerators}}{\text{Common denominator}}$$



$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$



$$\frac{5}{9} \qquad \frac{3}{9} \qquad \frac{2}{9}$$

What are the Operations on Fractions

Now, we have to learn, how to add and subtract the fractions. Certain methods are to be followed for doing these operations.

Addition and subtraction of like fractions

Example 1: Find the sum of

(a) $\frac{6}{11}$ and $\frac{9}{11}$ (b) $1\frac{1}{7}$ and $2\frac{2}{7}$

Solution:

(a) $$\frac{6}{11}+\frac{9}{11} = \frac{6+9}{11} = \frac{15}{11} = 1\frac{4}{11}$$

(b) $$1\frac{1}{7} \text{ and } 2\frac{2}{7} = \frac{8}{7}+\frac{16}{7} = \frac{8+16}{7}$$
$$= \frac{24}{7} = 3\frac{3}{7}$$

Example 2: Subtract

(a) $\frac{8}{15}$ from $\frac{13}{15}$ (b) $1\frac{1}{5}$ from $2\frac{3}{5}$

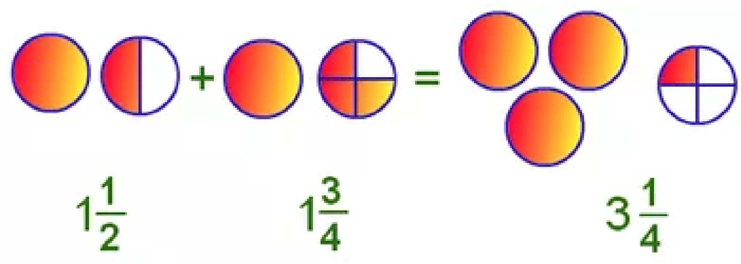
Solution:

(a) $$\frac{13}{15}-\frac{8}{15} = \frac{13-8}{15}$$
$$= \frac{5}{15} = \frac{1}{3}$$

(b) $$2\frac{3}{5}-1\frac{1}{5} = \frac{13}{5}-\frac{6}{5}$$
$$= \frac{13-6}{5}$$
$$= \frac{7}{5} = 1\frac{2}{5}$$

Addition and subtraction of unlike fractions

For adding/subtracting unlike fractions, we follow these steps:
1. Find the LCM of denominators of the given fractions.
2. Convert unlike fractions into like fractions by making LCM as its denominator.
3. Add/ subtract the like fractions.



Example 3: Add $\frac{9}{5}$ and $\frac{5}{6}$

Solution: LCM of 10 and 6 = 30

$$\frac{9}{10} = \frac{9\times 3}{10\times 3} = \frac{27}{30}$$

$$\frac{5}{6} = \frac{5\times 5}{6\times 5} = \frac{25}{30}.$$

$$\therefore \frac{9}{10}+\frac{5}{6} = \frac{27}{30}+\frac{25}{30} = \frac{27+25}{30} = \frac{52}{30} = \frac{26}{15} = 1\frac{11}{15}$$

Example 4:

Add $2\frac{1}{8},\ 2\frac{1}{2}$ *and* $\frac{7}{16}$.

Solution:

We have $2\frac{1}{8}+2\frac{1}{2}+\frac{7}{16}$

$$= \frac{17}{8}+\frac{5}{2}+\frac{7}{16} \quad \left(\begin{array}{l}\text{Convert mixed fractions}\\ \text{to improper fractions.}\end{array}\right)$$

$$= \frac{17\times 2}{8\times 2}+\frac{5\times 8}{2\times 8}+\frac{7\times 1}{16\times 1} \quad \left(\begin{array}{l}\because \text{LCM of}\\ 8, 2 \text{ and } 16 = 16\end{array}\right)$$

$$= \frac{34}{16}+\frac{40}{16}+\frac{7}{16}$$

$$= \frac{34+40+7}{16}$$

$$= \frac{81}{16}$$

$$= 5\frac{1}{16}$$

Example 5: Find $\frac{13}{5}$ – $\frac{4}{5}$

Solution: LCM of 15 and 5 = 15

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3}$$

$$= \frac{12}{15}$$

$$\therefore \frac{13}{15} - \frac{4}{5} = \frac{13}{15} - \frac{12}{15}$$

$$= \frac{13-12}{15}$$

$$= \frac{1}{15}$$

Example 6: Simplify $6\frac{1}{2} + 2\frac{2}{3} - \frac{1}{4}$

Solution: $6\frac{1}{2} + 2\frac{2}{3} - \frac{1}{4}$

$$= \frac{13}{2} + \frac{8}{3} - \frac{1}{4} \left(\begin{array}{l}\text{Converting mixed fractions} \\ \text{into impoper fractions.}\end{array}\right)$$

$$= \frac{13 \times 6}{2 \times 6} + \frac{8 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3} \left(\begin{array}{l}\because \text{LCM of 2,} \\ \text{3 and 4} = 12\end{array}\right)$$

$$= \frac{78}{12} + \frac{32}{12} - \frac{3}{12}$$

$$= \frac{78+32-3}{12}$$

$$= \frac{110-3}{12}$$

$$= \frac{107}{12}$$

$$= 8\frac{11}{12}$$

Multiplication of Fractions

Rule:

$$\text{Product of fractions} = \frac{\text{Product of their Numerators}}{\text{Product of their Denominators}}$$

(i) Whole number by a fraction
(ii) Fraction by a fraction

(iii) Whole number by a mixed fraction
(iv) Multiplication of two mixed fractions

Whole number by a fraction:
To multiply a whole number by a fraction, we simply multiply the numerator of the fraction by the whole number, keeping the denominator same.

Example 1: Find the product

(i) $3 \times \frac{2}{7}$ (ii) $3 \times \frac{1}{8}$ (iii) $\frac{7}{9} \times 6$

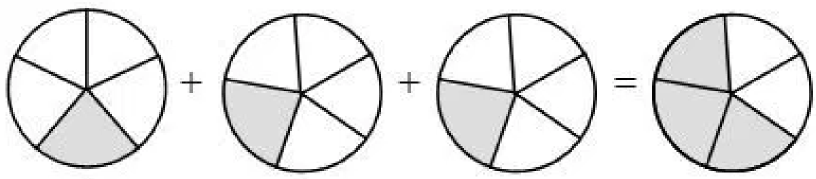
Solution:

$$\text{(i) } 3 \times \frac{2}{7} = \frac{3}{1} \times \frac{2}{7} = \frac{3 \times 2}{1 \times 7} = \frac{6}{7}$$

$$\text{(ii) } 3 \times \frac{1}{8} = \frac{3}{1} \times \frac{1}{8} = \frac{3 \times 1}{1 \times 8} = \frac{3}{8}$$

$$\text{(iii) } \frac{7}{9} \times 6 = \frac{7}{9} \times \frac{6}{1} = \frac{14}{3} = 4\frac{2}{3}$$

Example 2: Show $3 \times \frac{1}{5}$ by picture.
Solution:



$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $3 \times \frac{1}{5} = \frac{3}{5}$ (Shaded part)

Note : Multiplication is commutative i.e. ab = ba

Fraction by a fraction :

Example 3: Find the product

(i) $\frac{5}{8} \times \frac{3}{7}$ (ii) $\frac{6}{14} \times \frac{7}{9}$ (iii) $2\frac{4}{7} \times 2\frac{3}{4} \times 1\frac{2}{5}$

Solution:

$$\text{(i) } \frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56}$$

$$\text{(ii) } \frac{6}{14} \times \frac{7}{9} = \frac{6}{14} \times \frac{7}{9} = \frac{2 \times 1}{2 \times 3} = \frac{1 \times 1}{1 \times 3} = \frac{1}{3}$$

$$\text{(iii) } 2\frac{4}{7} \times 2\frac{3}{4} \times 1\frac{2}{5} = \frac{18}{7} \times \frac{11}{4} \times \frac{7}{5}$$

$$= \frac{18 \times 11 \times 7}{7 \times 4 \times 5} = \frac{9 \times 11}{2 \times 5} = \frac{99}{10} = 9\frac{9}{10}$$

Whole Number by a Mixed Fraction :
To multiply a whole number by a mixed fraction, we follow the following steps:

1. Convert the mixed fraction into an improper fraction.
2. Multiply the numerator by the whole number keeping the denominator same.
3. After multiplication, the fraction should be converted in its lowest form.
4. Convert the improper fraction (product so obtained) into a mixed numeral.

Example 4: Find $8 \times 5\frac{1}{6}$
Solution:

$= 8 \times \frac{31}{6}$ (Converting the mixed fraction into an improper fraction).

$= \frac{248}{6}$ (Multiplying the numerator by the whole number)

$= \frac{124}{3}$ (Simplifying into lowest term).

$= 41\frac{1}{3}$ (Converting the improper fraction into a mixed numeral).

Example 5: Find $6 \times 3\frac{1}{2}$
Solution:

Step 1. $3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$

Step 2. $6 \times 3\frac{1}{2} = 6 \times \frac{7}{2} = \frac{6 \times 7}{2} = \frac{42}{2}$

Step 3. $\frac{42}{2} = 21;$

Hence, $6 \times 3\frac{1}{2} = 21$

Multiplication of two Mixed Fractions:

1. To multiply two or more mixed numerals, we follow the following steps :
2. Convert the mixed fractions into improper fractions.
3. Multiply the improper fractions.
4. Reduce to lowest form.
5. If the product is an improper fraction, convert it into mixed fraction.

Example 6: Find the product of

(i) $3\frac{4}{5} \times \frac{10}{21}$ (ii) $\frac{15}{22} \times 4\frac{5}{7}$ (iii) $5\frac{2}{15} \times 3\frac{4}{7}$

Solution:

(i) $3\frac{4}{5}\times\frac{10}{21} = \frac{19}{5}\times\frac{10}{21} = \frac{38}{21} = 1\frac{17}{21}$

Thus, $3\frac{4}{5}\times\frac{10}{21} = 1\frac{17}{21}$

(ii) $\frac{15}{22}\times 4\frac{5}{7} = \frac{15}{22}\times\frac{33}{7} = \frac{15}{2}\times\frac{3}{7} = \frac{45}{14} = 3\frac{3}{14}$

(iii) $5\frac{2}{15}\times 3\frac{4}{7} = \frac{77}{15}\times\frac{25}{7} = \frac{11\times 5}{3\times 1} = \frac{55}{3} = 18\frac{1}{3}$

Facts:

1. It is not necessary first to multiply the fractions and then simplify. We may simplify first then multiply. For example,

   (i) $\frac{21}{25}\times\frac{45}{68} = \frac{21\times 45}{25\times 68} = \frac{21\times 9}{5\times 68} = \frac{189}{340}$

   (ii) $\frac{25}{7}\times\frac{12}{5}\times\frac{7}{4} = \frac{25\times 12\times 7}{7\times 5\times 4} = \frac{5\times 3\times 1}{1\times 1\times 1} = 15$
2. Cancellation could use only for fractions are multiplied and could not use for addition & subtraction of fractions.
3. Double of 3 or half of 7 can be written as 2 × 3 and 1/2 × 7 respectively.
   If word 'OF' is in between two fractions then multiply those fractions.
4. Product of two proper fractions < Each proper fraction.

   Ex. $\frac{2}{7}\times\frac{1}{3} = \frac{2}{21} \quad \therefore \quad \frac{2}{21} < \frac{2}{7}$ and $\frac{2}{21} < \frac{1}{3}$
5. Product of two improper fractions > Each improper fraction.

   Eg. $\frac{9}{4}\times\frac{7}{3} = \frac{63}{12} \quad \therefore \quad \frac{63}{12} > \frac{9}{4}$ & $\frac{63}{12} > \frac{7}{3}$;
6. Proper fraction < Product of proper and improper fraction < Improper fraction

   Eg. $\frac{1}{7}\times\frac{5}{2} = \frac{5}{14} \quad \therefore \quad \frac{1}{7} < \frac{5}{14} < \frac{5}{2}$
7. When the product of two fractional numbers or a fractional number and a whole number is 1, then either of them is the multiplicative inverse (or reciprocal) of the other. So the reciprocal of a fraction (or a whole number) is obtained by interchanging its numerator and denominator.
   Note : Reciprocal of zero (0) is not possible.

Division of Fractional Numbers

$\because$ We know Division = Dividend ÷ Divisor
When a fraction number (or whole no.) divide by fractional number (or whole no.) then we multiply dividend to reciprocal of divisor.

Example 1: Find the value of

(i) $\frac{5}{7}\div\frac{25}{21}$ (ii) $\frac{7}{8}\div\frac{15}{8}$ (iii) $1\frac{2}{7}\div 2\frac{1}{14}$

Solution:

(i) $\frac{5}{7} \div \frac{25}{21} = \frac{5}{7} \times \frac{21}{25} = \frac{3}{5}$

(ii) $\frac{7}{8} \div \frac{15}{8} = \frac{7}{8} \times \frac{8}{15} = \frac{7}{15}$

(iii) $1\frac{2}{7} \div 2\frac{1}{14} = \frac{9}{7} \div \frac{29}{14} = \frac{9}{7} \times \frac{14}{29} = \frac{18}{29}$

Facts:

1. (Fractional number) ÷ 1 = same fractional number

   $\frac{2}{3} \div 1 = \frac{2}{3} \times \frac{1}{1} = \frac{2}{3}$

2. 0 ÷ Fractional number = 0 (always)
3. non zero fractional number ÷ same number = 1 (always)

   $\frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = 1$

4. '0' cannot be a divisor (∵ reciprocal of zero is not possible)

Example 2:

Simplify : $\dfrac{2\frac{3}{4}}{1\frac{5}{7}}$

Solution:

$\dfrac{2\frac{3}{4}}{1\frac{5}{7}}$ is same as $2\frac{3}{4} \div 1\frac{5}{7}$

Now, $2\frac{3}{4} \div 1\frac{5}{7}$

$= \frac{11}{4} \div \frac{12}{7}$ ← (Rewrite the mixed numerals as improper fractions)

$= \frac{11}{4} \times \frac{7}{12}$ ← (Change ÷ to × and replace the divisor by its reciprocal.)

$= \frac{77}{48}$ ← (Reduce to lowest form and multiply the numerators and multiply the denominators)

$= 1\frac{29}{48}$ ←(Rewrite the improper fraction as mixed numeral)

Example 3:

(i) $12 \div \frac{3}{4}$ (ii) $2\frac{1}{5} \div 1\frac{1}{5}$

(iii) $\frac{2}{5} \div 1\frac{1}{2}$ (iv) $3\frac{1}{2} \div 4$

Solution:

$$\text{(i)} \quad 12 \div \frac{3}{4} = \frac{12}{1} \times \frac{4}{3} = \frac{4 \times 4}{1 \times 1} = \frac{16}{1} = 16$$

$$\text{(ii)} \quad 2\frac{1}{5} \div 1\frac{1}{5} = \frac{11}{5} \div \frac{6}{5} = \frac{11}{5} \times \frac{5}{6}$$

$$= \frac{11 \times 5}{5 \times 6} = \frac{11}{6} = 1\frac{5}{6}$$

$$\text{(iii)} \quad \frac{2}{5} \div 1\frac{1}{2} = \frac{2}{5} \div \frac{3}{2} = \frac{2}{5} \times \frac{2}{3} = \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$$

$$\text{(iv)} \quad 3\frac{1}{2} \div 4 = \frac{7}{2} \div \frac{4}{1} = \frac{7}{2} \times \frac{1}{4} = \frac{7 \times 1}{2 \times 4} = \frac{7}{8}$$

Simplifying brackets in fractions

Example 1:

$$\text{Simplify : } \frac{4}{7} + \left[\frac{1}{2} - \left\{\frac{3}{4} - \left(\frac{1}{5} + \overline{\frac{3}{7} - \frac{1}{5}}\right)\right\}\right]$$

Solution:

Let us first solve bar brackets :

$$\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\overline{\frac{3}{7}-\frac{1}{5}}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\overline{\frac{15-7}{35}}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\frac{8}{35}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{7+8}{35}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\frac{15}{35}\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\frac{3}{7}\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{21-12}{28}\right\}\right]=\frac{4}{7}+\left[\frac{1}{2}-\frac{9}{28}\right]$$

$$=\frac{4}{7}+\left[\frac{14-9}{28}\right]=\frac{4}{7}+\left[\frac{5}{28}\right]$$

$$=\frac{4}{7}+\frac{5}{28}=\frac{4\times4+5}{28}=\frac{21}{28}$$

Example 2:

Simplify : $\dfrac{\frac{2}{3}-\overline{\frac{1}{2}-\frac{1}{3}}\text{ of }\frac{1}{2}}{\overline{\frac{5}{6}-\frac{2}{3}}\text{ of }\frac{1}{3}+\frac{1}{9}}$

Solution:

$$\text{We have } \frac{\frac{2}{3}-\overline{\frac{1}{2}-\frac{1}{3}}\text{ of }\frac{1}{2}}{\overline{\frac{5}{6}-\frac{2}{3}}\text{ of }\frac{1}{3}+\frac{1}{9}}$$

$$=\frac{\frac{2}{3}-\overline{\frac{3-2}{6}}\text{ of }\frac{1}{2}}{\overline{\frac{5}{6}-\frac{2}{3}}\text{ of }\frac{1}{3}+\frac{1}{9}}$$

$$=\frac{\frac{2}{3}-\frac{1}{6}\text{ of }\frac{1}{2}}{\overline{\frac{5-4}{6}}\text{ of }\frac{1}{3}+\frac{1}{9}}$$

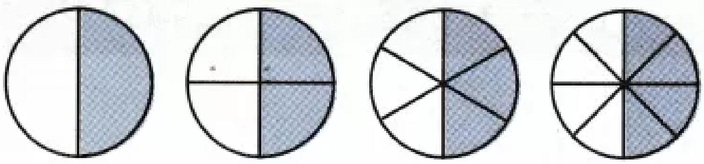
$$=\frac{\frac{2}{3}-\frac{1}{6}\times\frac{1}{2}}{\frac{1}{6}\times\frac{1}{3}+\frac{1}{9}}=\frac{\frac{2}{3}-\frac{1}{12}}{\frac{1}{18}+\frac{1}{9}}$$

$$=\frac{\frac{8-1}{12}}{\frac{1+2}{18}}=\frac{\frac{7}{12}}{\frac{3}{18}}=\frac{7}{12}\times\frac{18}{3}=\frac{7\times 6}{12\times 1}=\frac{7}{2}$$

What is an Equivalent Fraction

Equivalent Fraction

To understand the concept of equivalent fractions, let us take an example – Rama gave four cakes to his four children. The first child cut his cake into two equal halves and ate the first half. The second child cut his cake into four equal parts and ate two pieces out of four. The third child cut his cake into six equal parts and ate three of them, and the fourth child cut the cake into eight equal pieces and ate four of them.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

Do you think, they have eaten equal parts of the cake? Yes.

1^{st} child ate $\frac{1}{2}$ of the cake.

2^{nd} child ate $\frac{2}{4}$th of the cake. $\left(= \frac{1}{2} \text{ of the cake}\right)$.

3^{rd} child ate $\frac{3}{6}$th of the cake. $\left(= \frac{1}{2} \text{ of the cake}\right)$.

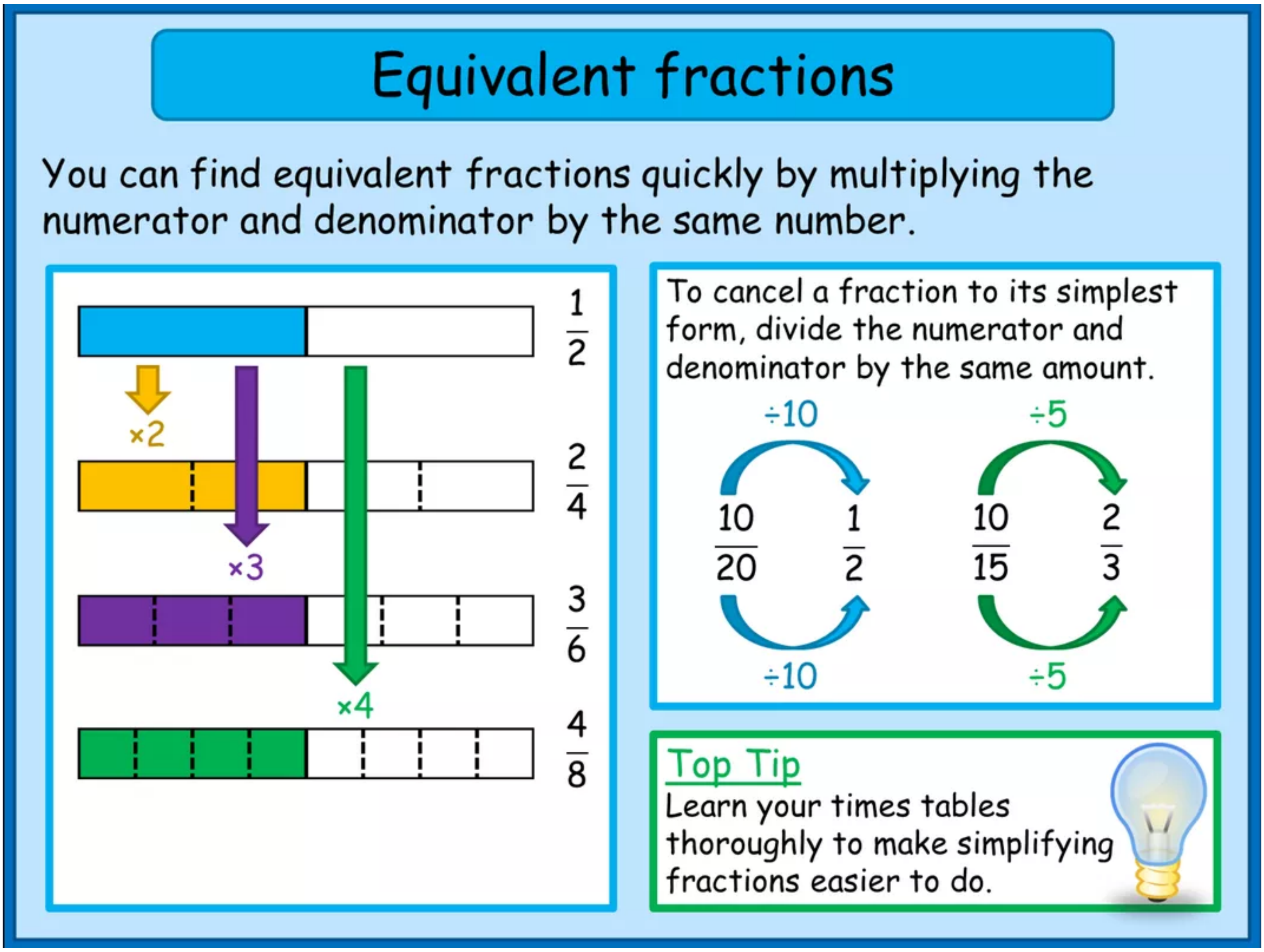
4^{th} child ate $\frac{4}{8}$th of the cake. $\left(= \frac{1}{2} \text{ of the cake}\right)$.

This means they all ate $\frac{1}{2}$ of the cake. Thus, fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ represent the same fraction $\frac{1}{2}$. These are called equivalent fractions. So, two or more fractions representing the same part (value) of the whole are called equivalent fractions.
To check if the fractions are equivalent or not, we do cross-multiplication.

For the fractions $\frac{a}{b}$ and $\frac{c}{d}$, if

(*i*) $a \times d = b \times c$ then $\frac{a}{b} = \frac{c}{d}$

(*ii*) $a \times d \neq b \times c$ then $\frac{a}{b} \neq \frac{c}{d}$



Note:
We can get as many equivalent fractions as we want, by multiplying/dividing the numerator and denominator of the given fraction by the same number.

Example 1: Write five equivalent fractions of $\frac{3}{5}$

Solution:

$$\frac{3}{5}=\frac{3\times 2}{5\times 2}=\frac{3\times 3}{5\times 3}=\frac{3\times 4}{5\times 4}=\frac{3\times 5}{5\times 5}=\frac{3\times 6}{5\times 6}$$

$$\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\frac{18}{30}$$

Hence, the five equivalent fractions of $\frac{3}{5}$ are

$$\frac{6}{10},\ \frac{9}{15},\ \frac{12}{20},\ \frac{15}{25},\ \frac{18}{30}.$$

Example 2: Which ofthefollowing pairs of fractions are equivalent:

(i) $\frac{5}{12},\frac{30}{72}$ (ii) $\frac{6}{7},\frac{14}{12}$

Solution:
(i) By cross-multiplying the terms of the fractions,

$$\frac{5}{12} \times \frac{30}{72}$$

5 × 72 = 12 × 30
360 = 360
∴ $\frac{5}{12}$ is equivalent to $\frac{30}{72}$
(ii) By cross-multiplying the terms of the fractions,

$$\frac{6}{7} \times \frac{14}{12}$$

$$6\times 12 \neq 14\times 7$$

$$72 \neq 98$$

∴ $\frac{6}{7}$ is equivalent to $\frac{12}{14}$

Simplest Form of a Fraction

A fraction is said to be in the lowest term or the simplest form, if its numerator and denominator do not have any common factor other than 1.

Methods to reduce a fraction into the simplest form:
(i) Find the HCF of the numerator and the yg denominator of the given fraction.
(ii) Divide its numerator and denominator by their HCF.

$$\frac{162\div 18}{90\div 18}=\frac{9}{5}$$

Example: $\frac{162}{90}$
HCF of 162 and 90 = 18

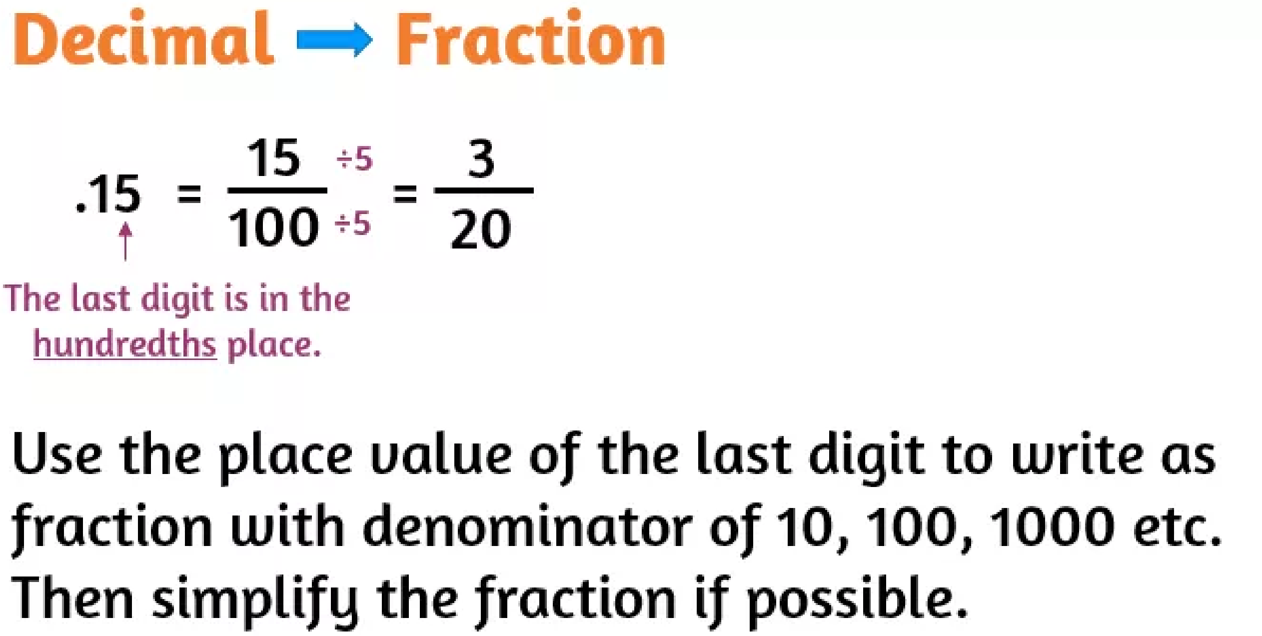
Dividing the numerator and denominator of the fraction by their HCF,

So, $\frac{9}{5}$ is the lowest form of the given fraction because common factor of 9 and 5 is only 1.

How do you Convert Fractions into Decimals and Vice Versa

Conversion of Decimal into Fraction

To change a decimal into a fraction, we have to follow the following steps:
Step 1: Write the given number without decimal point as the numerator of the fraction.
Step 2: Write 1 in the denominator followed by as many zeros as the number of decimal places in the given number.
Step 3: Reduce the fraction into the lowest form and if required change into mixed numeral.



Example 1: Convert 14.25 into a fraction.
Solution:
(i) Numerator of fraction = 1425
(ii) Denominator of fraction =100
(Because decimal places are 2, therefore we put 2 zeros after 1.)

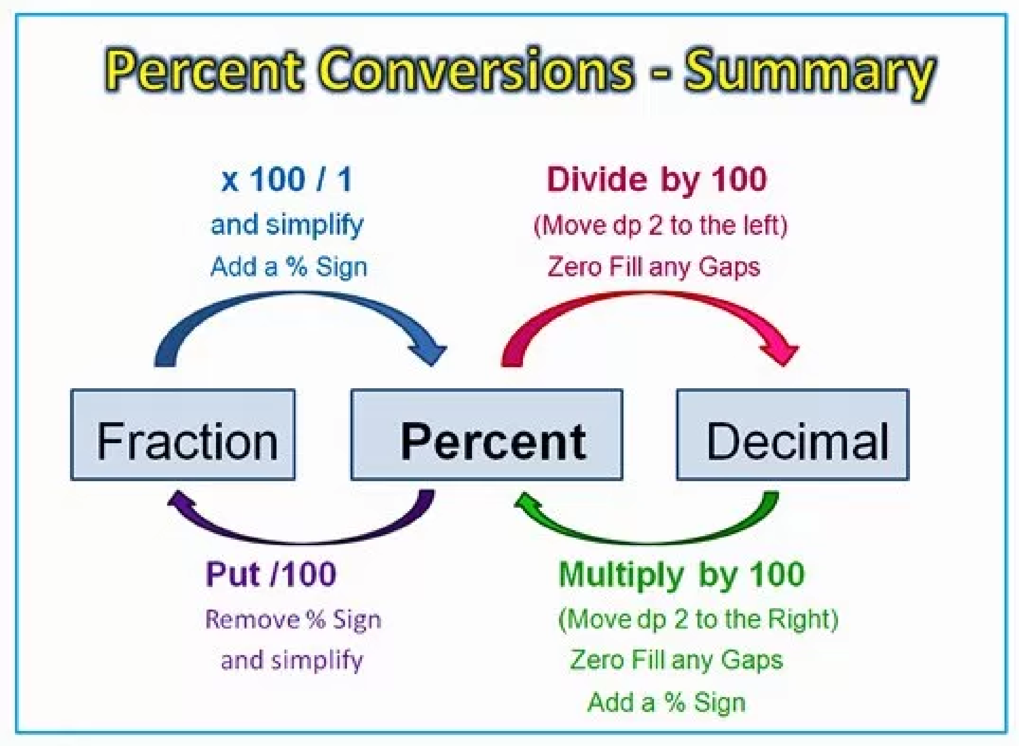
$$\text{So, } 14.25 = \frac{1425}{100} = 14\frac{25}{100} = 14\frac{1}{4}$$

Example 2: Convert 1.356 into a fraction.
Solution:
(i) Numerator of fraction = 1356
(ii) Denominator of fraction = 1000
(Because decimal places are 3, therefore we put 3 zeros ofter 1.)

(*ii*) 0.6 1.23 6.512
(Unlike decimals)
↓ ↓ ↓
0.600 1.230 6.512
(Like decimals with three decimal places)

Conversion of Fraction into Decimal

To change a fraction into decimal, we have to follow the following steps:
Step 1: First, change the given fraction into an equivalent fraction with denominators 10, 100, 1000, etc.
Step 2: Count the number of zeros in the denominator after 1. Put the decimal in the numerator, start from the extreme right, and move the decimal point to the left equal the number of zeros



Example 3: Convert the following into decimals.
(i) $\frac{3}{4}$ (ii) $5\frac{1}{2}$
Solution:

$$(i) \quad \frac{3}{4} = \frac{3\times 25}{4\times 25} = \frac{75}{100} = 0.75$$

$$(ii) \quad 5\frac{1}{2} = \frac{11}{2} = \frac{11\times 5}{2\times 5} = \frac{55}{10} = 5.5$$

Conversion by long division method
We can change a fraction into decimal by using the long division method. For that, we have to follow these steps:
Step 1: Convert the dividend to a suitable equivalent decimal.
Step 2: When a digit to the right of the decimal point is brought down, insert a decimal point in the quotient.

Example 4: Convert $\frac{3}{4}$ into decimals.

Solution: In $\frac{3}{4}$, since 3 is less than 4, it cannot be divided by 4.
But 3 = 3.00, which can be divided by 4.
Now,

```
4)3.00(0.75
  28↓ ←—— Insert decimal in
   20     quotient at this step
   20
    0
```

Thus $\frac{3}{4}$ = 0.75

Percents

Percents are used to describe parts of a whole base amount. When one of the parts of the relationship is unknown, we can solve an algebraic equation for the unknown quantity.
(The solution methods shown on this page are of an algebraic, sentence translation nature. Of course, other methods of solution are also possible.)

There are two main types of problems dealing with percents:

1. In the first type of problem, **the percent is given.**
In these problems, you will change the percent to a decimal.
To change a percent to a decimal, divide the number by 100.
This will move the decimal point two places to the left.

Example 1: Find 7% of 250.
(This can also be read "What number is 7% of 250?")
Solution: Let x = the answer
x = 7% • 250
x = 0.07 • 250   changing 7% to a decimal
x = 17.5
Remember that the word "of " means to multiply!

Example 2: 30 is 15% of what number?
Solution: Let n = the answer ("what number")
30 = 15% • n
30 = 0.15 • n   changing 15% to a decimal
30/0.15 = n   dividing both sides by 0.15
200 = n
n = 200
Remember that the word "of " means to multiply!

2. In the second type of problem, you are **looking for the percent.**
In these problems, you will represent the % as a fraction.

Example 1: 3 is what percent of 12?

Let $x\%$ = the percent.
Writing this problem literally we would get:
$3 = x\%$ of 12

$$3 = \frac{x}{100} \cdot 12$$

$$3 = \frac{12x}{100}$$

$300 = 12x$

$25 = x$
The answer is 25%.

To change a percent to a fraction, divide the percent value by 100.

Application Problems

Example 1: If 120 million roses were sold on Valentine's Day, and 75% of the roses were red, how many red roses were sold on Valentine's Day?
Solution: Let x = the number of red roses sold
x = 75% of 120
x = 0.75 • 120
x = 90 million red roses sold

Example 2: Juan missed 6 out of 92 questions on a test. To the nearest percent, what percent of the questions did he solve correctly?

Solution:

If he missed 6 questions, he got 86 questions correct. Re-word the question to be "86 is what percent of 92"?

$86 = x\%$ of 92

$$86 = \frac{x}{100} \cdot 92$$

$$86 = \frac{92x}{100}$$

$$8600 = 92x$$

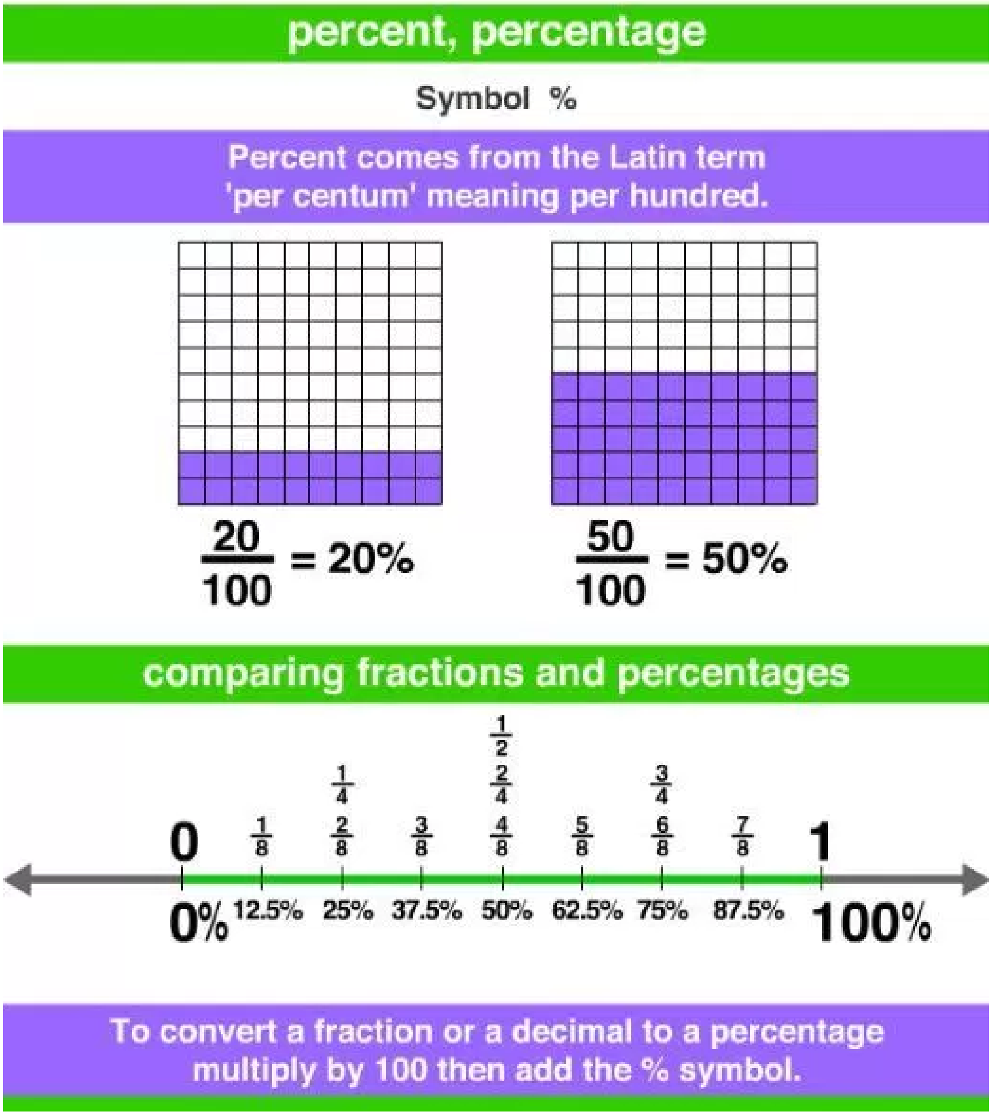
$$93.47826087 = x$$

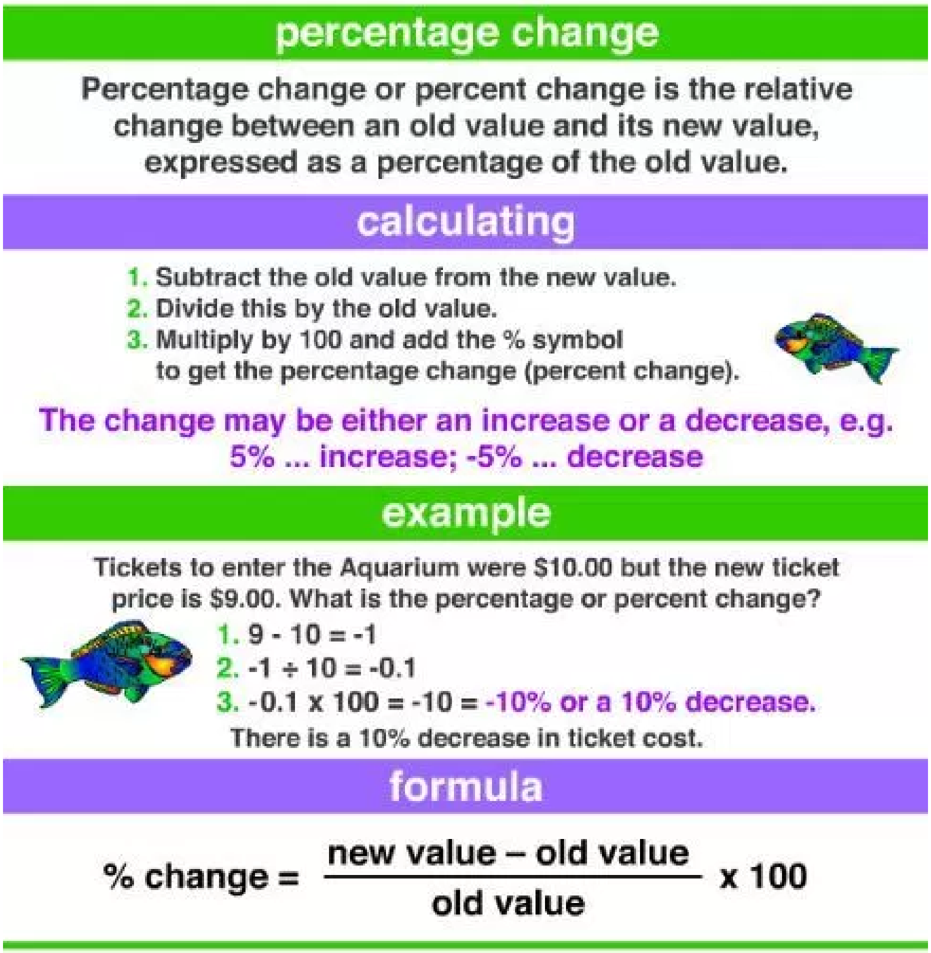
To the nearest percent, he got 93% correct.

How do you Calculate Percentages?

Percentage

When we take 100 as the denominator of fractions, the numerators are called percentages. For convenience, the symbol % is used for percent. When we take 100 as the denominator of fractions, the numerators are called percentages. For convenience, the symbol % is used for percent.
Or
"A percentage is simply a ratio in which the second term is arranged to be 100". Also percent is an abbreviation of the Latin phrase per centum, meaning per hundred or hundredths.





1. A fraction may be converted into a percentage by multiplying that fraction by 100%. This does not change its value, since 100% is 1.
2. A decimal may be converted into a percentage by multiplying it by 100%.

Uses of Percentages

1. Interpreting percentages.
2. Converting percentage to 'How many'.
3. Converting ratio to percentage.
4. Increase or decrease as percent.

Eg: Raju invests 10% of his pocket money in buying toffees means ₹10 out of ₹100 are invested by Raju in buying the toffees.

Eg: A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win ?
Here, the total number of matches played are 20. Out of these 25% are won by the team.

I method (direct):
Out of 100, 25 matches are won by the team. So, out of 20, number of matches won by the team
$= \frac{25}{100} \times 20$
= 5 matches.

II method (using percentage):

$$25\% \text{ of } 20 = \frac{25}{100} \times 20 = 5.$$

Percentages Problems with Solutions

1. Express $\frac{7}{20}$ as a percentage.
Solution:

$$\frac{7}{20} = \frac{7}{20} \times 100\% = 35\%$$

2. Express 0.625 as a percentage.
Solution: 0.625 = 0.625 × 100% = 62.5%

3.

Write (a) $\frac{1}{4}$ (b) $\frac{22}{44}$ (c) $\frac{4}{25}$ as percent.

Solution:

(a) We have $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = \left(\frac{100}{4}\right)\% = 25\%$

(b) $\frac{22}{44} = \left(\frac{22}{44} \times 100\right)\% = 50\%$

(c) $\frac{4}{25} = \left(\frac{4}{25} \times 100\right)\% = 16\%$

4. Out of 50 students in a class, 15 like to play cricket. What is percentage of students who like to play cricket ?
Solution:
Total students = 50
Students who like to play cricket = 15
So, % age of students who like to play cricket

$$= \left(\frac{15}{50} \times 100\right)\% = 30\%.$$

5. Convert the given decimals to percent:
(a) 0.6
(b) 0.75
(c) 0.08
(d) 0.56
Solution:
We have
(a) 0.6 = (0.6 × 100)% = 60%
(b) 0.75 = (0.75 × 100)% = 75%
(c) 0.08 = (0.08 × 100)% = 8%
(d) 0.56 = (0.56 × 100)% = 56%

6. Convert a percentage into fraction
(i) 45%
(ii) 65%
(iii) 42.5%

Solution: We have

$$(i)\ 45\% = \frac{45}{100} = \frac{9}{20}$$

$$(ii)\ 65\% = \frac{65}{100} = \frac{13}{20}$$

$$(iii)\ 42.5\% = \frac{42.5}{100} = \frac{425}{1000} = \frac{85}{200} = \frac{17}{40}.$$

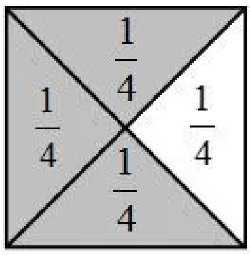
7. Convert each of the following into decimal fraction :
(a) 53%
(b) 0.38%
(c) 4.7%
Solution:

$$(a)\ 53\% = \frac{53}{100} = 0.53$$

$$(b)\ 0.38\% = \frac{0.38}{100} = 0.0038$$

$$(c)\ 4.7\% = \frac{4.7}{100} = \frac{47}{1000} = 0.047.$$

8. What percentage of the adjoining figure is shaded and what percentage is unshaded ? Find it.
Solution:
First we find the fraction of the figure that is shaded or unshaded. From this fraction we will find the percentage of the shaded and unshaded regions.



$$\text{So, shaded region} = \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) = \frac{3}{4}$$

Now, percentage of shaded region

$$= \left(\frac{3}{4} \times 100\right)\% = 75\%$$

$$\text{Unshaded region} = \frac{1}{4}$$

Now, percentage of unshaded region

$$= \left(\frac{1}{4} \times 100\right)\% = 25\%$$

9. Convert each of the following ratios into a percentage :Convert each of the following ratios into a percentage :
(i) 15 : 45 (ii) 3 : 5
Solution: We have,

$$\text{(i) } 15 : 45 = \frac{15}{45} = \left(\frac{15}{45} \times 100\right)\%$$

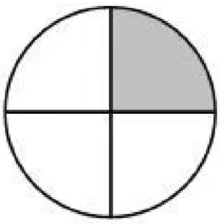
$$= \left(\frac{3}{9} \times 100\right)\%$$

$$= \left(\frac{1}{3} \times 100\right)\%$$

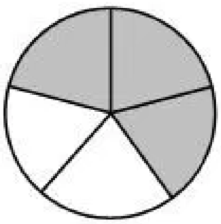
$$= \frac{100}{3}\% = 33\frac{1}{3}\%$$

$$\text{(ii) } 3 : 5 = \left(\frac{3}{5} \times 100\right)\% = 60\%$$

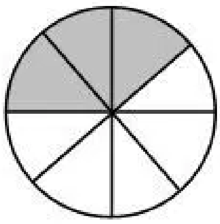
10. Estimate what region of the following figures is shaded and hence find percentage of that shaded region.



(i)



(ii)



(iii)

Solution: We have,

(i) Shaded region = $\frac{1}{4}$

$$\% \text{ of shaded region} = \left(\frac{1}{4}\times 100\right)\% = 25\%$$

(ii) Shaded region = $\frac{3}{5}$

$$\% \text{ of shaded region} = \left(\frac{3}{5}\times 100\right)\% = 60\%$$

(iii) Shaded region = $\frac{3}{8}$

$$\% \text{ of shaded region} = \left(\frac{3}{8}\times 100\right)\%$$

$$= \left(\frac{3}{2}\times 25\right)\% = \frac{75}{2}\% = 37.5\%$$

11. Convert given percents to decimal fractions and also to fractions in simplest form :Convert given percents to decimal fractions and also to fractions in simplest form :
(i) 25% (ii) 150% (iii) 20% (iv) 5%
Solution:

S.No.	Percentage	Fraction	*Decimal*
(i)	25%	$\frac{25}{100} = \frac{1}{4}$	*0.25*
(ii)	150%	$\frac{150}{100} = \frac{3}{2}$	*1.50*
(iii)	20%	$\frac{20}{100} = \frac{1}{5}$	*0.20*
(iv)	*5%*	$\frac{5}{100} = \frac{1}{20}$	*0.05*

12. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.

Solution:

Percentage decrease

$$= \frac{\text{Decrease in population}}{\text{Initial population}} \times 100$$

$$= \frac{25000 - 24500}{25000}$$

$$= \left(\frac{500}{25000} \times 100\right)\% = 2\%$$

13. The population of India is 113 crore. If it increases by 1.7% every year, Find India's population after one year.

Solution:

India's population = 113 crore

Increased by 1.7%

$$= 113 \text{ crore} + \left(\frac{1.7}{100} \times 113\right) \text{crore}$$

$$= 113 \text{ crore} + \frac{192.1}{100} \text{crore}$$

$$= 113 \text{ crore} + 1.921 \text{ crore}$$

$$= 14.921 \text{ crore}$$