CBSE Test Paper 04
CH-07 Permutations & Combinations

1. The number of distinguishable ways in which the 4 faces of a regular tetrahedron can be painted with 4 different colours is
   a. 24
   b. 2
   c. 4
   d. none of these
2. The number of three digit numbers having atleast one digit as 5 is
   a. 648
   b. 225
   c. 252
   d. 246
3. 20 students can compete for a race. The number of ways in which they can win the first three places is (given that no two students finish in the same place)
   a. 1140
   b. 8000
   c. 6840
   d. none of these.
4. The number of ways in which 4 red ,3 yellow and 2 green discs be arranged if the discs of the same colour and indistinguishable
   a. 1260
   b. 999
   c. 1512
   d. 2260
5. The total number of numbers from 1000 to 9999 (both inclusive) that do not have 4 different digits
   a. 9000
   b. 4464
   c. 4536
   d. none of these.

6. Fill in the blanks:

   The values of P(15, 3) is ________.

7. Fill in the blanks:

   If there are two events such that they can be performed independently in m and n ways respectively, then either of the two events can be performed in ________ ways.

8. If ${}^nC_8 = {}^nC_2$. find ${}^nC_2$.

9. How many 3 letter words can be made, using the letters of the word 'ORIENTAL?

10. In how many ways, can a cricket team of 11 players be selected out of 16 players, If two particular players are always to be included?

11. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

12. How many 4-letter codes can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

13. In an examination a question paper consist of 12 questions divided into two parts i.e. part I and part II containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

14. If $\frac{1}{7!} + \frac{1}{9!} = \frac{x}{10!}$, find x

15. Find the number of ways in which 5 boys and 5 girls be seated in a row, so that

    i. no two girls may sit together.
    ii. all the girls and all the boys sit together.
    iii. all the girls are never sitting together.

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Solution

1. (b) 2

   Explanation: We have a regular tetrahedron has 4 faces and we have to colour it with 4 different colours in $4! = 24$ ways.

   But in this we will be getting many overcountings .

   We have there are 12 ways in which we can orient a regular tetrahedron

   Hence the number of distinct ways of colouring a regular tetrahedron with 4 different colours is $\frac{24}{12} = 2$

2. (c) 252

   Explanation: First we will find the number of three digit numbers (i.e, numbers from 100 to 999)which can be formed using the digits 0,1,2,3,4,5,6,7,8and 9 with repitition allowed.
   Now we have the first place can be filled by any of the 9 digits other than 0 and since repetition is allowed the second and third can be filled by any of the ten digits.

   Hence the total number of three digit numbers will be = $9 \times 10 \times 10 = 900$

   Now we will consider the case that the number does not have the digit 5.

   Now the first place can be filled by any of the 8 digits other than 0 and 5 and since repetition is allowed the second and third can be filled by any of the 9 digits other than 5.

   Hence the total number of ways we can form a three digit number with out 5 will be =$8 \times 9 \times 9 = 648$

   Therefore the number of three digit numbers with atleast one 5=$900 - 648 = 252$

3. (c) 6840

   Explanation: for first place we have 20 students, for second we have 19 and for the third we have 18

$^{20}P_3$=20*19*18

4. (a) 1260

   Explanation: Total number of discs =9

   Out of which red-4 ,yellow -3 and green -2 are of the same kind.

   Hence required number of permutations=$\frac{9!}{4!.3!.2!} = \frac{5\times6\times7\times8\times9}{1\times2\times3\times1\times2} = 1260$

5. (b) 4464 Explanation:

   First we will find the number of four digit numbers that can be formed using the digits 0,1,2,3,4,5,6,7,8,9 with repetition .

   The first place can be filled by any of the 9 digits other than 0, and the second, third and the fourth places each can be filled by any of the ten digits

   Hence the total number of ways of forming a four digit number = $9 \times 10 \times 10 \times 10 = 9000$

   Now we will find the number of four digit numbers in which nall the digits are distinct

   The first place can be filled by any of the 9 digits other than 0, and the second, can be filled by any of the remaining 9 digits since repetition is not possible

   Similarly third and the fourth places each can be filled by 8 and 7 digits respectively

   Hence the total number of ways of forming a four digit number with distinct digits b= $9 \times 9 \times 8 \times 7 = 4536$

   The total number of numbers from 1000 to 9999 (both inclusive) that do not have 4 different digits=$9000 - 4536 = 4464$

6. 2730

7. (m + n)

8. Here ${}^nC_8 = {}^nC_2 \Rightarrow {}^nC_8 = {}^nC_{n-2}\ [\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow 8 = n - 2 [\because {}^nC_x = {}^nC_y \Rightarrow x = y]$
$\Rightarrow n = 10 \therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!8!} = 45$

9. We have word 'ORIENTAL'

Total number of letters, n = 8

Number of letters to be used in forming a word, r = 3

Thus, total number of words thus formed = ${}^8P_3$

$= \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\times7\times6\times5!}{5!}$ = 336

10. When two players are always to be included, then 9 more players are to be selected out of the remaining 14 players, which can be done in ${}^{14}C_9$ ways

$= \frac{14!}{9!5!} = \frac{14\times13\times12\times11\times10}{5\times4\times3\times2\times1}$

= 2002 ways

11. There are 9 courses and number of courses to be selected are 5 in which 2 specific courses are compulsory.

We have to select 3 courses out of remaining 7 courses.

$\therefore$ Number of ways of selection $= {}^7C_3$

$\frac{7!}{3!4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!} = 35$

12. Here the unit place can be filled by any one of the first 10 letters of the English alphabet. So the unit place can be filled in 10 ways. The tens place can be filled in 9 ways by remaining 9 letters of the English alphabet. The hundreds place can be filled in 8 ways by remaining 8 letters of the English alphabet.

The thousands place can be filled in 7 ways by the remaining 7 letters of the English alphabet.

$\therefore$ Total number of 4-letter code numbers $= 7 \times 8 \times 9 \times 10 = 5040.$

13. Here number of questions in part I are 5 and number of questions in part II are 7. We have to select 8 questions at least 3 questions from each section. So we have required selections are 3 from part I and 5 from part II or 4 from part I and 4 from part II or 5 from part I and 3 from part II.

$\therefore$ Number of ways of selection $= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$

$= \frac{5!}{3!2!} \times \frac{7!}{5!2!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{5!0!}$

$$= \frac{5\times4\times3!}{3!\times2\times1} \times \frac{7\times6\times5!}{5!\times2\times} + \frac{5\times4!}{4!\times1} \times \frac{7\times6\times5\times4!}{4!\times3\times2\times1} + 1 \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$$
$$= 10 \times 21 + 5 \times 35 + 1 \times 35$$
= 210 + 175 + 35 = 420

14. Here $\frac{1}{7!} + \frac{1}{9!} = \frac{x}{10!}$,
$$\Rightarrow \frac{1}{7!} + \frac{1}{9\times8\times7!} = \frac{x}{10\times9\times8\times7!}$$
$$\Rightarrow \frac{1}{7!}\left[1 + \frac{1}{72}\right] = \frac{1}{7!}\left[\frac{x}{10\times9\times8}\right]$$
$$\Rightarrow \frac{73}{72} = \frac{x}{10\times9\times8}$$
$$\Rightarrow x = \frac{73}{72} \times 10 \times 9 \times 8 = 730$$

15. We have given 5 boys and 5 girls.

i. 5 boys can be selected in $^5P_5$ = 5! ways

Now, in the 6 seats, 5 girls can be arranged in $^6P_5$ ways.

Hence, the number of ways in which no two girls sit together
$$= 5! \times ^6 P_5 = 5! \times \frac{6!}{(6-5)!} = 5! \times 6!$$

ii. The two groups of girls and boys can be arranged in 2! ways.

3 girls can be arranged among themselves in 3! ways.

Similarly, 5 boys can be arranged among themselves in 5! ways.

$\therefore$ The total number of seating arrangements

$= 2!(5! \times 5!)$ = $2(5!)^2$

iii. The total number of ways in which all the girls are never together

= (Total number of arrangements) - (Total number of arrangements in which all the girls always sit together)

= 10! - 5! $\times$ 6!

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5! - 5! \times 6!$$

= 5!(30240 - 720)

$= 120 \times 29520$ = 3542400